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Key Points:

- Tsunami source uncertainty estimation with unknown spatial discretization and gauge weights
- Results locally adapt to source complexity without requirement for regularization
- Tohoku tsunami source is elongated trench parallel, well-resolved bimodal feature in northern part

Supporting Information:

- Supporting Information S1

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Tsunami source uncertainty estimation: The 2011 Japan tsunami

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Abstract This paper studies the initial sea surface displacement and its uncertainty after an earthquake based on tsunami waveforms. The spatial distribution is inferred with a Bayesian approach that provides probabilities that are interpreted as uncertainties of the displaced sea surface. The parameterization is nonlinear and treats apparent rupture velocity as unknown but assumes rise time to be fixed at 30 s. Importantly, the spatial complexity of the source is constrained by observations using a transdimensional algorithm based on a wavelet decomposition of the displacement field. In this approach, the number of wavelet coefficients is an unknown random variable that is also estimated as part of the inversion. The resulting parameterization is parsimonious in that it can adapt to the spatially varying source complexity while being consistent with the information in the tsunami waveforms. In this way, the resolution of displacement varies across the source region with more parameters introduced for parts of the source that are resolved well by the data and/or have significant complexity. The noise level (standard deviation) at each gauge is initially treated as unknown to estimate data covariance matrices. These matrices are applied in subsequent inversion and include unknown scaling which eliminates the requirement to assume station weights and accounts for temporally correlated waveform noise. The method is applied to waveforms recorded during the 2011 Japan Tsunami and results show high resolution (low uncertainty) in most parts of the source region and a previously unreported level of source detail. In particular, the main peak of the source is elongated trench parallel and shows a well-resolved bimodal finger-like feature in the northern source region that closely follows the trench.

1. Introduction

Tsunamis are among the most devastating natural hazards and have tragically caused large numbers of fatalities, displaced people, and resulted in substantial economic losses. They are most commonly generated by large seafloor deformation due to rupture of seismogenic faults at convergent margins [Abe, 1973]. However, other events, such as submarine mass failures [Ward, 2001], can also contribute to the source mechanism. The seafloor deformation produces an initial vertical displacement of the sea surface which propagates throughout ocean basins causing hazardous waves in both the near and far field. Here this initial vertical displacement of the sea surface is termed the tsunami source.

The tsunami source can extend over large areas (lateral scales of >100 km), reach several meters in magnitude, and have a complex spatiotemporal evolution [Saito et al., 2010; Satake et al., 2013]. However, no direct observations of the source process exist and it must be inferred from seismological, geodetic, and tsunami waveform data. To date, these inversion results (the spatiotemporal information about the source) depend on many subjective choices that are commonly applied in the inversion. This work addresses some of these choices and provides a robust quantification of the uncertainty in sea surface displacement using an objective framework.

Many studies have considered the tsunami source by inferring the spatiotemporal slip distribution on the seismogenic fault (Figure 1a), from which the sea surface displacement can be derived [e.g., Satake, 1989; Fujii and Satake, 2006; Fujii et al., 2011; Satake et al., 2013; An et al., 2014; Melgar and Bock, 2015]. While advantageous when considering seismic data, this approach suffers from the requirement to specify a fault geometry and makes assumptions about rupture and translation of deformation to displacement [Cummins et al., 1998], which may be subjective and difficult to justify. Consequently, the fault slip estimates depend on these

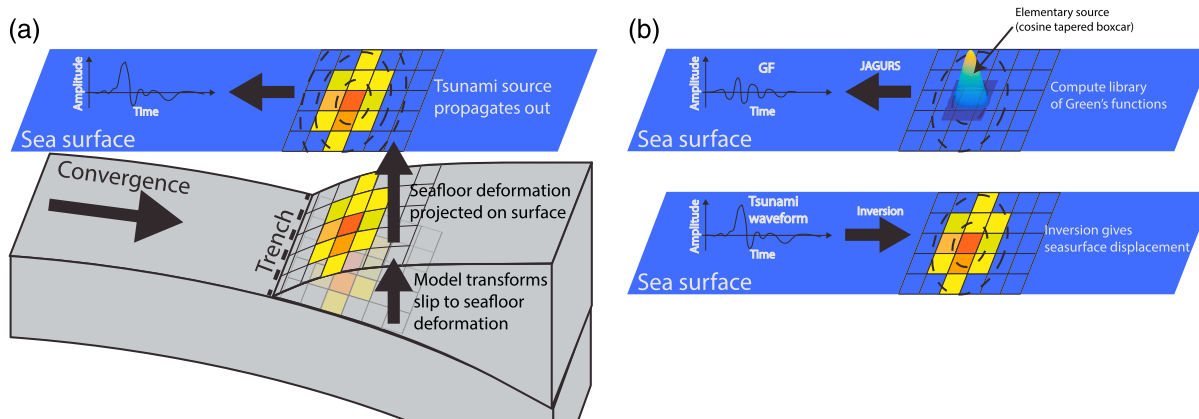


Figure 1. Two approaches to tsunami source inversion: (a) Slip on a fault is assumed and translated to surface deformation, requiring several assumptions about fault geometry, material properties, and dislocation modeling for the computation of Green's functions (GFs). (b) The approach applied here computes GFs directly for elementary waves at the sea surface and inverts for the tsunami source directly without assumptions about the fault and seafloor deformation.

subjective choices [e.g., Beresnev, 2003] which can lead to inaccurate source estimates. An alternative method (Figure 1b) is based on only tsunami waveforms and requires fewer assumptions about the seismic source by parameterizing the sea surface directly (Figure 1b) [Aida, 1972; Baba and Cummins, 2005; Saito et al., 2010]. To date, these inversion methods assume a linear problem [e.g., Saito et al., 2011; Tsushima et al., 2012; Hossen et al., 2015] and apply regularization to stabilize the inverse problem.

While computationally efficient, linear and linearized approaches require many strong/subjective assumptions that can have undesirable effects. For example, smoothest regularization is typically a global process, controlled by just one or two unknowns, the selection of which results in a smooth slip/displacement image. Such regularization is always a compromise between regions which are overregularized or under regularized. Most regularization methods are also associated with nonlinearity in that a scaling parameter must be estimated. Some form of grid search in conjunction with L curve tests [Hansen, 1992], discrepancy principle [Constable et al., 1987], Akaike's Bayesian Information Criterion [Yabuki and Matsu'ura, 1992], or generalized cross validation [Golub et al., 1979] is typically applied to address the lack of independent knowledge about a value for this parameter that is consistent with data information. The parameter is then assumed as fixed at an optimal value in the final inversion which produces the parameter estimates. Poor choices for these parameters can profoundly affect the solution. More recently, fault slip models with uncertainty estimation have been considered [e.g., Fukuda and Johnson, 2008; Monelli and Mai, 2008; Fukuda and Johnson, 2010; Simons et al., 2011; Hooper et al., 2013; Minson et al., 2014; Dettmer et al., 2014; Jolivet et al., 2014] and extended to account for some aspects of environmental mismatch (e.g., uncertainty in seismic velocity model [Yagi and Fukahata, 2011; Duputel et al., 2014]). These studies do not consider uncertainty estimates for tsunami sources (although some predict sea surface displacements from posterior mean or median slip models). To our knowledge, uncertainties of tsunami source parameters have only been considered in linear inversion [Takagawa and Tomita, 2014].

The impact of model selection on inversion results is often not considered, including studies where Bayesian sampling is applied for uncertainty estimation. Model selection refers to the quantitative process of identifying those parameterizations which are appropriate for a given data set based on the data information itself. It is well known that overparameterized 1-D models (e.g., utilizing many layers of fixed thicknesses below the resolution power of the data) require some form of regularization to avoid spurious, unconstrained structure in the inversion result [e.g., Constable et al., 1987]. The simplest such regularization, sometimes referred to as damping, uses a diagonal matrix with a scaling which, in a Bayesian approach, is equivalent to applying an uncorrelated Gaussian prior of standard deviation proportional to the scaling. The extension to 2-D with regular grids is straightforward [e.g., De Groot-Hedlin and Constable, 1990] and widely applied in linear slip inversion [e.g., Yabuki and Matsu'ura, 1992].

It is important to note that nonlinear Bayesian approaches suffer from similar problems (unconstrained structure, poor uncertainty estimates) when an ad hoc regular grid is chosen [Bodin and Sambridge, 2009] without model selection or regularization. While priors can be considered a form of regularization, they may not be

sufficient to avoid dependence on parameterization choice. *Fukuda and Johnson* [2008] include regularization parameters as unknowns which addresses model selection via the regularization prior but results in a globally smooth slip distribution. However, most geophysical data exhibit spatially variable sensitivity to geophysical properties. Therefore, objective model selection should include a parameterization that can adapt locally to the resolution power of the data.

This work applies Bayesian model selection with uncertainty estimation to the tsunami source (the initial sea surface displacement) while avoiding most assumptions associated with the earthquake rupture. We apply numerical sampling to obtain a nonlinear estimate of the tsunami source that treats the discretization and the waveform noise as unknowns and accounts for temporal noise correlations. The unknown discretization is addressed by defining a transdimensional (trans-D) model [Malinverno, 2000; Sambridge *et al.*, 2006; Dosso *et al.*, 2014].

Trans-D models have been applied over the last two decades to relax parameterization assumptions in Bayesian inference in many fields of science [e.g., Green, 1995; Brooks *et al.*, 2011]. They provide a more general answer than studies applying fixed parameterizations (including arbitrarily fine discretizations), since the solution includes the effect of limited independent knowledge about the parameterization. They avoid both overparameterization and under parameterization problems which can be shown to lead to poor uncertainty estimates and biased or spurious features that may lead to erroneous interpretation (see Appendix A for details).

Trans-D models were recently applied to slip inversion [Dettmer *et al.*, 2014] by employing an irregular grid of Voronoi cells with an unknown and uncertain number of nodes [Bodin and Sambridge, 2009] to partition slip on the fault. The resulting parameterization is efficient in that it captures slip patterns with few parameters. The approach adapts locally to rupture complexity by placing more nodes where the data require rupture properties to vary spatially. A limitation is the piecewise-constant nature of Voronoi cells that is not an ideal representation of smooth geophysical properties such as sea surface displacement.

Here we follow recent work by Hawkins and Sambridge [2015] and employ a trans-D tree method with wavelet parameterization for the sea surface. The wavelet parameterization is based on a hierarchical decomposition of the model into wavelet coefficients and the maximum number of coefficients is equal to the number of source patches: The sea surface is divided by a regular grid of source patches of 15 km discretization in both latitude and longitude (the smallest spacing our computing resources could support). Tsunami waveform predictions are based on a library of Green's functions (GFs) which is precomputed for these source patches. The elementary sources (ES) used to compute GFs are cosine-tapered boxcar functions which overlap with neighboring source cells by 1/2 of the grid spacing. This is an important distinction; from previous studies based on Gaussian-shaped ES [e.g., Baba and Cummins, 2005; Saito *et al.*, 2010; Hossen *et al.*, 2015] which result in intrinsic smoothing/regularization that is difficult to justify and can obfuscate the meaning of uncertainty estimates. The cosine-tapered ES are sufficiently smooth to allow stable tsunami propagation modeling [Baba *et al.*, 2015] but also have spatially limited support to avoid arbitrary smoothness of the tsunami source estimate.

The method is applied to the 2011 Tohoku-Oki earthquake which triggered one of the most destructive and lethal tsunamis ever recorded. Due to the available instrumentation in Japan, it is one of the best recorded events in history and is therefore an ideal candidate for which to study the tsunami source. The inversion results are based on 16 high-quality tsunami recordings in the near field and midfield. The results do not require any form of smoothing, and weights of the 16 gauges are estimated as part of the inversion by treating the noise at each gauge as independent and unknown while accounting for temporal noise correlations. Results are interpreted in terms of posterior median models and various marginal distributions that quantify the uncertainty of displacement as a function of space.

2. Method

2.1. Parameterization of the Sea Surface and Green's Functions

To obtain waveform predictions in an efficient manner, GFs need to be precomputed from each source patch to each observation point before the inversion is carried out. Strictly, these GFs should be for instantaneous point sources of unit amplitude. However, some approximations are required for numerical stability which results in approximate GFs which include the effect of a finite rise time for seafloor deformation and a finite spatial extent of deformation. We employ the tsunami propagation algorithm JAGURS [Baba *et al.*, 2015] to numerically simulate the tsunami propagation for each elementary source.

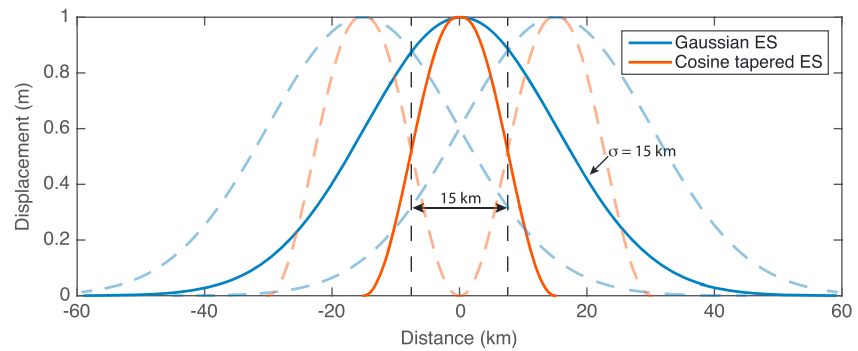


Figure 2. Comparison of a widely applied Gaussian shape elementary source with 15 km standard deviation (blue and dashed blue) to the cosine-tapered shape used in this work (red and dashed red) for three source positions along distance (−15, 0, and 15 km). The width of the source patch is 15 km (dashed black). The more compact support of the cosine source which results in significantly less overlap and less arbitrary smoothing. Note that this comparison is scale invariant since both types of sources must always sum to a uniform surface across the source area.

In particular, each elementary source η_i is centered on patch $\mathbf{s}_i$ and defined by an initial sea surface displacement that is a cosine-tapered boxcar function (Figure 2)

$$\eta_i(\mathbf{s}_i) = 0.25[1 + \cos(\pi\xi_{1i}/L)][1 + \cos(\pi\xi_{2i}/L)], L \leq \xi_{1i}, \xi_{2i} \leq L, \quad (1)$$

where ξ_{1i} and ξ_{2i} are strike parallel and perpendicular distances along the Earth surface, respectively, and the origin of ξ_i coincides with $\mathbf{s}_i$. This cosine-tapered basis function results in more localized support (Figure 2) compared to previous inversion work which applied Gaussian ES [e.g., Saito *et al.*, 2011; Hossen *et al.*, 2015]. Compact support is important for inversion applications since the spatial extent in the approximate GFs causes intrinsic smoothing that is difficult to justify and quantify. Since one goal of this work is objective estimation of source complexity, arbitrary smoothness is not desirable.

Here the temporal evolution of rupture is treated with a nonlinear parameterization that requires few variables and assumes a single ramp function with a rise time of 30 s. This rise time is consistent with previous studies [e.g., Satake *et al.*, 2013], but we note that treating rise time as an unknown would be preferable and results in more complete uncertainty estimates. Currently, the added computational cost prohibits such parameterizations. Apparent rupture velocity is assumed to be unknown but constant throughout the source area.

The GFs $G_i(\mathbf{x}_j, t)$ at observation point $\mathbf{x}_j$ is based on solving the linear shallow water wave equations with the initial condition given by source η_i . JAGURS solves the shallow water equations in spherical coordinates with a linear dispersion term (nonlinear effects are small for the water depths considered here), by implementing a staggered, leap-frog finite difference scheme. The computational domain is from 140–158°E to 27–46°N. Bathymetry data are from the GEBCO_08 30'' grid over the whole domain with more accurate bathymetry in the source region given by the M7000 map 7 series provided by the Marine Information Research Center, Japan Hydrographic Association. All GF time series are simulated for 9000 s with 1 s resolution. The spatial grid is nested such that the source region uses 20'' resolution in both latitude and longitude and the remainder uses 60''. Since the time evolution in JAGURS uses an iterative scheme to achieve convergence of the dispersive waves, we carried out a convergence test for a single patch and determined that a convergence parameter [Baba *et al.*, 2015] of 10^{-9} is sufficient to compute stable dispersive GFs. Computation expense for all 512 GFs was ~80,000 CPU hours on the Australian National Computational Infrastructure facility Raijin.

2.2. Bayesian Inference

Our method is based on Bayesian inference, where parameters are considered to be random variables characterized by probability density functions (PDFs). The probabilities express degrees of belief and can be understood as a generalized logic [Jaynes, 2002]. In the context of this work, the probabilities are applied to express degrees of belief that a parameter value (e.g., sea surface displacement of a source patch) is consistent with the value of the geophysical property of interest (the actual sea surface displacement at that point in space at the time of seafloor deformation). Therefore, the PDFs can be interpreted as uncertainties. The inference process itself can be viewed as probability updating: First prior probabilities are assigned to parameters and then updated by data information which is expressed by the likelihood function. Mathematically,

this process is governed by Bayes' Theorem [Brooks *et al.*, 2011; Gelman *et al.*, 2013]

$$p(\mathbf{m}|\mathbf{d}) = \frac{p(\mathbf{m})p(\mathbf{d}|\mathbf{m})}{p(\mathbf{d})}, \quad (2)$$

where $\mathbf{m}$ is a vector of M parameters, $\mathbf{d}$ is a vector of N observed data, and $p(\mathbf{m})$ is the prior assigned to $\mathbf{m}$ which is based on independent information about the parameters (e.g., geological constraints). Since the data are considered fixed, $p(\mathbf{d}|\mathbf{m})$ can be interpreted as a function of only $\mathbf{m}$, the likelihood function $L(\mathbf{m})$. The normalizing constant $p(\mathbf{d})$, also known as evidence, is the basis for Bayesian model selection and represents the likelihood of the model (how well a parameterization is supported by the observed data). The solution to the inference problem is given by $p(\mathbf{m}|\mathbf{d})$, which quantifies our knowledge about the parameters given data and prior information. The likelihood function $L(\mathbf{m})$ quantifies the data information and can be of complicated (not convex) form for nonlinear problems even though it is typically based on an assumption of Gaussian noise. Here the nonlinearities arise from the process of data predictions which include changes in spatial discretization, noise parameters, and apparent rupture velocity. Therefore, the posterior is not Gaussian and can take on forms for which no analytic representation exists. Consequently, it must be characterized by numerical integration, often referred to as sampling.

This study assumes waveform noise to be Gaussian distributed and to include both measurement error and theory error due to model limitations. Waveform noise is assumed to be independent between different gauges but is represented by a covariance matrix including off-diagonal terms and variations on the diagonals which account for temporal dependence. The covariance matrices are estimated iteratively by initially assuming uncorrelated errors of unknown magnitude [e.g., Dettmer *et al.*, 2007]. For this initial step, the logarithm of the likelihood function is

$$\log L(\mathbf{m}) \propto \sum_{j=1}^S \left(-N \log \sigma_j - \frac{1}{2\sigma_j^2} \mathbf{r}_j^T \mathbf{r}_j \right), \quad (3)$$

where S is the number of gauges and σ_j the noise standard deviation parameter at gauge j . Vector $\mathbf{r}_j = \mathbf{d}_j - \mathbf{d}_j^{\text{pred}}$ is the residual error at each gauge defined as the difference between observed data $\mathbf{d}_j$ and prediction $\mathbf{d}_j^{\text{pred}}$. Note that the σ_j are unknowns in the inversion, which provide automated weighting for each gauge so that various gauges contribute to the solution with their significance given by that waveform's noise level. For the simulations considered in this work, equation (3) is sufficient. However, for the Japan tsunami data, the inversion results obtained with equation (3) provide residual errors for the most probable model which are used to estimate covariance matrices for each waveform [Dettmer *et al.*, 2014]. These matrices account for both measurement and theory errors and are applied in a subsequent inversion with likelihood function

$$\log L(\mathbf{m}) \propto \sum_{j=1}^S \left(-\frac{N_j}{2} \log \xi_j - \frac{1}{2\xi_j} \mathbf{r}_j^T \mathbf{C}_d^{-1} \mathbf{r}_j \right). \quad (4)$$

which includes unknown parameters ξ_j for each gauge that scale the covariance magnitude according to data information to reduce dependence of the results on this iterative estimation.

Note that $\mathbf{m}$ need not be limited to a single choice of parameter vector. Rather, the parameterization can be relaxed to any number of choices for which relative inferences are desired. This allows for a more general inversion which additionally accounts for uncertainty due to our inability to specify a single parameterization with perfect confidence. In that case $\mathbf{m}$ is referred to as a trans-D model and obtaining a posterior for it intrinsically applies model selection: Relative probabilities can be obtained for each choice of parameter vector that quantify data support. However, absolute comparison (e.g., if other models are considered in the future) requires computation of Z , a computationally daunting process [Dettmer *et al.*, 2010]. Here the trans-D parameterization is in terms of a hierarchy of wavelet coefficients which result in inferences that adapt model complexity based on data information/resolution.

The numerical sampling of the posterior is by Markov chain Monte Carlo methods. We employ the reversible-jump or Metropolis-Hastings-Green (MHG) algorithm [Geyer and Moller, 1994; Green, 1995] which uses a random walk to move through trans-D parameter space. The current state of the algorithm is given by a set of parameter values $\mathbf{m}$ (e.g., a realization of displacement values for source patches) and a new state $\mathbf{m}'$ is

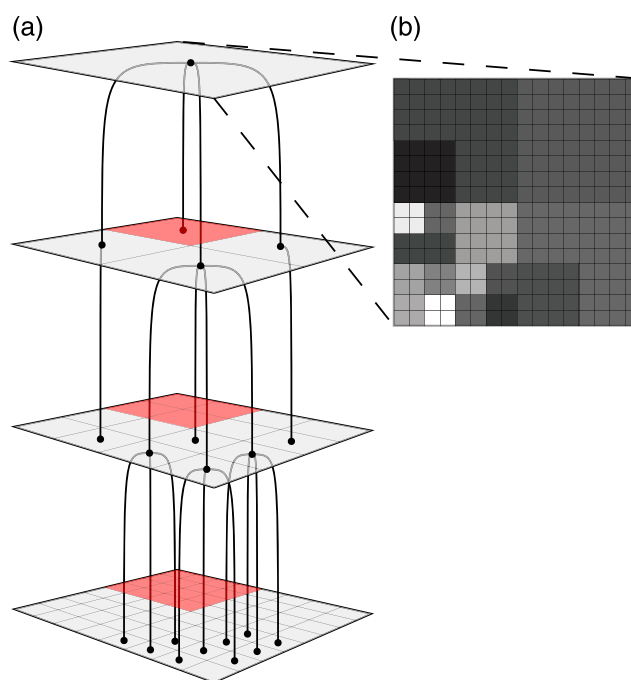


Figure 3. Schematic of the trans-D tree method with wavelet parameterization of the sea surface displacement: (a) The displacement is decomposed into a hierarchy of coefficients, with a single coefficient for the highest level and the finest discretization given by the spacing of GFs. Note that the trans-D parameterization results in a parsimonious quality, where large areas of the model can be described by few coefficients (highlighted in red). (b) The various levels contribute to the final image of the displacement distribution for which waveforms are predicted. The trans-D structure facilitates adaptation to local source features.

proposed based on a distribution q , centered on the current state (a Cauchy distribution in this case, see *Dosso and Wilmut* [2008]). The proposed $\mathbf{m}'$ is accepted or rejected as a new state of the chain based on probability

$$\alpha = \min \left(1, \frac{p(\mathbf{m}')}{p(\mathbf{m})} \left(\frac{L(\mathbf{m}')}{L(\mathbf{m})} \right)^\beta \frac{q(\mathbf{m}|\mathbf{m}')}{q(\mathbf{m}'|\mathbf{m})} |J| \right), \quad (5)$$

where $|J|$ is a Jacobian that accounts for potential changes in parameterization between $\mathbf{m}$ and $\mathbf{m}'$ and β is a tempering parameter ≤ 1 . The numerical approach includes parallel tempering [Geyer, 1991] with a large number of Markov chains that are computed concurrently on a computer cluster and exchange information. Parallel tempering has the effect of turning the intrinsically serial MHG algorithm into a parallel algorithm that scales to large numbers of CPUs and addresses high-dimensional problems efficiently. Details of our particular implementation can be found in previous work [e.g., *Dettmer and Dosso*, 2012; *Dettmer et al.*, 2013, 2014].

A significant difference to our previous work is that the trans-D model does not use a partitioning based on Voronoi cells. Rather, the spatial distribution of displacement is decomposed by a hierarchy of wavelet coefficients which efficiently capture the source complexity with few coefficients [Hawkins and Sambridge, 2015]. The hierarchical decomposition is sampled via a tree structure (Figure 3) and the complexity of the tree is probabilistically based on equation (5). This results in a parsimonious parameterization which adapts locally to the spatially varying source complexity based on data information. Therefore, large areas with little structure are described by only few parameters, while complex areas are described by more coefficients. This is an important quality of a parameterization and crucial to address under parameterization and overparameterization. A parameterization based on a fixed uniform grid is a poor match for tsunami source inversion since tsunami gauges are spatially nonuniform resulting in observations that have varying sensitivity to spatial length scales across the domain. Forcing a globally fixed resolution grid and/or globally fixed smoothness, no matter how well chosen, will result in some regions being overparameterized and others under parameterized and this will in turn negatively affect the parameter and uncertainty estimates.

The details of this trans-D tree method with wavelet parameterization can be found in *Hawkins and Sambridge* [2015]. All results in this work are based on the CDF 9/7 wavelet basis [Cohen et al., 1992] which has been

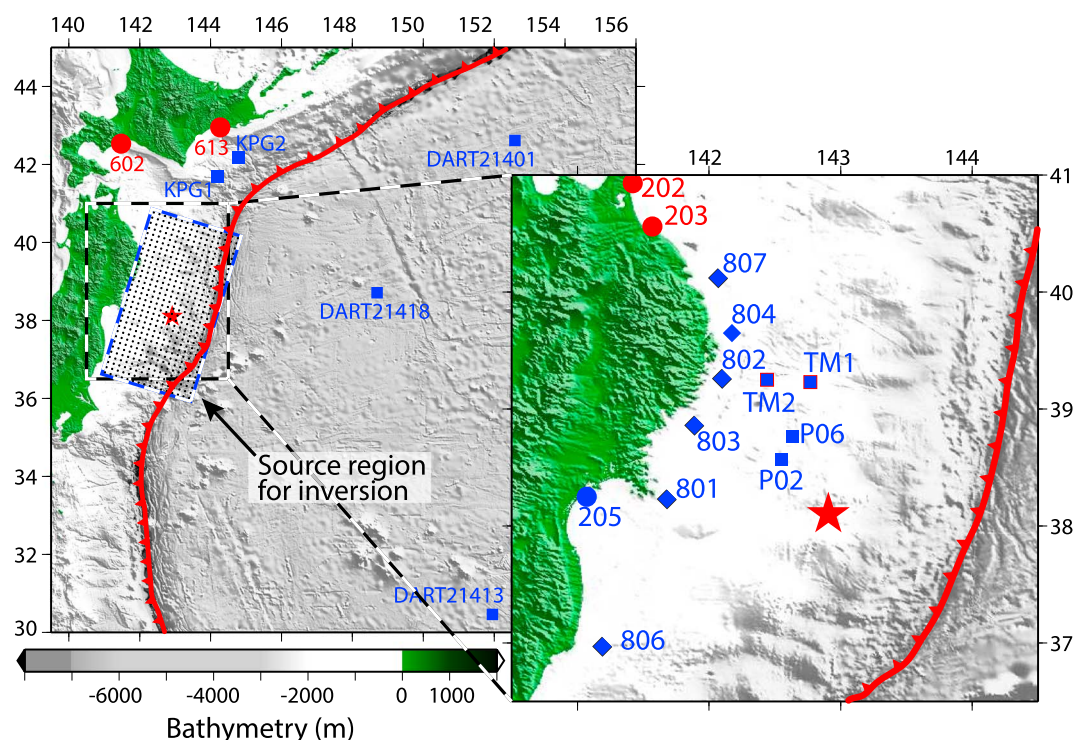


Figure 4. Tohoku tsunami source region considered in this study (epicenter—red star). Tsunami gauges used in the inversion (OBP—blue squares, GPS—blue diamonds, CWG—blue circle). The source area is also shown (blue dashed and black dots for locations of elementary sources). The source area is approximately aligned with the Japan trench (red sawtooth). Four additional CWG locations are shown as red circles. These data were not included in the inversion but are employed to verify tsunami predictions.

shown to work well for properties expected to exhibit some spatial smoothness [Hawkins and Sambridge, 2015]. We note that a major advantage of the wavelet approach is the computational efficiency of the Bayesian tree structure. In contrast, Voronoi cells can be challenging for the reversible-jump algorithm. In particular, adding and deleting cells is highly inefficient and computational cost increases significantly with the number of Voronoi nodes and the number of GFs.

Disadvantages of the wavelet parameterization are the intrinsically rectangular decomposition (which can cause challenges for diagonal features) and the specification of prior bounds on the properties of interest. Here we follow the simple approach of applying uniform, wide priors for all coefficients. While this approach works well in this application, in other cases it may be desirable to apply more prior information. This is an area requiring additional research.

However, we consider the advantages to far outweigh the disadvantages. In particular, computational efficiency allows consideration of problems that were previously not addressable with the Voronoi-based approach. In addition, other basis functions can in principle be applied, in combination with the trans-D tree framework, to address more sophisticated priors and avoid other problems specific to wavelets.

3. Application to the 2011 Tohoku Tsunami

3.1. Observations

The magnitude $M_w \sim 9.0$ 2011 Tohoku earthquake (11 March, 05:46:23 UTC) is the largest recorded Japanese earthquake and generated one of the worst tsunamis in Japan's history (Figure 4). Direct observations of seafloor deformation [Iinuma et al., 2012; Fujiwara et al., 2011] indicate large fault slip (>50 m) near the trench axis, consistent with the strong tsunami genesis. Large near-trench slip is also supported by several studies that estimated fault slip based on seismic, geodetic, and tsunami data sets [e.g., Ozawa et al., 2011; Fujii et al., 2011; Satake et al., 2013; Melgar and Bock, 2015]. However, the various results show significant differences which are difficult to interpret and sometimes inconsistent. While more objective inversion approaches with uncertainty estimation could reconcile such inconsistencies, only a few studies estimate uncertainties of

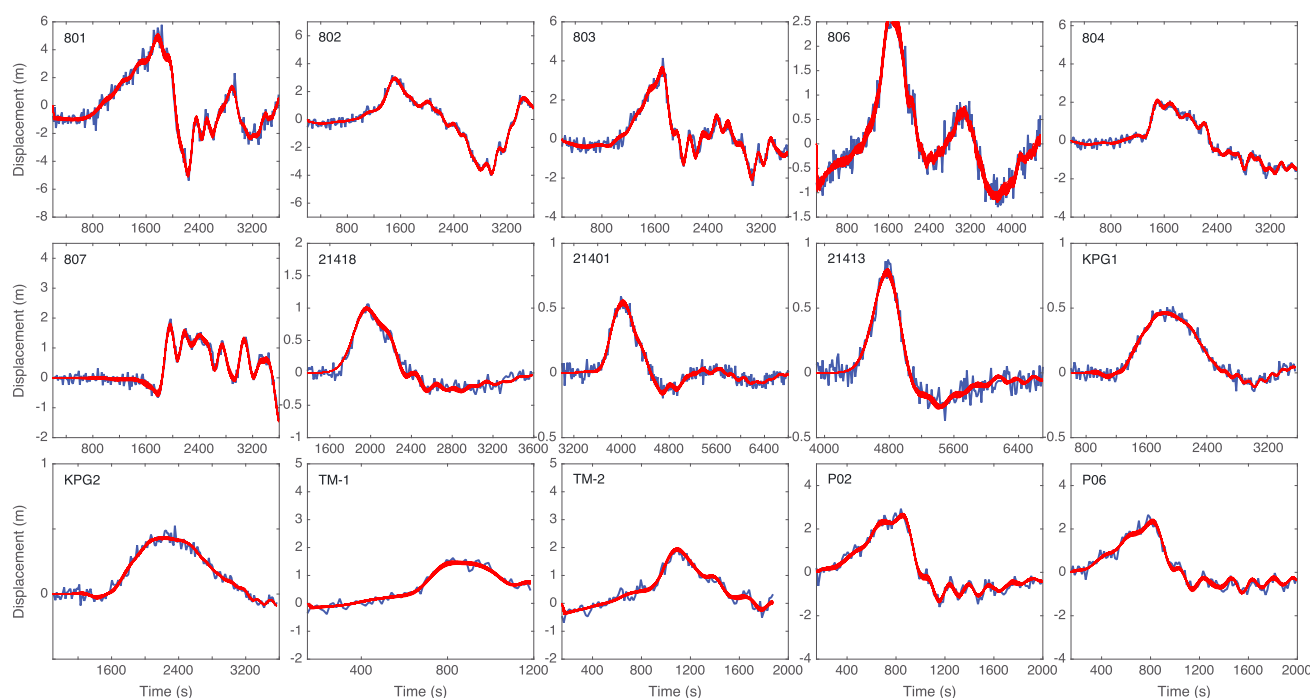


Figure 5. Simulated noisy data (blue) and data predictions (red) for a simulation test case. Data predictions are many superimposed lines for a large random subset of the posterior ensemble. To reduce computational cost, the simulation is based on a 30 km Green functions grid. The simulation geometry is identical to the 2011 Japan tsunami study.

slip for this event [e.g., *Simons et al.*, 2011; *Minson et al.*, 2014]. Even less focus has been on studying sea surface displacement without making assumptions about the underlying fault [e.g., *Saito et al.*, 2011; *Tsushima et al.*, 2012; *Saito et al.*, 2014; *Hossen et al.*, 2015]. To date, only *Takagawa and Tomita* [2014] considered sea surface displacement uncertainty in linear inversion without studying nonlinearity and the impact of model assumptions on results (i.e., model selection).

Nevertheless, the abundant instrumentation and recording of the 2011 Japan tsunami make it an ideal candidate to study tsunami source physics and parameter uncertainty/resolution. The various Japanese and international networks include coastal tide gauges, coastal wave gauges, offshore Global Positioning System (GPS) buoys, and cabled ocean bottom pressure gauges (OBP). These recordings have resulted in an unprecedented, high-quality data set with extensive spatial sampling of seafloor deformation and tsunami waveforms. Several OBP and seafloor GPS sites are within the source region (Figure 4) and have resulted in new understanding of the rupture process and the deformation cycle [*Sun et al.*, 2014]. We consider recorded waveforms from six GPS buoys (operated by Nationwide Ocean Wave Information Network for Ports and Harbors [*Kawai et al.*, 2013]), six OBPs (KPG1 and KPG2 operated by the Japan Agency for Marine–Earth Science and Technology [*Hirata et al.*, 2002]; P02 and P06 operated by Tohoku University [*Hino et al.*, 2013]; TM-1 and TM-2 operated by the Earthquake Research Institute (ERI), the University of Tokyo [*Fujii et al.*, 2011]), and Deep-ocean Assessment and Reporting of Tsunamis buoys (DART, operated by the National Oceanic and Atmospheric Administration [*Mungov et al.*, 2012]). All recording points and the epicenter (38.1035°N 142.861°E) are shown in Figure 4. Tide and wave gauges are not considered to avoid the more complicated (nonlinear) wave propagation near the coast due to shallow water and coastline effects. One exception is coastal wave gauge 205 for which we include the early part of the waveform to add additional constraints to the Sendai area for which few OBPs exist. However, we carry out waveform predictions for several coastal wave gauges to judge the predictive ability of our source estimate.

The time windows for the waveforms are chosen such that the main tsunami waves are included in the record but later parts that can be complicated by reflections and other effects are not included. In addition, we do not include OBP data for times <5 min after the earthquake origin time, to avoid complications of time-dependent subsidence for stations in the source region [*Tsushima et al.*, 2012]. The records at TM-1, TM-2, P02, and P06 are corrected for the permanent seafloor deformations which offsets the time series [*Iinuma et al.*, 2012].

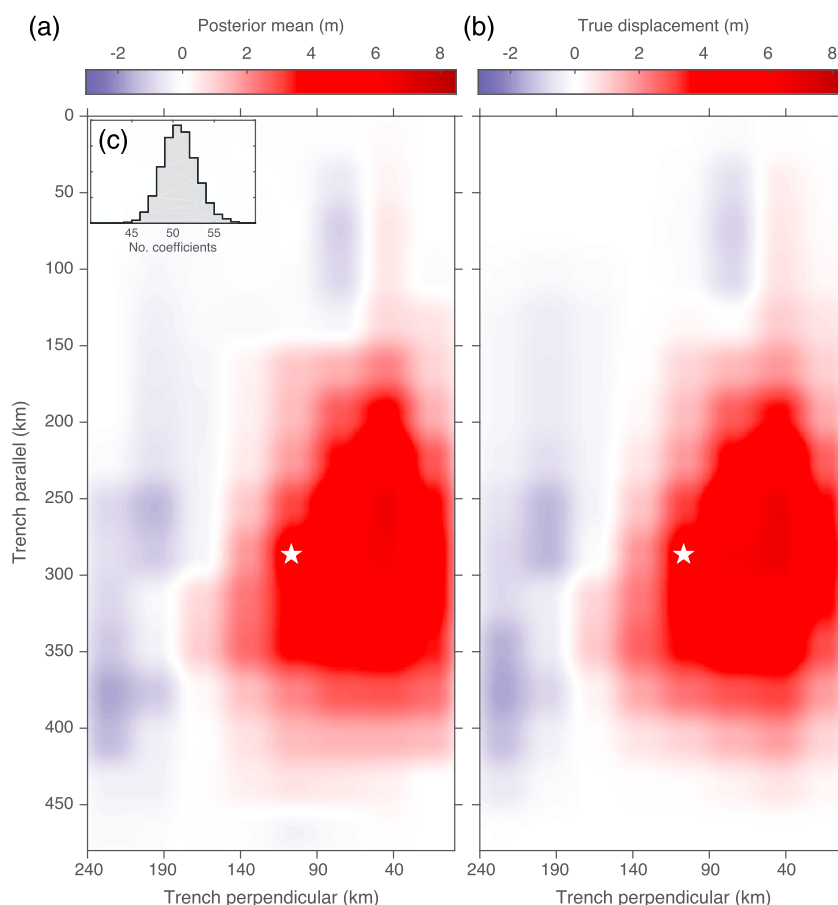


Figure 6. Simulation results in terms of (a) posterior median displacement compared to (b) the true displacement that was used to simulate the data in Figure 5. The true displacement distribution was chosen to resemble previous inversion results in broad terms. Note that the inversion recovers the true distribution well and shows almost no signs of artifacts. The (c) source complexity is parsimoniously captured by ~45–55 coefficients (the maximum number of coefficients is 128).

3.2. Simulation Results

First, we briefly present simulation results for the method. The simulation is based on the same bathymetry and observation geometry as the 2011 Japan tsunami. Waveforms were simulated at 15 stations (Figure 5) for a simulated tsunami source that mimics the actual tsunami source (Figure 6). To reduce computational cost, the simulation is carried out for a larger source patch size of 30 km. In this case, the finest discretization of the sea surface consists of 8×16 patches and the wavelet decomposition has five levels (1, 1×2 , 2×4 , 4×8 , and 8×16 patches, see *Hawkins and Sambridge* [2015]). The rectangular power-of-two grid is required by the wavelet transform and results in 128 grid nodes. However, to avoid systematic errors where some source patches overlaid landmass or are significantly past the trench, 10 patches are excluded from data predictions and likelihood computation. We apply uniform prior bounds of $[-8.5, 8.5]$ for each level. The prior on the number of wavelet coefficients is uniform over $[1, 128]$. Gaussian noise was added to each waveform with a standard deviation of 7.5% of its maximum displacement. Therefore, the noise level is different for each gauge which is accounted for by treating the standard deviation of the noise at each gauge as unknown according to equation (3). The unknown standard deviations provide weights for each gauge in that noisy gauges will contribute less to the displacement estimate. Other weighting is not necessary and, in fact, not desirable since it is less objective. Priors for noise standard deviations are chosen to be uniform from 0.05 to 2 m.

Figure 6 considers the simulation result in terms of the posterior median model. The posterior median captures all features of the source well and resolves both subsidence near the coast and large displacement near the trench. The complex source pattern is quantified with only ~45–55 wavelet coefficients (Figure 6c). Uncertainty estimates are considered in terms of the spatial distribution of the width of 95% credibility intervals (CI, Figure S1 in the supporting information) and 1-D marginal distributions (Figure S2, supporting information).

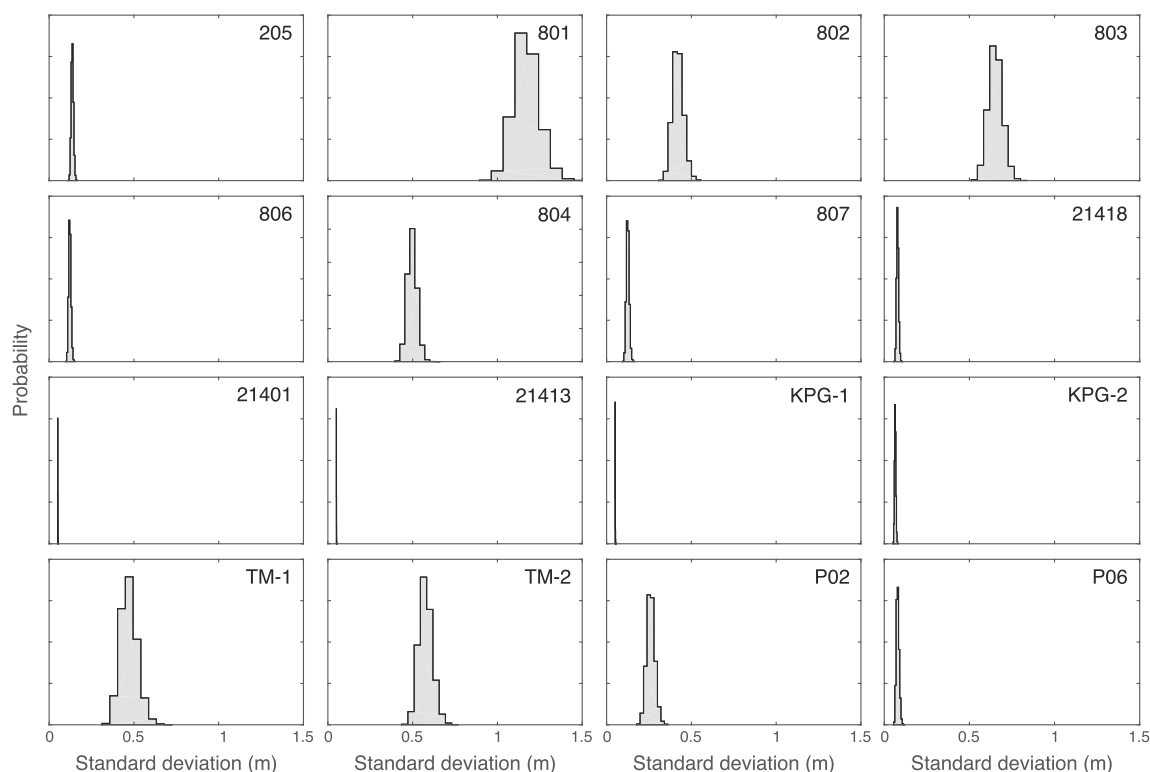


Figure 7. Inferred noise standard deviations for all stations considered in the inversion. The standard deviations weight the data importance of a station in an automated fashion. The results show that the DART buoys (21401, 21413, and 21418) exhibit the lowest measurement and theory errors, followed by some of the OBPs (KPG1, KPG2, P02, and P06). The OBPs TM-1 and TM-2 exhibit somewhat larger error levels, likely due to theory error. Priors are uniform from 0 to 2 m.

The displacement is well resolved in all source regions with 95% credibility intervals <0.5 m in most areas north of the epicenter. However, uncertainties are noticeably larger in the south, which is likely due to less sensor coverage. Some marginal distributions show nonlinear features (multiple modes, tails) but notably large areas with zero displacement are quantified well with small uncertainties and no artifacts or unconstrained structure.

3.3. The 2011 Japan Tsunami Results

For the observations, we work with a grid of 16 (trench perpendicular) by 32 (trench parallel) source patches with equidistant 15 km spacing for a total number of 512 patches. The total source area is 240×480 km and the strike is approximately trench parallel (Figure 4). In this case, the wavelet decomposition has six levels and we use prior bounds of $[-8.5, 8.5]$ for each level (see Appendix B for details). We include at least one source patch beyond the trench, since some sea surface displacement is possible in this region. Other parts of the area which fall significantly beyond the trench and overlap with Japan's landmass do not contribute to the inversion. The total number of source patches is thus reduced to 459. Note that while the wavelet parameterization applies to the full rectangular area, the masked parts do not contribute to the likelihood computation. The prior on the number of wavelet coefficients is uniform over $[1, 459]$.

An initial inversion is carried out by applying equation (3) as the likelihood function to obtain residual errors for the most probable solution under the assumption of uncorrelated noise of unknown standard deviation. The prior for the standard deviation parameters was uniform over $[0, 2]$ m. From these residuals, data covariance matrices are estimated [Dettmer *et al.*, 2014, see Figure S3]. Figure 7 shows the noise levels inferred by the initial inversion and indicates that DART buoys have low noise levels which allows inference even at low wave heights. The OBPs and GPS buoys show variable noise levels. These variations are likely due to theory error rather than high measurement noise since some features in the waveform that are of larger magnitude than the presignal noise are not fit.

The final inversion includes the covariance matrix estimates (Figure S3) from the initial step by applying equation (4). The ability of the covariance matrices to account for time dependence in the waveform noise is

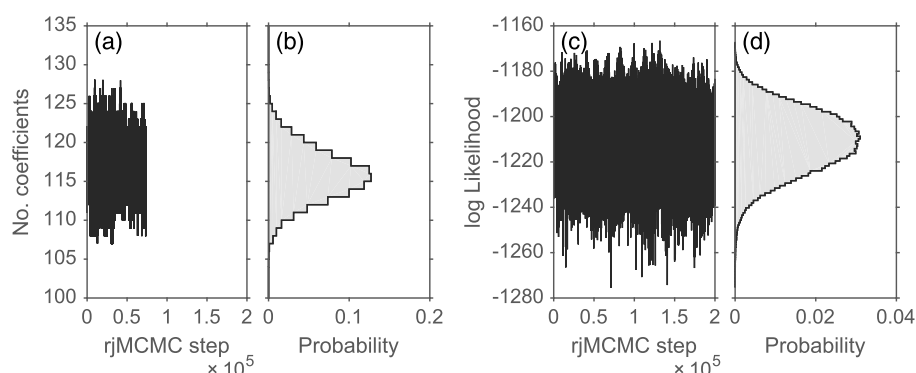


Figure 8. Inversion results in terms of chain history and marginal distributions for the (a, b) number of wavelet coefficients and for the (c, d) values of the logarithm of the likelihood function. The prior on the number of coefficients was uniform from 1 to 459.

considered in Figures S4 and S5 of the supporting information. Figure 8 shows chain histories and marginal distributions for the number of wavelet coefficients and the logarithm of the likelihood function. The chain histories are for the portion of the inversion that was considered to be stable (i.e., there is no trend in the likelihood as a function of MCMC step). All inferences are based on this part of the inversion, while the preceding part (sometimes referred to as burn-in) is discarded. The lack of a trend in the likelihood and the number of coefficients indicates that the inversion has converged. In addition, marginal distributions of several nonoverlapping sections of the chain show no significant change, further supporting that convergence has been reached (see supporting information Figure S6). The computation time for these results is approximately 48 h with parallel tempering carried out on 240 CPUs of which 20 are used to collect samples (with tempering parameter $\beta = 1$, see equation (5)) and every tenth sample is recorded. Therefore, the total ensemble of the parallel algorithm was 2×10^6 samples. The tempering schedule for the other chains is logarithmic such that the minimum β is 0.0002. The marginal for the number of coefficients is interpreted to quantify the model complexity and shows that between 105 and 130 coefficients are required by the data to infer the source. This histogram quantifies the relative probabilities of the various parameterization choices and shows that the result is parsimonious since the maximum number of coefficients is 459.

Figure 9 shows the observed and predicted data for the 16 gauges considered in the inversion. The axes limits are consistent with the data used in the inversion. Since inferences are based on a large ensemble of models, we show the range of data predictions produced by the ensemble rather than a single prediction (e.g., for the best fit parameters). It is evident that all DART waveforms are fit well by the inversion which agrees with the simpler wave propagation physics expected for deep-ocean gauges. The GPS gauges—near the coast but in deep water (between 125 and 204 m)—are fit reasonably well but show discrepancies at later times which may be due to complicated effects from coast line and bathymetry not modeled by our GFs. The waveforms at the two OBP gauges P02 and P06 have excellent fit. Poorer fit exists for the waveforms at OBPs TM-1 and TM-2. The reasons for the poor fit are unclear and have also been observed previously in linear source inversions [Hossen *et al.*, 2015]. The challenges may be related to these two gauges being located near a region that shows detailed and complex near-trench sea surface displacement which is discussed in more detail later in this section.

The excellent waveform fits in Figure 9 are obtained with a small number of parameters, including two epicenter parameters and a single parameter for apparent rupture velocity (Figure 10) and <130 coefficients that describe the displacement (Figure 8). This is a significant contrast to linear inversions which require much larger parameter counts (in large part due to multiple time windows) and, consequently, smoothing in both time and space. The prior for apparent rupture velocity was uniform from 0.5 to 3.5 km/s and is estimated with low uncertainty (Figure 10). However, we do not think that the parameter is truly well resolved. Rather, the parameterization we choose limits the values this parameter can take while fitting the data. A better approach that is not considered here may be to treat spatial variability of apparent rupture velocity which would likely result in much larger uncertainty. The median value of apparent rupture velocity is 1.41 km/s. The epicenter location is also unknown with a uniform prior around the initial epicenter (± 15 km in latitude and longitude around 38.1035°N 142.861°E). Figure 10 shows that the epicenter location is estimated to be ~ 11 km south

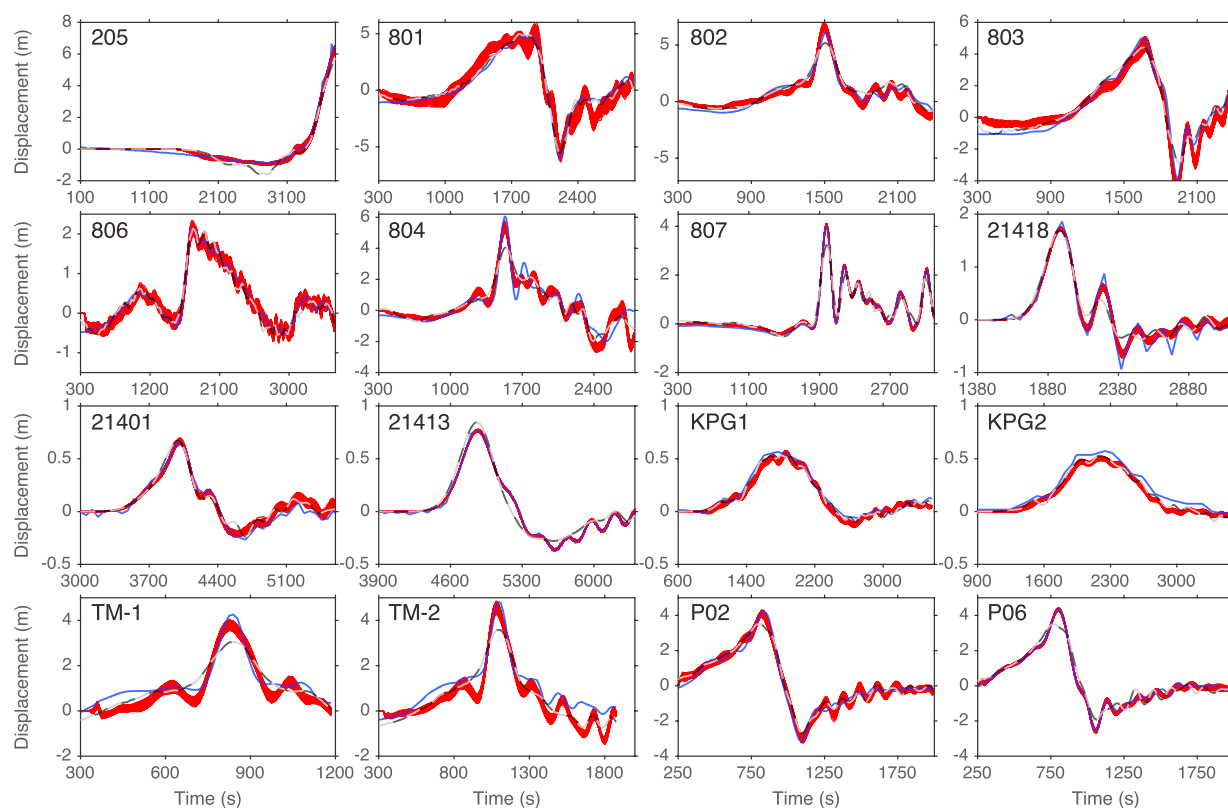


Figure 9. Observed (blue) and predicted (red) tsunami waveforms for the 16 observation points considered in the inversion. The data predictions are shown superimposed for a large (10^4) random subset of the posterior ensemble. Data fits for a linear inversion (dashed, single time window with optimal apparent rupture velocity and smoothing determined by L curve) are also shown for comparison.

and 6 km to the west of the initial location. We regard these as “nuisance” parameters which we include in the inversion to avoid dependence on the specification of the epicenter. The rupture evolution as a function of time for posterior median displacement and apparent rupture velocity is shown in the supporting information (Figure S4).

The main result of the inversion are estimates of the spatial distribution of sea surface displacement and are considered in several ways. First, Figure 11 shows the posterior median model which is based on the large ensemble of models which approximate the posterior. We chose the median model (over the mean) since it is less likely to be affected by heavy tails in the uncertainty distribution. In addition, Figure 11 shows the spatial distribution of the width of the 95% CIs. These indicate significantly lower uncertainty in the northern part

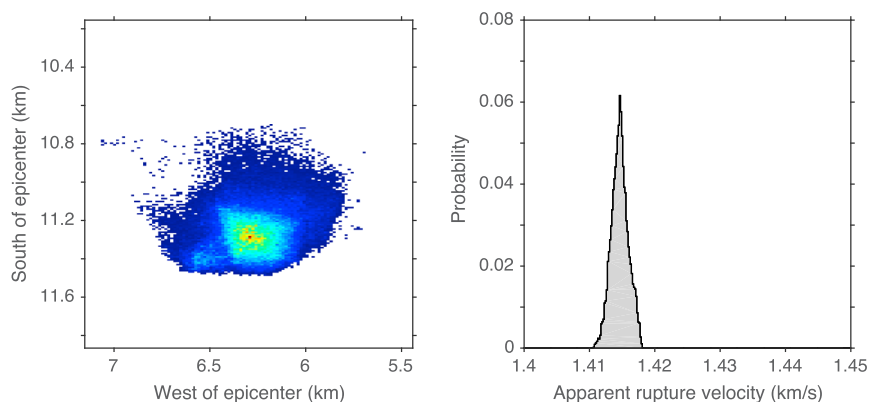


Figure 10. Posterior marginal distribution for epicenter location and apparent rupture velocity. Note that priors for the epicenter are ± 15 km around the initial position and those for apparent velocity are uniform from 0.5 to 3.5 km/s.

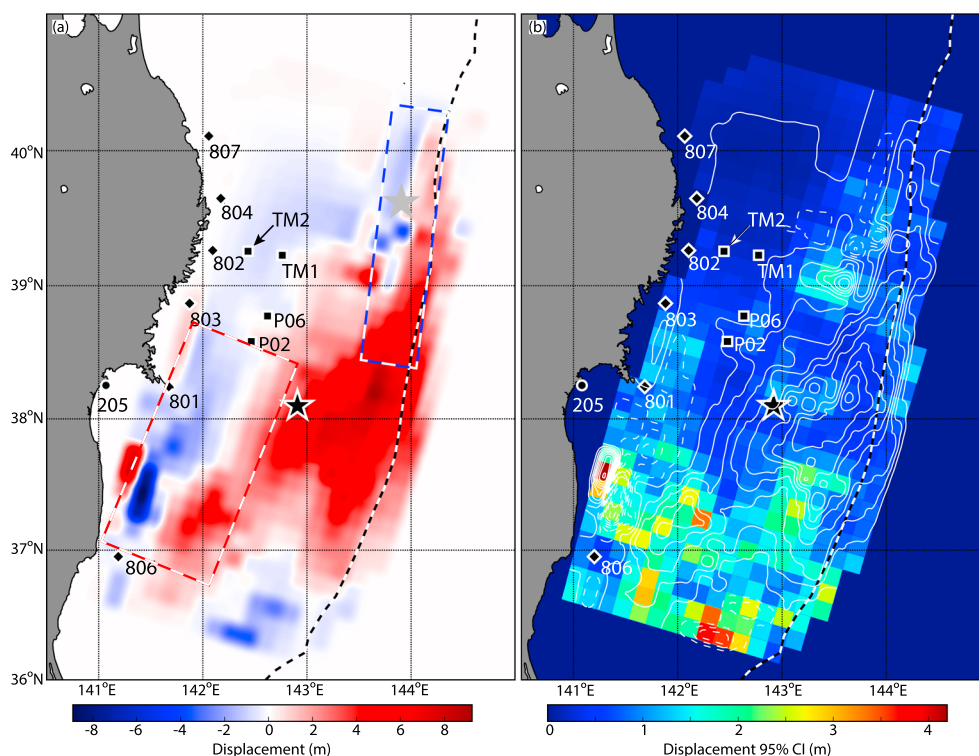


Figure 11. Inversion results in terms of (a) the posterior median model for sea surface displacement and (b) the width of 95% credibility intervals. Hypocenter (star), trench (dashed), and gauge locations in the source region are also shown. The contour lines (white) in Figure 11b are for our posterior median result and negative contours are dashed. The source areas for the 869 Jogan (red dashed) [Namegaya and Satake, 2014] and 1896 Sanriku (blue dashed, gray star) [Tanioka and Satake, 1996] earthquakes are also shown in Figure 11a.

of the source region, where better sensor coverage helps constrain the results. The largest uncertainties are in the southwestern part (Sendai region) where a lack of OBPs results in poor constraints. The median model shows more detailed features in several parts of the source area than previously reported. At the same time, large regions show an extensive smooth surface without any requirement to specify smoothing and for GFs that are based on more compact elementary sources than previously considered. Other sea surface displacement studies used Gaussian basis functions which can introduce significantly more smoothing (Figure 2). For example, a Gaussian basis function on a 10 km discretization has significant support over a radius of over 20 km. The cosine-tapered basis we employ has finite support within a 30×30 km box.

The inversion adapts the parameterization locally to increased resolution in the near-trench region from 37 to 40.5°N . The main peak of the tsunami source is trenchward and north of the epicenter and extends in a clear finger-shaped feature to the north which closely follows the trench. Subsidence is resolved in a broad north-south region along the coast with uncertainty lowest where the source region OBPs are situated. The region between GPS gauges 801 and 806 shows the largest uncertainties and also a feature that is unrealistic: an area of four source patches with extreme positive and negative displacements next to each other and near the coast. It is not certain what causes this issue but it is likely due to a combination of incomplete data and theory/epistemic errors stemming from an incomplete model of the environment. We also note that our previous studies based on Gaussian-tapered elementary sources did show similar features, although they were more difficult to notice due to the smoothing intrinsic to that approach (see Figure S12).

In addition to the 95% CIs, Figure 12 considers displacement marginal distributions for a subset of the faults in the region of the main source peak. The marginals visualize the uncertainty of the displacement estimate for each source patch. A full account is provided in the supporting information (Figures S10–S13). From Figure 12, it is evident that some of the distributions are nonlinear (i.e., do not exhibit Gaussian shape). However, overall displacement is well constrained and spatial variation is clearly resolved.

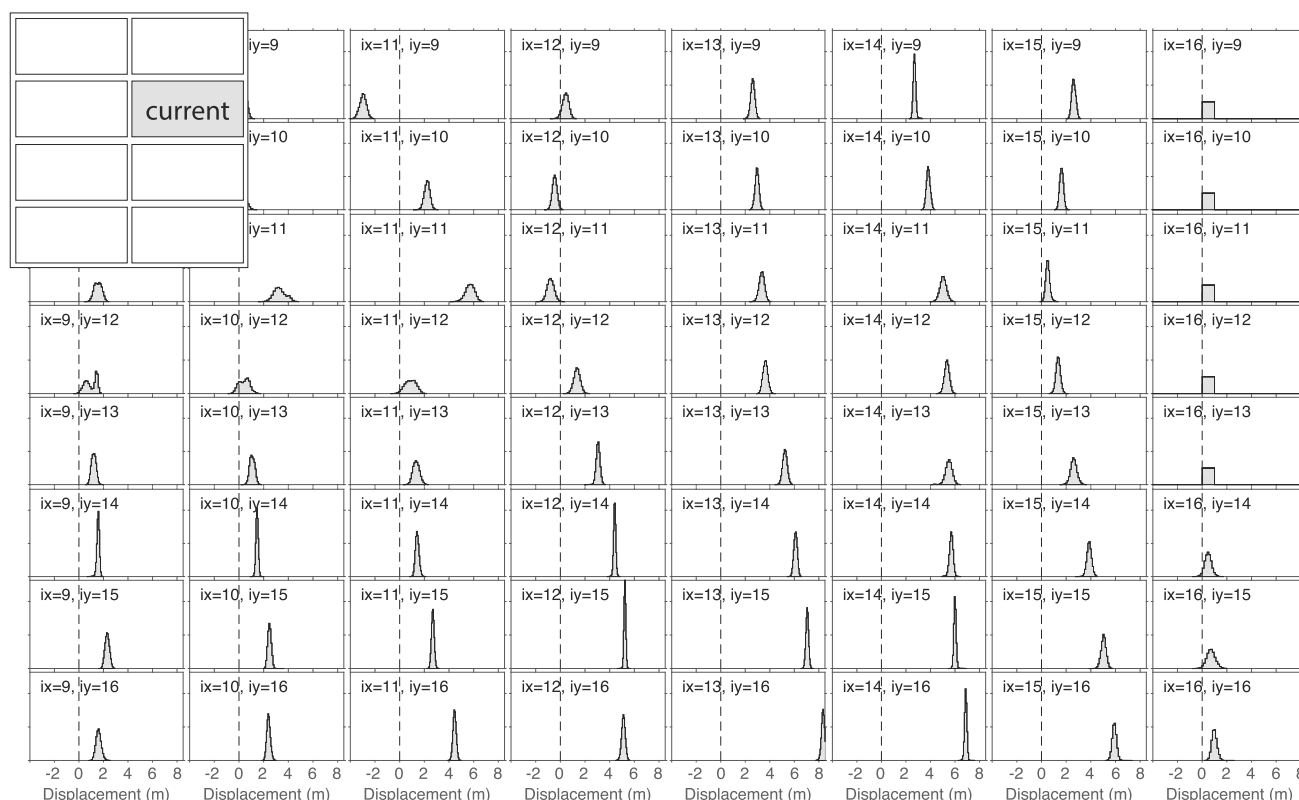


Figure 12. Displacement uncertainty estimates in terms of 1-D marginal distributions for each source patch. A subset of source patches is shown as indicated by the gray box and indices in trench-parallel and trench-perpendicular directions are shown as iy and ix with the origin in the northwestern corner of the surface (see Figure 4 for geographical location of source patches). The complete set of eight panels is provided in the supporting information.

A fascinating feature of our results can be seen in the northern part of the source region. The median model (Figure 11) shows a clear bimodal feature of significant magnitude landward of the trench (39.5° – 40.5° N). The finger feature closely follows the trench, has a magnitude of ± 2 m, and a trench-perpendicular extent of ~ 50 km. This part of the source aligns closely with the source of the 1896 Sanriku earthquake [Tanioka and Satake, 1996]. The uncertainties for this region are among the lowest in the model (Figure 11). Figure 13 shows a detailed view of the displacement marginals for this near-trench region. Both the peak and the trough are well resolved with uncertainties significantly smaller than the magnitude of the signal and well-separated distributions.

4. Summary and Discussion

We developed a new approach to infer the initial sea surface displacement after an earthquake from tsunami waveform recordings based on trans-D Bayesian sampling. The approach provides estimates of both the spatial distribution of sea surface displacement as well as uncertainties. The sea surface displacement is parameterized in terms of a hierarchical wavelet decomposition with an unknown number of wavelet coefficients. The number of coefficients is also treated as uncertain (quantified by a probability distribution), and the distribution adapts to the data information about the source. This parameterization results in a description of the source that is parsimonious in that it captures source complexity with an efficient number of parameters. A critical part of parsimony is that the parameterization adapts locally to source features as supported by the data. Some large areas may be described by few parameters while other areas are resolved with much more detail. Since the trans-D parameterization intrinsically balances data fit with model complexity according to data and prior information, no regularization is required (as opposed to Bayesian inversion on regular grids which still require regularization). The noise on waveforms is initially treated to be uncorrelated and have unknown standard deviation. The results from that initial inversion are used to estimate covariance matrices that account for temporal dependence. A subsequent inversion applies these matrices to produce the final result. This process results in an automated and objective weighting of gauges.

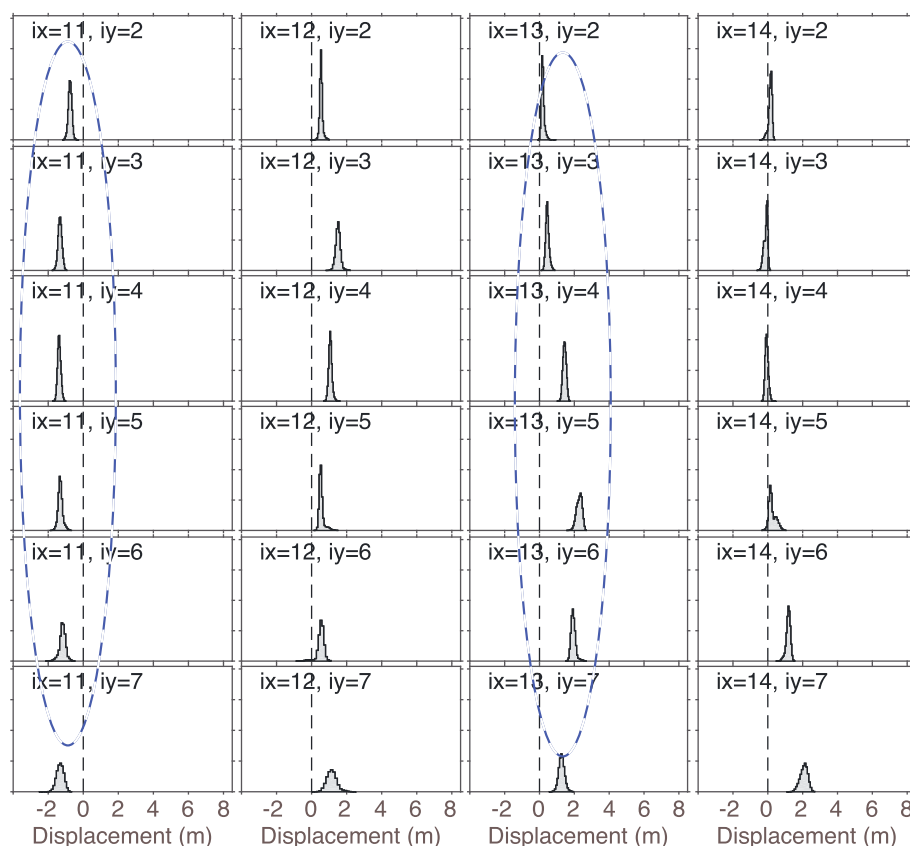


Figure 13. Detailed view of uncertainty estimates for the northern part of the model that exhibits a well-resolved bimodal displacement pattern (highlighted by dashed blue). Indices in trench-parallel and trench-perpendicular directions are shown as iy and ix (with the origin in the northwestern corner of the surface), zero displacement is highlighted as dashed.

The temporal discretization in our approach is based on a single time window and a rise time of 30 s. The apparent rupture velocity is treated as unknown so that dependence of the inversion on this parameter is reduced. Previous approaches to sea surface displacement have considered instantaneous displacement [Saito *et al.*, 2011] and multiple time window parameterizations which assume a finite apparent rupture velocity. Instantaneous displacement has been shown to produce poor data fits [Hossen *et al.*, 2015] and is not considered here. Our approach includes finite apparent rupture velocity but neglects that seafloor deformation may propagate faster than the rupture front. Treating apparent rupture velocity as unknown and spatially variable is desirable, but it is currently not clear how a trans-D parameterization of apparent rupture velocity can be achieved in this method.

The method was applied to tsunami recordings of the 2011 Japan tsunami at 16 high-quality gauges. The results show previously unreported detail of the source, quantify uncertainty spatially, and produce excellent rigorous data fits. The source estimate shows an elongated peak trenchward from the epicenter that closely follows the trench, indicating significant seafloor deformation near the trench. Particularly notable is a bimodal (negative to positive) displacement feature in the northern part of the source near the trench. The feature has significant amplitude and is clearly resolved by the data with low uncertainties. Satake *et al.* [2013] reported very shallow fault slip in this region that resulted in focused seafloor deformation of several meters. The slip pattern is similar to the 1896 Sanriku earthquake source [Tanioka and Satake, 1996] that our results closely align with. The results of Saito *et al.* [2011] also show features in the northern source region, but they do not align with the trench. Rather, those source estimates show a positive displacement significantly seaward of the trench and negative displacement at the trench, followed by positive displacement farther landward (Figure S17). Our results can be explained by a rupture area similar to that of the Sanriku event. The location of the displacement peak which extends slightly seaward may support previous assertions about deformation in the sedimentary wedge [Ito *et al.*, 2011; Strasser *et al.*, 2013]. In addition, the horizontal movement of the

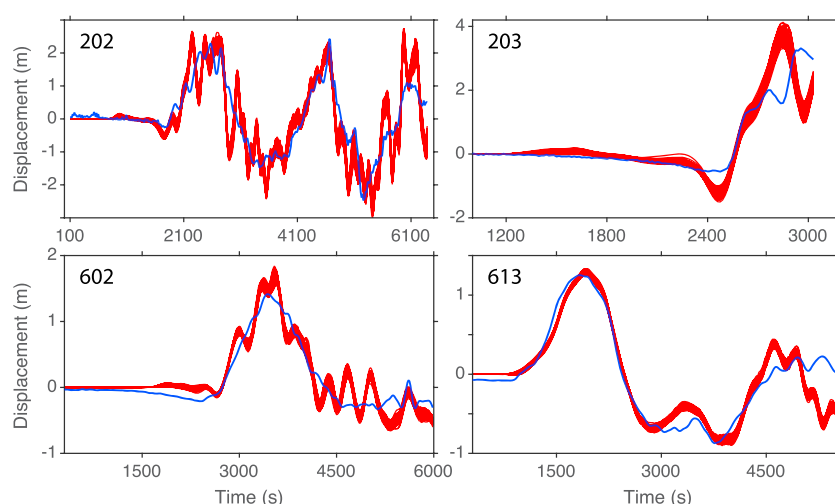


Figure 14. Observed (blue) and predicted (red) tsunami waveforms for five coastal wave gauges that were not included in the inversion process. The predictions are for the complete ensemble that is inferred by the Bayesian sampling.

crust in very shallow rupture can cause additional uplift in the accretionary wedge due to backstop scraping of sediments [Tanioka and Satake, 2001]. Gusman et al. [2012] showed that such uplift contributed significantly to the tsunami genesis. Our results corroborate those conclusions and add higher resolution throughout the source region and extend the inference northward to 40.3°N along the trench. Other studies assert that a submarine mass failure [Tappin et al., 2014] may have contributed in this area. Several seabed scars are located in this northern source region, and it is possible that a significant mass failure could have contributed. However, the large extent along the trench supports a significant seismic source.

The OBPs that are closest to this region are the most difficult to fit. Note that our method applies no subjective station weights and no smoothing but requires fit that is consistent across all gauges. This is an important distinction; a hierarchical Bayesian model fits all waveforms jointly according to the information content of all waveforms. Therefore, the relatively large noise standard deviations we estimate in the initial inversion for TM-1 and TM-2 (Figure 7), show that some waveform features are to some degree inconsistent with the other waveforms which may hint that our model complexity is insufficient to fully explain TM-1 and TM-2. For example, these data may require higher resolution than we can computationally provide at present.

In contrast to most other work, we employ GFs computed for a basis of elementary waves that are cosine tapered with 15 km source patch size. In comparison to a Gaussian basis, the cosine has more compact support and mostly removes subjective smoothness from the source estimate. For comparison, inversions were also carried out for a discretization of 8×16 patches (30 km source patch size with cosine taper) and for 15 km source patch size but with the more commonly used Gaussian basis functions. The 30 km cosine results (Figures S5–S7) show a cruder source estimate with less resolution along the trench and a broader main peak. The reduced resolution results in a generally poorer fit to the data. In particular, some of the shorter wavelength features are not fit. Similar issues exist for the results with the 15 km Gaussian basis (Figures S8 and S9). The spatial extent and overlap (required to produce a near-even displacement surface across the source region) of the Gaussian basis results in smoothness that is not supported by the data but rather prescribed by the approach. Figure S9 also illustrates that the actual inversion parameters (amplitudes for each source patch) produce a complicated image of the source that can only be interpreted after applying the smoothing effect of the Gaussian basis. This is undesirable since many inversion approaches regularize the source complexity based on these parameters, which can be highly irregular while still producing a smooth image due to the intrinsic GF smoothing. In addition, the Gaussian basis results in higher computational cost for trans-D sampling since it requires a higher degree of local focus from the parameterization which is more difficult to estimate.

Figure 14 shows waveform predictions for several coastal wave gauges. These data were not included in the inversion process and are shown here to illustrate the near-field predictive capability of our results. Since the GFs in this prediction do not include nonlinear effects, discrepancies are expected. In a broad sense, the

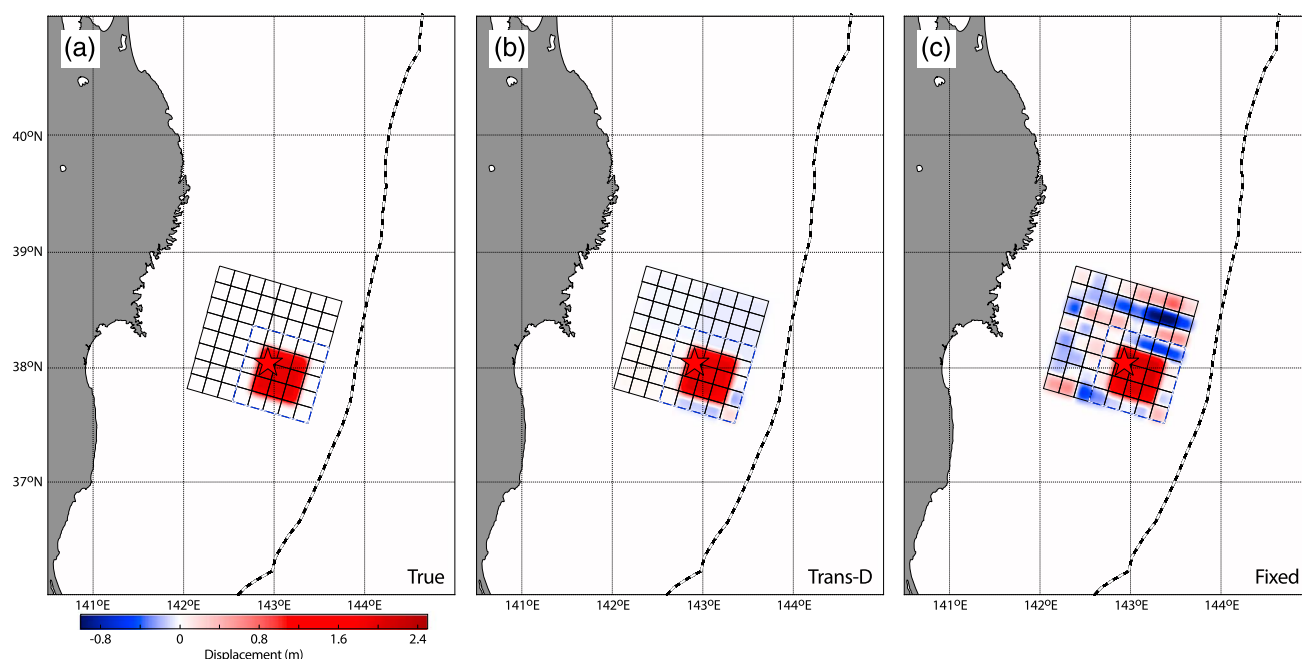


Figure A1. Source patches and (a) true source for the simulation comparing trans-D and overparameterized but not regularized Bayesian sampling. The highlighted source area (blue dashed) in Figure A1a is considered for more detailed uncertainty comparison in Figure A2. The posterior median models for the (b) trans-D and (c) fixed (over parameterized) inversions are also shown.

predictions match the observed tsunami well. In all cases, the predictions are more extreme than the observations and are of larger magnitude (i.e., a conservative prediction). The predictions also exhibit higher-frequency oscillations that are not visible in the observations, which may be due to dispersion starting to show nonlinearity for the water depths at the coastal gauges. However, in summary, the prediction quality for our inferred source is excellent.

Appendix A: Comparison With Overparameterized Approach

This section considers a simple tsunami source estimation example to illustrate the effect of parameterization choice on the source estimate and uncertainties. In particular, this example exposes problems that fixed overparameterized parameterizations cause for inversion results and that trans-D parameterizations alleviate these problems. The simulation is based on the Tohoku geometry (Figure 4) using gauges 801, 803, 804, 21418, 21413, KPG2, TM-2, and P06. Data are simulated for the source shown in Figure A1 assuming instantaneous displacement. The source area spans 120×120 km and is discretized by 8×8 patches. The tsunami source has a 2 m amplitude over an area of 3×3 patches. Gaussian noise with a standard deviation of 20% of the maximum waveform amplitude is added to the simulated data.

The data are inverted twice assuming two different parameterizations. First, we parameterize the fault with 64 wavelet coefficients, which is equal to the number of source patches (fixed dimensional case). Second, the source is parameterized with our trans-D tree method with wavelet parameterization. The priors on wavelet coefficients are uniform over $[-3, 3]$ in both cases.

Notably, the trans-D sampling includes 64 different parameterizations, to each of which we assign equal prior probability. The 64 parameterizations have increasing complexity with one coefficient for the first/simplest member, two coefficients for the second, three coefficients for third, and so on up to the most complex parameterization with 64 coefficients. Therefore, the most complex member in the trans-D model is identical to the fixed dimensional case. However, the Bayesian sampling algorithm we employ for the trans-D case jumps between the 64 members with quantitative rigor. Each member is visited with a frequency that is proportional to the data support which aids in efficiency required to obtain a solution for 64 parameterizations concurrently. However, most importantly, the trans-D posterior is a more general answer to the inverse problem than the fixed dimensional case since it addressed both under parameterization and overparameterization issues which the results in this section illustrate.

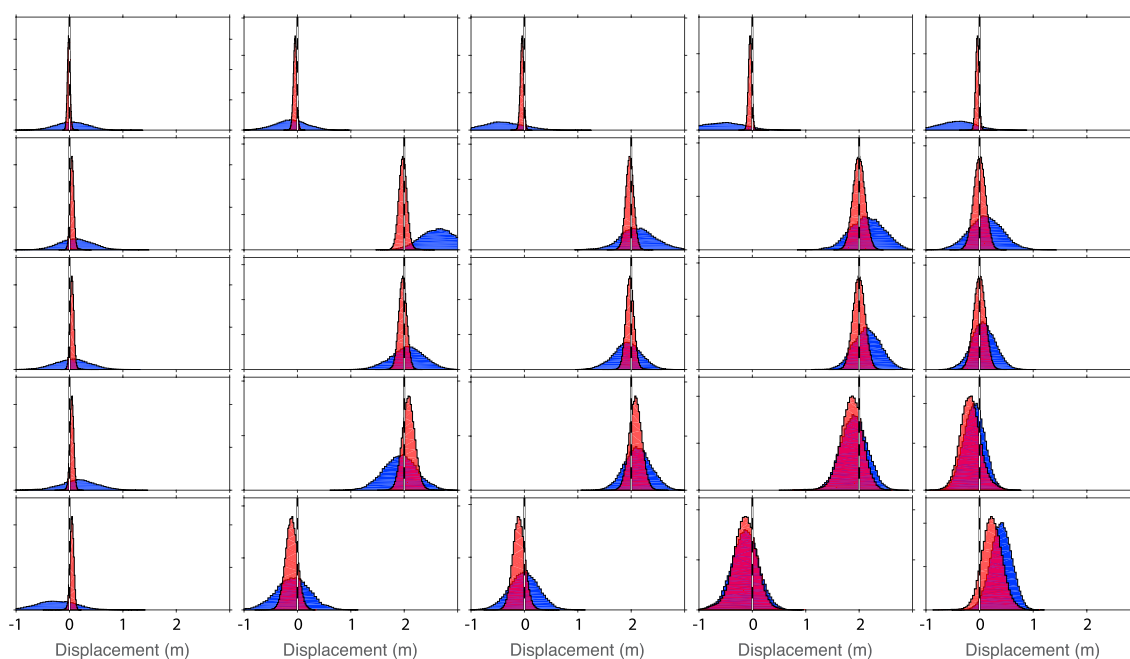


Figure A2. Comparison of marginal probabilities in a 5×5 source patch area for trans-D source estimation (red) and overparameterized estimation (blue). True values (dashed) are also shown. The trans-D results are more meaningful and exhibit much lower uncertainty, and the overparameterized results show unconstrained structure. The top left panel corresponds to the northwestern corner of the highlighted source area in Figure A1.

In the trans-D case, the source is inferred parsimoniously with only ~ 12 coefficients and the posterior median model is much better resolved than in the overparameterized case (Figure A1). Further, Figure A2 compares the two inversion results in terms of marginal probabilities. The results are shown for a 5×5 subset (indicated in Figure A1) to improve clarity. The fixed dimensional estimate exhibits signs of overparameterization with unrealistically large uncertainties and unconstrained structure. The trans-D results are far superior and provide a better source estimate. These problems are likely exacerbated for more complex/realistic sources.

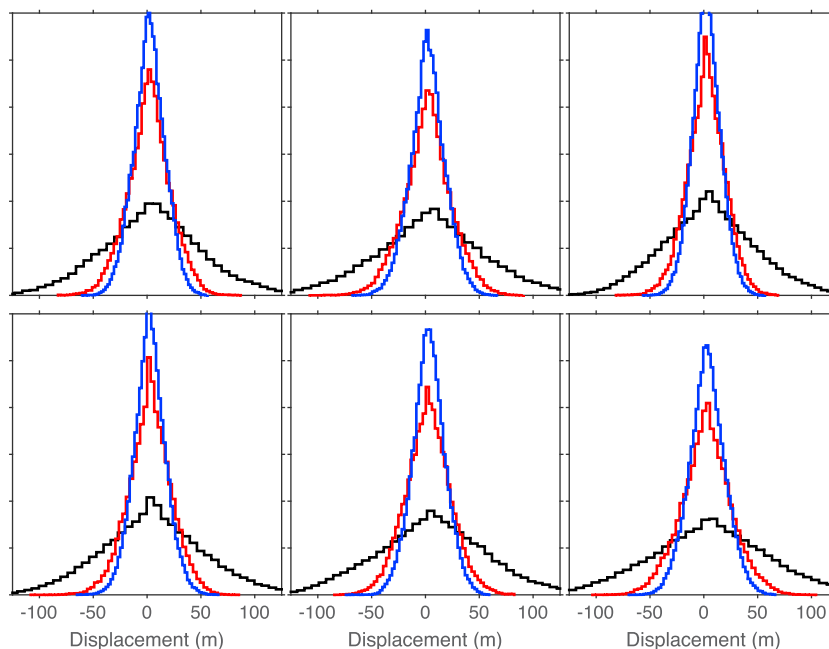


Figure B1. Comparison of three wavelet priors in terms of displacement marginal probabilities for six representative source patches when sampling with no data. The results are for widths of ± 8.5 (blue), ± 12 (red), and ± 28 (black).

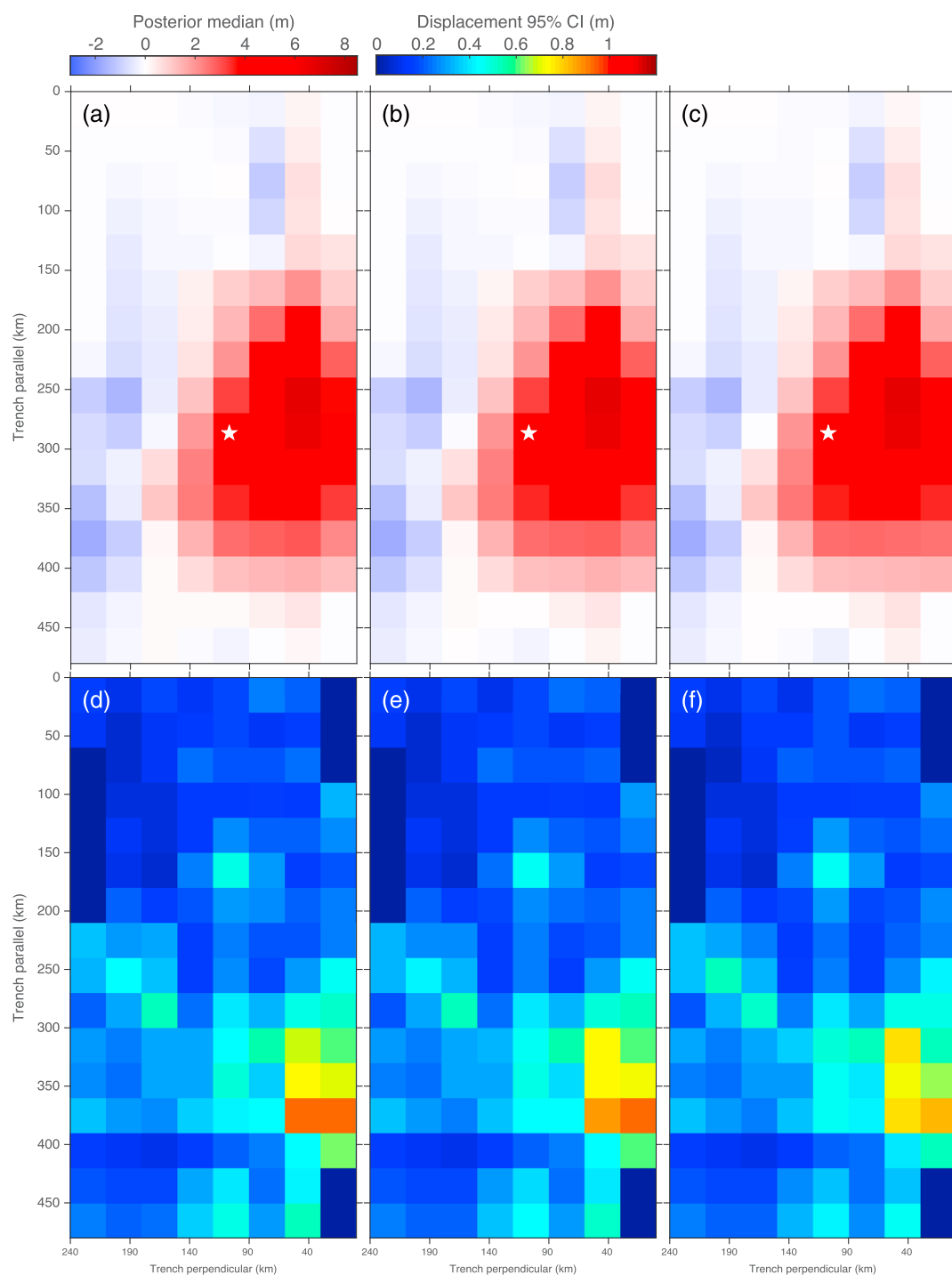


Figure B2. Trans-D simulation results in terms of (a–c) posterior median model and (d–f) 95% credibility intervals for the three prior choices (± 8.5 (Figures B2a and B2d); ± 12 (Figures B2b and B2e); and ± 28 (Figures B2c and B2f)). The differences in the results are small compared to the resolved features.

These undesirable effects are expressions of over parameterization commonly observed in many inverse problems. The results show that application of Bayesian sampling without quantitative parameterization selection is problematic. Trans-D inference is one solution to address these issues since it can adapt parameterization complexity spatially based on data information and provides more meaningful results. It is not a method intended to address the computational burden of source estimation at fixed, fine-scale discretizations.

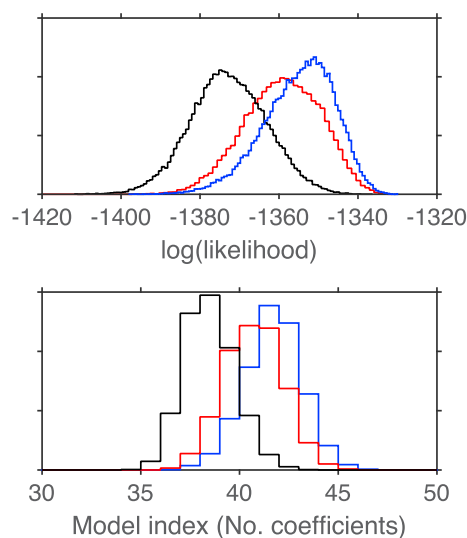


Figure B3. Trans-D simulation results in terms of marginal distributions for the logarithm of the likelihood function and for the number of coefficients for the three prior choices considered in this section: ± 8.5 (blue), ± 12 (red), and ± 28 (black). Note that the distribution for the number of coefficients changes only slightly for large changes in the prior.

Detmer et al. [2014] carried out an analogous study for finite fault inversion and shows similar results in Figure 3 of that article.

Appendix B: Prior and Prior Sensitivity

The sea surface priors in this study are in terms of wavelet coefficients and it is not immediately obvious how these translate to displacement. This section shows numerical estimates of the priors in terms of displacement

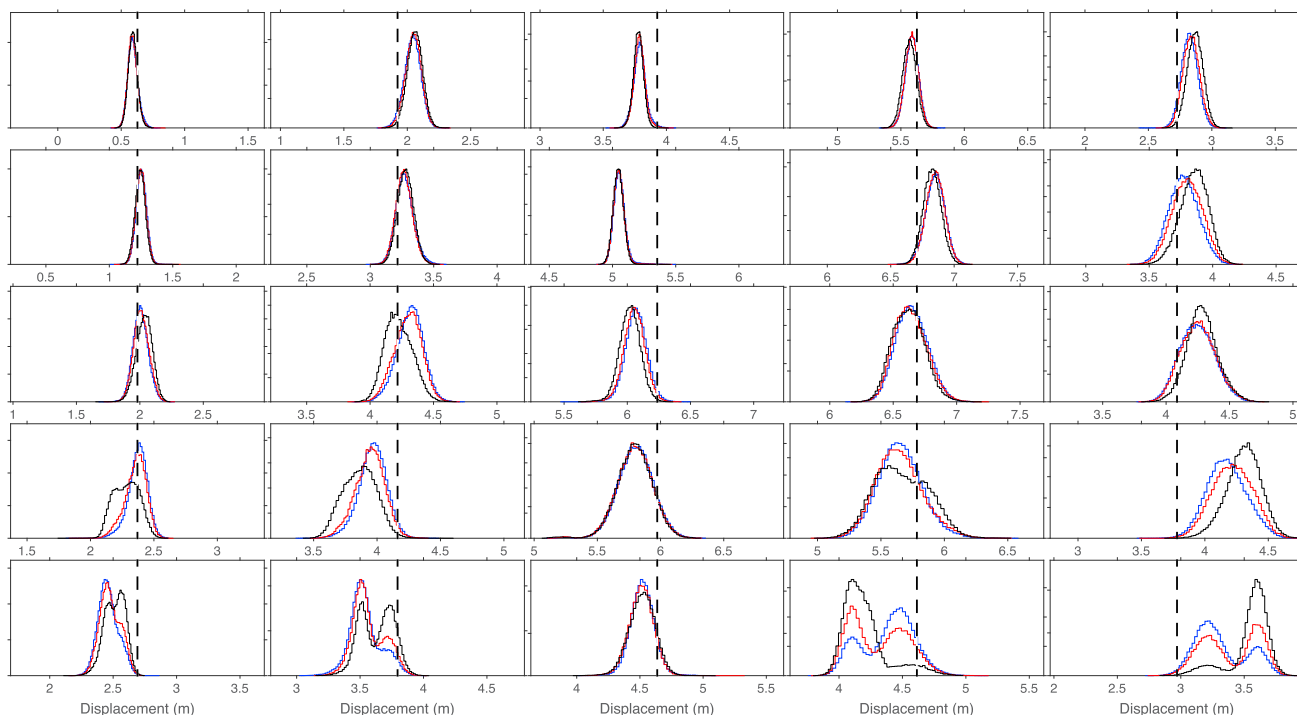


Figure B4. Trans-D simulation results in terms of displacement marginal distributions. True displacement values are shown as dashed lines. Results for the three prior choices are shown: ± 8.5 (blue), ± 12 (red), and ± 28 (black). The source patches are a representative subset including patches that show the largest discrepancy. The marginals agree closely in most cases even though the priors are very different. The discrepancies are largest for the widest prior which is over 3 times wider than what was identified as a reasonable prior choice.

for three different prior choices. In addition, we examine the sensitivity of trans-D inferences to prior choices for a specific simulated case.

In this work, we choose wavelet coefficient priors to be uniform and symmetric around zero. In the case where the finest discretization of the sea surface consists of 8×16 patches (30×30 km source patches) the wavelet decomposition has five levels (1×2 , 2×4 , 4×8 , and 8×16 patches, see Hawkins and Sambridge [2015]) and we apply identical priors for all levels. The bounds for the uniform distributions are chosen wide so that they translate to displacement values larger than the anticipated magnitude of the source and also ensure that coefficient values are not sampled near the bounds, as this can cause poor model fitting and adversely influence uncertainty estimates. Both the simulated data and the Japan tsunami data are analyzed with priors of ± 8.5 for all levels which was found to be wide enough so that no coefficients are sampled at the bounds: The smallest coefficient value in the entire ensemble for the Japan tsunami results is > -8.02 and the largest < 7.4 . We conclude that this prior does not restrict the trans-D sampling and allows parsimonious adaptation.

To further study the prior for wavelet coefficients in terms of displacement, we apply our sampling algorithm without data to sample only the prior. Figure B1 shows results as marginalized displacements (m) for six representative patch locations and for three choices of prior width: ± 8.5 , ± 12 , and ± 28 . The ± 8.5 prior supports a displacement range of $\sim [-50, 50]$ m which is large compared to the anticipated source magnitude. The ± 12 prior supports displacement values of $\sim [-100, 100]$ m and the ± 28 prior supports displacement values of $\sim [-250, 250]$ m.

To study the sensitivity of our trans-D results to prior choice, we repeat the simulation study for each prior while assuming that noise and apparent rupture velocity are known. Note that all three priors are chosen such that the range of true source displacements are well within the prior bounds. This aims to isolate the effect of prior width on the trans-D algorithm. Figure B2) considers the inversion results for the three choices of prior in terms of displacement and uncertainty values. The inferences are similar and indicate that the dependence on the choice of prior is not strong. Figure B3 shows marginals for likelihood values and the number of wavelet coefficients and shows that the inversion results differ only slightly for these dramatically different priors. Likewise, the detailed view of estimated displacement marginals (Figure B4) shows similar results for most patches. In some cases, the posterior exhibits multiple modes and these patches show the largest differences. However, these differences are exaggerated by the choice of the extreme range in prior values. We conclude that in this case, the trans-D result does not depend strongly on prior choice as long as bounds are chosen sufficiently wide.

Acknowledgments

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