WHY ROTATION SEISMOLOGY:
CONFRONTATION BETWEEN CLASSIC AND ASYMMETRIC THEORIES

→ Need of Modern Seismological World Network

Roman Teisseyre
Institute of Geophysics, Polish Academy of Sciences,
ul. Ksiecia Janusza 64, 01-452 Warszawa, Poland
e-mail: rt@igf.edu.pl
• Classical elasticity has many insufficiencies.
• Angular motions are not incorporated in it; only artificially we can introduce it defining a point and arm of the moment

Numerous attempts to improve this theory:
• Cosserat brothers’ theory with displacements and rotations (Cosserat, E. and F. 1909)
• The micropolar and micromorphic theories (Eringen and Suhubi, 1964; Mindlin, 1965; Nowacki, 1986; Eringen 1999, 2001): a powerful tool for many complicated material problems (Jones 1973); these theories seem to be sometimes too complicated and therefore are not in common use in the seismological studies (some examples of seismological application, see: Teisseyre, 1973; 1974).

Example of some unsolved problems faced by the classical elasticity:
• Angular motions introduced artificially with length and reference rotation point (angular momentum balance ↔ symmetric stresses)

• Fault slip solutions rely on the additional friction constitutive law along a fault

• Advanced deformations, like granulation and fragmentation - not included

• Earthquake fracture geometry reveals an asymmetric pattern with the main slip plane (premonitory micro-fractures are probably also asymmetric)

• Edge dislocations present asymmetry in relation to the strain components

• Direct differential relation for the density of edge dislocations and stresses cannot be derived in the classical theory
ASYMMETRIC THEORY: Conventions used

• NOTE: vector field could be replaced by antisymmetric tensor field:

\[ \omega_{[ik]} = \varepsilon_{iks} \omega_s ; \quad \text{or in vector notation, this tensor is: } \omega_{[i]} = \nabla \times \omega \]

• \( \varepsilon_{iks} = \{1, -1, 0\} \) when indexes are ordered \((1, 2, 3; 2, 3, 1; 3, 1, 2)\), missordered or repeating

• NOTE: any symmetric tensor can be split into the axial and deviatoric tensors.

\[ D_{(ns)} = \delta_{ns} D^A_{kk} + D^D_{ns} : \quad D^A_{(kk)} = D_{kk} \quad \text{and} \quad D^D_{ns} = D_{ns} - \frac{1}{3} \delta_{ns} D_{kk} \]

• Summation convention for repeating indexes:

\[ D_{kk} = \sum_k D_{kk} \]
Material destruction is a very complicated surely nonlinear process; many fields are released, some are inter-related.
However, it is much more complicated to understand the earthquake events as, in this case, our knowledge of the external conditions is still very limited.

Therefore, it is very important to refer to the boundary or initial conditions: these are related to strains: axial, deviatoric and moment related (rotational).

Seismometers record not directly displacements but deformations, which may be related either to real displacements or to strains:

\[ \Delta u_k = \frac{\partial u_k}{\partial x_i} \Delta x_i \rightarrow u_k = \int \Delta u_k \]

where \( \Delta x_i \) is a length of a seismometer platform much more rigid than soil layer

Displacements, \( u \), appear as integrating effect.
Fundamental Point Motions and Deformations

The problem of rotation waves becomes again actual due to the recent observations based on very precise instruments able to measure very small rotation motions and due to development and new approach to the continuum theory.

We start our consideration recalling that between serious defaults of the classical approach, like the known problem how to include into continuum theory the angular motions and related moments, there is a necessity to include there the constitutive laws to account for response to the applied moments and angular motions in continuum.

WE SHALL POSTULATE THAT EACH DEFORMATION FIELD SHALL BE DESCRIBED BY THE INDEPENDENT BALANCE RELATION !!!
Axial point deformation (scalar), (in 3D, 2D, 1D):

Shear point deformation: string-string (vector ⊥ to the plane)
LEFT: **ROTATION** motion, \((\text{vector} \perp \text{to the plane})\); its velocity means spin;

RIGHT: **SHEAR AXES OSCILLATIONS** (vector \(\perp\) to the plane) this **SHEAR_TWIST** is equivalent to **STRING - STRING** basic deformation
**Shear-Twist** means the **rotational oscillation** of the main shear axes with the changes of shear magnitude as caused by the internal fractures.
Three Reference Motions (3): \textbf{displacements}; (or rotations 3) are equivalents: we will refer to displacements $u$): remains 3

Seven STRAINS (7): \textbf{Axial Strain} – 1; \textbf{Shears} (strike-strike) – 3 \textbf{antisymmetric} strains - \textbf{Rotations} – 3;

All deformations appear due to specific load conditions: rotation strains (external moment or internal friction); axial strain – to external pressure (or dilatancy); strike-strike deformation – to external shears

\textbf{All together we have 10 independent fields:}

\textbf{All} these fields shall have the independent balance relations !!

\textbf{Some of them may be inter-related} as all deformations could be refer to the \textbf{unique diaplaacement} (or rotation) field or related to a \textbf{number of independently generated displacements} or \textbf{directly appearing due to load stress conditions} (symmetric or, and antisymmetric – moment related)
Basic point motions (related to the Planck dimension) - given by vectors:

DISPLACEMENTS, \( u \), and ROTATIONS, \( \omega \)

present the reference fields; we may refer to DIFFERENT \( u \) !!!

Strain deformations:

**AXIAL STRAIN** (refering to: \( \bar{u} \))

\[
E_{(ss)} = \bar{E} = \frac{\partial \bar{u}_s}{\partial x_s}
\]

**SHEAR STRAIN** (refering to: \( \hat{u} \)):

\[
E_{(ik)} = \hat{E} = \frac{1}{2} \left( \frac{\partial \hat{u}_k}{\partial x_i} + \frac{\partial \hat{u}_i}{\partial x_k} \right) - \frac{1}{3} \delta_{ik} \frac{\partial \hat{u}_s}{\partial x_s}
\]

**ROTATION STRAIN** (refering to: \( \hat{\theta} \)):

\[
E_{[ik]} = \frac{1}{2} \left( \frac{\partial \hat{u}_k}{\partial x_i} - \frac{\partial \hat{u}_i}{\partial x_k} \right)
\]

Displacement rotation means the antisymmetry strain!
ELEMENETARY FRACTURE PROCESS

can join some different reference displacements:

\[ \bar{u}, \hat{u}, \tilde{u} \]

RELEASED CONSECUTIVELY WITH PHASE SHIFTS:

\[ \bar{u}_s = \xi^0 u_s , \quad \hat{u}_s = e^0 u_s , \quad \tilde{u}_s = \chi^0 u_s \]

\[ \{ \xi^0, \chi^0, e^0 \} = \{ 0, \pm 1, \pm i \} \]

We may refer these motions to a unique slip displacements, \( u \), released in a final stage of an earthquake process:

The phase shift constants include both the advanced or delayed motions in an elementary source process.
Strain deformations - unique reference field

\[ E_{(ss)} \rightarrow \bar{E}_{(ss)} = \frac{\partial u_s}{\partial x_s} = \xi^0 \frac{\partial u_s}{\partial x_s} \]

\[ E^D_{(ik)} = \hat{E}_{(ik)} = \frac{1}{2} \left( \frac{\partial \hat{u}_k}{\partial x_i} + \frac{\partial \hat{u}_i}{\partial x_k} \right) - \frac{1}{3} \delta_{ik} \frac{\partial \hat{u}_s}{\partial x_s} = \]

\[ e^0 \left( \frac{1}{2} \left( \frac{\partial u_k}{\partial x_i} + \frac{\partial u_i}{\partial x_k} \right) - \frac{1}{3} \delta_{ik} \frac{\partial u_s}{\partial x_s} \right) \]

\[ E^{[ik]} = \frac{1}{2} \left( \frac{\partial u_k}{\partial x_i} - \frac{\partial u_i}{\partial x_k} \right) = \chi^0 \frac{1}{2} \left( \frac{\partial u_k^-}{\partial x_i} - \frac{\partial u_i}{\partial x_k} \right) \]

double rebound process:

\[ i i = -1 \]

\[ \{ e^0, \chi^0, \xi^0 \} = \{ 0, \pm 1, \pm i \} \]
ASymmetric Theory: Stresses and Strains:

\[
S_{kl} = S_{(kl)} + S_{[ik]}
\]

\[
E_{ik} = E_{(ik)} + E_{[ik]}
\]

\[
E_{(ss)} \rightarrow \overline{E} ; \quad E_{(ik)}^{D} \rightarrow \hat{E}_{(ik)} ; \quad E_{[ik]} \rightarrow \hat{E}_{[ik]}
\]

Symmetric part of stresses: we take the classical constitutive law and supplement with the Shimbo law for the antisymmetric part:

\[
S_{(kl)} = \lambda \delta_{kl} E_{ss} + 2\mu E_{kl} , \quad S_{ss}^{A} = (3\lambda + 2\mu) \overline{E}
\]

\[
S_{(ik)}^{D} = 2\mu \hat{E}_{(ik)} , \quad S_{[ik]} = 2\mu \hat{E}_{[ik]}
\]

In these constitutive relations we put – for simplicity - the same constants as for shears and rotations.
FOR ANY REFERENCE DISPLACEMENTS

\[(\lambda + \mu) \frac{\partial^2 u_n}{\partial x_n \partial x_s} + \mu \frac{\partial^2 u_n}{\partial x_s \partial x_s} = \rho \frac{\partial^2}{\partial t^2} u_n + F\]

After differentiating: for derivatives:

\[\mu \frac{\partial^2}{\partial x_k \partial x_k} \left( \frac{\partial u_l}{\partial x_n} + \frac{\partial u_n}{\partial x_l} \right) - \rho \frac{\partial^2}{\partial t^2} \left( \frac{\partial u_l}{\partial x_n} + \frac{\partial u_n}{\partial x_l} \right) = -2(\lambda + \mu) \frac{\partial^3 u_k}{\partial x_n \partial x_l \partial x_k} + \left( \frac{\partial F_k}{\partial x_l} + \frac{\partial F_l}{\partial x_n} \right)\]

Adding (or subtracting) another with interchanged indexes \(n, l\)

\[\mu \frac{\partial^2}{\partial x_k \partial x_k} \frac{\partial u_n}{\partial x_l} - \rho \frac{\partial^2}{\partial t^2} \frac{\partial u_n}{\partial x_l} = -(\lambda + \mu) \frac{\partial^2}{\partial x_n \partial x_l \partial x_k} \frac{\partial u_s}{\partial x_s} + \frac{\partial F_n}{\partial x_l}\]

\[\mu \frac{\partial^2}{\partial x_k \partial x_k} \left( \frac{\partial u_l}{\partial x_n} - \frac{\partial u_n}{\partial x_l} \right) - \rho \frac{\partial^2}{\partial t^2} \left( \frac{\partial u_l}{\partial x_n} - \frac{\partial u_n}{\partial x_l} \right) = \left( \frac{\partial F_n}{\partial x_l} - \frac{\partial F_l}{\partial x_n} \right)\]
AXIAL STRAINS: \[
(\lambda + 2\mu) \frac{\partial^2 E_{(ss)}}{\partial x_s \partial x_s} - \rho \frac{\partial^2 E_{(ss)}}{\partial t^2} = \frac{\partial F_s}{\partial x_s}
\]

SYMMETRIC STRAINS: \[
\mu \frac{\partial^2 E_{(nl)}}{\partial x_s \partial x_s} - \rho \frac{\partial^2 E_{(nl)}}{\partial t^2} = \frac{s}{p} \quad \text{ple!}
\]

DEVIA\-TORIC STRAINS: \[
\frac{\mu}{\partial x_s \partial x_s} \frac{\partial^2 \hat{E}_{(nl)}}{\partial x_s \partial x_s} - \frac{\rho}{\partial t^2} \frac{\partial^2 \hat{E}_{(nl)}}{\partial x_s \partial x_s} + (\lambda + \mu) \left( \frac{\partial^2 E_{(ss)}}{\partial x_n \partial x_l} - \frac{\delta_{nl}}{3} \frac{\partial^2 E_{(ss)}}{\partial x_s \partial x_s} \right)
\]

ROTATION STRAINS: \[
\mu \frac{\partial^2 E_{[nl]}}{\partial x_s \partial x_s} - \rho \frac{\partial^2 E_{[nl]}}{\partial t^2} = \frac{1}{2} \left( \frac{\partial F_n}{\partial x_l} - \frac{\partial F_l}{\partial x_n} \right)
\]
\[ (\lambda + 2\mu) \frac{\partial^2}{\partial x_s \partial x_s} \overline{E}_{(ss)} - \rho \frac{\partial^2}{\partial t^2} \overline{E}_{(ss)} = \frac{\partial F_s}{\partial x_s} \]

AXIAL STRAINS:

\[ V = V_P \]

\[ \mu \frac{\partial^2}{\partial x_s \partial x_s} E_{(nl)} - \rho \frac{\partial^2}{\partial t^2} E_{(nl)} = - (\lambda + \mu) \frac{\partial^2}{\partial x_n \partial x_l} E_{(ss)} + \frac{1}{2} \left( \frac{\partial F_n}{\partial x_l} + \frac{\partial F_l}{\partial x_n} \right) \]

STRAINS:

\[ V = V_S \]

\[ \mu \frac{\partial^2}{\partial x_s \partial x_s} E_{[nl]} - \rho \frac{\partial^2}{\partial t^2} E_{[nl]} = \frac{1}{2} \left( \frac{\partial F_n}{\partial x_l} - \frac{\partial F_l}{\partial x_n} \right) \]

ROTATION STRAINS:

\[ V = V_S \]
REPEAT

AXIAL STRAIN:

\[
(\lambda + 2\mu) \frac{\partial^2}{\partial x_s \partial x_s} \bar{E}_{(ss)} - \rho \frac{\partial^2}{\partial t^2} \bar{E}_{(ss)} = \frac{\partial F_s}{\partial x_s} : \\
(\lambda + 2\mu) \frac{\partial^2}{\partial x_s \partial x_s} \frac{\partial \hat{u}_s}{\partial x_s} - \rho \frac{\partial^2}{\partial t^2} \frac{\partial \hat{u}_s}{\partial x_s} = \frac{\partial F_s}{\partial x_s}
\]

DEVIATORIC STRAINS:

\[
\mu \frac{\partial^2 \hat{E}_{(nl)}}{\partial x_s \partial x_s} - \rho \frac{\partial^2 \hat{E}_{(nl)}}{\partial t^2} = -(\lambda + \mu) \left( \frac{\partial^2 \bar{E}_{(ss)}}{\partial x_n \partial x_l} - \frac{1}{3} \delta_{nl} \frac{\partial^2 \bar{E}_{(ss)}}{\partial x_s \partial x_s} \right) + \frac{1}{2} \left( \frac{\partial F_n}{\partial x_l} + \frac{\partial F_l}{\partial x_n} - \frac{2}{3} \delta_{nl} \frac{\partial F_s}{\partial x_s} \right) : \\
\mu \frac{\partial^2}{\partial x_s \partial x_s} \left( \frac{\partial \hat{u}_n}{\partial x_l} + \frac{\partial \hat{u}_l}{\partial x_n} - \frac{2}{3} \delta_{nl} \frac{\partial \hat{u}_s}{\partial x_s} \right) - \rho \frac{\partial^2}{\partial t^2} \left( \frac{\partial \hat{u}_n}{\partial x_l} + \frac{\partial \hat{u}_l}{\partial x_n} - \frac{2}{3} \delta_{nl} \frac{\partial \hat{u}_s}{\partial x_s} \right) = \\
-2(\lambda + \mu) \left( \frac{\partial^2}{\partial x_n \partial x_l} - \frac{1}{3} \delta_{nl} \frac{\partial^2}{\partial x_s \partial x_s} \right) \frac{\partial \hat{u}_s}{\partial x_s} + \left( \frac{\partial F_n}{\partial x_l} + \frac{\partial F_l}{\partial x_n} - \frac{2}{3} \delta_{nl} \frac{\partial F_s}{\partial x_s} \right)
\]

ANTISYMMETRIC (ROTATION) STRAINS:

\[
\mu \frac{\partial^2}{\partial x_s \partial x_s} \hat{E}_{[nl]} - \rho \frac{\partial^2}{\partial t^2} \hat{E}_{[nl]} = \frac{1}{2} \left( \frac{\partial F_n}{\partial x_l} - \frac{\partial F_l}{\partial x_n} \right) : \\
\mu \frac{\partial^2}{\partial x_s \partial x_s} \left( \frac{\partial \hat{u}_n}{\partial x_l} - \frac{\partial \hat{u}_l}{\partial x_n} \right) - \rho \frac{\partial^2}{\partial t^2} \left( \frac{\partial \hat{u}_n}{\partial x_l} - \frac{\partial \hat{u}_l}{\partial x_n} \right) = \left( \frac{\partial F_n}{\partial x_l} - \frac{\partial F_l}{\partial x_n} \right)
\]
Release Rebound Fracture Theory:

- e.g.,: first: break of molecular bonds (rotation)
- then: phase shifted delayed slip
- Theory shall help to discriminate such inter-related motions
- Elementary process shall follow this release rebound theory:
Three wave fields (at $F=0$):

\[
\Delta \bar{E} - \frac{\partial^2 \bar{E}}{V_p^2 \partial t} = 0
\]

\[
\Delta \hat{E}_{(nl)} - \frac{\partial^2 \hat{E}_{(nl)}}{V_s^2 \partial t} = - (\lambda + \mu) \left( \frac{\partial^2}{\partial x_n \partial x_l} - \frac{1}{3} \delta_{nl} \frac{\partial^2}{\partial x_s \partial x_s} \right) \bar{E}_{(ss)}
\]

\[
\Delta \hat{E}_{[nl]} - \frac{\partial^2 \hat{E}_{[nl]}}{V_s^2 \partial t^2} = 0
\]
In 4D The homogeneous wave fields equivalent to Maxwell – like relations

At: \( \overline{E} = 0 \)

\[
| \Delta \hat{E}_{(nl)} \rangle - \frac{\partial^2 \hat{E}_{(nl)}}{V_s^2 \partial t} = 0, \quad \Delta \hat{E}_{[nl]} \rangle - \frac{\partial^2 \hat{E}_{[nl]}}{V_s^2 \partial t^2} = 0
\]

Remark: These two relations are equivalent when the RELEASE-REBOUND process in source runs with the phase shift \( \pm \pi/2 \):

\[
\hat{E} = \pm i \hat{E}
\]
• shear-rotation interaction in waves
Experimental data

- Some experimental data confirm appearance of the correlated motions
- (especially between shears and rotations)
- with immediate correlation or with phase shift $\pi/2$

\[ E \propto \hat{E} \quad \text{or} \quad \hat{E} \propto \pm i \hat{E} \]

or at the double rebound process:

\[ \hat{E} \propto i i \hat{E} = -\hat{E} \]

POSTER by Krzysztof P. Teisseyre
Defect Induced Stresses (rearrangement of stresses)

• The important relation (M. Peach and J.S. Koehler, 1950, The Forces Exerted on Dislocations and the Stress Fields Produced by Them, Phys. Rev. 80, 436–439) defines the forces exerted on dislocations (Peach and Koehler, 1950)

• Considering the continuous defect fields we define the induced stresses under the applied stress system and internal defects

• Rearrangement of the stress system
For the axial, $\overline{S}$, shear, $\hat{S}$, and rotation stresses, $\epsilon$, applied we obtain the generalized forces action on defects das defined by the Burgers vector and versor of defect edge: $b_k \nu_q$

$$\overline{F}_n = \frac{1}{3} \epsilon_{n sq} \overline{S} \delta_{sk} b_k \nu_q = \epsilon_{nkq} \overline{S} b_k \nu_q$$

$$\hat{F}_n = \epsilon_{n sq} \hat{S}_{(sk)} b_k \nu_q$$

$$F_n \rightarrow M_n = \epsilon_{n sq} S_{[sk]} b_k \nu_q$$
For the defect density $\alpha_{qk} = \nu_q b_k$

We define the INDUCED stresses

$$S^{IND}_{np} \equiv F_n n_p = \varepsilon_{nsq} \alpha_{qk} n_p S^{LOAD}_{sk}$$

$$S^{IND}_{np} = \frac{1}{3} \varepsilon_{nkq} \bar{S} \alpha_{qk} n_p ,$$

$$S^{IND}_{np} = \varepsilon_{nsq} \hat{S}_{(sk)} \alpha_{qk} n_p ,$$

$$S^{IND}_{np} = \varepsilon_{nsq} \hat{S}_{[sk]} \alpha_{qk} n_p$$
Particular CASES

DEFECT PLANES PARALLEL TO EARTH’ SURFACE

\[ S_{n3}^{\text{IND}} = \frac{1}{3} \varepsilon_{nkq} \bar{S} \alpha_{qk} \]

\[ S_{n3}^{\text{IND}} = \varepsilon_{nsq} \hat{S}(sk) \alpha_{qk} \]

\[ S_{n3}^{\text{IND}} = \varepsilon_{nsq} \hat{S}[sk] \alpha_{qk} \]
Planes parallel to surface

\[
S_{xz}^{IND} = \frac{1}{3} \hat{S} \alpha_{zy} - \frac{1}{3} \hat{S} \alpha_{yz}
\]

\[
S_{yz}^{IND} = \frac{1}{3} \hat{S} \alpha_{xz} - \frac{1}{3} \hat{S} \alpha_{zx}
\]

\[
S_{zz}^{IND} = \frac{1}{3} \hat{S} \alpha_{yx} - \frac{1}{3} \hat{S} \alpha_{xy}
\]

\[
S_{xz}^{IND} = \hat{S}_{(yx)} \alpha_{zx} - \hat{S}_{(zx)} \alpha_{yx} + \hat{S}_{(yy)} \alpha_{zy} - \hat{S}_{(zk)} \alpha_{yy} + \hat{S}_{(yz)} \alpha_{zz} - \hat{S}_{(zz)} \alpha_{yz}
\]

\[
S_{yz}^{IND} = \hat{S}_{(zx)} \alpha_{xx} - \hat{S}_{(xx)} \alpha_{zx} + \hat{S}_{(zy)} \alpha_{xy} - \hat{S}_{(xy)} \alpha_{zy} + \hat{S}_{(zz)} \alpha_{xz} - \hat{S}_{(xz)} \alpha_{zz}
\]

\[
S_{zz}^{IND} = \hat{S}_{(xx)} \alpha_{yy} - \hat{S}_{(yx)} \alpha_{xx} + \hat{S}_{(xy)} \alpha_{yy} - \hat{S}_{(yy)} \alpha_{xx} + \hat{S}_{(xz)} \alpha_{yz} - \hat{S}_{(yz)} \alpha_{xz}
\]
\[
S_{xz}^{\text{IND}} = \frac{1}{3} S\alpha_{zy} - \frac{1}{3} S\alpha_{yz}
\]
\[
S_{yz}^{\text{IND}} = \frac{1}{3} S\alpha_{xz} - \frac{1}{3} S\alpha_{zx}
\]
\[
S_{zz}^{\text{IND}} = \frac{1}{3} S\alpha_{xy} - \frac{1}{3} S\alpha_{yx}
\]
\[
S_{rz}^{\text{IND}} = \hat{S}(\varphi_k)\alpha_{zk} - \hat{S}(z_k)\alpha_{\varphi k}
\]
\[
S_{\varphi z}^{\text{IND}} = \hat{S}(z_k)\alpha_{rk} - \hat{S}(r_k)\alpha_{zk}
\]
\[
S_{\varphi k}^{\text{IND}} = \hat{S}(r_k)\alpha_{\varphi k} - \hat{S}(\varphi k)\alpha_{rk}
\]
For rotational squeeze we put

\[ S^{IND}_{r\phi} = \frac{1}{3} \overline{S} \alpha_{z\phi} n_\phi - \frac{1}{3} \overline{S} \alpha_{\phi z} n_\phi \]

\[ S^{IND}_{\phi\phi} = \hat{S}_{(zr)} \alpha_{rr} n_\phi - \hat{S}_{(rr)} \alpha_{zr} n_\phi + \hat{S}_{(z\phi)} \alpha_{r\phi} n_\phi - \hat{S}_{(r\phi)} \alpha_{z\phi} n_\phi + \hat{S}_{(zz)} \alpha_{rz} n_\phi - \hat{S}_{(rz)} \alpha_{zz} n_\phi \]

\[ S^{IND}_{z\phi} = \hat{S}[rr] \alpha_{\phi r} n_\phi - \hat{S}[\phi r] \alpha_{rr} n_\phi + \hat{S}[r\phi] \alpha_{\phi \phi} n_\phi - \hat{S}[\phi \phi] \alpha_{r\phi} n_\phi + \hat{S}[rz] \alpha_{\phi z} n_\phi - \hat{S}[\phi z] \alpha_{rz} n_\phi \]

AND FOR LOAD:

\[ p = -\frac{1}{3} \overline{S} = -\frac{1}{3} \left( \hat{S}_{(rr)} + \hat{S}_{(zz)} \right) \]

\[ S^{IND}_{r\phi} = \frac{1}{3} \overline{S} \alpha_{z\phi} n_\phi - \frac{1}{3} \overline{S} \alpha_{\phi z} n_\phi \]

\[ S^{IND}_{\phi\phi} = \hat{S}_{(zz)} \alpha_{rz} n_\phi - \hat{S}_{(rr)} \alpha_{zr} n_\phi \]

\[ S^{IND}_{z\phi} = \hat{S}[rr] \alpha_{\phi r} n_\phi \]
We have in particular

\[ \bar{S} = S_{rr} + \bar{S}_{\varphi\varphi} + S_{zz} \]

We note an appearance of the significant component

\[ E_{\varphi\varphi} \]

in strain measurement related to earthquake events (Gomberg and Agnew, 1996). The authors original statement was that this squeeze strain angular squeeze component can be estimated from the solution for the displacement potentials. However, the obtained asymptotic solution of the wave equations, for the scalar and vector potentials of displacements, indicated that such field according to theory is negligible.
However, according to the asymmetric theory this component

\[ E_{\phi\phi} \]

shall be estimated from wave equation for the independent wave equation for the axial strains. For the case with a pressure gradient (z-direction) we may have an angular symmetry and angular squeeze in the plane parallel to the Earth surface. We can obtain the following asymptotic solution of the wave equations for this component in the cylindrical coordinates \((r, \phi, z)\)
The required solution for the axial strain may be expressed directly by an expansion of the Bessel functions and the cylindrical harmonics; we obtain significant result:

$$E_{\varphi\varphi} \approx \frac{S^{LOAD}}{3\lambda + 2\mu} \sqrt{\frac{2}{\pi k_r r}} \exp\left[i\left(k_r r + k_z z - \omega t - \pi / 4\right)\right]$$

assuming the same constants for angular squeeze.
Reasumming our theretical assumptions related to the Asymmetric Continuum Theory explains also the discovered experimentaly significant squeeze cylindrical strains.

\[ E_{\phi\phi} \]
• Our final remarks concern a need to create the modern worldwide seismological network to record the important and independent strain fields;

• To this end we present sensitivity and discrimination ability of the different sensors.
### Sensitivity of the different sensors

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- Rotation seismometer: two pendulums used to record the rotational motions: two perpendicularly oriented rotation seismometers detect the intensity of rotations.
### Discrimination ability for motions or deformations

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<td>Rotation Strain</td>
<td>-</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>-</td>
</tr>
<tr>
<td>Shear Strain</td>
<td>-</td>
<td>x</td>
<td>-</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Axial Strain</td>
<td>-</td>
<td>x</td>
<td>-</td>
<td>-</td>
<td>x</td>
</tr>
</tbody>
</table>
• These tables show how poor equipment we have in seismology !!!
• Seismometer records do not discriminate and explain exactly to what deformations the recorded displacement may belong: some information come from recorded time (P, or S or other seismic phases) but still we do not know if these motions belong to real displacements or to axial stresses or shears or rotational strains !
Example: Element of Simple Network

The 4 – squares mean the 3 – component seismometer system; circle - control rotation sensor.