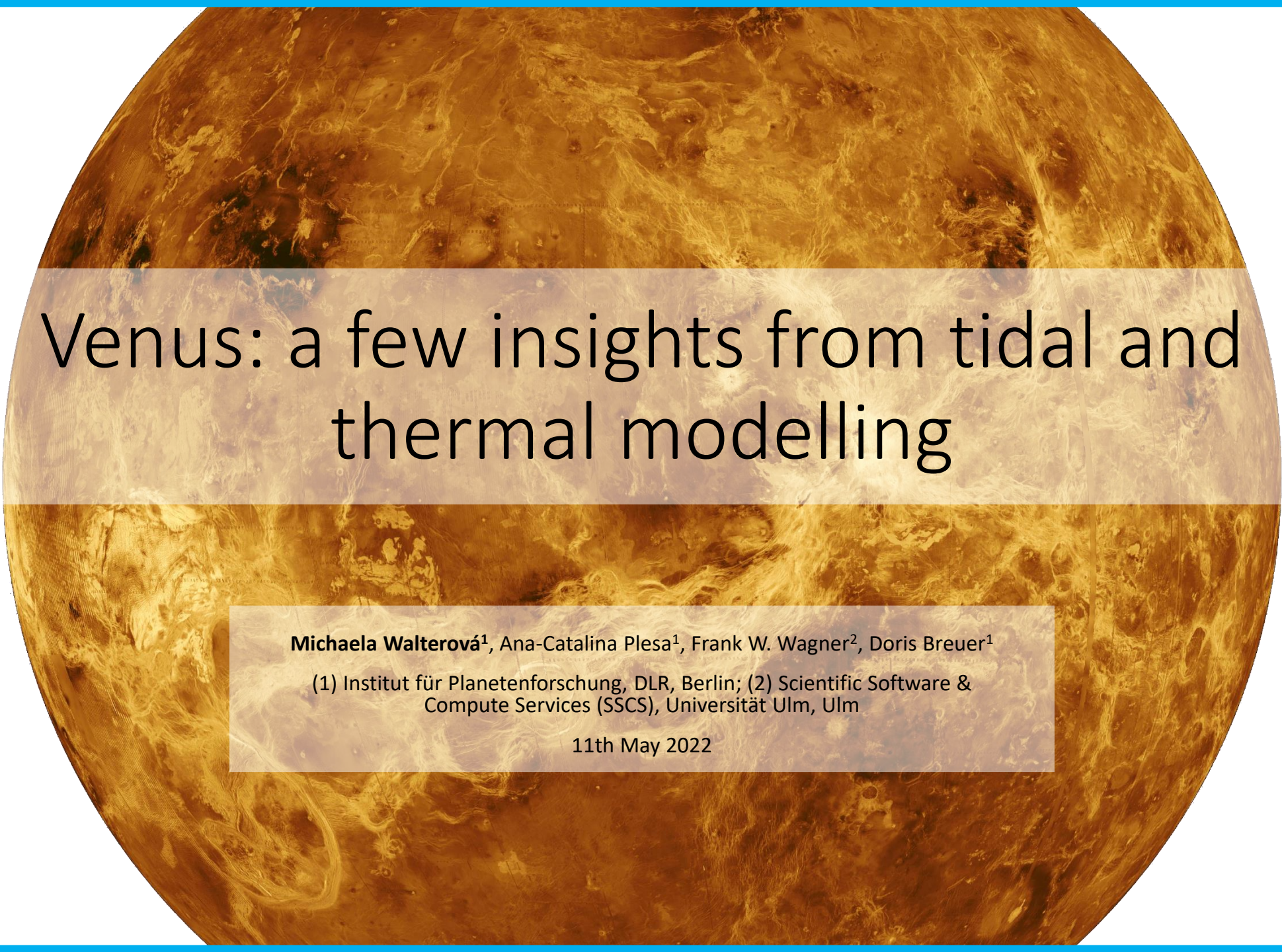




Venus: a few insights from tidal and thermal modelling

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Overview

Moment of inertia and the tidal response of planets depend on their interior structure

Interior structure depends on the thermal history

What can we say about **past and present Venus** from k_2 and MoIF measurements?

A combination of 1-d thermal evolution modelling of a stagnant-lid planet and the calculation of tidal parameters, assuming anelastic response

Additional motivation: **planned missions EnVision and VERITAS** promise considerable increase in the accuracy of the essential constraints

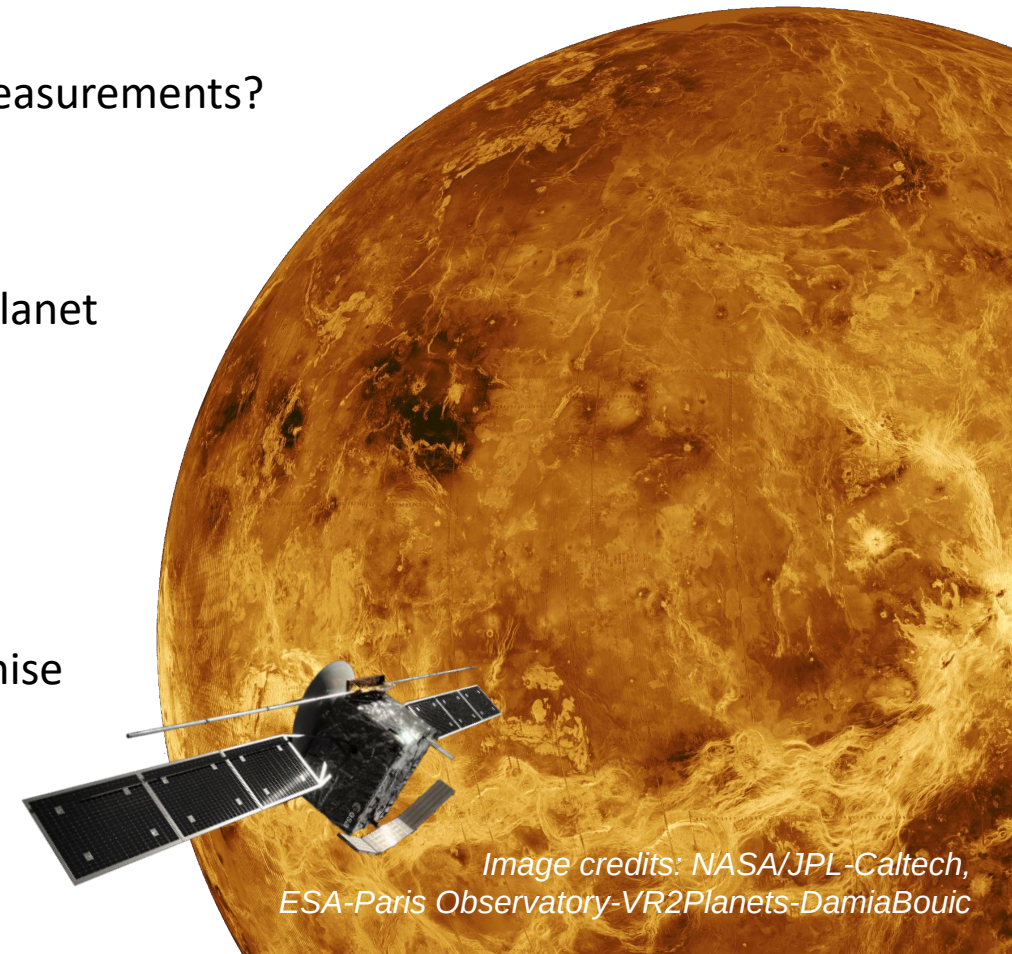


*Image credits: NASA/JPL-Caltech,
ESA-Paris Observatory-VR2Planets-DamiaBouic*

Tidal deformation

Direct consequence of the differential gravitational field in an extended body

Nonrotating planet: a prolate ellipsoid with the long axis pointing toward the perturber

Additional external potential due to deformation:

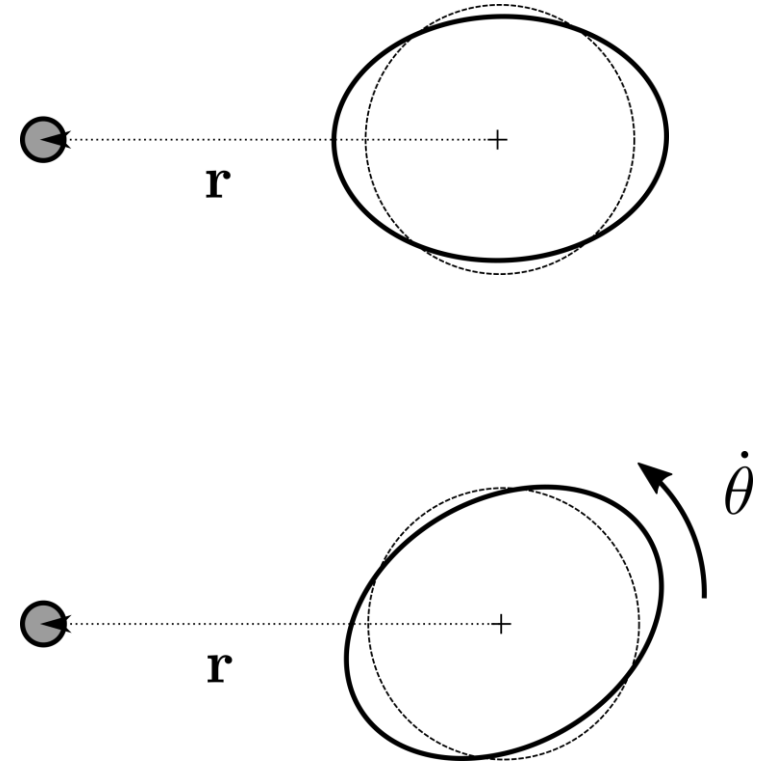
$$\mathcal{V}(r, \vartheta, \varphi) = \sum_{n=2}^{\infty} \mathcal{V}_n = \sum_{n=2}^{\infty} \left(\frac{r}{R}\right)^{n+1} k_n \mathcal{U}_n(R, \vartheta, \varphi)$$

additional potential

tidal potential

Rotating planet: tidal lagging due to anelasticity

Tidal phase lag ε_n or quality factor Q



Moment of inertia factor

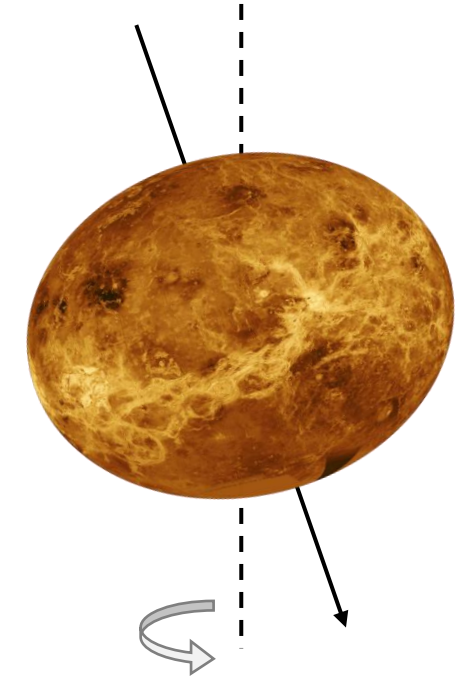
Measure of mass concentration: $\text{MoIF} = \frac{C}{MR^2}$, where $C = \int_V \rho r_{\perp}^2 dV$

$\text{MoIF} = 0.4$ corresponds to a homogeneous (constant-density) sphere

$\text{MoIF} > 0.4$ corresponds to a density decreasing with depth

For Venus, the polar MoIF is inferred from obliquity and precession of the spin axis (both can be measured from the Earth)

Period of precession: $\sim 29,000$ years (*Margot et al., 2021*)



$$\frac{d\psi}{dt} = \frac{3}{2} \left(\frac{n^2}{\dot{\theta}} \right) J_2 \left(\frac{MR^2}{C} \right) \cos \beta$$

Empirical values

k_2 was estimated from Doppler tracking of Magellan and Pioneer Venus Orbiter (PVO)

- an analysis by *Konopliv and Yoder (1996)* resulted in the following estimate:

$$k_2 = 0.295 \pm 0.066$$

MoIF is estimated from Earth-based radar observations – the newest estimate of *Margot et al. (2021)* is:

$$\text{MoIF} = 0.337 \pm 0.024$$

Tidal dissipation in Venus (i.e., the quality factor Q or the phase lag ϵ) has not yet been measured.

Frequency-dependent tidal response

$$k_2 = 0.295 \pm 0.066$$

Konopliv & Yoder (1996)

Reaction of planet-forming materials to tidal (or other) loading is viscoelastic or even anelastic, and thus frequency-dependent

For Venus, **anelasticity is important** and anelastic models (*Dumoulin et al., 2017*) indicate greater tidal deformation than elastic ones (*Yoder, 1995*)

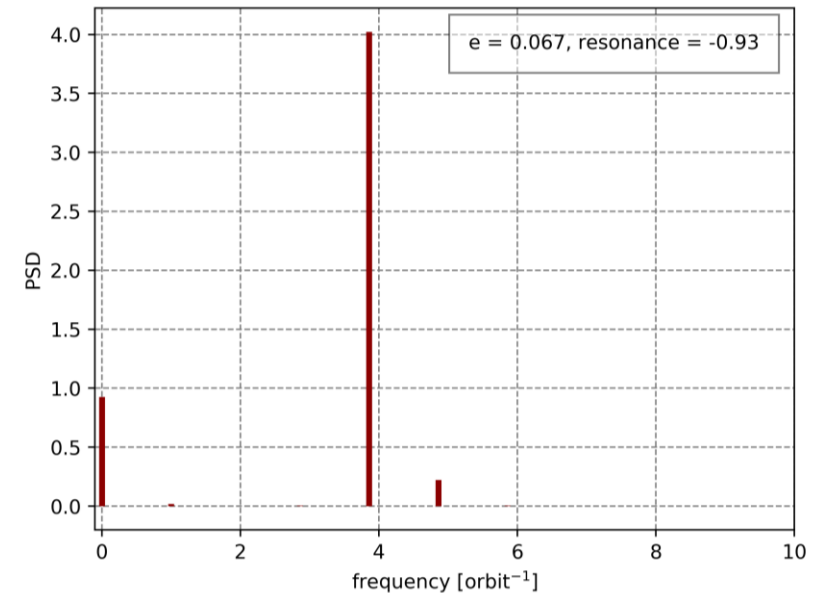
Yoder (1995): $k_2 \sim 0.17$ for a **solid core**

$k_2 \in (0.22, 0.31)$ for a **liquid core**

Dumoulin et al. (2017): $k_2 < 0.27$ would indicate a fully **solid core**

Leading term of the solar tide sensed by Venus is semi-diurnal, the **loading frequency** is thus twice the synodic rotation frequency

$$\omega = 2 \times \frac{2\pi}{T_{\text{syn}}} = 4\pi \left(\frac{1}{T_{\text{orb}}} - \frac{1}{T_{\text{rot}}} \right) \approx 1.245 \times 10^{-6} \text{ rad/s}$$



Periodogram of (solar) tidal potential sensed at the Cytherean equator

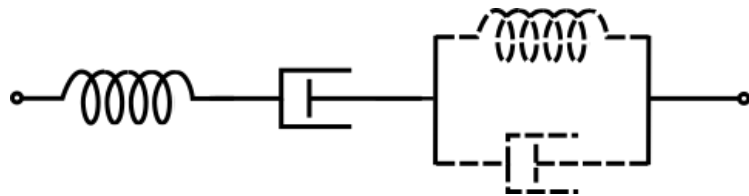
Why Andrade rheology?

Laboratory data: “high-temperature background”, attenuation in polycrystalline solids at high temperatures well described by Andrade or extended Burgers rheologies

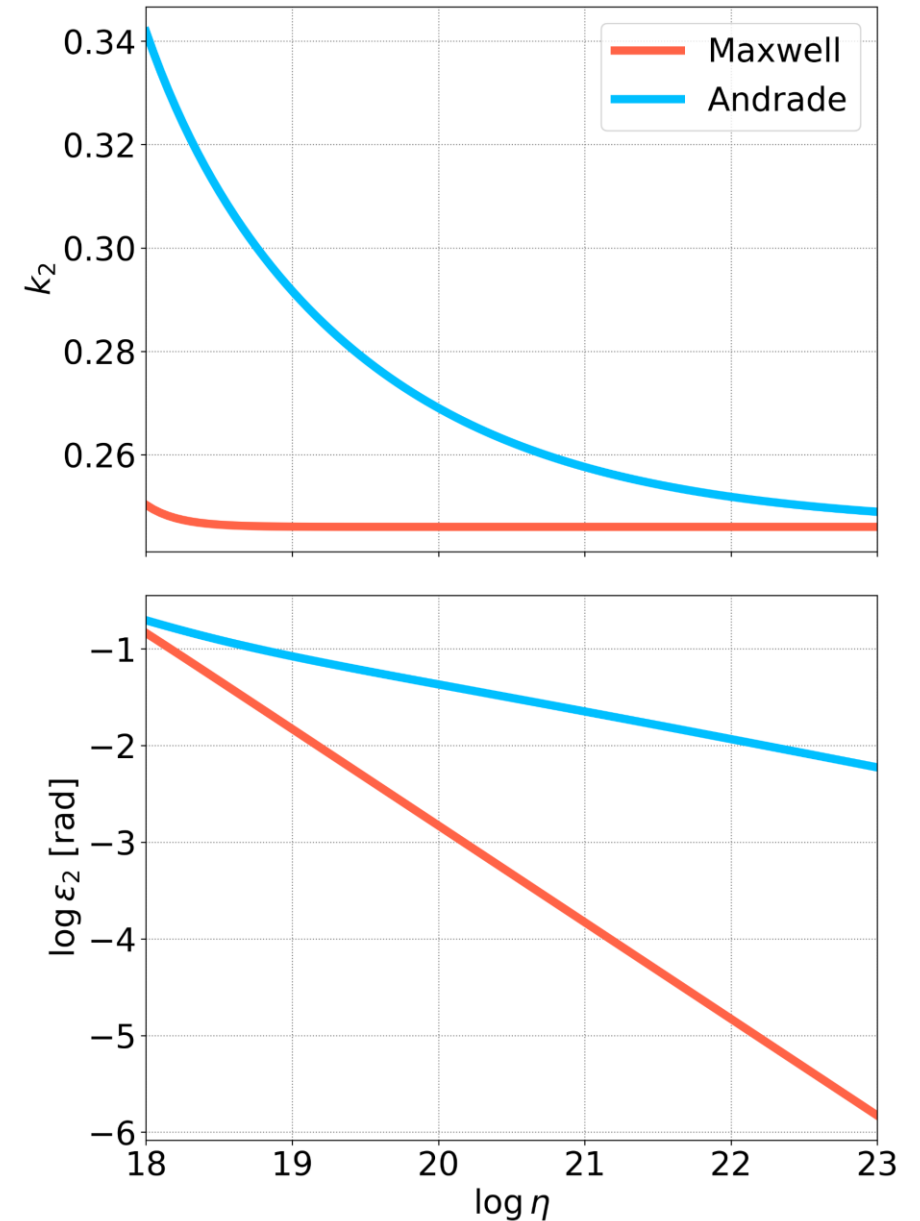
Seismology: “absorption-band model”, attenuation (Q^{-1}) over a broad range of frequencies is *almost* constant

Andrade (1910): deformation of metal wires

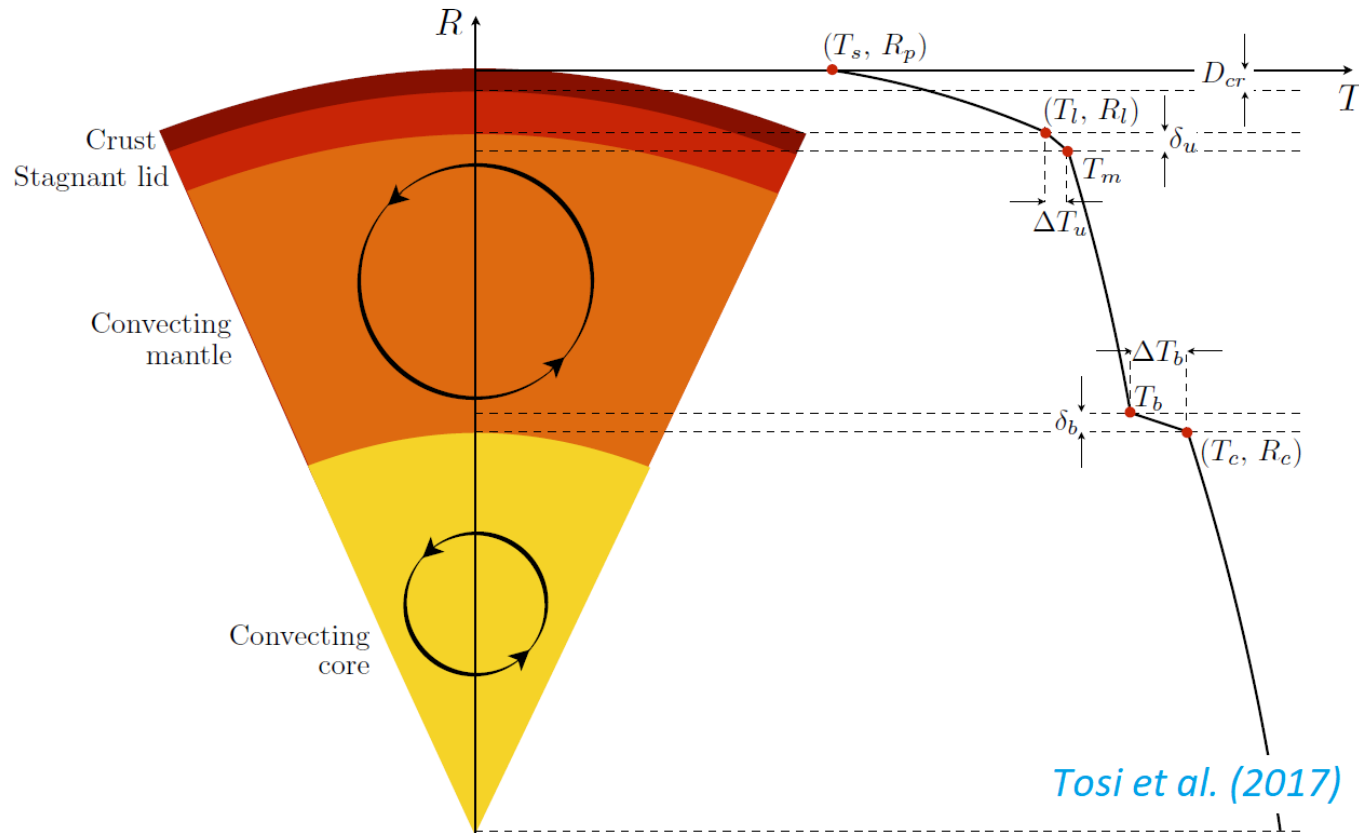
1. elasticity, 2. transient creep, 3. steady-state creep



$$\bar{J}(\omega) = \frac{1}{\mu} - \frac{i}{\eta\omega} + \frac{\mu^{\alpha-1}}{(i\zeta\eta\omega)^{\alpha}} \Gamma(1 + \alpha)$$



Thermal evolution model



Python model of Philipp Baumeister, based on *Morschhauser et al. (2011)* and *Tosi et al. (2017)* – 1-d parameterized convection

Input: outer radii of the main layers, initial T , thermal and material properties

Output: evolution of T , D_{cr} , outgassing and many other quantities

Updates: core solidification (*Stevenson et al., 1983*), depth-dependent V^* , calculation of MoIF and tidal parameters

Thermal evolution model

$$\eta(P, T) = \eta_0 \exp \left\{ \frac{E^* + PV^*}{RT} \right\}$$

1. parameters of the Arrhenius law, which determine the viscosity profile (η_0 , E^* , V^*)

Parameter	Min	Max	Unit
Reference viscosity η_0	10^{18}	10^{23}	Pa s
Activation energy E^*	150	550	kJ/mol
Activation volume V^*	0	10	cm ³ /mol
Initial mantle temperature $T_{m,0}$	1,600	1,900	K
Initial CMB temperature drop ΔT_{cmb}	0	700	K
Crustal thickness D_{cr}	10	50	km
Enrichment factor	5	20	—
Core radius R_c	2,600	3,200	km
Core density ρ_c	9,000	15,000	kg/m ³
Thermal conductivity of the mantle k_m	3	5	W m ⁻¹ K ⁻¹
Thermal conductivity of the crust k_{cr}	1.5	3	W m ⁻¹ K ⁻¹
Specific heat of the core c_c	600	800	J K ⁻¹ kg ⁻¹
Andrade parameter α	0.1	0.4	—
Andrade parameter ζ	10^{-1}	10^1	—
Initial sulfur content in the core	4	16	wt%

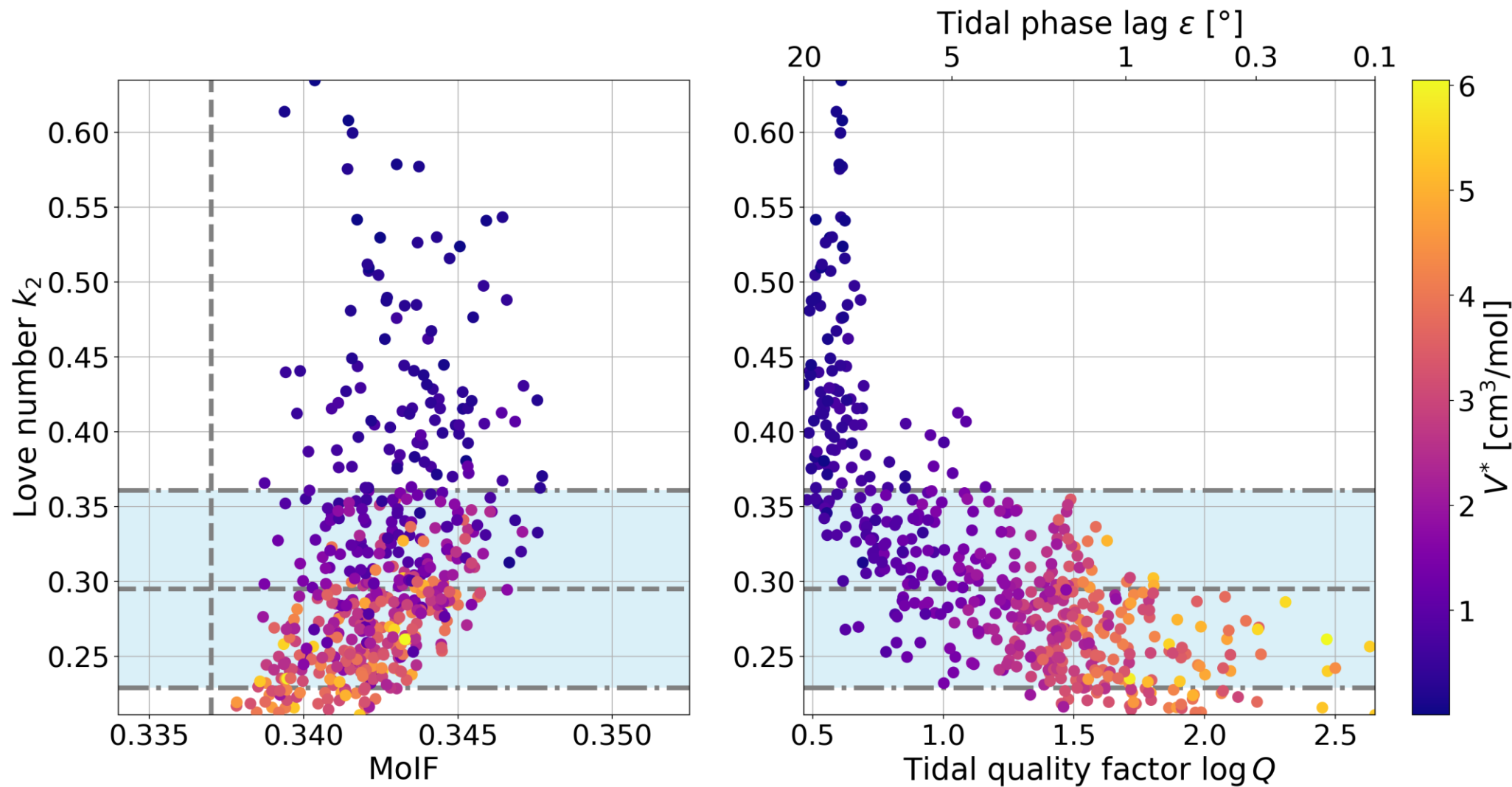
V4	
<i>Mantle + Crust</i>	
SiO ₂	40.4
TiO ₂	0.24
Al ₂ O ₃	3.4
Cr ₂ O ₃	0.3
MgO	33.3
FeO	18.7
MnO	0.2
CaO	3.4
Na ₂ O	0.15
H ₂ O	-
K(ppm)	185
U(ppm)	0.019
Th(ppm)	0.066

“Fe-rich
end-member”

(Basaltic Volcanism Study Project, 1981)

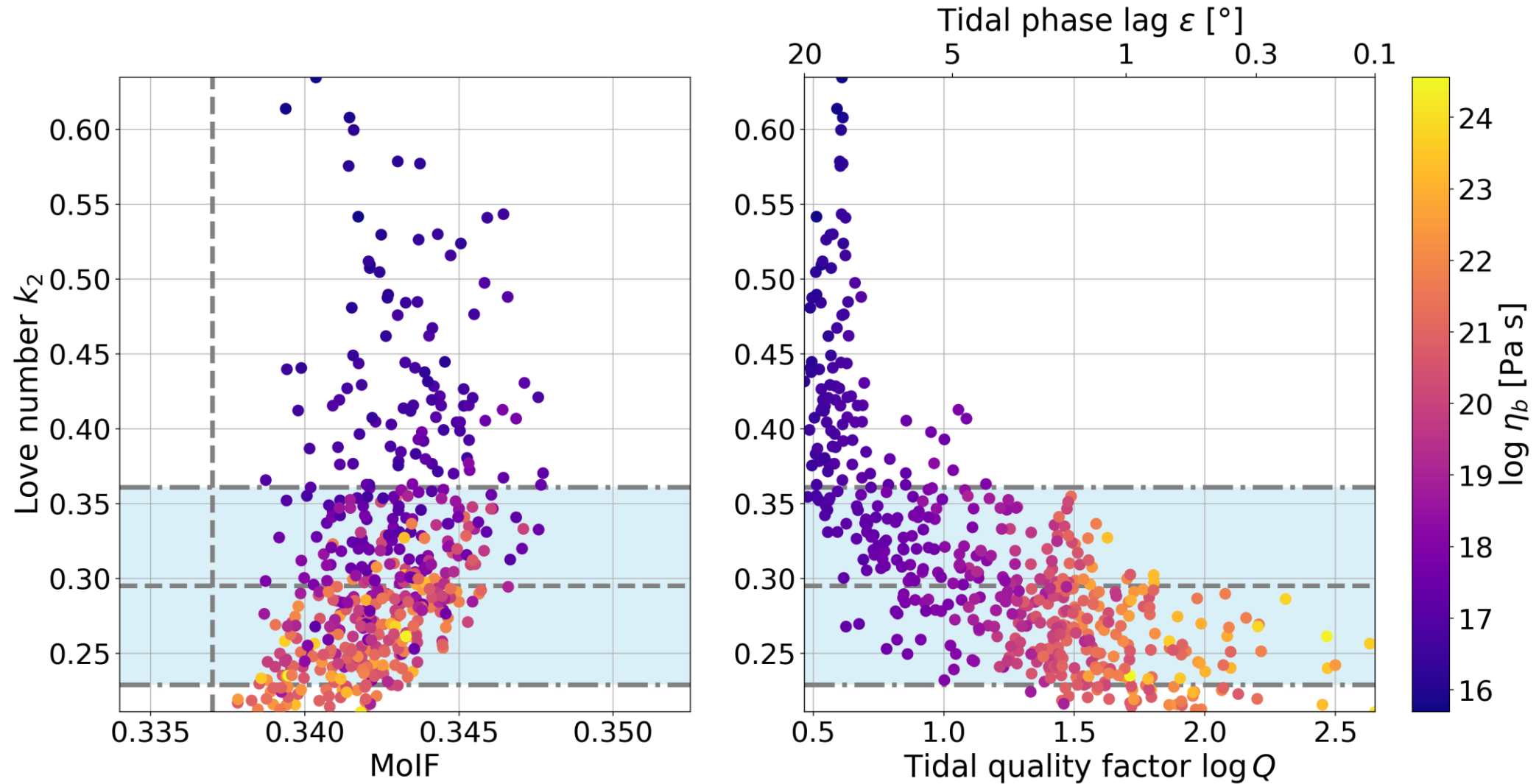
Arrhenius law - parameters: V^*

$$\eta_0 = 10^{21} \text{ Pa s}$$
$$E^* = 300 \text{ kJ/mol}$$



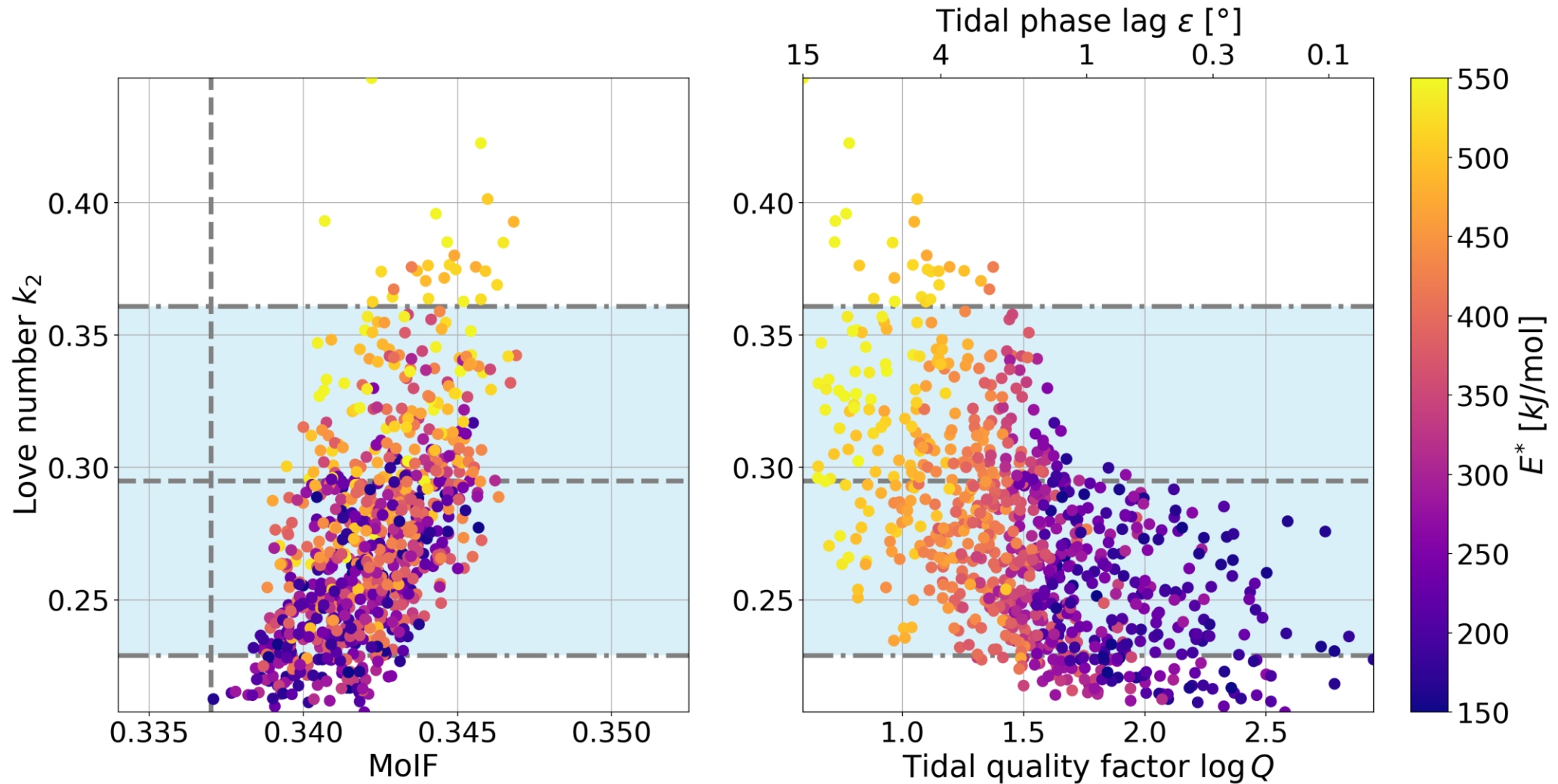
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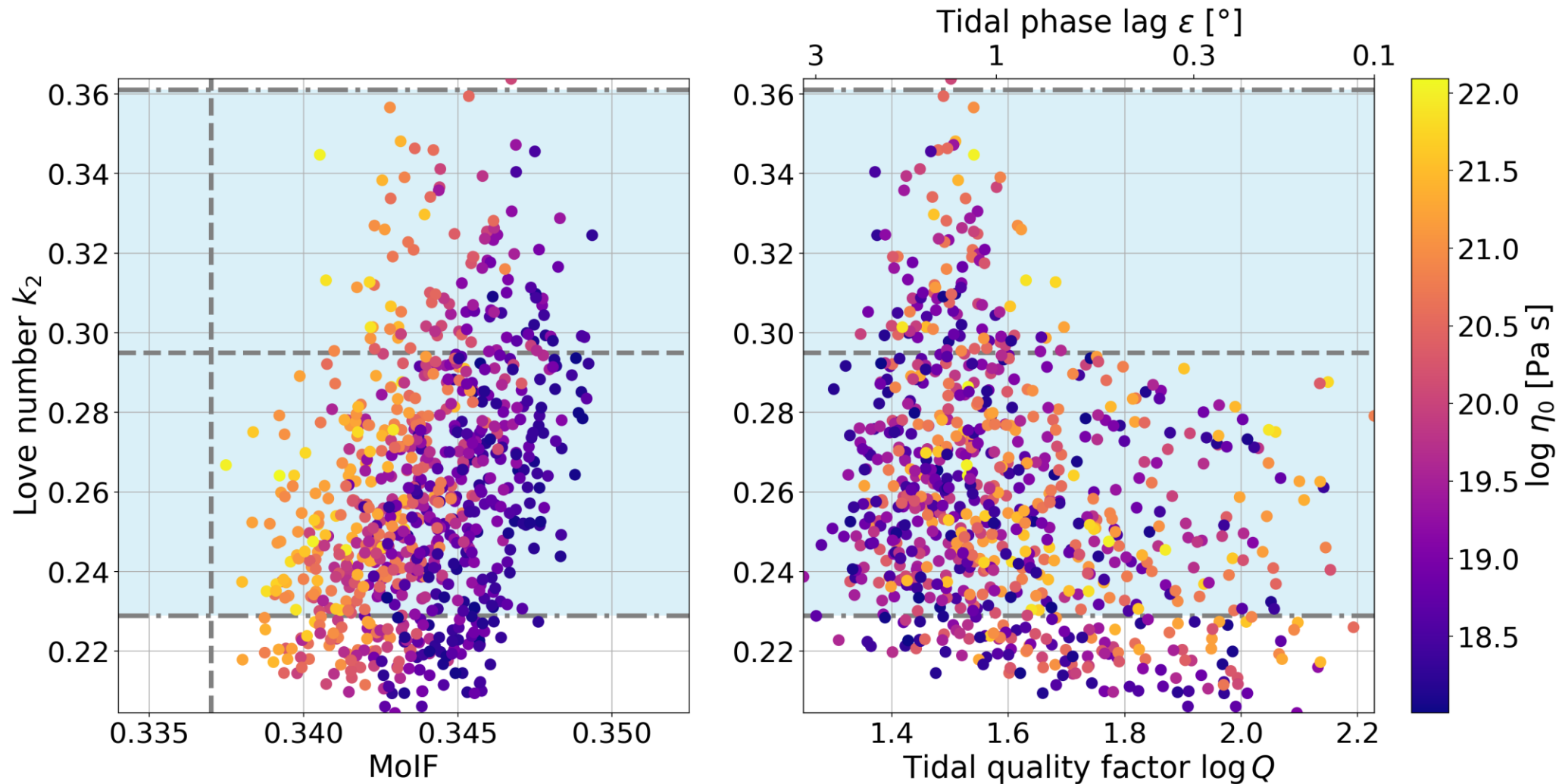
Arrhenius law - parameters: E^*

$$V^* = 3.5 \text{ cm}^3/\text{mol}$$
$$\eta_0 = 10^{21} \text{ Pa s}$$



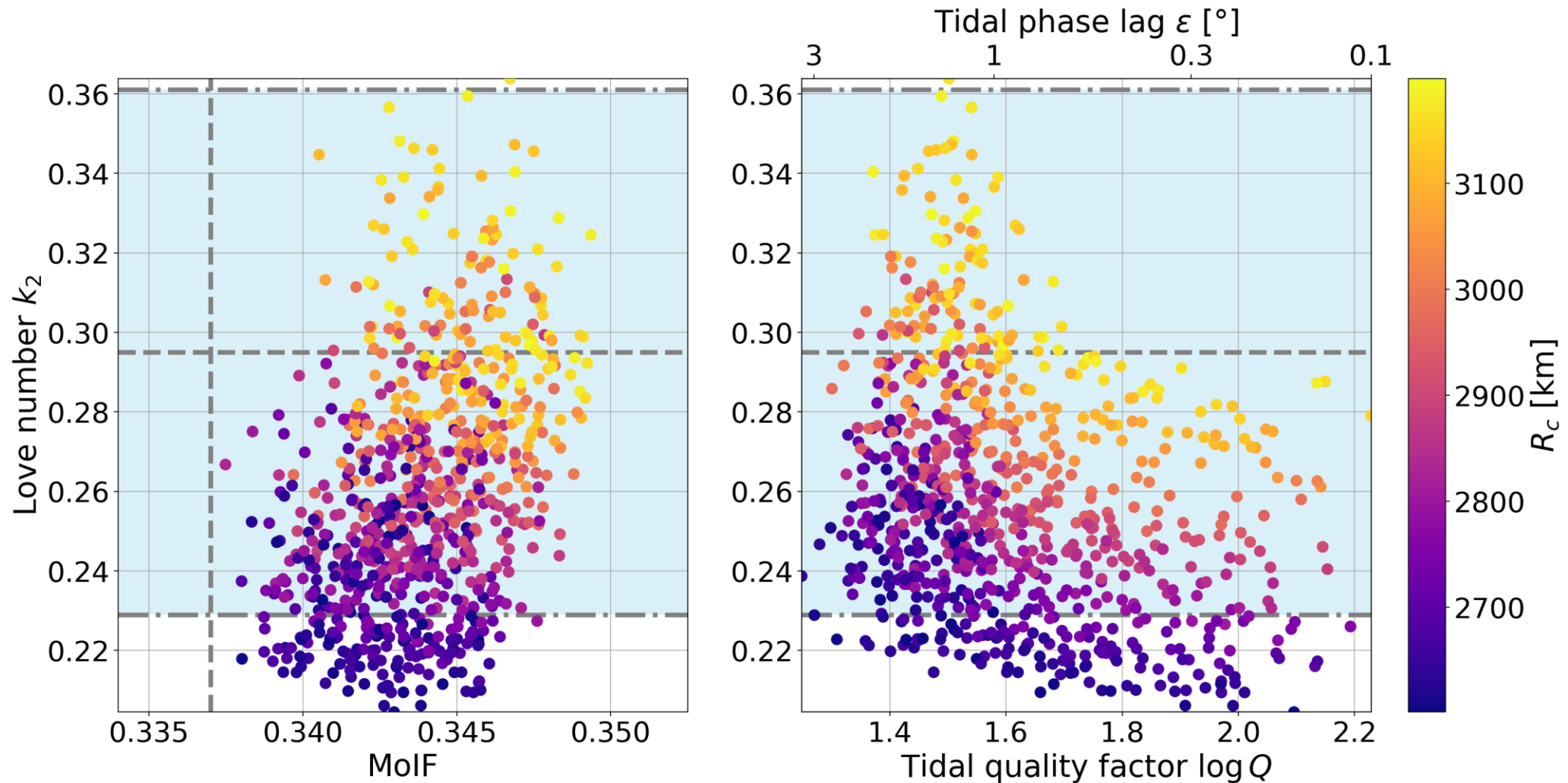
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Arrhenius law - parameters: η_0

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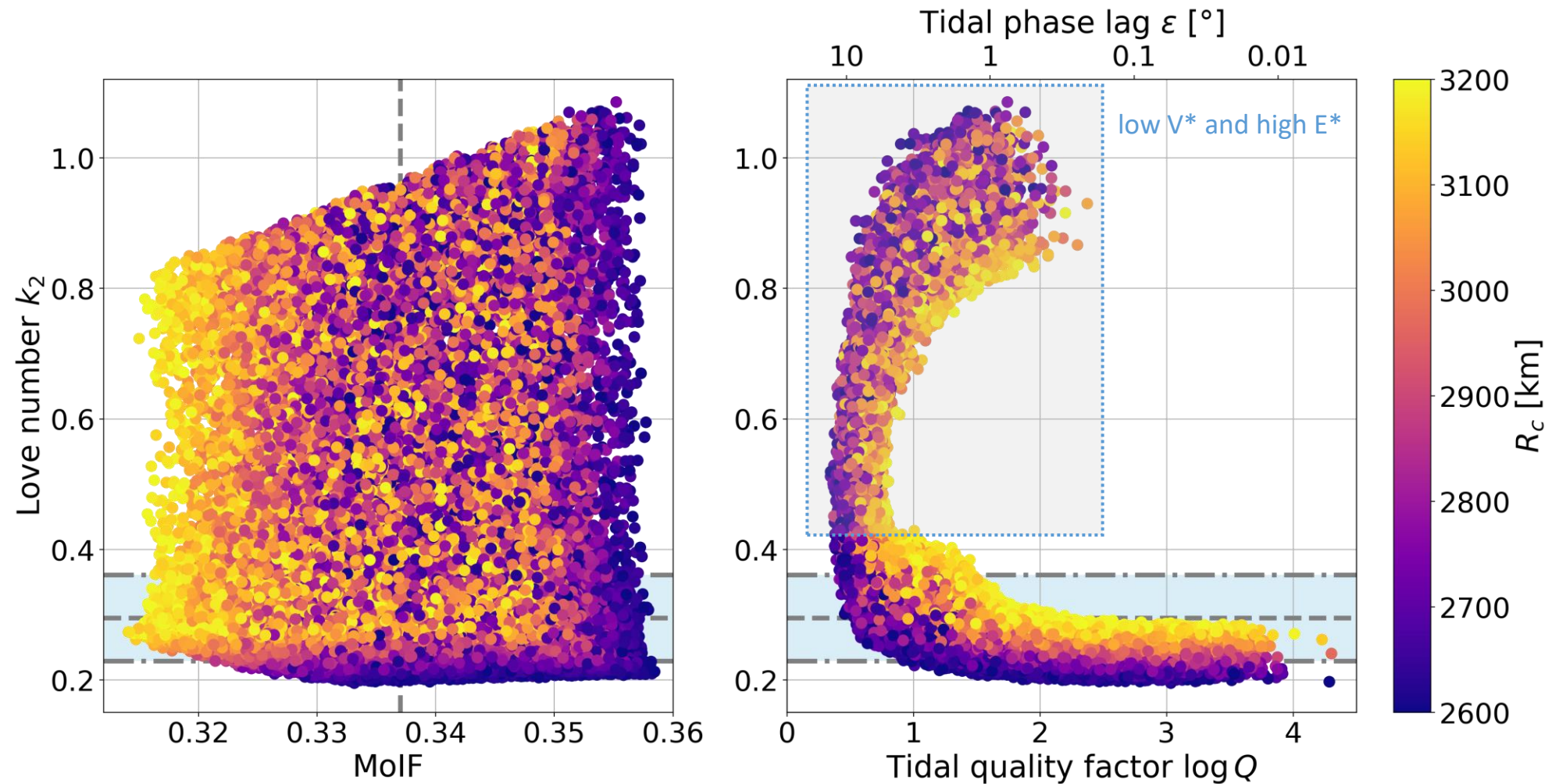
1. parameters of the Arrhenius law, which determine the viscosity profile (η_0 , E^* , V^*)

2. results of the full parameter study, with all constraints available

100,000 samples

All results: too many samples

→ let's apply several constraints



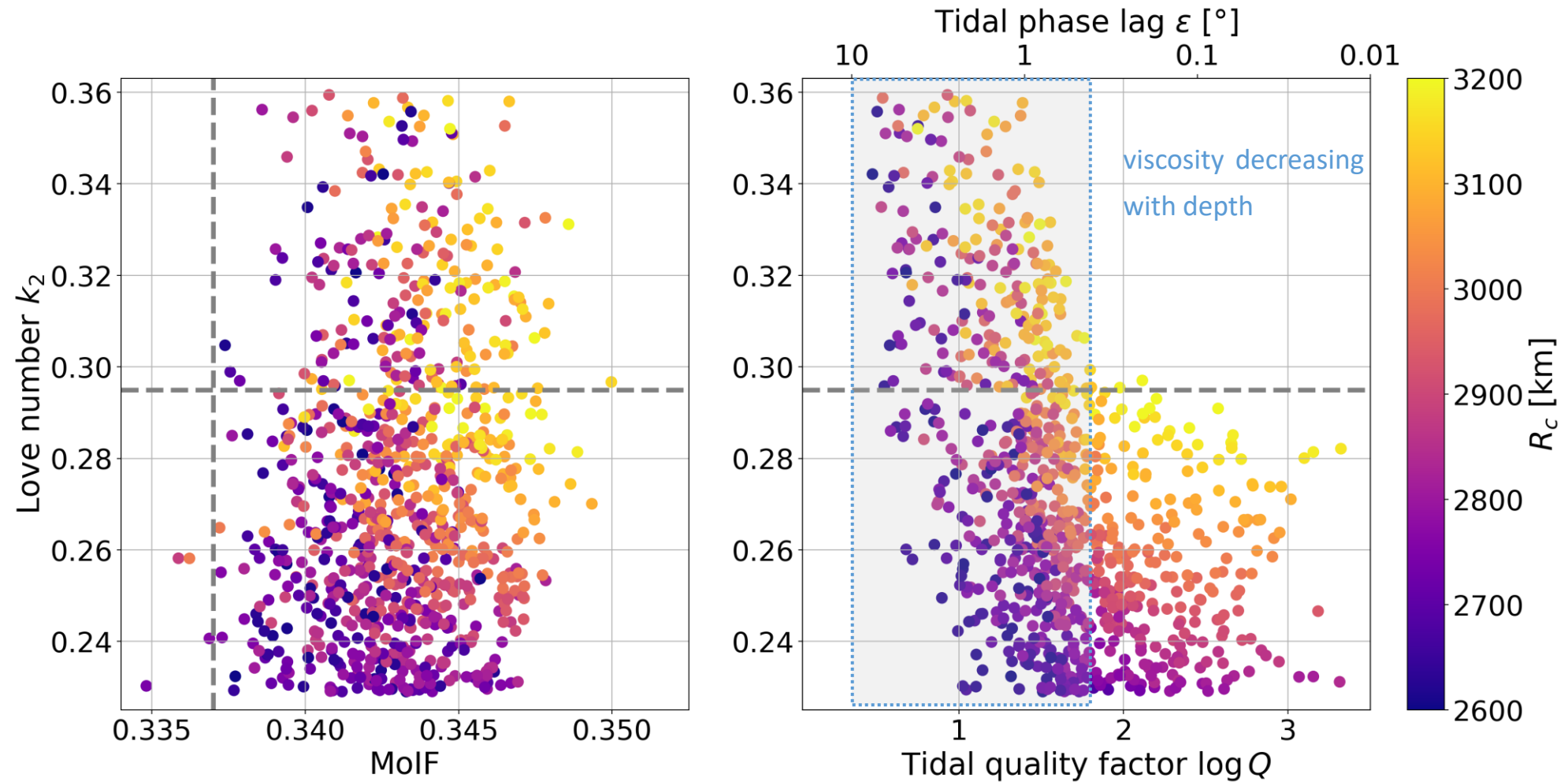
Observational (and other) constraints

Convection vs. conduction:	convecting mantle after 4.5 Gyr		50,512 samples
Surface gravity:	$g_s = 8.87 \text{ m s}^{-2}$		6,724 samples
Moment of inertia factor:	$\text{MoIF} = 0.337 \pm 0.024$	<i>Margot et al. (2021)</i>	6,724 samples
Tidal Love number:	$k_2 = 0.295 \pm 0.066$	<i>Konopliv & Yoder (1996)</i>	3,927 samples
CMB heat flux:	lower than q_{adiab}	<i>Smith et al. (1963)</i>	4,958 samples
Elastic thickness:	$D_{\text{el}} < 20 \text{ km}$	<i>Anderson and Smrekar (2006), O'Rourke and Smrekar (2018)</i>	4,110 samples
Lithospheric thickness:	greater than D_{cr}		3,680 samples

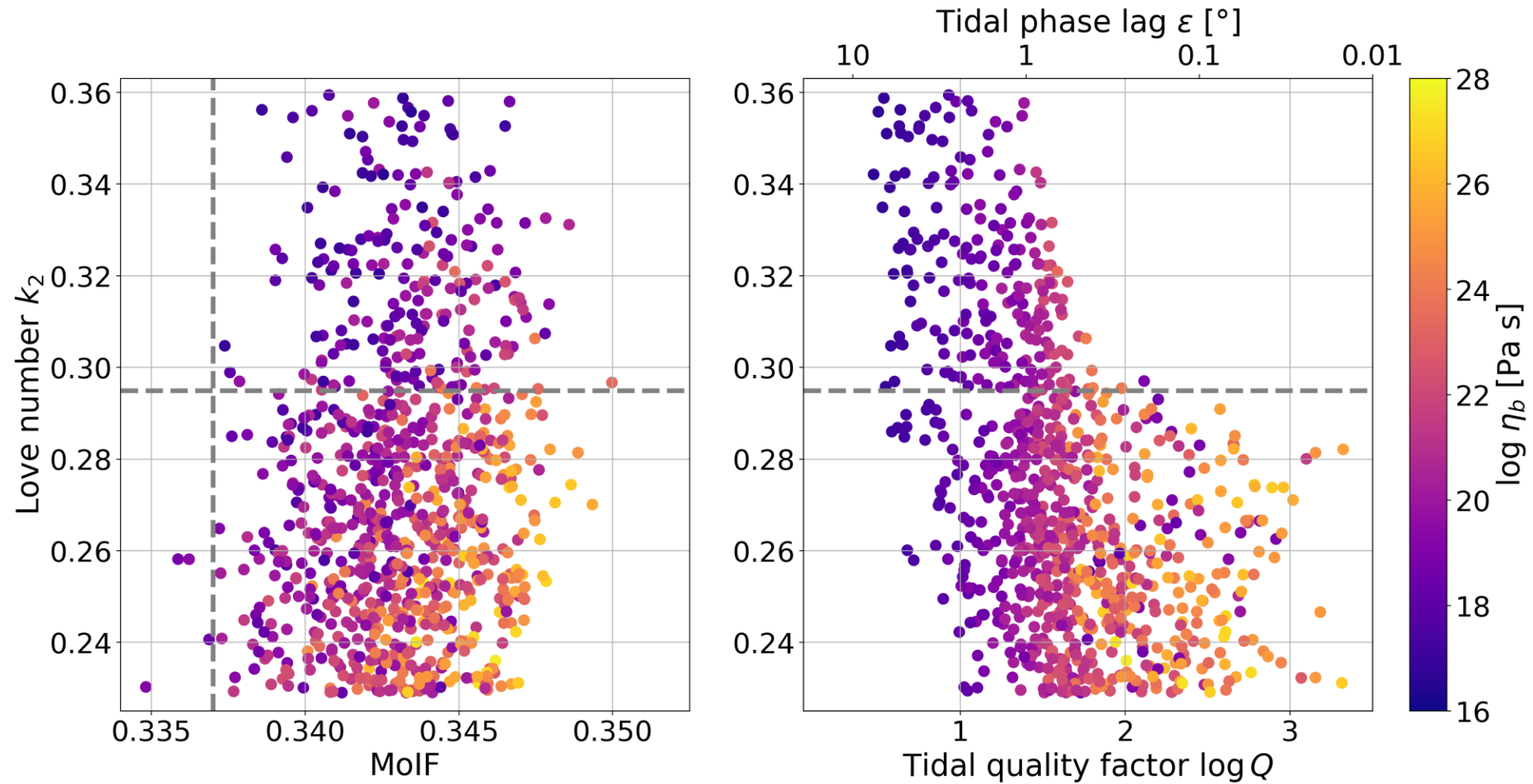
To include more samples, an arbitrary 1% errorbar was added to the surface gravity

891 samples in total

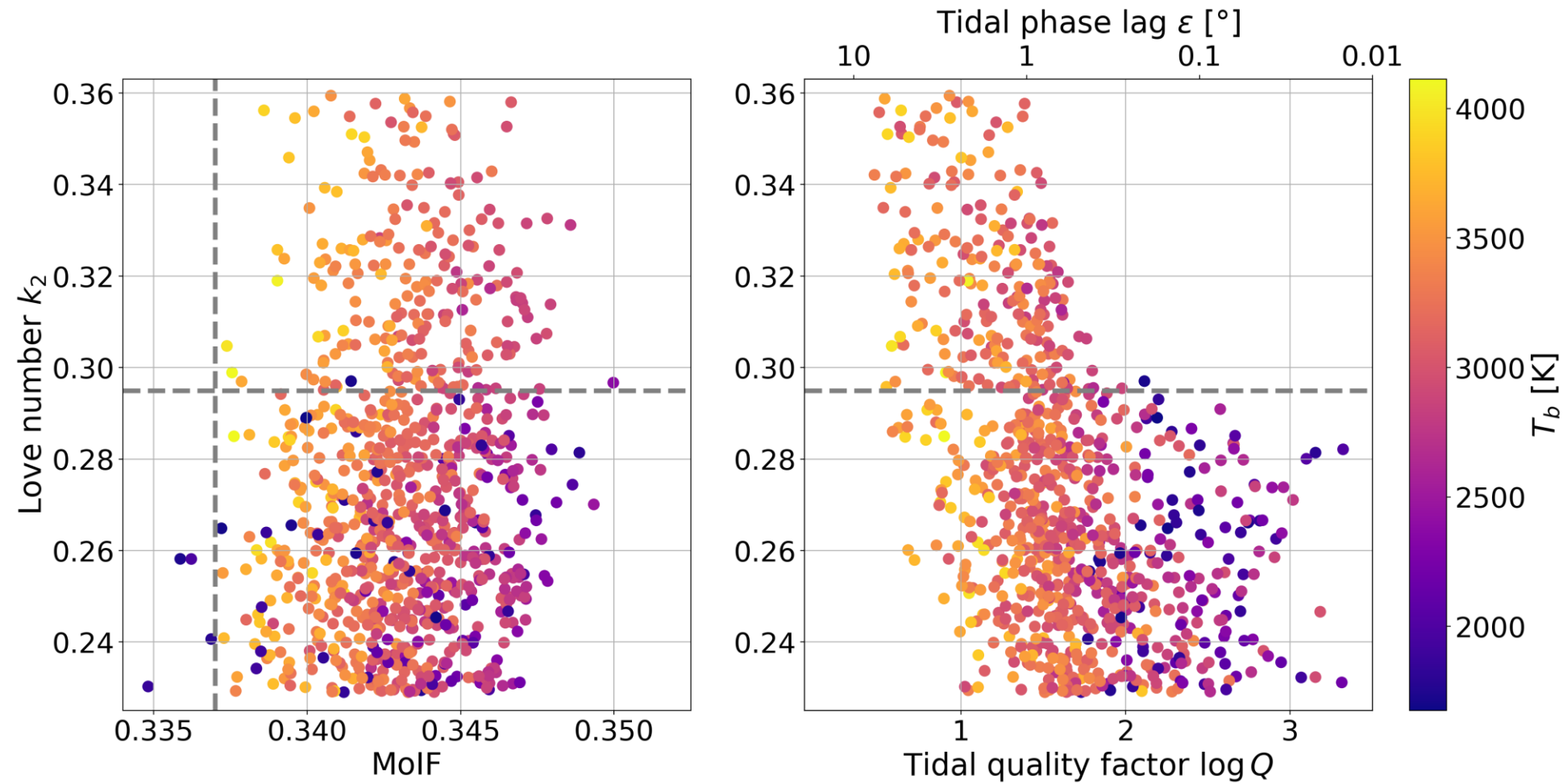
Constrained results



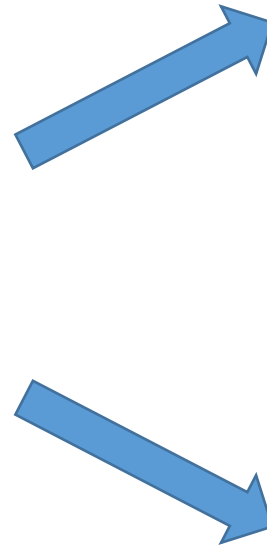
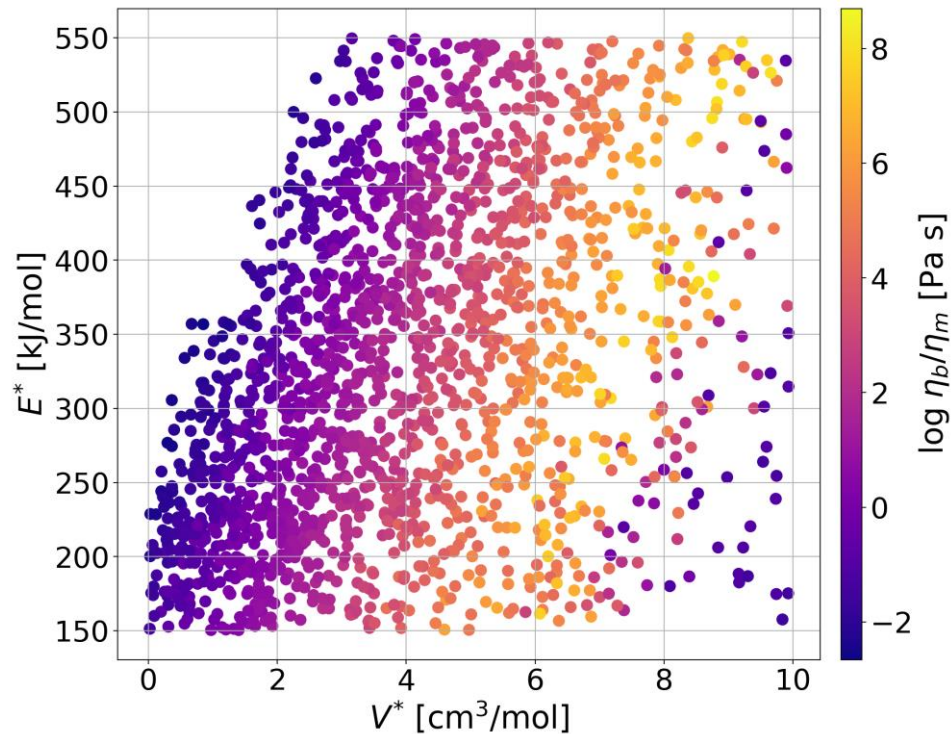
Constrained results



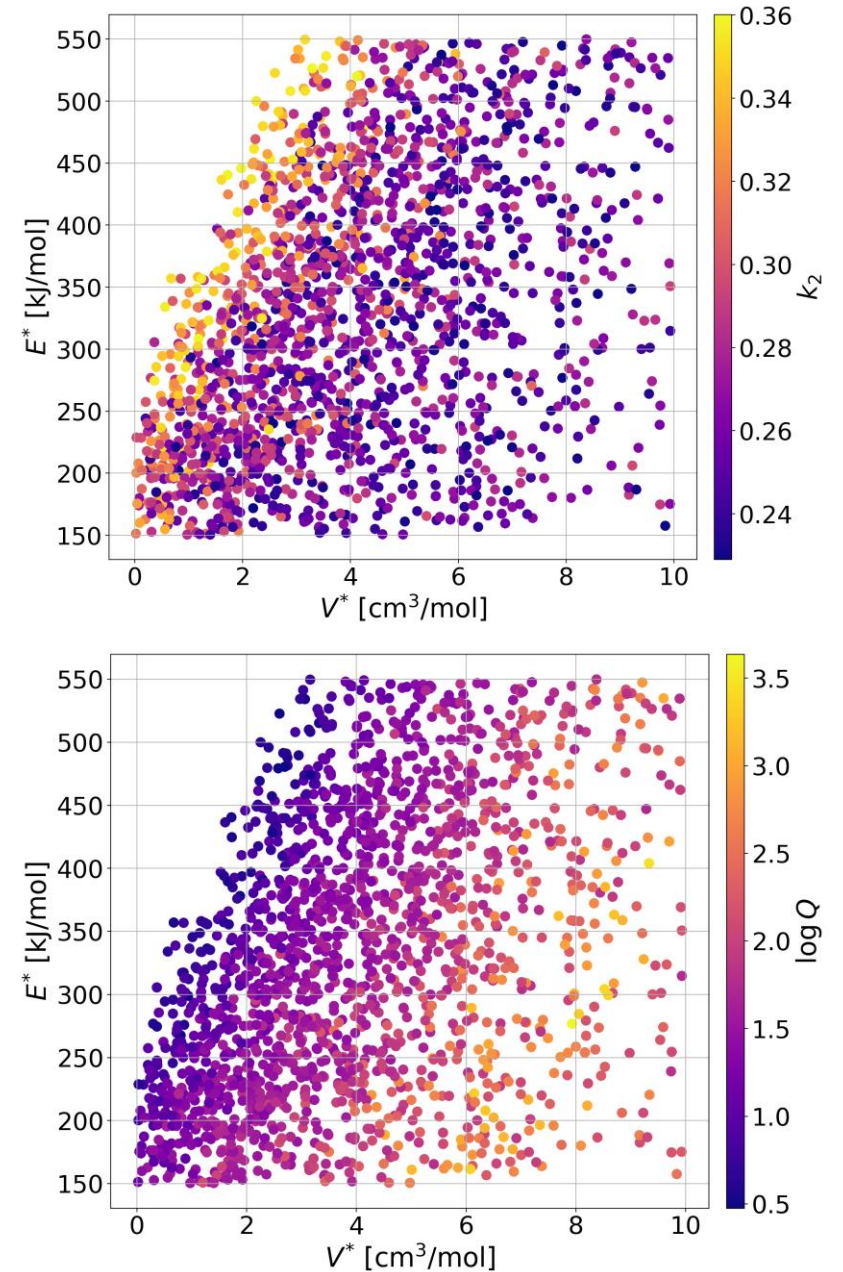
Constrained results



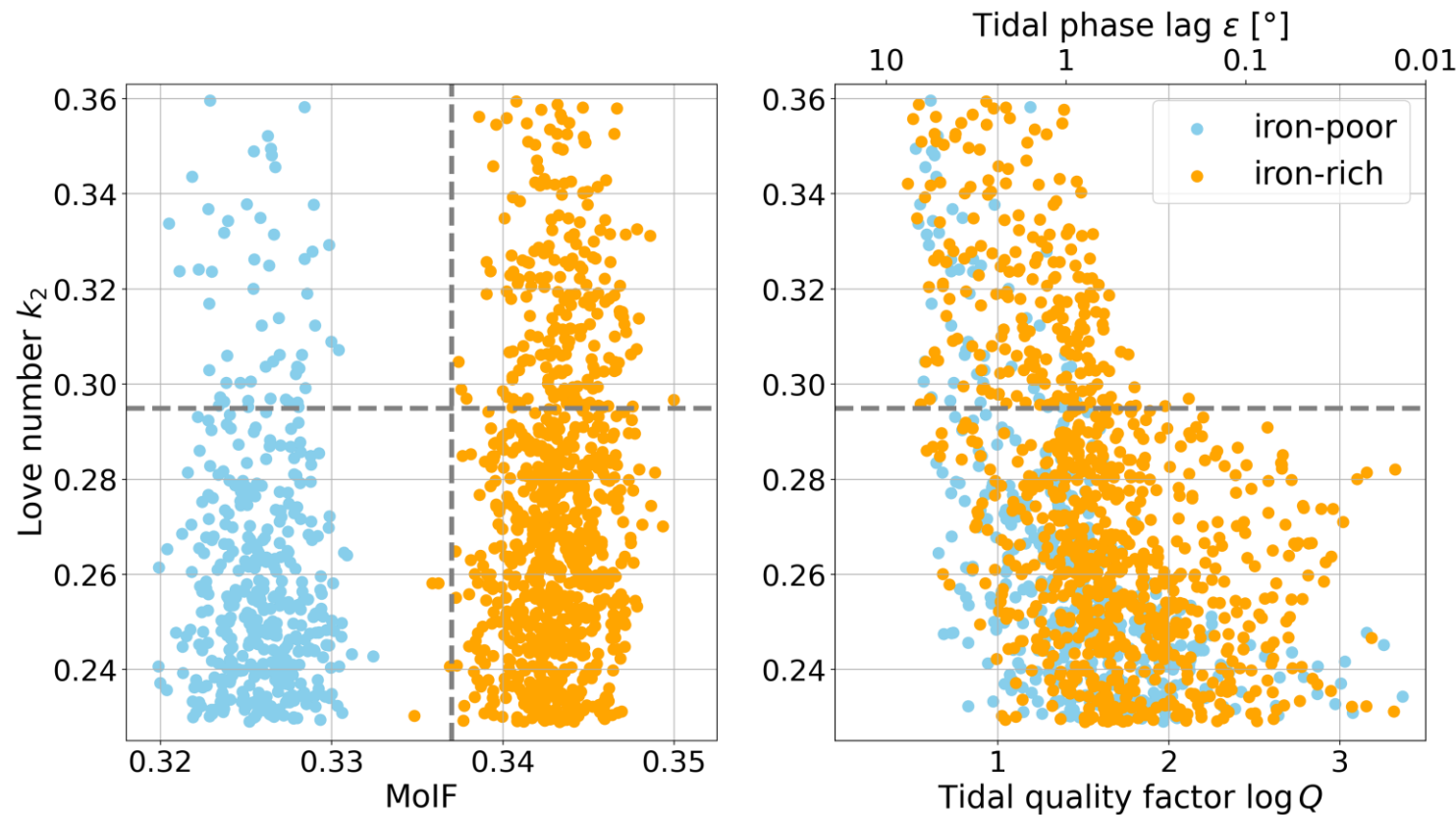
Constrained results



Viscosity profile has a profound effect on the resulting tidal parameters: high k_2 (> 0.3) combined with low Q (< 20) might indicate viscosity decreasing with depth...



Iron-rich vs. iron-poor mantle mineralogy



	V1	V4
<i>Mantle + Crust</i>		
SiO ₂	52.9	40.4
TiO ₂	0.20	0.24
Al ₂ O ₃	3.8	3.4
Cr ₂ O ₃	-	0.3
MgO	37.6	33.3
FeO	0.24	18.7
MnO	-	0.2
CaO	3.6	3.4
Na ₂ O	1.6	0.15
H ₂ O	0	-
K(ppm)	1442	185
U(ppm)	0.021	0.019
Th(ppm)	0.073	0.066

(Basaltic Volcanism Study Project, 1981)

Both compositions are consistent with the MoIF constraint; Fe-rich model is closer to the mean value

Conclusions and outlook

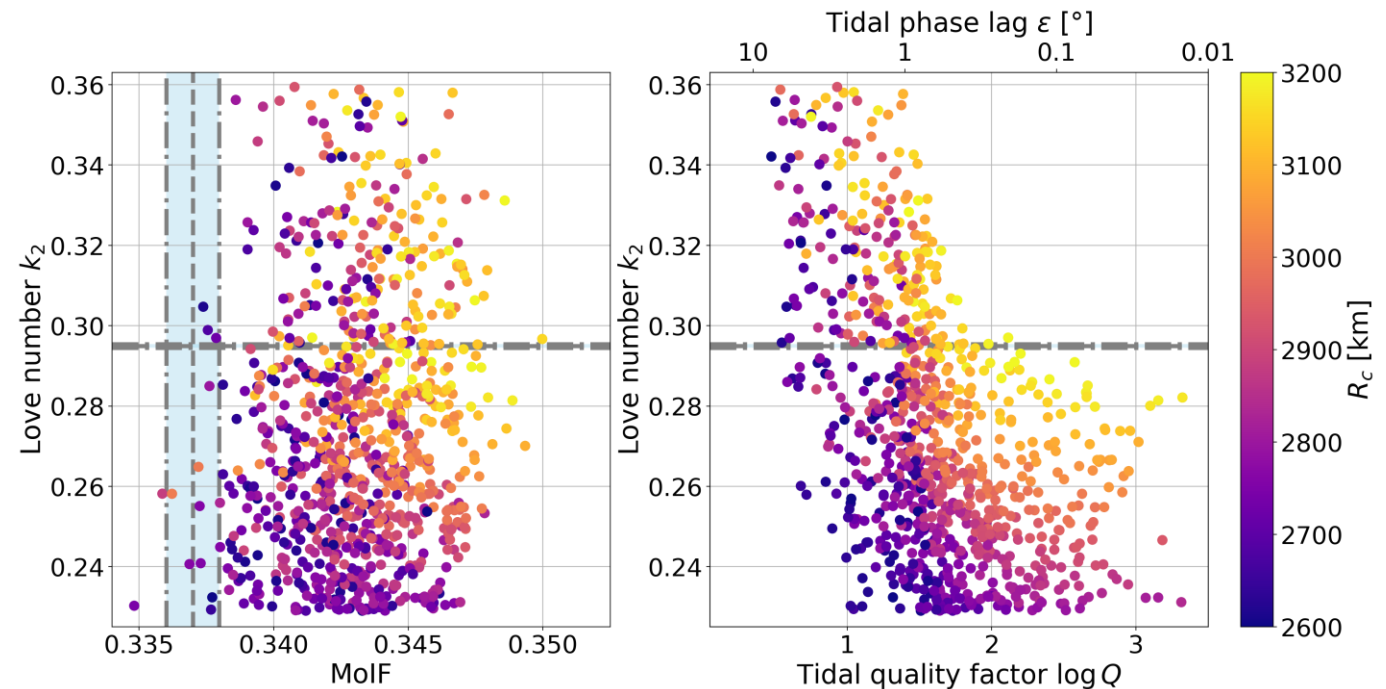
Tidal parameters are very sensitive to the viscosity structure

Tidal quality factor Q (or the phase lag ε) probes the lower-mantle viscosity and the corresponding temperature

Current estimate of the mean value of MoIF (*Margot et al., 2021*) might indicate iron-rich mantle

Upcoming missions VERITAS and EnVision are expected to greatly reduce the uncertainties (*Cascioli et al., 2021; Rosenblatt et al., 2021*)

Phase lag: accuracy $\sim 0.05^\circ$ or 0.1°
 k_2 and MoIF: $\sim 10^{-3} - 10^{-4}$



Conclusions and outlook

In the present study, we only solved the forward problem and we did not perform a probability analysis of the (constrained) samples

Nevertheless, some interior models appear to be more frequent than others – a possible direction of our next steps

