

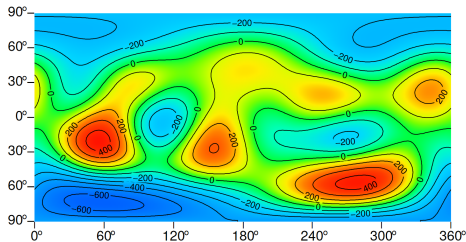
Modeling uneven heat transfer in the ocean of Titan

1st year PhD presentation

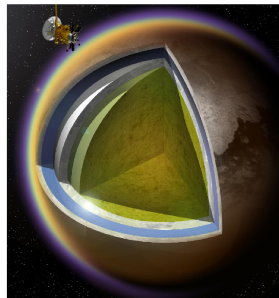
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31.03.2021

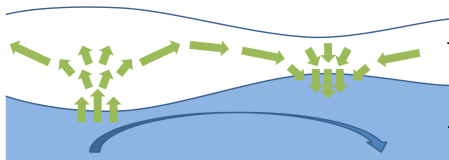
Titan



Topographic
measurement



Consequence of an oceanic flow?



Kvorka et al., 2018

Topography supported by phase changes on the lower boundary

Are large timescale suitable
flow patterns possible?

Convection in a rotating spherical shell - basics



Figure: Impact of rotation on the temperature variations, source: Gastine et al., 2016

Convection in a rotating spherical shell - basics

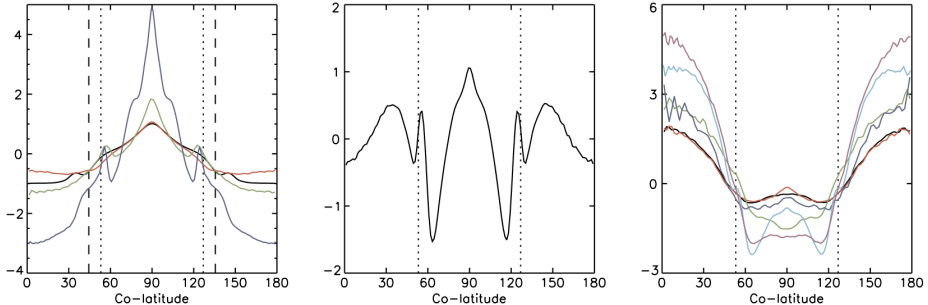


Figure: Impact of rotation on the surface heat flux, source: Amit et al., 2020

Rotating convection

$$\partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0$$

$$\rho(\partial_t \vec{v} + (\vec{v} \cdot \nabla) \vec{v}) = \nabla \cdot \mathbf{T} + \rho \vec{g} - 2\rho \vec{\Omega} \times \vec{v} - \rho \vec{\Omega} \times (\vec{\Omega} \times \vec{r})$$

$$\rho c_p (\partial_t \theta + \vec{v} \cdot \nabla \theta) = \nabla \cdot (k \nabla \theta) - \alpha \rho \theta \vec{v} \cdot \vec{g} + \mathbf{T}^d : \nabla \vec{v}$$

$$\mathbf{T} = -p \mathbf{I} + \eta [(\nabla \vec{v}) + (\nabla \vec{v})^T]$$

$$\rho = \rho(p, \theta)$$

$\vec{v}$ - velocity, ρ - density, p - pressure, $\mathbf{T}$ - stress, $\vec{g}$ - gravity profile, $\vec{\Omega}$ - rotation rate, θ - temperature, c_p - heat capacity, k - thermal conductivity, η - viscosity, α - thermal expansivity

No need for physical units, non-dimensional formalism

Non-dimensional equations

$$\nabla \cdot \vec{v} = 0$$

$$\frac{1}{Pr}(\partial_t \vec{v} + \vec{v} \cdot \nabla \vec{v}) = \nabla \cdot \mathbf{T} - Ra(\theta - \theta_0)g\vec{e}_r - \frac{2}{Ek}\vec{e}_z \times \vec{v}$$

$$\mathbf{T} = -(p - p_0)\mathbf{I} + (\nabla \vec{v} + (\nabla \vec{v})^T)$$

$$\partial_t \theta - \vec{v} \cdot \vec{q} = -\nabla \cdot \vec{q}$$

$$\vec{q} = -\nabla \theta$$

$\vec{v}$ - velocity, p - pressure, g - gravity, θ - temperature, $\mathbf{T}$ - stress

Control parameters

- **Prandtl number** - ratio of thermal to diffusion time scales
- **Rayleigh number** - ratio of buoyancy to viscous forces (larger Ra means stronger buoyant driving)
- **Ekman number** - the ratio of viscous to Coriolis forces (larger Ek means stronger rotational effects)

$$Pr = \frac{\nu}{\kappa}, \quad Ra = \frac{\alpha g_0 \Delta \theta D^3}{\nu \kappa}, \quad Ek = \frac{\nu}{\Omega D^2}, \quad \chi = \frac{r_d}{r_u}$$

Model

- spherical, isothermal, impermeable boundaries

<i>no-slip</i>	<i>free-slip</i>
$\vec{v} = \vec{0}$	$\vec{n} \cdot \vec{v} = 0$ $\mathbf{T} \cdot \vec{n} - (\vec{n} \cdot \mathbf{T} \cdot \vec{n}) \vec{n} = \vec{0}$
$\theta_d = 1, \theta_u = 0$	

- no-slip - physically correct
- free-slip - numerically correct

Nusselt number

- traditional measure of heat transport efficiency

$$Nu = \frac{Q}{Q_{conductive}} = \frac{1}{4\pi r_o r_i} \frac{QD}{\rho C_p \kappa \Delta\theta}$$

- possible determination of an appropriate Rayleigh number for no-slip models

$$Nu_{RR} \propto Pr^{-1/2} Ra^{3/2} Ek^2, \quad Nu_{NR} \propto Ra^{1/3}$$

Modified Nusselt and flux-based Rayleigh number

- more appropriate for free-slip models

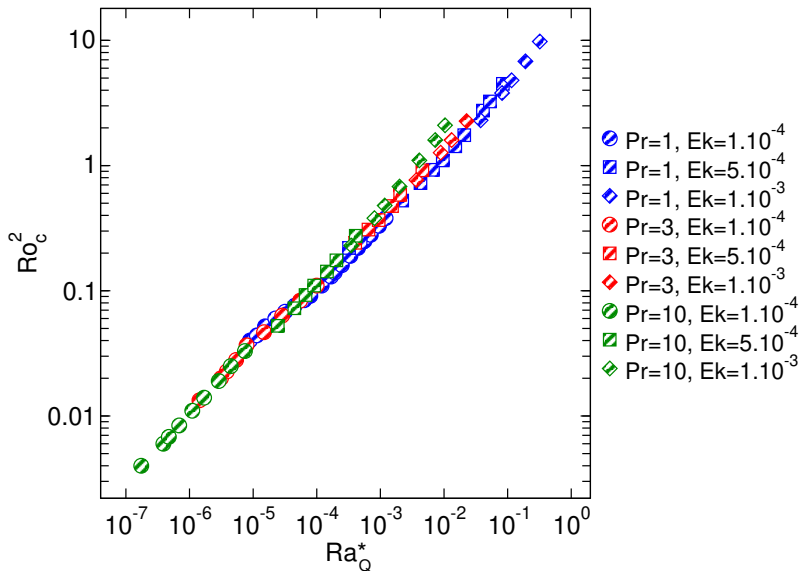
$$Nu^* = \frac{1}{4\pi r_o r_i} \frac{Q_{adv}}{\rho C_p \Omega D \Delta\theta} = (Nu - 1) Ek / Pr$$

$$Ra_q^* = \frac{1}{4\pi r_o r_i} \frac{\alpha g_o Q_{adv}}{\rho C_p \Omega^3 D^2} = Ra (Nu - 1) Ek^3 / Pr^2$$

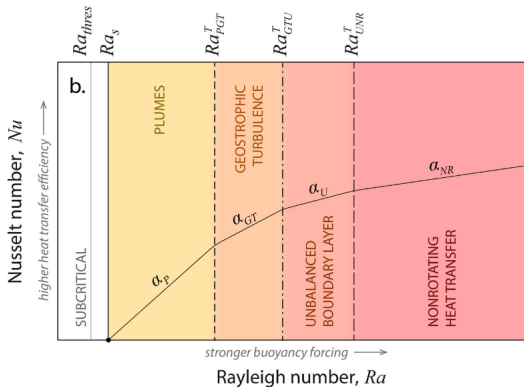
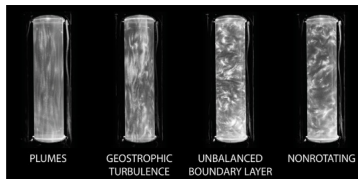
- numerically confirmed exponential relation

$$Nu^* = \text{const} (Ra_q^*)^\gamma \implies Ro_c^2 = Ra Ek^2 / Pr = \frac{1}{\text{const}} (Ra_q^*)^{1-\gamma}$$

Modified Nusselt and flux-based Rayleigh number



Diagnostic parameters



Diagnostic parameters

- geostrophic turbulence breakdown

$$Ra_{GTU}^T \propto Ek^{-1.8} - Ek^{-1.65} \text{ Ecke and Niemela, 2014}$$

$$Ra_{GTU}^T \propto Ek^{-5/3} \text{ Julien et al., 2012a}$$

$$Ra_{GTU}^T \propto Pr^{3/5} Ek^{-8/5} \text{ Julien et al., 2012b}$$

$$Ra_{GTU}^T \propto Ek^{-7/4} \text{ King et al., 2013}$$

- pool of possible diagnostic quantities

$$Ro_{loc} = Pr^{-3/5} Ra Ek^{8/5} \approx 1$$

$$R_G^* = Ra Ek^{12/7} / Pr \approx 1$$

$$R_K^* = Ra Ek^{7/4} / Pr \approx 1$$

Diagnostic parameters

- beginning of the nonrotating heat transfer

$$Ra_{UNR}^T \propto Ek^{-2} \text{ Gillman, 1977}$$

$$Ra_{UNR}^T \propto Ek^{-2} \text{ Ecke and Niemela, 2014}$$

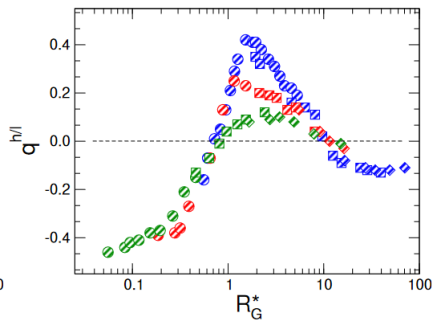
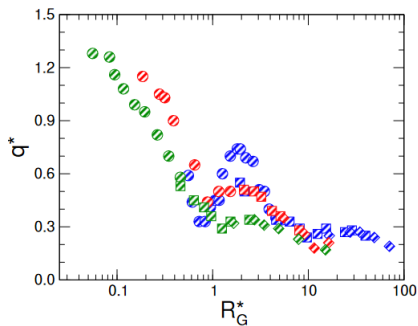
- pool of possible diagnostic quantities

$$Ro_c = \sqrt{RaEk^2/Pr} \approx 1$$

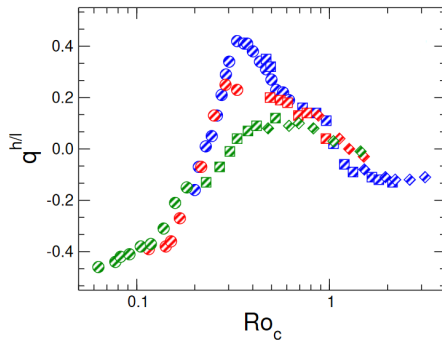
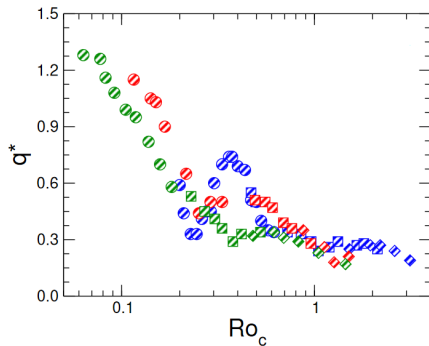
Parameters range

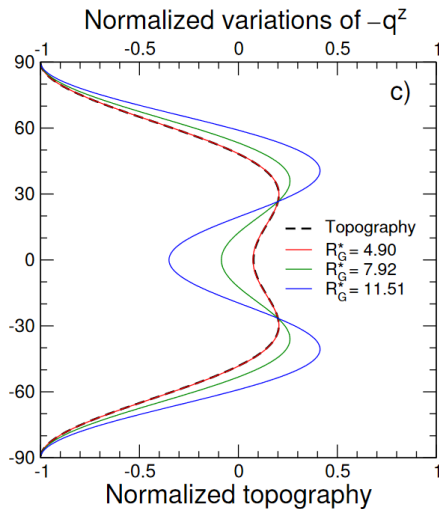
Parameter	Realistic range		
D [km]	200	300	400
Ra_Q^*	$1.2 - 2.6 \cdot 10^{-7}$	$0.6 - 1.2 \cdot 10^{-7}$	$3.2 - 7.1 \cdot 10^{-8}$
Ek	$9.8 \cdot 10^{-12}$	$4.3 \cdot 10^{-12}$	$2.4 \cdot 10^{-12}$
Ra	$3.5 - 5.4 \cdot 10^{20}$	$1.2 - 1.8 \cdot 10^{21}$	$2.9 - 4.4 \cdot 10^{21}$
Ro_{loc}	217-326	200-302	191-288
Ro_c	0.06-0.07	0.05-0.06	0.04-0.05
R_G^*	4.8-7.2	4.0-6.0	3.6-5.4
R_K^*	1.9-2.9	1.6-2.4	1.4-2.1

Results

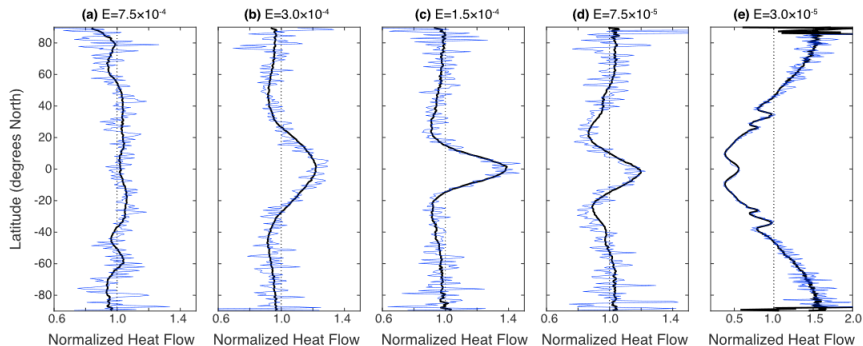


Results





freeslip vs noslip



freeslip vs noslip

