Barotropní oceánský slapový model 3

DEBOT model

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Outline

1. DEBOT overview
2. Comparison with tide gauge measurements
3. Satellite altimetry data assimilation into ocean tide models
4. DEBOT Assimilative
5. Magnetic field

Section 1

DEBOT overview

The DEBOT model

- **D**avid **E**inšpigel's **B**arotropic **O**cean **T**ide model
- Development since 2010
- Barotropic = pressure is dependent on the density only (not temperature neither salinity...) = *2-D problem*
- Modelling of *tides* — tidal potential of the second and third order

Mathematical model – shallow water equations

- Continuity equation = equation of the surface elevation

$$\frac{\partial \zeta}{\partial t} + \frac{1}{a \cos\phi}\left(\frac{\partial (hu)}{\partial \lambda} + \frac{\partial (\cos\phi\, hv)}{\partial \phi}\right) = 0$$

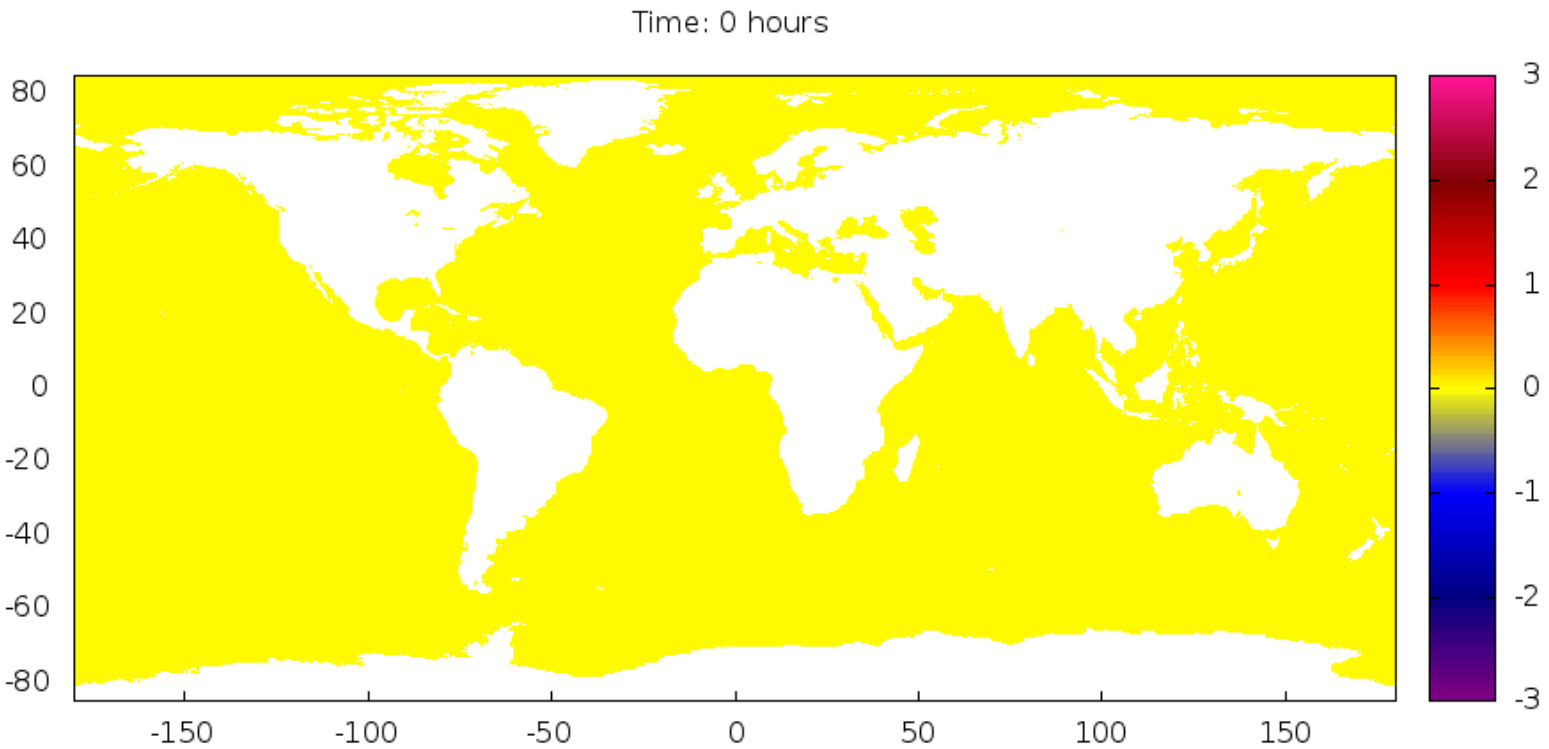
- Equation of motion in zonal direction

$$\frac{\partial (hu)}{\partial t} = -\frac{g_\epsilon h}{a \cos\phi}\frac{\partial \zeta}{\partial \lambda} + fhv - ku\sqrt{u^2+v^2} + \mathrm{Adv}_\lambda + A_H \mathrm{Vis}_\lambda + T_\lambda - C_D u$$

- Equation of motion in meridional direction

$$\frac{\partial (hv)}{\partial t} = -\frac{g_\epsilon h}{a}\frac{\partial \zeta}{\partial \phi} - fhu - kv\sqrt{u^2+v^2} + \mathrm{Adv}_\phi + A_H \mathrm{Vis}_\phi + T_\phi - C_D v$$

Simulation of global ocean tides



Section 2

Comparison with tide gauge measurements

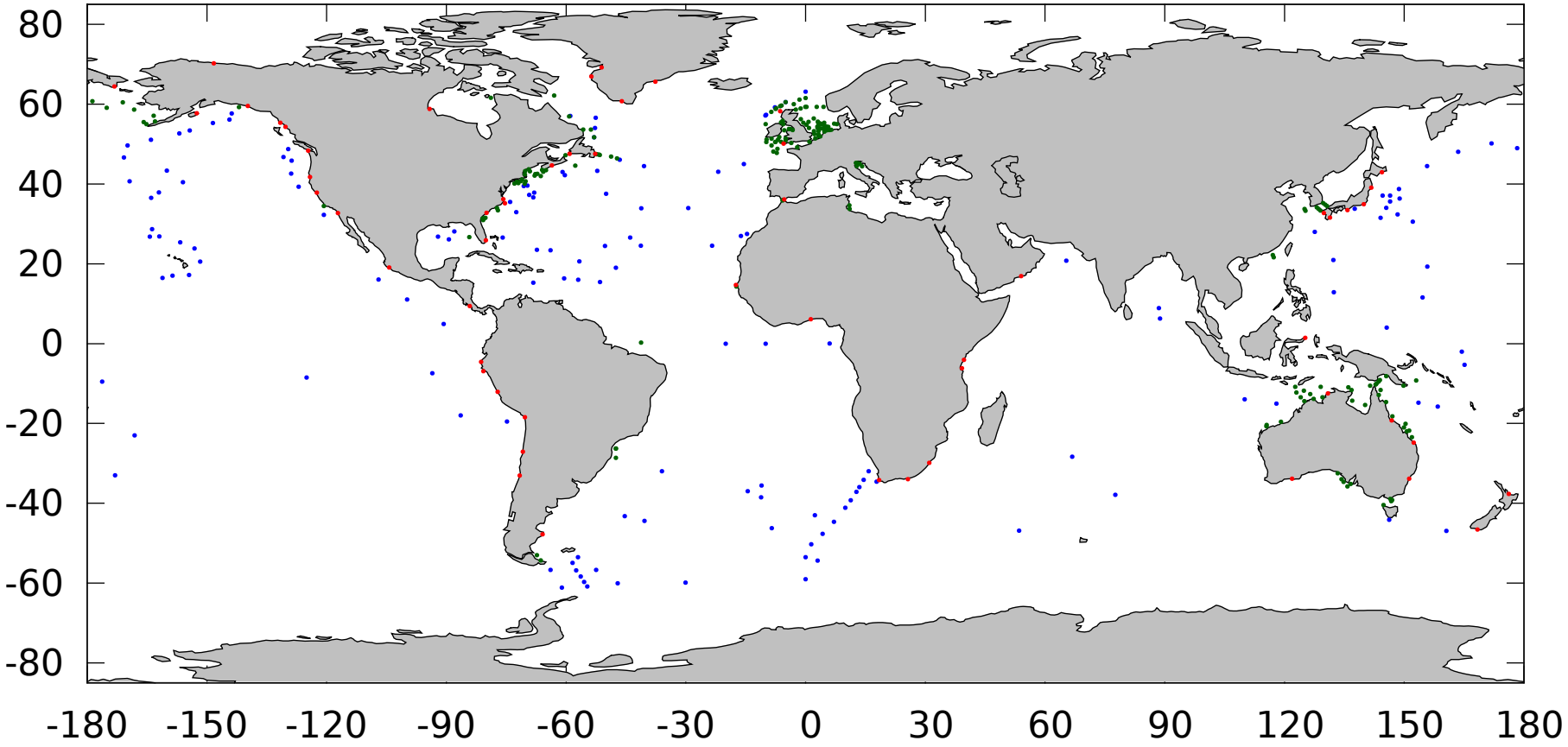
Island-tide gauge data vs. Bottom-pressure stations

Bottom-presure data provide better information for assessment of barotropic tide models than island-tide gauge:

- background noise is markedly lower,
- less sensitive to internal tides, which can not be simulated by a barotropic model but a fully three-dimensional model,
- tide-gauge data are influenced by local tide effects, which can occur even at small islands.

Ground truth data

151 deep ocean, 195 shelf seas (76 NW Eur. shelf, 119 elsewhere), 56 coast



Harmonic analysis

- Time-series can be expressed as a summation of K partial tides:

$$\zeta(\phi,\lambda,t) = \sum_{k}^{K} f_k(t) H_k(\phi,\lambda) \cos(\Theta_k(t) + \chi_k + \mu_k(t) - G_k(\phi,\lambda)) + \zeta_r(t)$$

$$\zeta(\phi,\lambda,t) = \sum_{k}^{K} f_k(t) A_k(\phi,\lambda) \cos(\Theta_k(t) + \chi_k + \mu_k(t)) + f_k(t) B_k(\phi,\lambda) \sin(\Theta_k(t) + \chi_k + \mu_k(t)) + \zeta_r(t)$$

- Amplitude and Greenwich phase lage of k^{th} tide

$$H_k = \sqrt{A_k^2 + B_k^2}, \qquad G_k = \arctan\left(\frac{B_k}{A_k}\right)$$

- A_k and B_k are computed by the least square method
- Lenght of time series: $T = 401$ days
- $K = 267$, tides were selected to fulfil the condition: $T > \frac{1}{|f_1 - f_2|}$

Comparative methods

- Root mean square differences:

$$\mathrm{RMS}_m = \sqrt{\overline{\left(H_{\text{station}} \cos(f_m t - G_{\text{station}}) - H_{\text{model}} \cos(f_m t - G_{\text{model}})\right)^2}}$$

  Average over all stations and over one full cycle of the constituent, e.g. $f_m t$ varies from 0 to 2π

- $\mathrm{Signal}_m = \sqrt{\overline{H^2_{\text{station}}}/2}$
- Relative RMS_m = Signal_m / RMS_m
- Residual sum of squares:

$$\mathrm{RSS} = \sqrt{\sum_m \mathrm{RMS}_m^2}$$

- Relative RSS = $\sqrt{\sum_m \mathrm{Signal}_m^2}$ / RSS

Comparative methods

- Median absolute differences::

$$\mathrm{MAD}_m = \mathrm{median}(|A_{\mathrm{station}} - A_{\mathrm{model}}|, |B_{\mathrm{station}} - B_{\mathrm{model}}|)$$

- $\mathrm{Signal}_m = \mathrm{median}(A_{\mathrm{station}}, B_{\mathrm{station}})$
- Relative $\mathrm{MAD}_m = \mathrm{Signal}_m$ / MAD_m
- Residual sum of squares:

$$\mathrm{RSS} = \sqrt{\sum_m \mathrm{MAD}_m^2}$$

- Relative RSS $= \sqrt{\sum_m \mathrm{Signal}_m^2}$ / RSS

Tidal constituents used in the comparison

Constituent	Speed [cpd]	Origin
Q_1	0.8932	Larger elliptical diurnal lunar
O_1	0.9295	Principal diurnal lunar
P_1	0.9973	Principal diurnal solar
K_1	1.0027	Principal diurnal lunisolar
N_2	1.8960	Larger elliptical semidiurnal lunar
M_2	1.9322	Principal semidiurnal lunar
S_2	2.0000	Principal semidiurnal lunar
K_2	2.0055	Declinational lunisolar

Important parameters of the model

- Internal wave drag
  - Conversion of barotropic tides into baroclinic (internal) waves.
- Self-attraction and loading of the seawater
  - Change of the gravitational potentional due to change in mass distribution of the seawater.
- Eddy viscosity
  - Turbulences on very short scales cause energy losses in the large-scale motions.
- Bottom friction

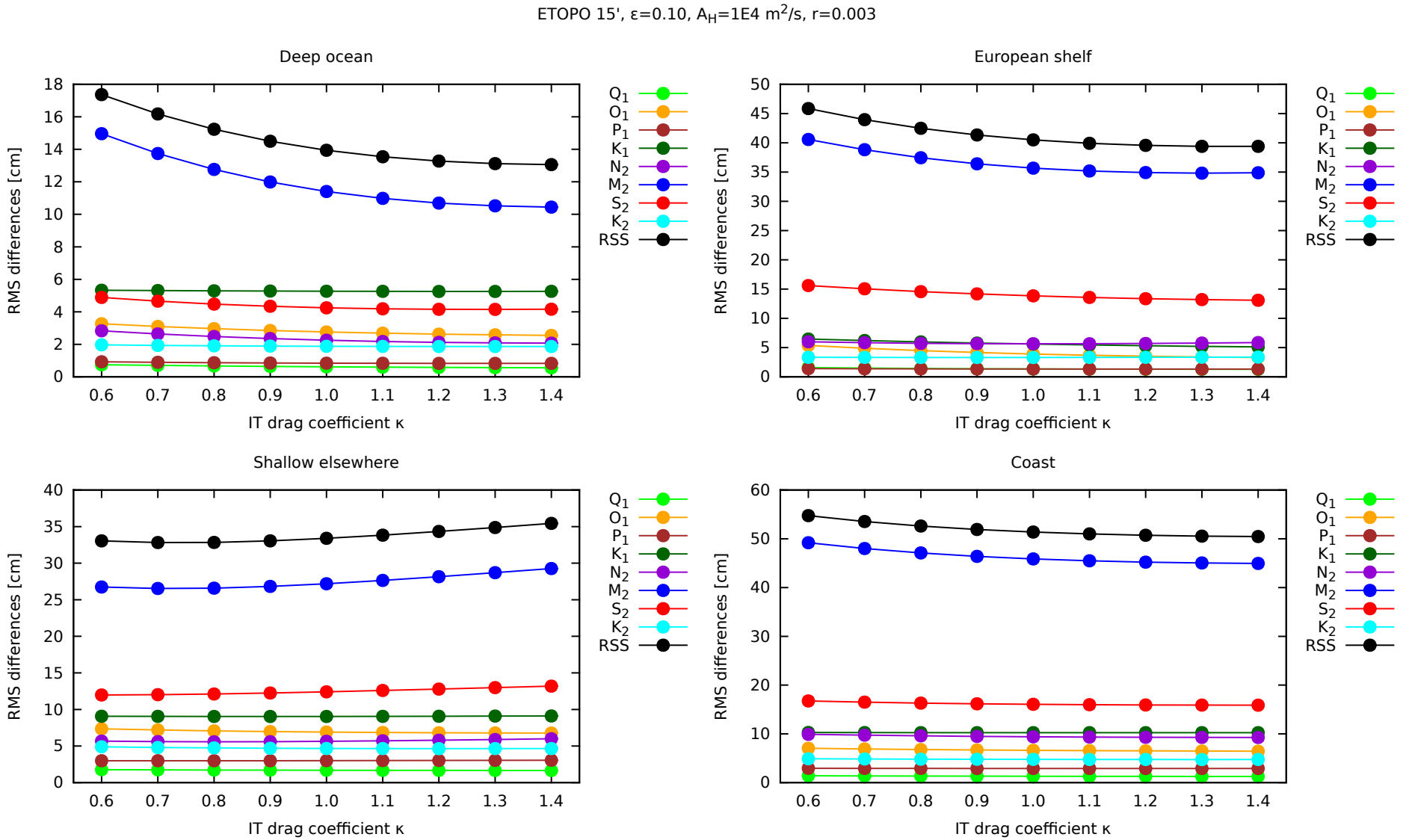
Effect of the internal wave drag

- Internal wave drag = conversion of barotropic tidal currents into baroclinic (internal) waves through the scattering by ocean-bottom topography of surface tides into internal waves.
- A formulation by Jayne & St. Laurent (2001):

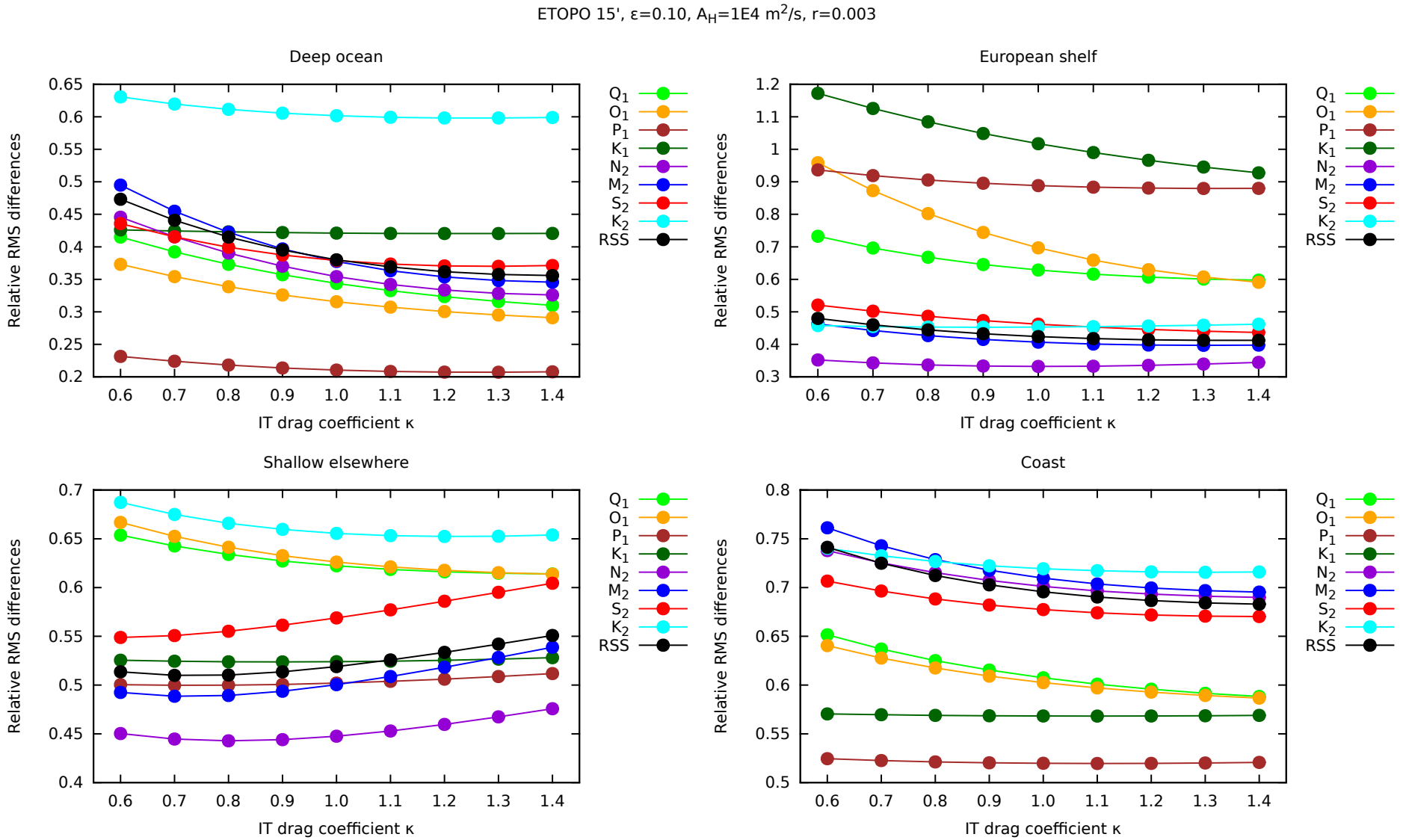
$$\vec{\tau}_{int} = \kappa \frac{\pi}{10000} \hat{h}^2 N_b \vec{v}$$

  - $\hat{h}$ ... the bottom roughness = the standard deviation of the bathymetric data
  - N_b ... the observed buoyancy frequency at the seabed, provided by Michael Schindelegger from TU Vienna, computed from salinity and temperature data of World Ocean Atlas 2005, $1^\circ \times 1^\circ$ grid
  - $N = \sqrt{-\frac{g}{\rho}\frac{\partial \rho}{\partial z}}$

Effect of the internal wave drag



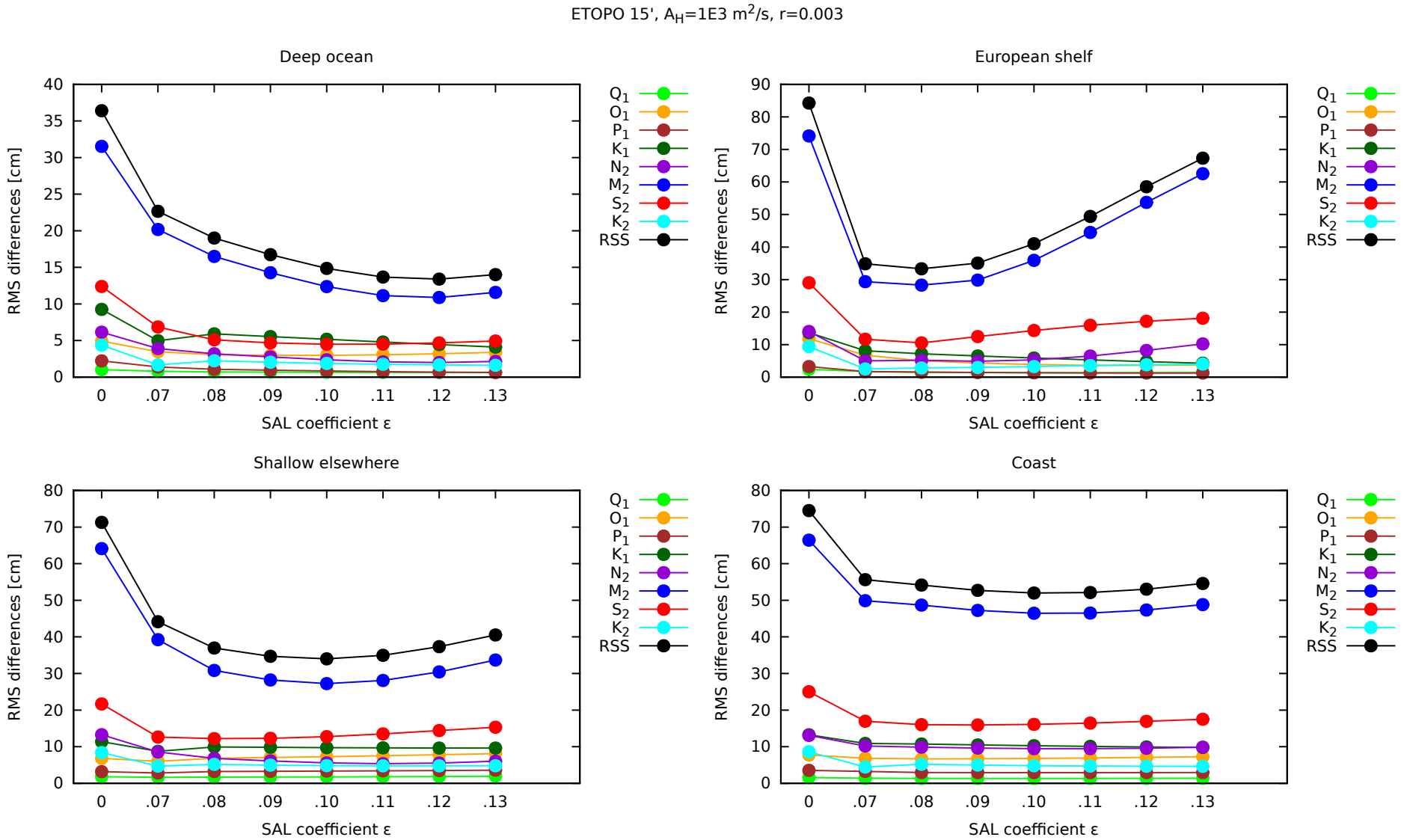
Effect of the internal wave drag



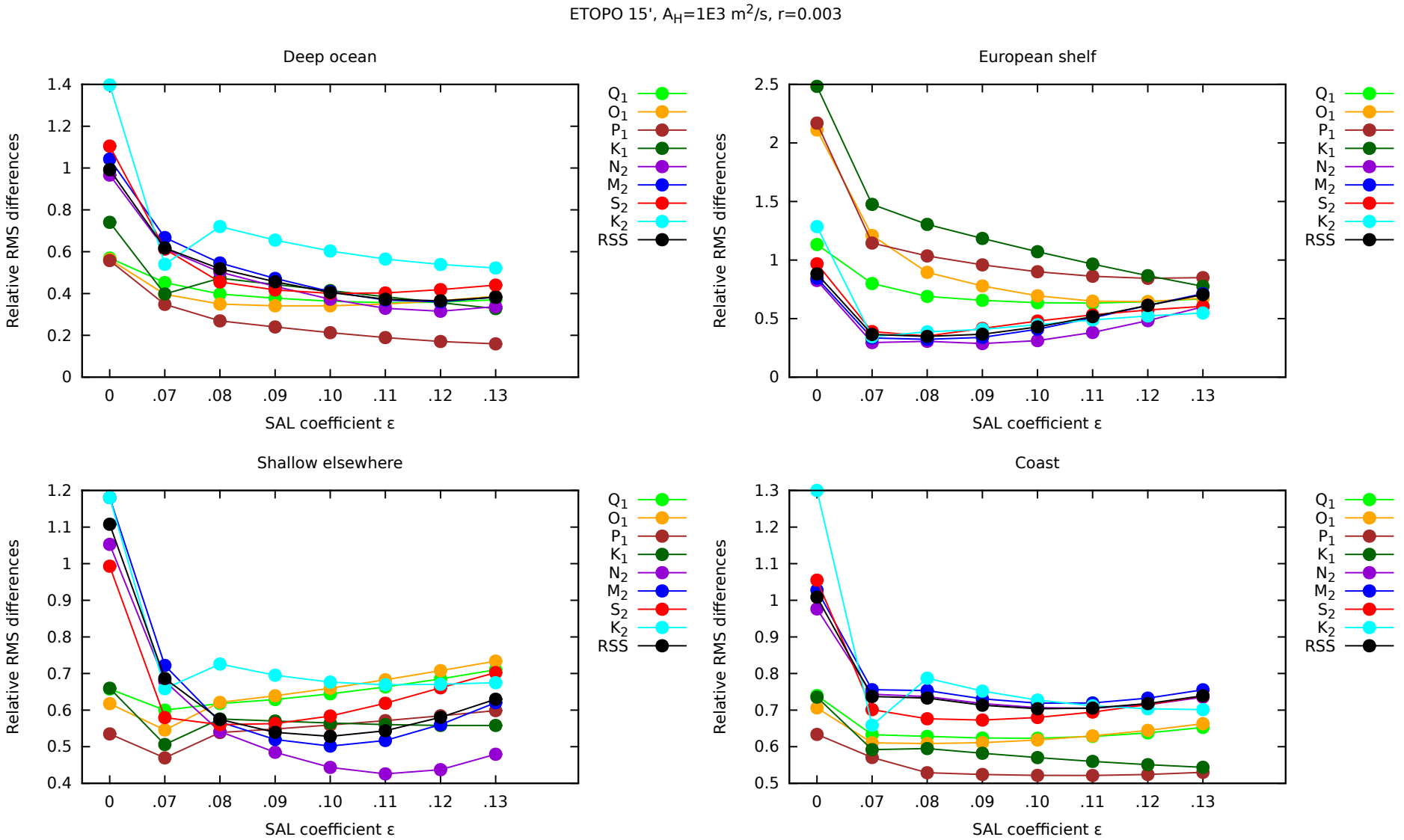
Effect of the self-attraction and loading of the water

- The change in mass distribution of the water column by the tidal elevation itself causes a change of the potential by the gravitational self-attraction of the water mass and evokes a potential change due to the surface load.
- RHS of the equation of motion: $+gh\nabla\zeta_{\mathrm{SAL}}$
- Calculation of the full SAL-effect is time-consuming (it requires to solve an integro-differential equation by iteration).
- Scalar approximation: $\zeta_{SAL} = \epsilon\zeta$
- This leads to the reduced gravity $\Rightarrow g \rightarrow g_\epsilon = g(1-\epsilon)$

Effect of the self-attraction and loading of the water



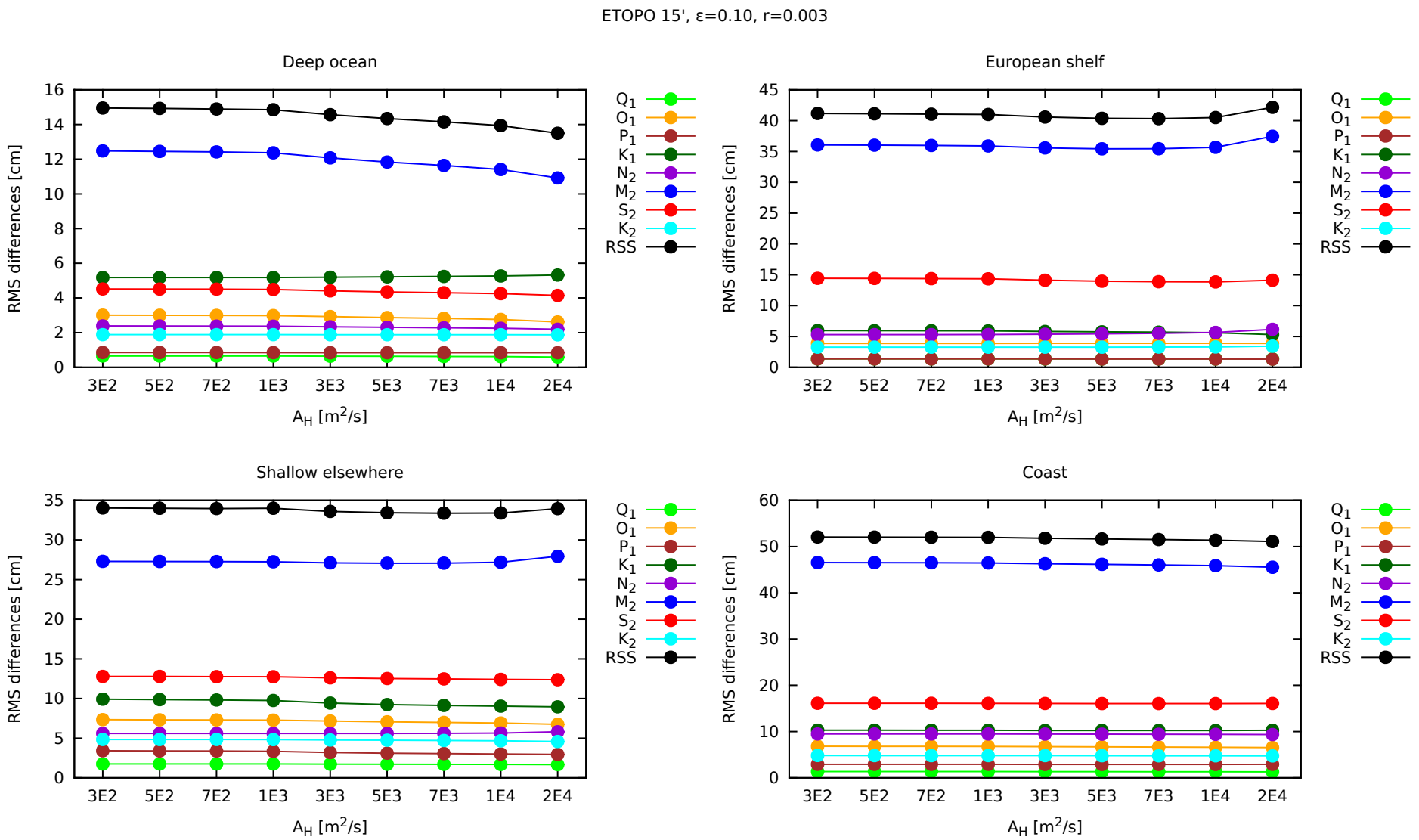
Effect of the self-attraction and loading of the water



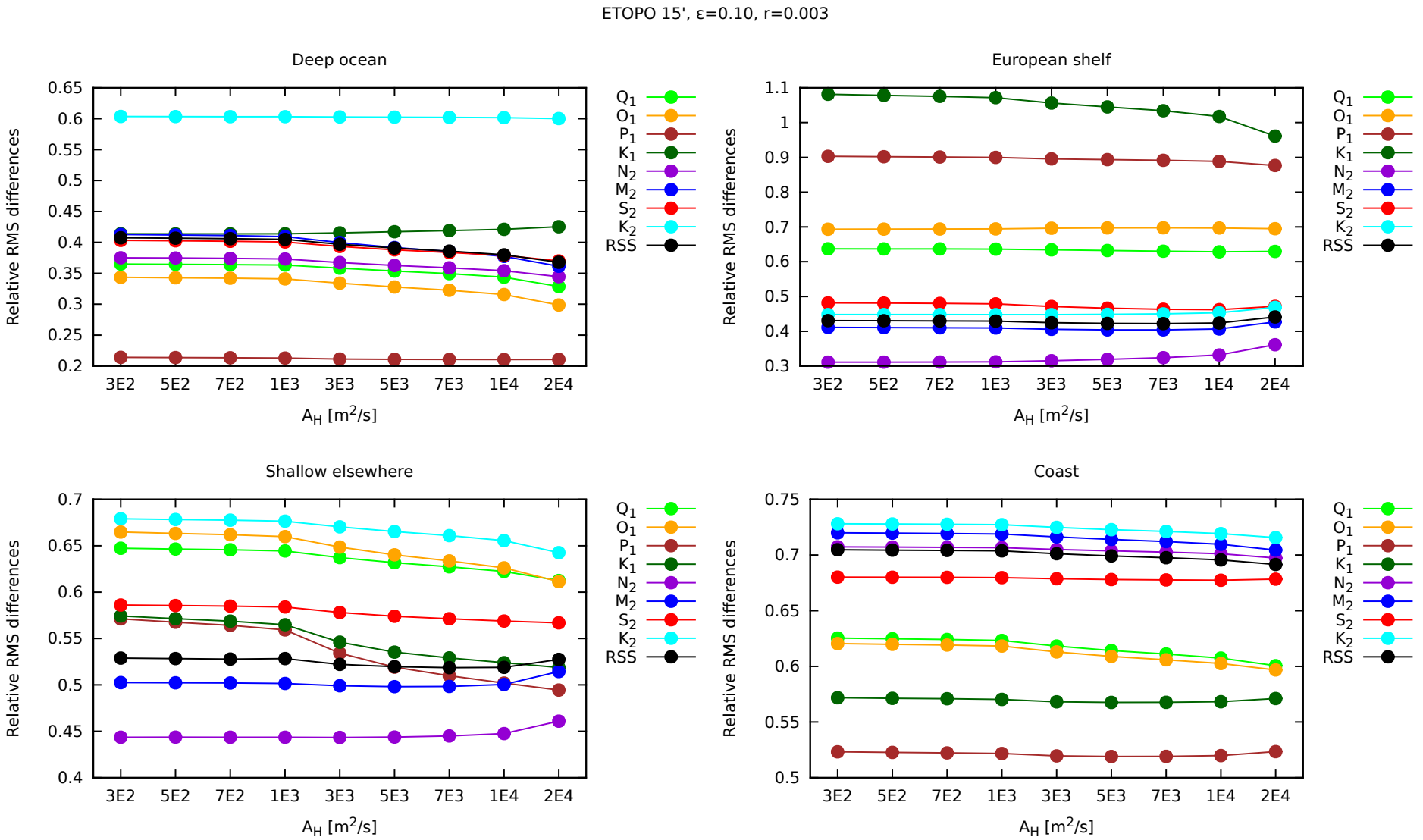
Effect of the eddy viscosity

- Turbulences on very short scales cause energy losses in the large-scale motions.
- Parameterizied by eddy viscosities: $\vec{s}_{vis} = A_H \nabla_{\mathrm{H}} \cdot \boldsymbol{\sigma}$
  - A_H ... horizontal eddy viscosity coefficient
  - $\boldsymbol{\sigma}$ ... Reynolds tensor, a combination of the spatial derivatives of the velocity field

Effect of the eddy viscosity



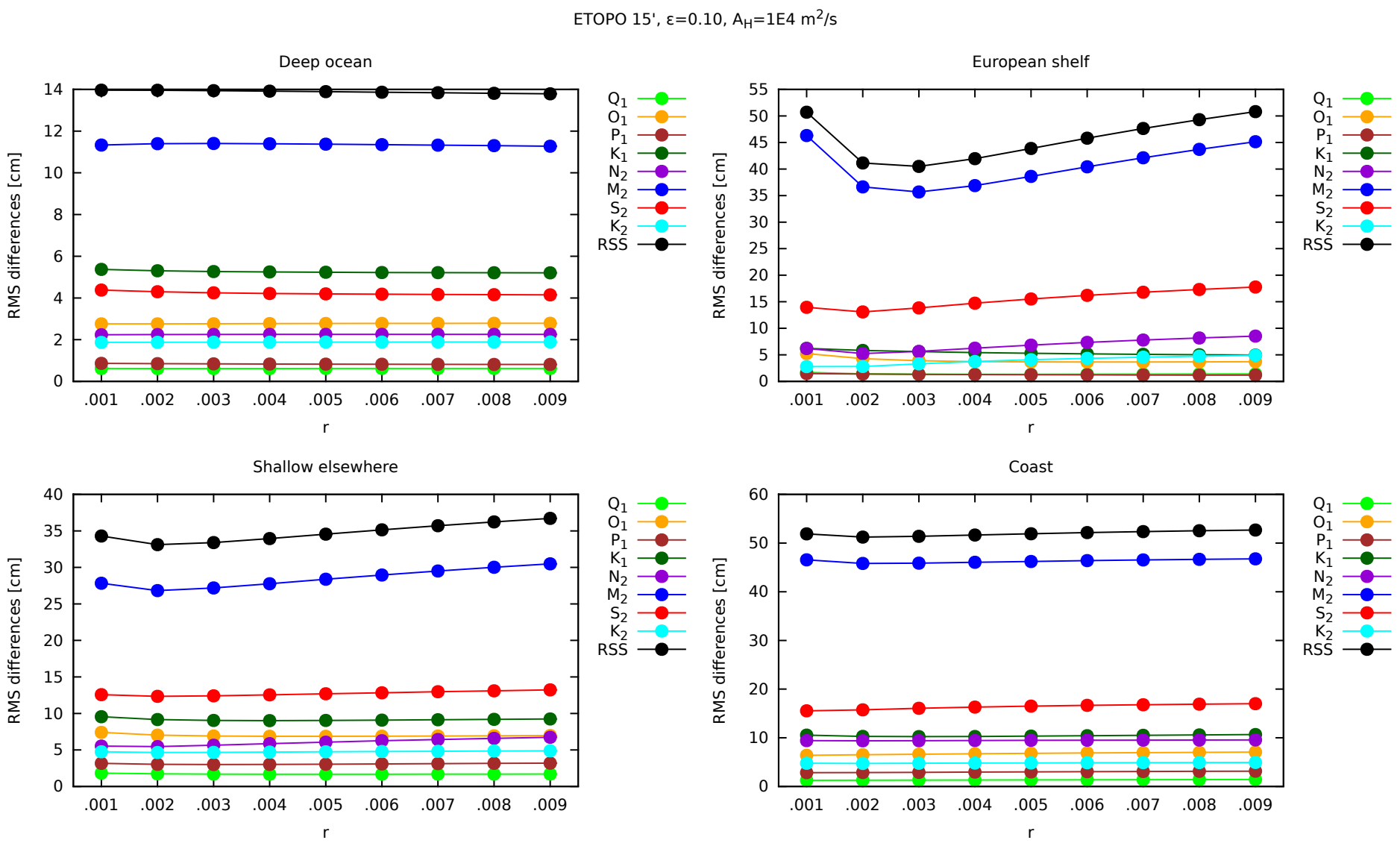
Effect of the eddy viscosity



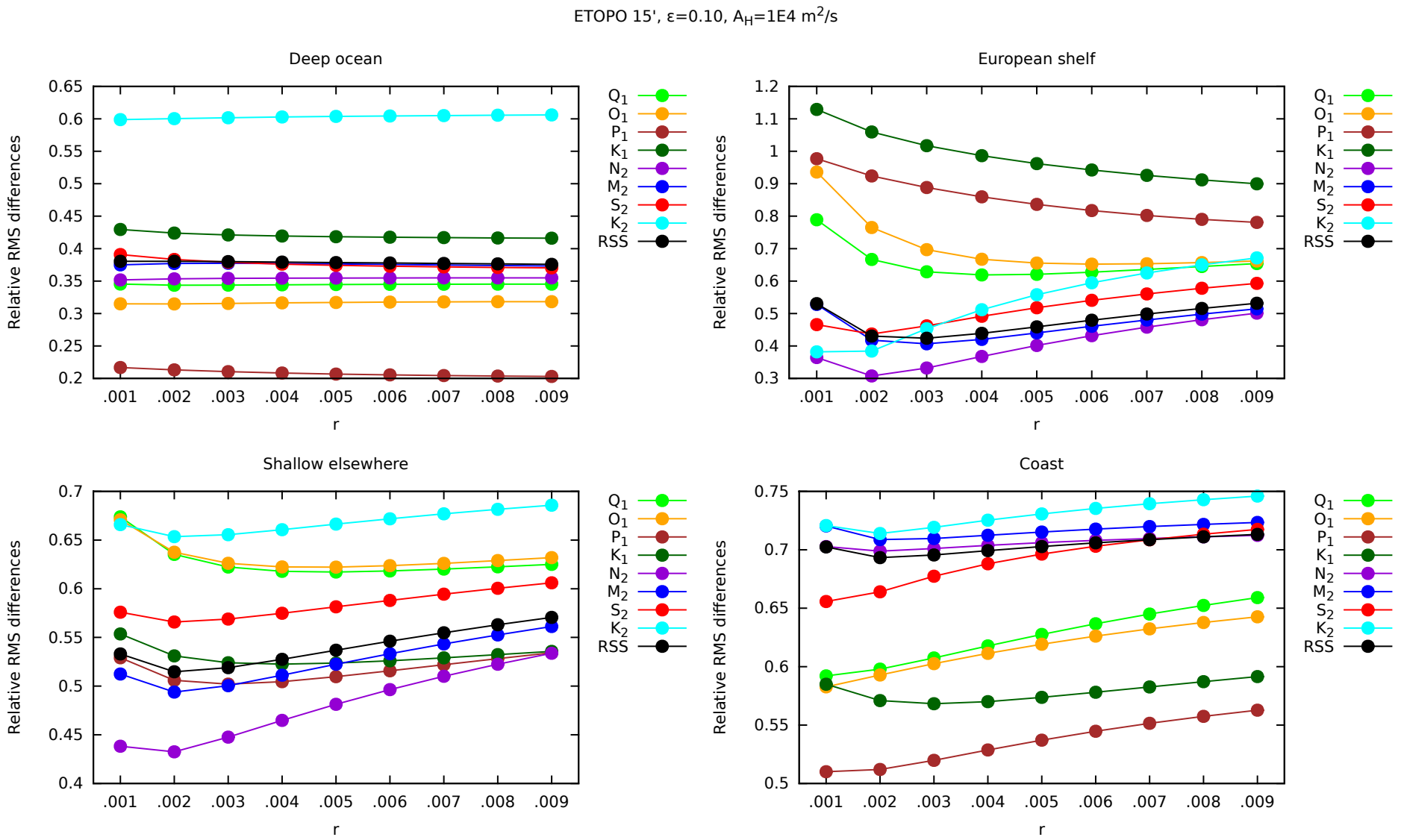
Effect of the bottom friction

- Energy losses due to friction at the bottom
- $\vec{s}_{fr} = -r\vec{v}|\vec{v}|$

Effect of the bottom friction



Effect of the bottom friction



Section 3

Satellite altimetry data assimilation into ocean tide models

Modern ocean tide models

1. Assimilative models
   - Assimilation of altimetric data into a barotropic hydrodynamic model
   - TPXO8, FES2012, HAMTIDE
2. Empirical models
   - Purely empirical models = directly derived from satellite data: OSU12
   - Semi-empirical models = empirical adjustments to an a priori model: GOT4.8, DTU10, EOT11a

Assimilative models

- Based on linearised momentum and continuity equation which are solved in spectral domain

$$i\omega\hat{\zeta} + \nabla \cdot \hat{\boldsymbol{U}} = 0,$$
$$\boldsymbol{\Omega}\hat{\boldsymbol{U}} + gh\nabla\hat{\zeta} = \boldsymbol{F},$$

where

$$\boldsymbol{\Omega} = \begin{pmatrix} i\omega + \kappa & -f \\ f & i\omega + \kappa \end{pmatrix},$$
$$\zeta = \hat{\zeta} e^{-i\omega t},$$
$$h\boldsymbol{u} = \boldsymbol{U} = \hat{\boldsymbol{U}} e^{-i\omega t},$$

Assimilative models

- Linear model:

$$\mathbf{S}\boldsymbol{u} = \boldsymbol{F}$$

- Altimetric data:

$$\boldsymbol{d} = \mathbf{L}\boldsymbol{u}$$

- A general inversion approach = to minimize the functional

$$\boldsymbol{J} = (\mathbf{S}\boldsymbol{u} - \boldsymbol{F})^H \boldsymbol{\Sigma}_m^{-1} (\mathbf{S}\boldsymbol{u} - \boldsymbol{F}) + (\boldsymbol{d} - \mathbf{L}\boldsymbol{u})^H \boldsymbol{\Sigma}_d^{-1} (\boldsymbol{d} - \mathbf{L}\boldsymbol{u})$$

Empirical models

- Derived by an analysis of satellite altimetry data = no equations to be solved
- Only the sea surface elevations
- Tidal currents can be estimated by a least-square method

$$\nabla \cdot \hat{\boldsymbol{U}} = -i\omega\hat{\zeta},$$

$$\begin{pmatrix} i\omega+\kappa & -f \\ f & i\omega+\kappa \end{pmatrix} \hat{\boldsymbol{U}} = \boldsymbol{F} - gh\nabla\hat{\zeta}$$

Section 4

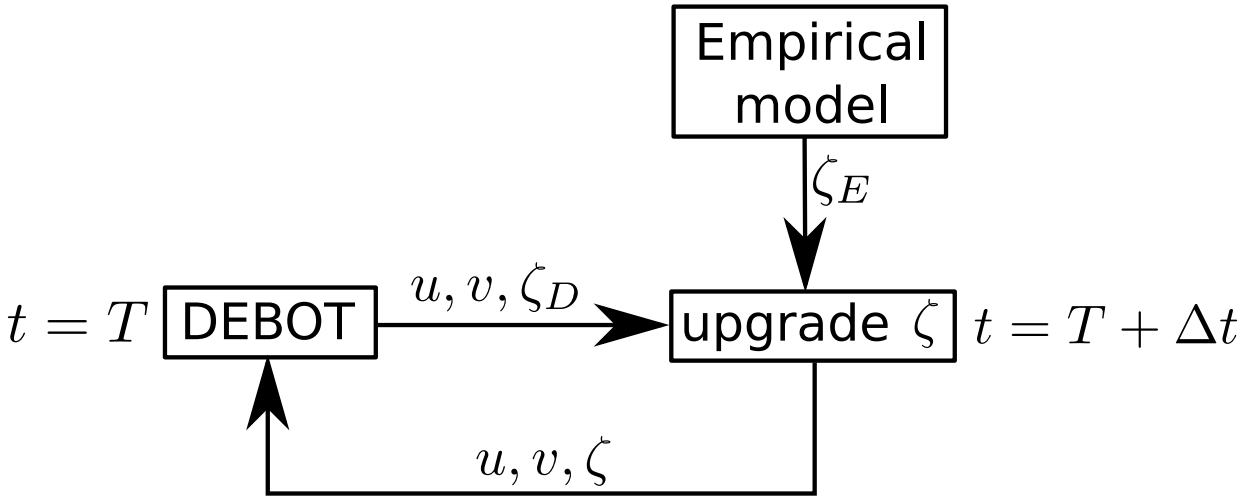
DEBOT Assimilative

What to do with DEBOT

- Modern assimilative models are linear spectral-domain
- DEBOT is non-linear time-domain.
- How to assimilate data into DEBOT?

DEBOT assimilation scheme

- DEBOT: ζ_D
- Empirical model: $\zeta_E = \sum_{i=1}^{N} \zeta_i$



- Upgrade: $\zeta = w\zeta_E + (1 - w)\zeta_D$

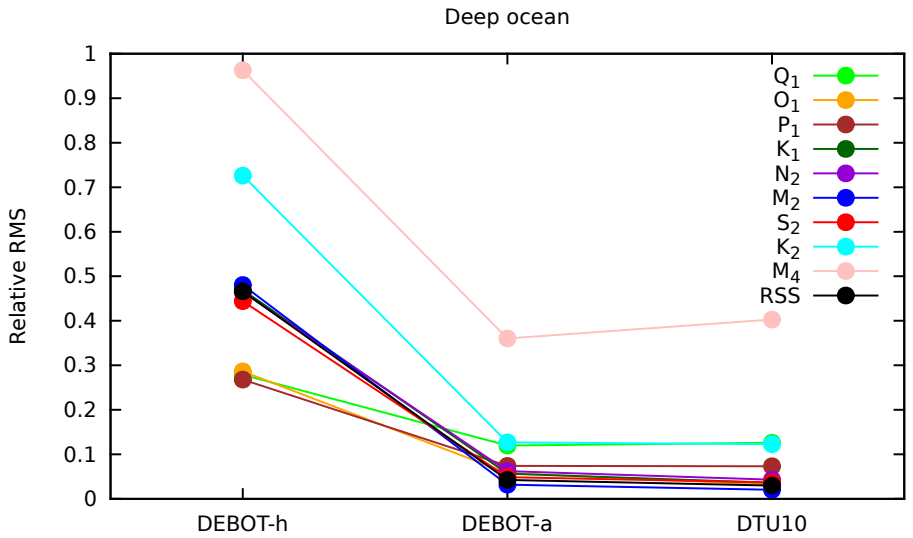
DEBOT assimilation scheme: problem

- ζ_D = generated by the full tidal potential = all tidal constituents are included
- ζ_E = only major tidal constituents
- Signals from minor constituents in ζ_D are destroyed during assimilation
- Solution:
  1. Launching DEBOT without the assimilation and computing time series of the elevation for all gridpoints
  2. Extracting the signals of the major tidal constituents from the time series
  3. Residual time series can be used as another "data" in the next DEBOT launch with the assimilation

Assimilation: first attempt

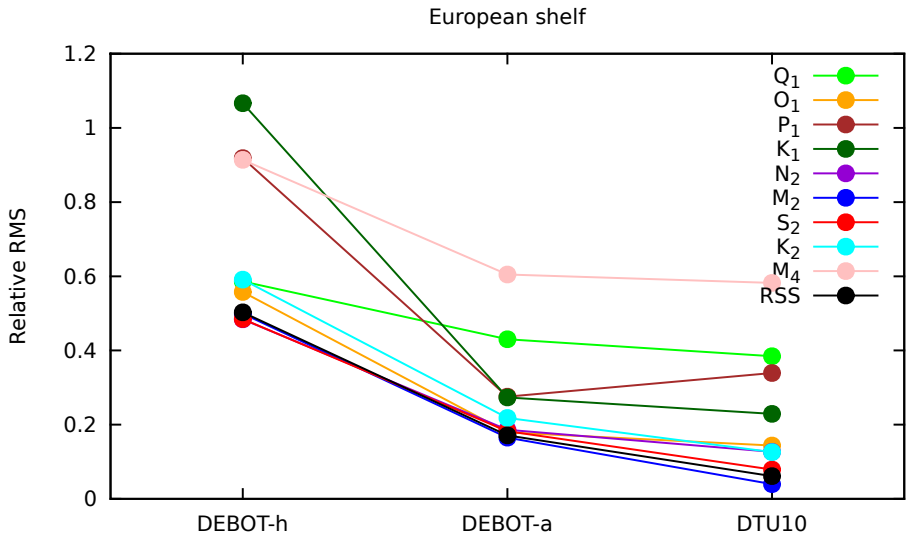
- DEBOT: $30'$ resolution, time step 15 s
- Data: DTU10, a $7.5'$ model, bilinearly interpolated to the DEBOT grid
- Tidal constituents: Q_1, O_1, P_1, K_1, N_2, M_2, S_2, K_2, M_4
- S_1 constituent was not used, S_1 is a radiational tide
- Assimilation process: every 15 minutes
- Data are assimilated in four time steps with the increasing weight $w = 0.15, 0.30, 0.45, 0.60$
- Comparison against "ground truth" tidal measurements: deep ocean, shelf and coastal tidal station
- RMS differences of tidal constituents and their RSS

Deep ocean



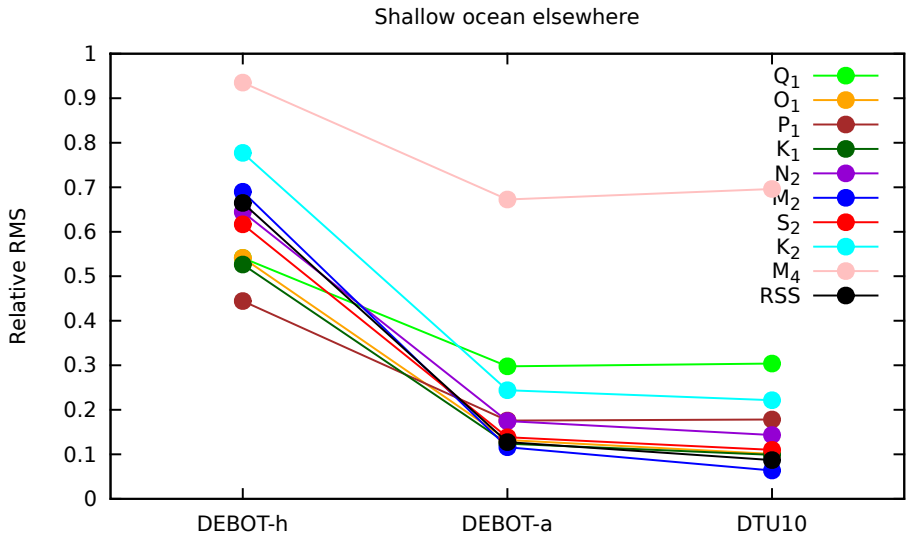
	Q_1	O_1	P_1	K_1	N_2
Signal	1.794	8.755	3.993	12.506	6.360
DEBOT-h	0.500	2.506	1.070	5.872	2.954
DEBOT-a	0.215	0.506	0.295	0.707	0.396
DTU10	0.226	0.277	0.292	0.449	0.274
	M_2	S_2	K_2	M_4	RSS
Signal	30.222	11.209	3.120	0.221	36.626
DEBOT-h	14.520	4.977	2.265	0.213	17.079
DEBOT-a	0.961	0.546	0.396	0.080	1.559
DTU10	0.613	0.415	0.383	0.089	1.092

European shelf



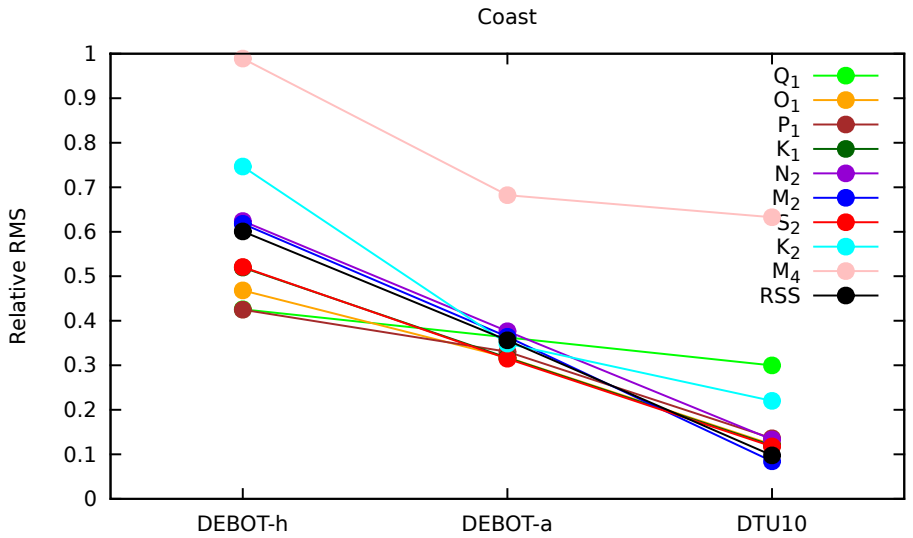
	Q_1	O_1	P_1	K_1	N_2
Signal	2.16	5.63	1.50	5.55	17.13
DEBOT-h	1.26	3.14	1.38	5.91	8.29
DEBOT-a	0.93	1.01	0.41	1.52	3.19
DTU10	0.83	0.81	0.51	1.27	2.17
	M_2	S_2	K_2	M_4	RSS
Signal	88.32	30.16	7.30	4.71	95.65
DEBOT-h	44.11	14.62	4.32	4.30	48.10
DEBOT-a	14.57	5.50	1.59	2.85	16.36
DTU10	3.50	2.38	0.92	2.74	5.85

Shelf seas elsewhere



	Q_1	O_1	P_1	K_1	N_2
Signal	2.70	10.99	5.95	17.19	12.56
DEBOT-h	1.46	5.95	2.64	9.05	8.09
DEBOT-a	0.80	1.45	1.05	2.13	2.19
DTU10	0.82	1.11	1.06	1.70	1.80
	M_2	S_2	K_2	M_4	RSS
Signal	54.09	21.73	7.08	1.81	63.78
DEBOT-h	37.32	13.40	5.50	1.69	42.40
DEBOT-a	6.28	3.01	1.73	1.22	8.13
DTU10	3.44	2.39	1.57	1.26	5.55

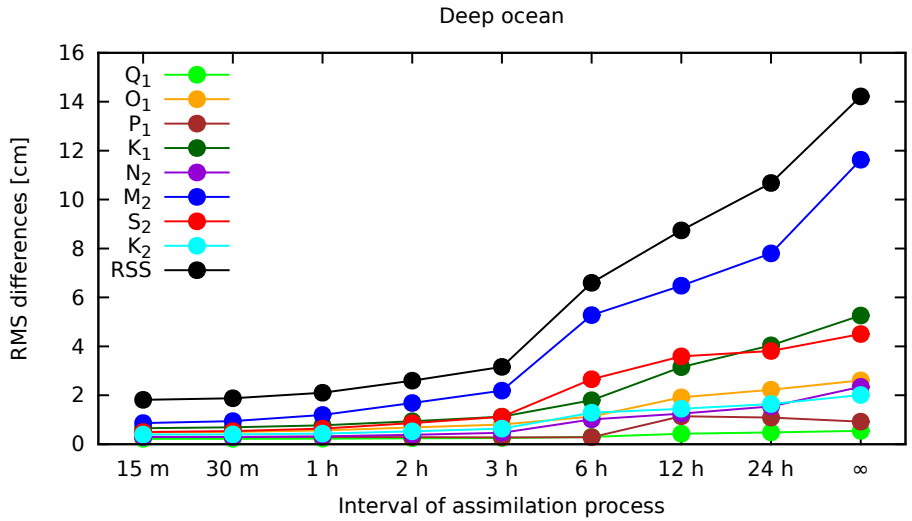
Coast

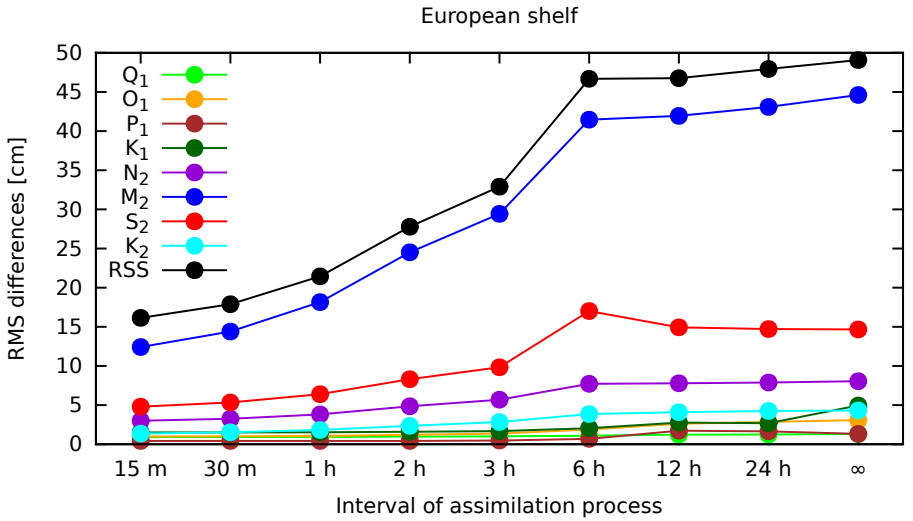


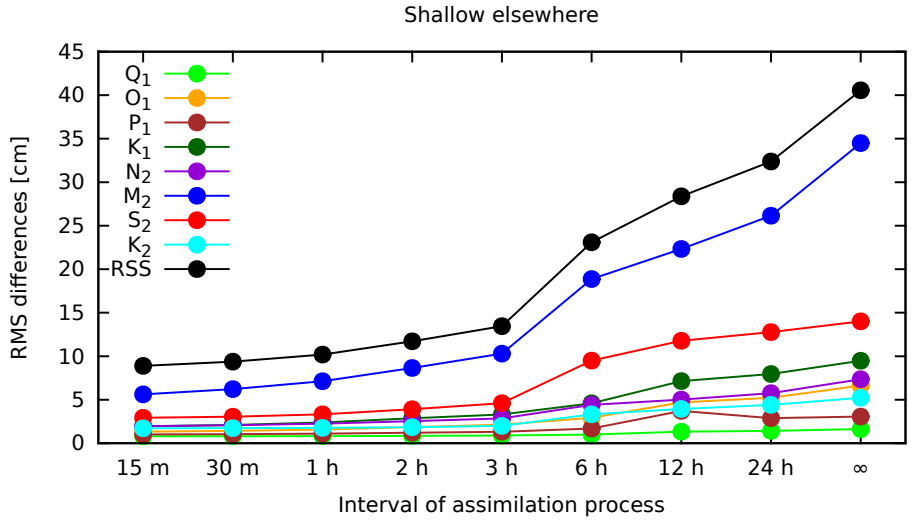
	Q_1	O_1	P_1	K_1	N_2
Signal	2.07	10.55	5.36	17.32	12.89
DEBOT-h	0.88	4.94	2.28	9.00	8.05
DEBOT-a	0.75	3.35	1.77	5.49	4.85
DTU10	0.62	1.29	0.73	2.08	1.72
	M_2	S_2	K_2	M_4	RSS
Signal	62.09	22.77	6.36	1.94	70.91
DEBOT-h	38.37	11.86	4.75	1.92	42.60
DEBOT-a	22.62	7.17	2.22	1.33	25.26
DTU10	5.24	2.68	1.40	1.23	6.93

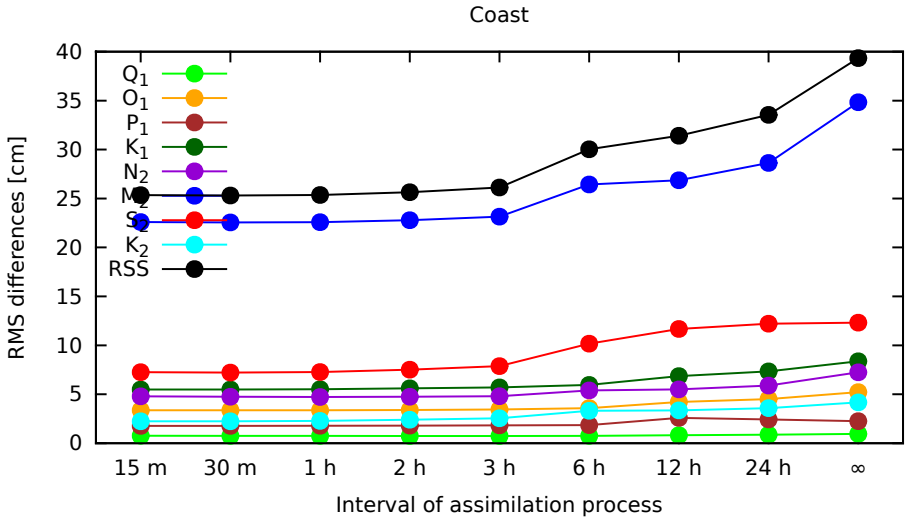
Effect of the assimilation interval

ETOPO 30', ε=0.10, A_H=1E4 m^2/s, r=0.003

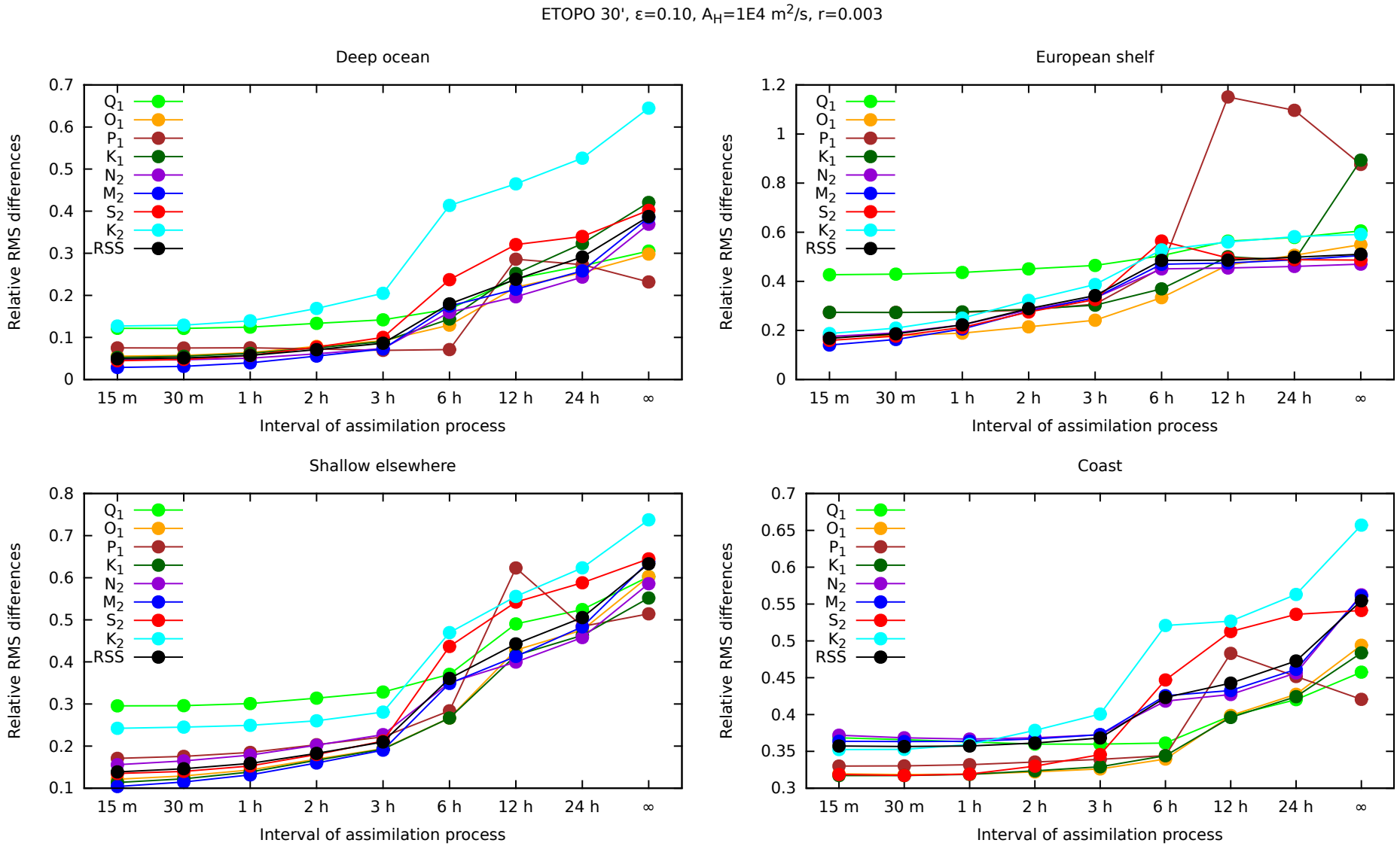




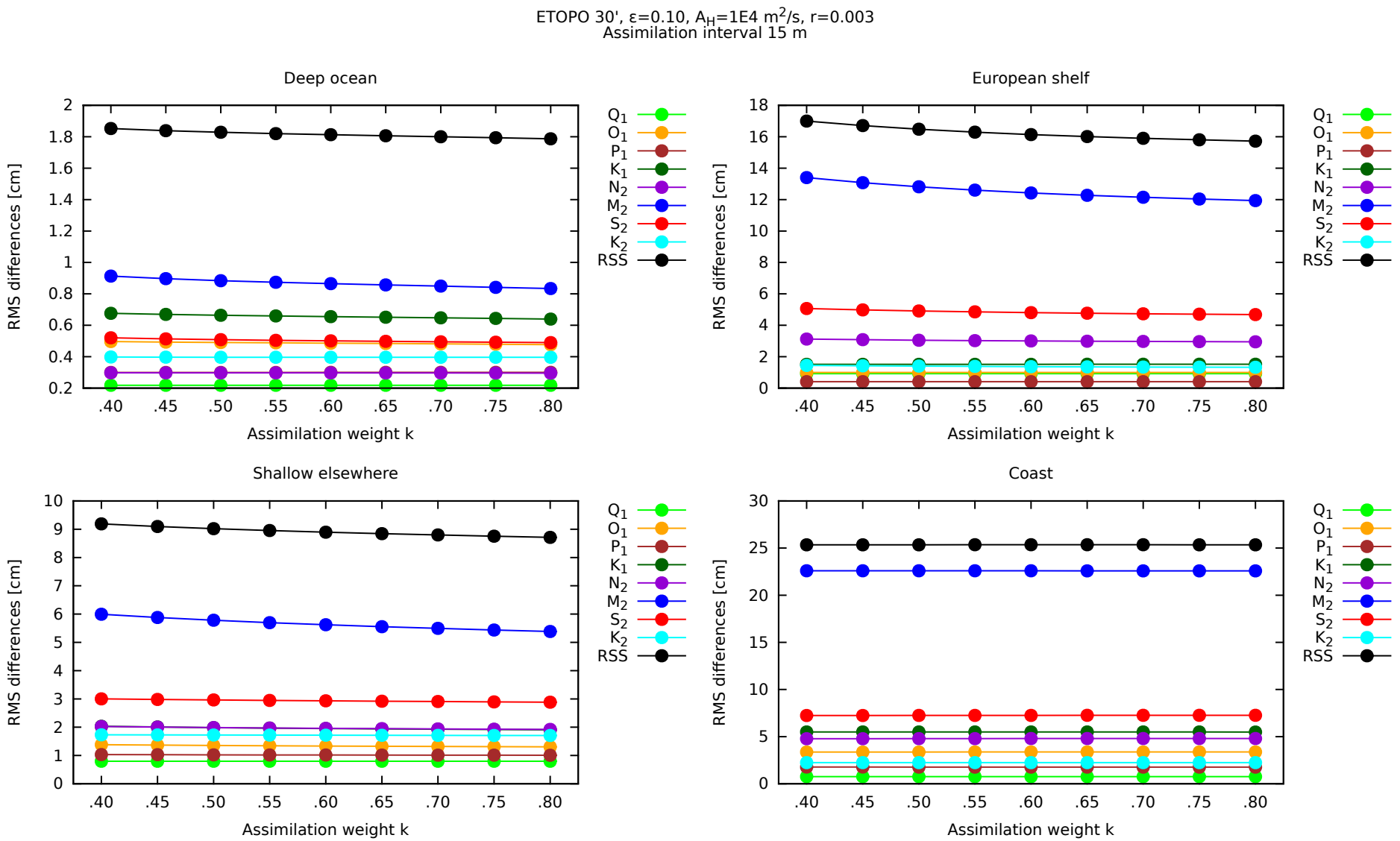




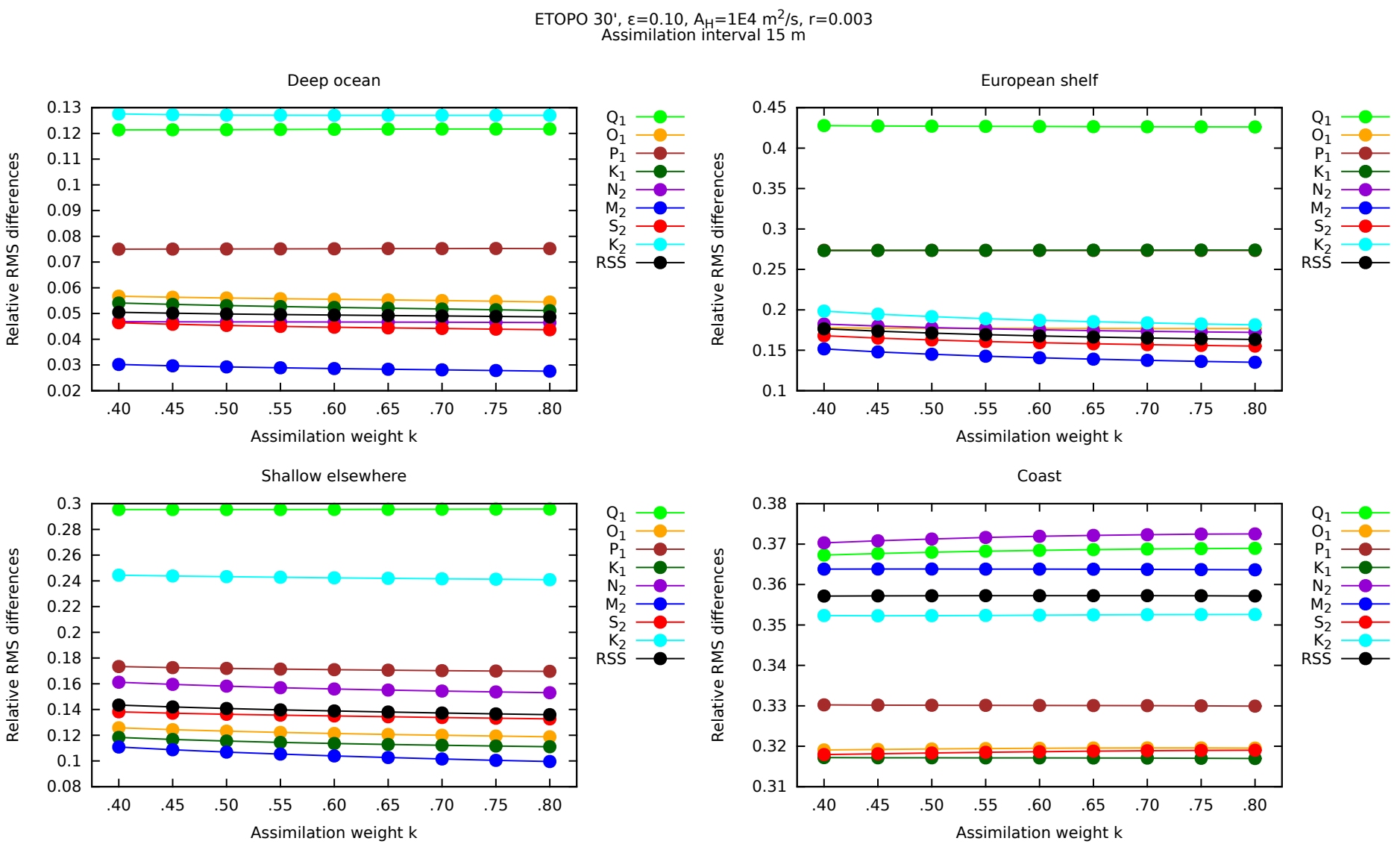
Effect of the assimilation interval



Effect of the assimilation weight



Effect of the assimilation weight



Section 5

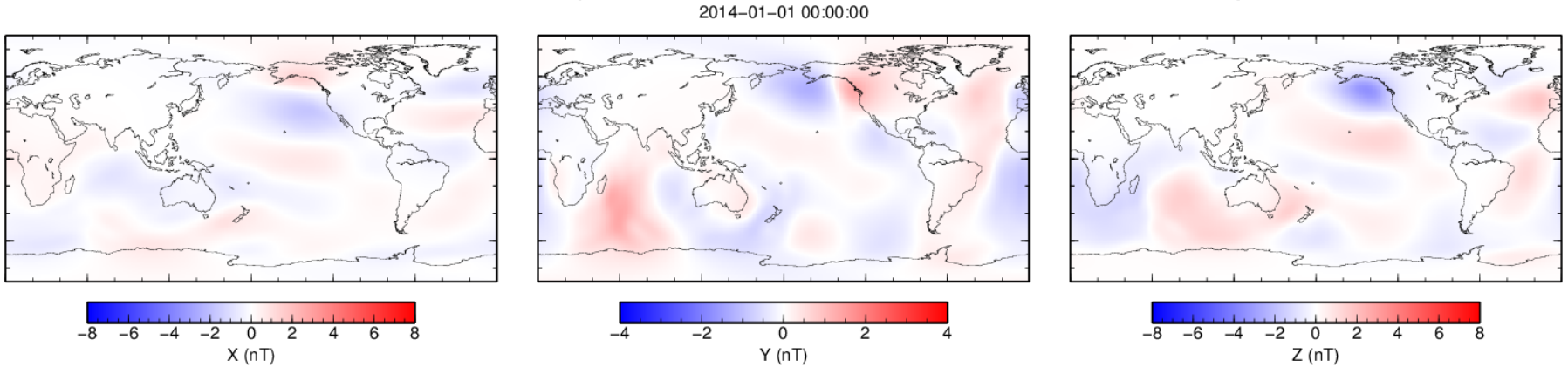
Magnetic field

Ocean-induced magnetic field

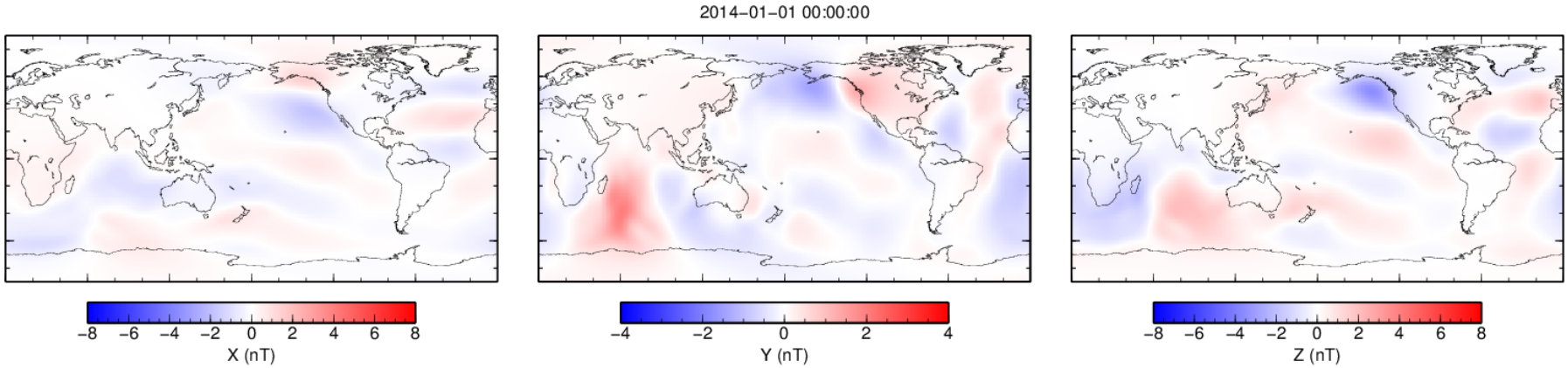
- EM induction model by Jakub Velímský
- Swarm data analysis by Zdeněk Martinec

Magnetic signals induced by DEBOT at 480 km

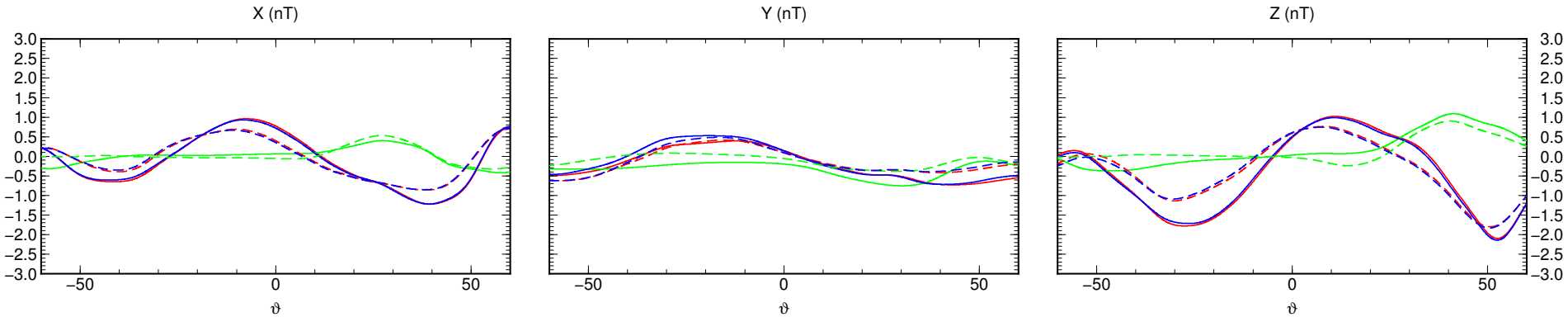
DEBOT-h (hydrodynamical, non-assimilated)



DEBOT-a (with altimetry assimilation)

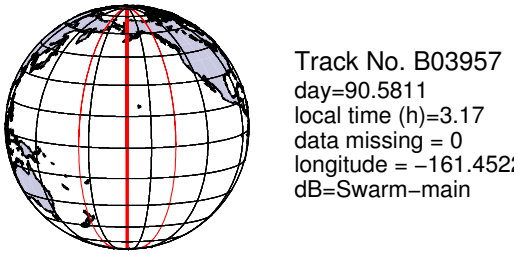


Magnetic signals induced by DEBOT along the satellite tracks

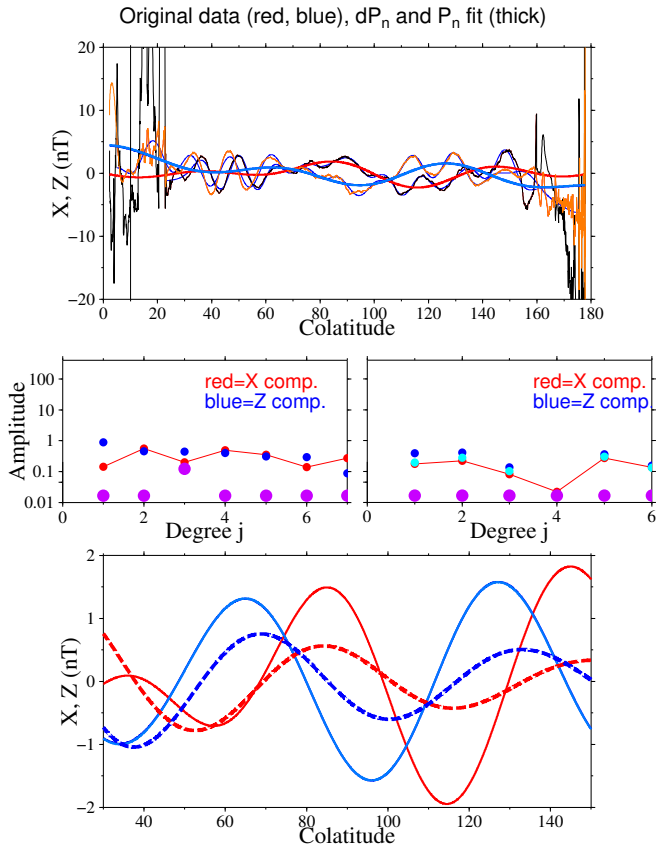


Swarm A 3963.0, Swarm B 3938.5, Swarm C 3959.0

DEBOT-h dashed lines, DEBOT-a solid lines

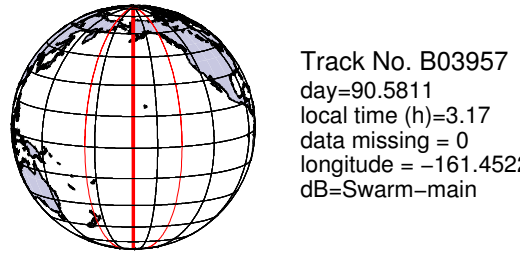


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data missing = 0
longitude = −161.452
dB=Swarm−main

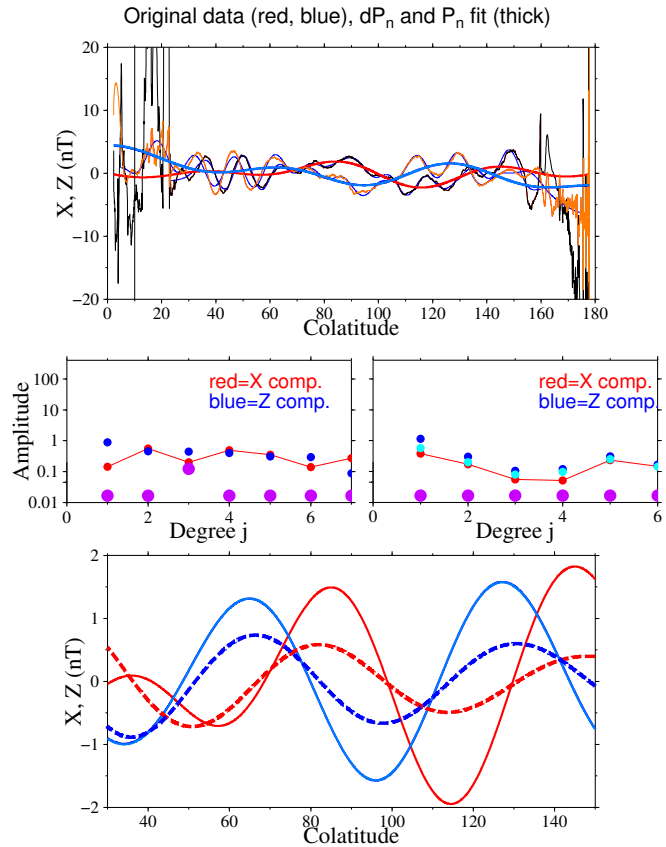


Swarm: left spectrum, solid lines

DEBOT hydrodynamical: right spectrum, dashed lines

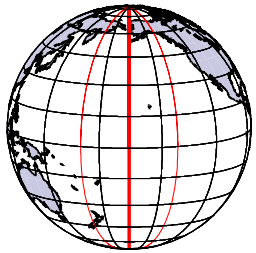


Track No. B03957
day=90.5811
local time (h)=3.17
data missing = 0
longitude = −161.452
dB=Swarm−main

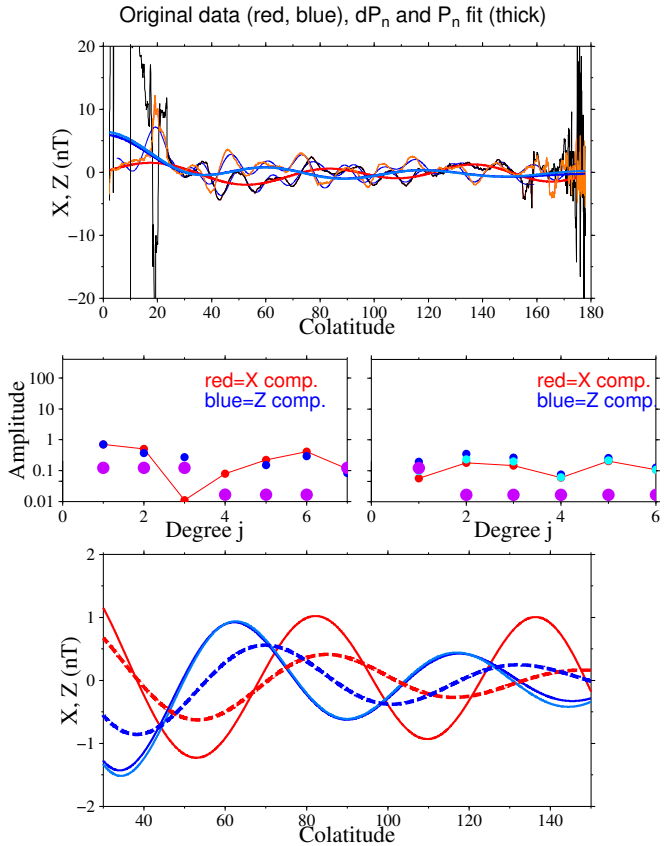


Swarm: left spectrum, solid lines

DEBOT assimilative: right spectrum, dashed lines

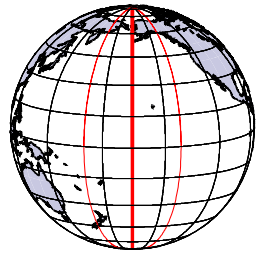


Track No. B04743
day=116.501
local time (h)=0.95
data missing = 0
longitude = −165.685
dB=Swarm−main

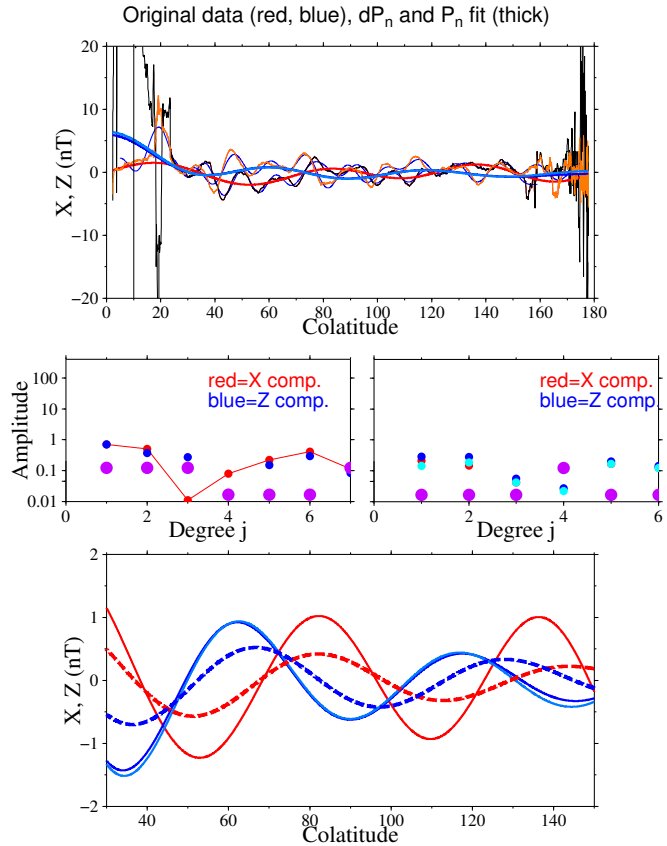


Swarm: left spectrum, solid lines

DEBOT hydrodynamical: right spectrum, dashed lines



Track No. B04743
day=116.501
local time (h)=0.95
data missing = 0
longitude = −165.685
dB=Swarm−main



Swarm: left spectrum, solid lines

DEBOT assimilative: right spectrum, dashed lines

Thank you for your attention!

