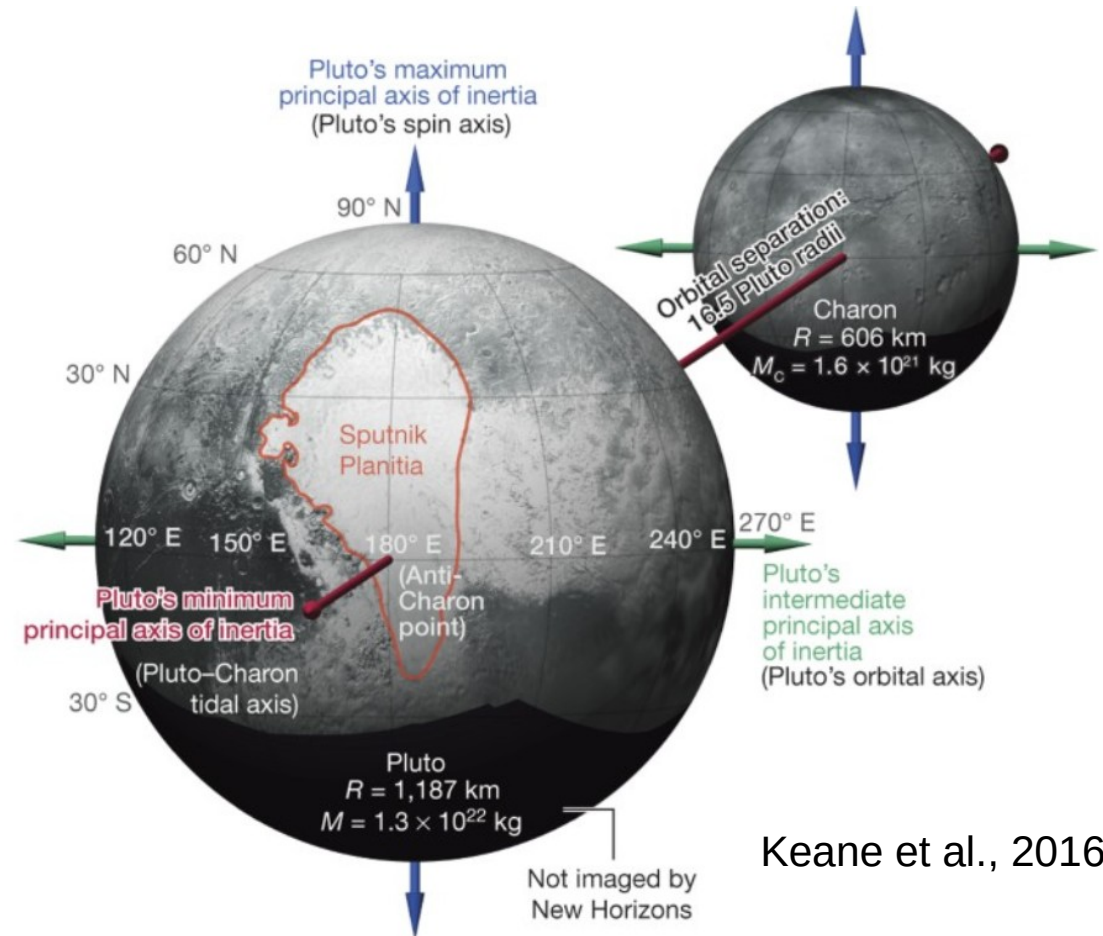


Dynamic reorientation of tidally locked bodies

Vojtěch Patočka

Application to Pluto

Martin Kihoulou
Ondřej Čadek

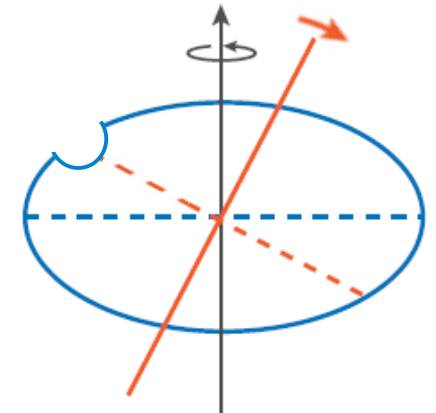
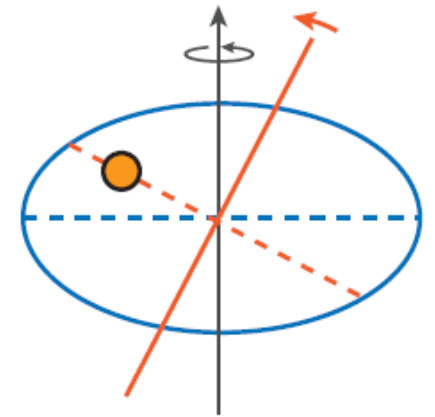


Keane et al., 2016

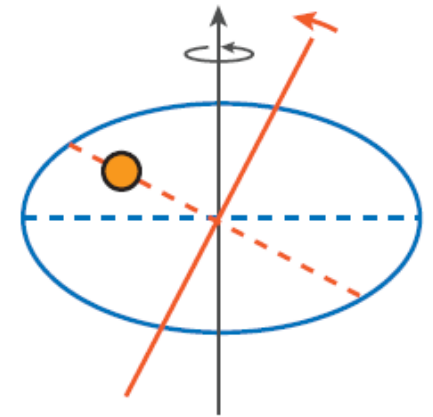
Contents:

- Mechanism (basics, LE, tides, wobble)
- $\omega||m$ MIA method
- Dynamics: straight or curved paths?
- Pluto – Sputnik Planitia

Mechanism: basic concept



Mechanism: basic concept



$$E_{\text{rot}}^0 = \frac{1}{2} \boldsymbol{\omega}_0 \cdot \mathbf{I}_0 \cdot \boldsymbol{\omega}_0 = \frac{1}{2} C_0 \omega_0^2$$

$$\mathbf{H} = \mathbf{I} \cdot \boldsymbol{\omega} = C_0 \omega_0 = \text{const}$$

$$\omega_{\text{new}} = \omega_0 \frac{C_0}{C_{\text{new}}}$$

$$E_{\text{rot}}^{\text{new}} = \frac{1}{2} C_{\text{new}} \omega_{\text{new}}^2 = E_{\text{rot}}^0 \frac{C_0}{C_{\text{new}}} < E_{\text{rot}}^0$$

Key vocabulary: the LE, inertia, bulge

Liouville equation (LE): $M = \frac{d\mathbf{H}}{dt}$

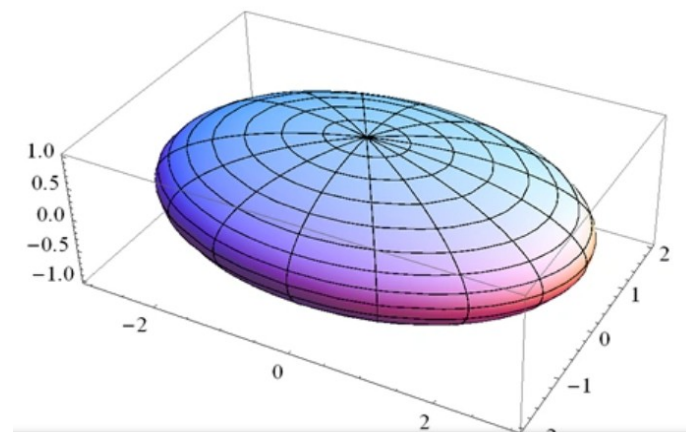
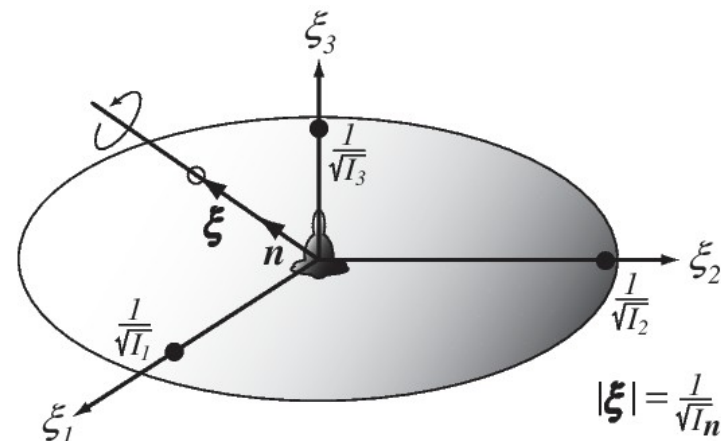
Key vocabulary: the LE, inertia, bulge

Liouville equation (LE):
$$\mathbf{M} = \frac{d\mathbf{H}}{dt} + \boldsymbol{\omega} \times \mathbf{H} = \frac{d(\mathbf{I} \cdot \boldsymbol{\omega} + \mathbf{h})}{dt} + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega} + \mathbf{h})$$

Tisserand frame:
$$\mathbf{h} = \int_{v(t)} \mathbf{r} \times (\rho \mathbf{v}) dv = \mathbf{0}$$

Inertia tensor:
$$\mathbf{I} = \int_{v(t)} ((\mathbf{r} \cdot \mathbf{r}) \mathbb{1} - \mathbf{r} \otimes \mathbf{r}) \rho dv,$$

$$\frac{d}{dt}(\mathbf{I} \cdot \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) = \mathbf{M}$$



Key vocabulary: potentials, torques

centrifugal: $\Psi = \frac{1}{2} ((\boldsymbol{\omega} \cdot \mathbf{r})^2 - \omega^2 r^2)$

tidal: $\Theta = \frac{1}{6} o^2 r^2 - \frac{1}{2} (\mathbf{o} \cdot \mathbf{r})^2$ $o = \frac{3 G M_h}{a^3}$

locked body: $\omega = \sqrt{\frac{G(M_h + M)}{a^3}} = o \sqrt{\frac{M_h + M}{3 M_h}}$

Key vocabulary: potentials, torques

centrifugal: $\Psi = \frac{1}{2} ((\boldsymbol{\omega} \cdot \mathbf{r})^2 - \omega^2 r^2)$

tidal: $\Theta = \frac{1}{6} o^2 r^2 - \frac{1}{2} (\mathbf{o} \cdot \mathbf{r})^2 \quad o = \frac{3 G M_h}{a^3}$

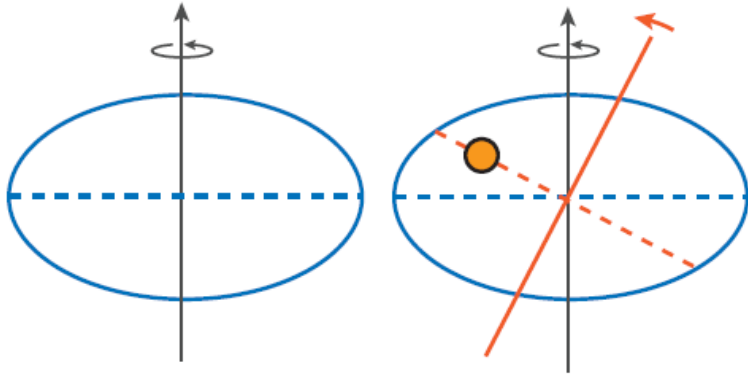
locked body: $\omega = \sqrt{\frac{G(M_h + M)}{a^3}} = o \sqrt{\frac{M_h + M}{3 M_h}}$

$$\int_{v(t)} \mathbf{r} \times (-\nabla \Psi) \rho \, dv = \int_{v(t)} \mathbf{r} \times (\mathbf{r} \omega^2 - \boldsymbol{\omega} (\boldsymbol{\omega} \cdot \mathbf{r})) \rho \, dv = \boldsymbol{\omega} \times \int_{v(t)} \mathbf{r} (\boldsymbol{\omega} \cdot \mathbf{r}) \rho \, dv = -\boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}).$$

$$\mathbf{M} = \int_{v(t)} \mathbf{r} \times (-\nabla \Theta) \rho \, dv = \mathbf{o} \times (\mathbf{I} \cdot \mathbf{o})$$

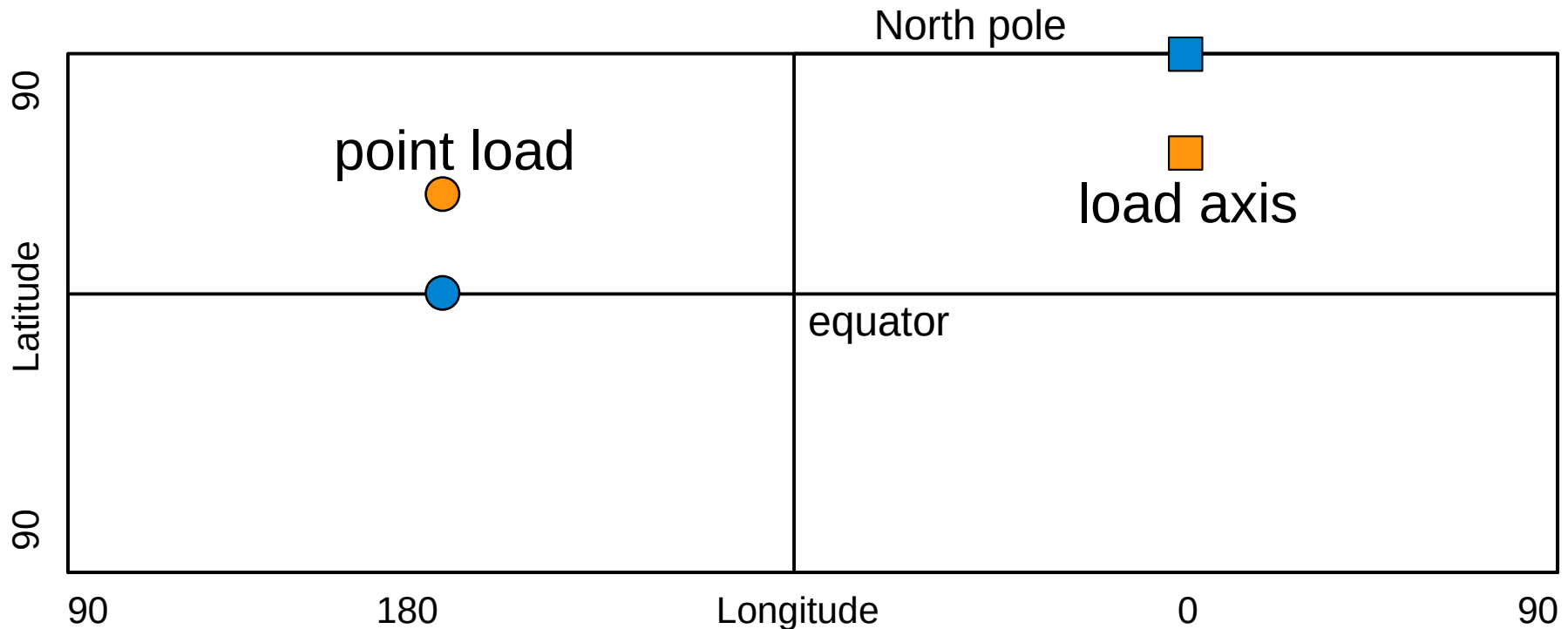
$$\frac{d}{dt}(\mathbf{I} \cdot \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) = \mathbf{o} \times (\mathbf{I} \cdot \mathbf{o}).$$

Mechanism: step by step (Earth)



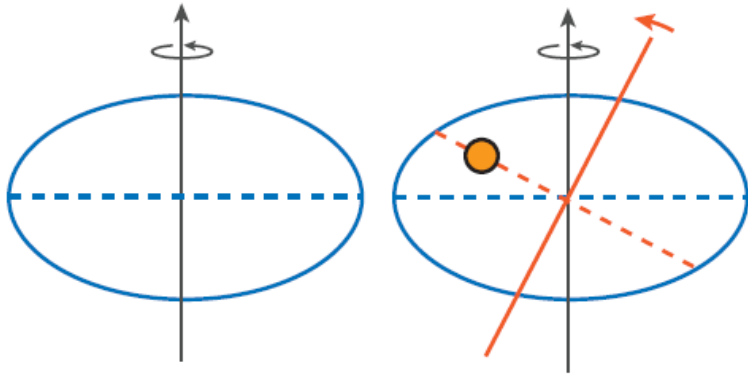
$$0 = \frac{d}{dt}(\mathbf{I} \cdot \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega})$$

Matsuyama I., Nimmo F., and Mitrovica J.X. (2014): Planetary reorientation. *Ann. Rev. Earth Planet. Sci.*



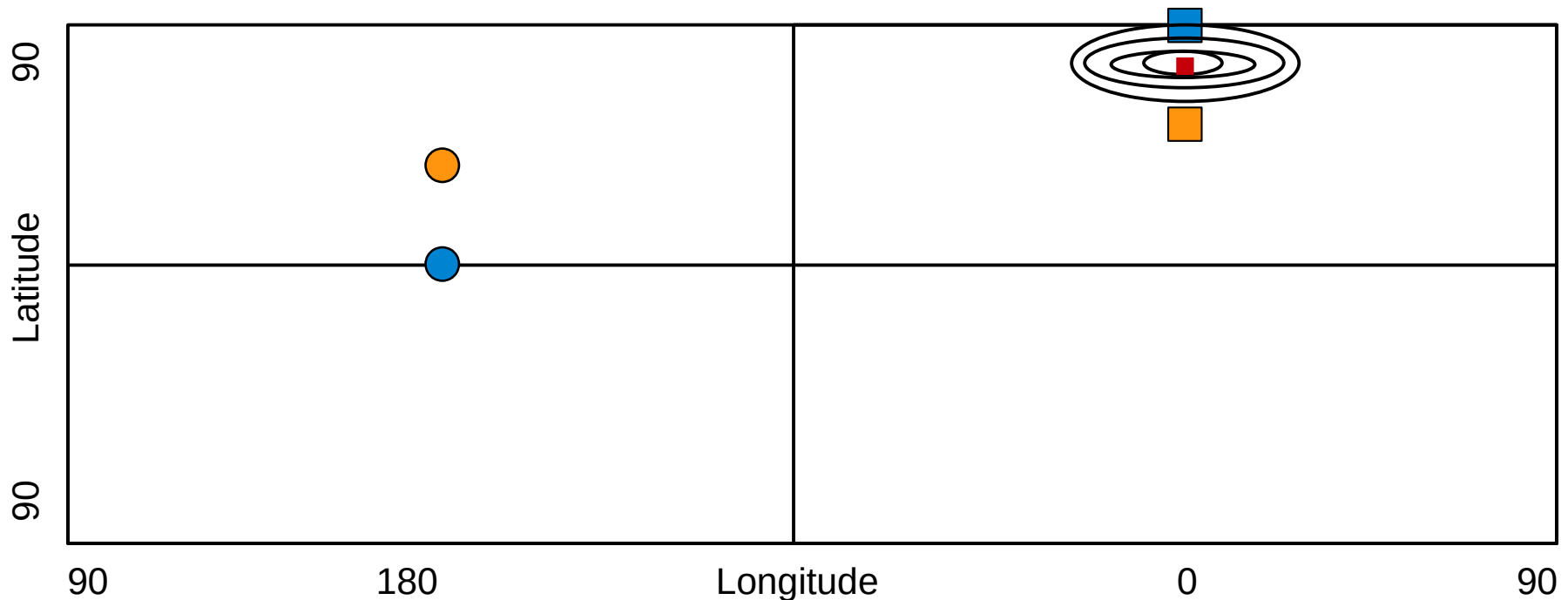
Mechanism: wobble

$\omega \parallel \text{MIA}$

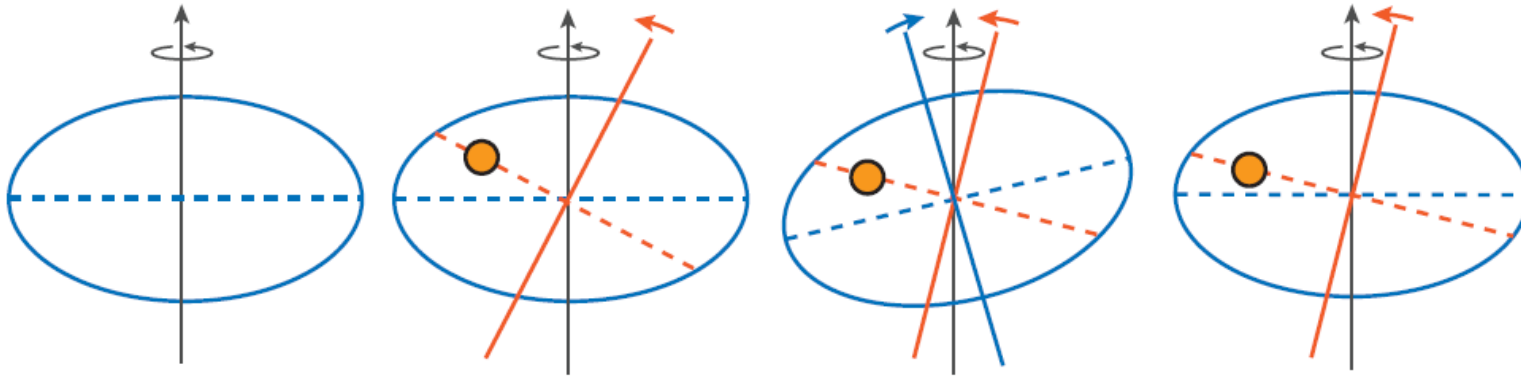


$$0 = \frac{d}{dt}(\mathbf{I} \cdot \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega})$$

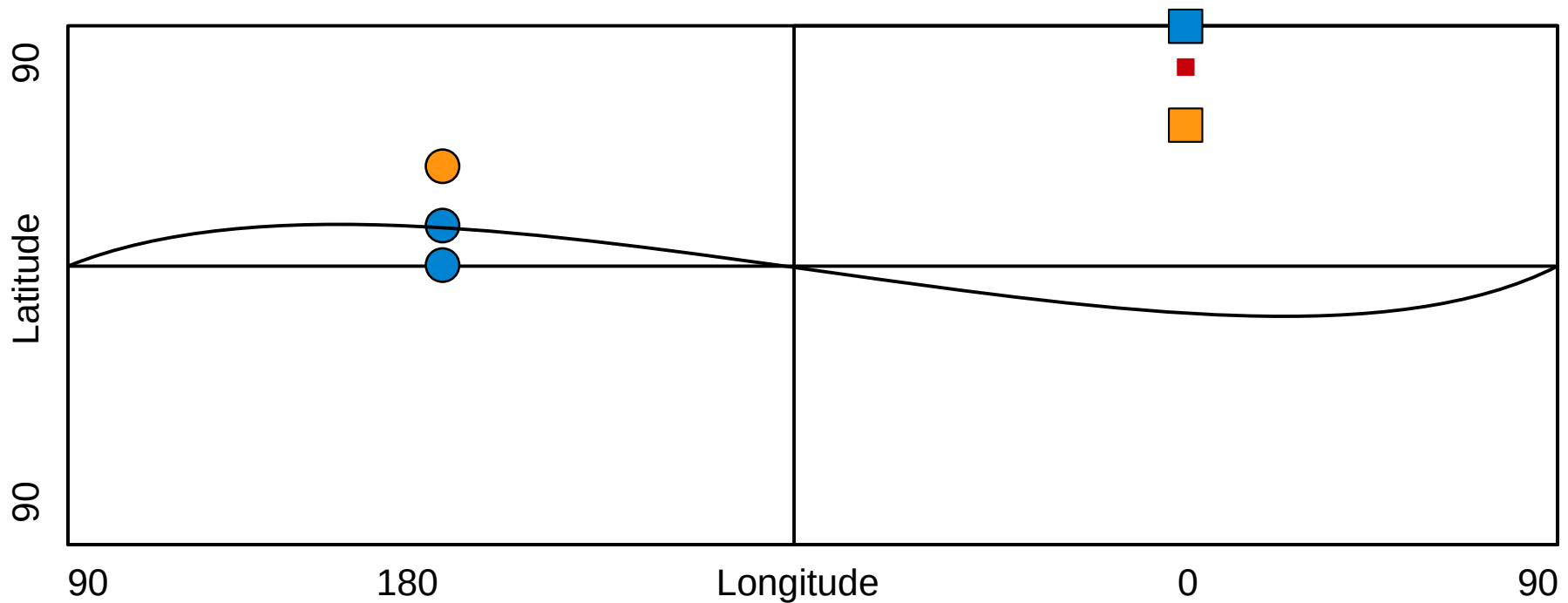
Matsuyama I., Nimmo F., and Mitrovica J.X. (2014): Planetary reorientation. *Ann. Rev. Earth Planet. Sci.*



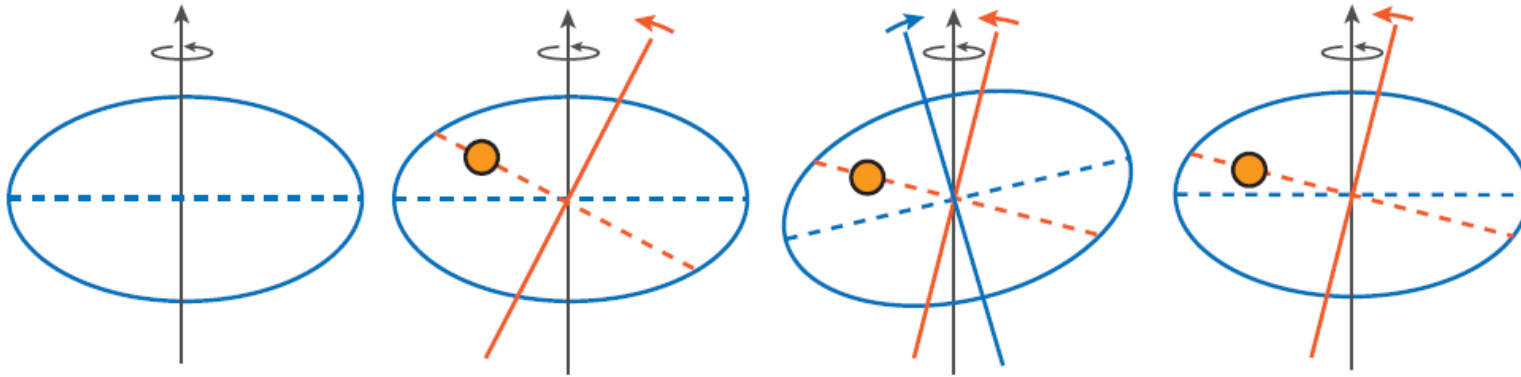
Mechanism: bulge readjustment



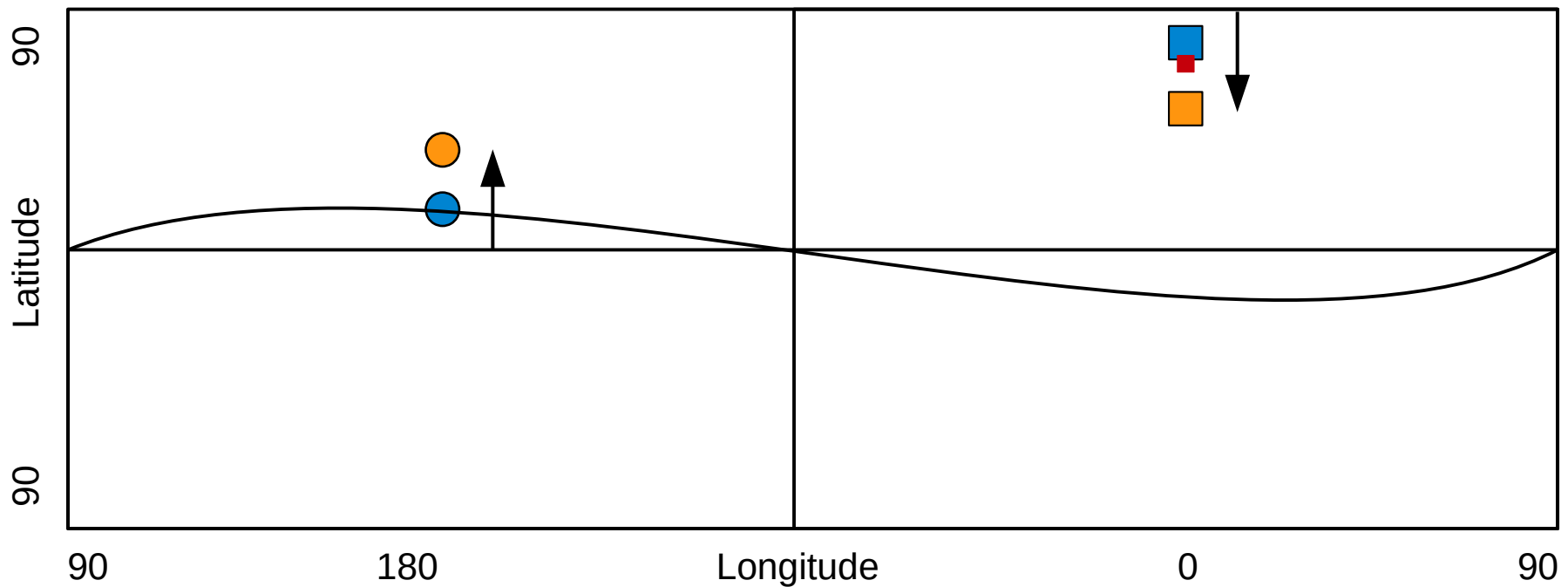
Matsuyama I., Nimmo F., and Mitrovica J.X. (2014): Planetary reorientation. *Ann. Rev. Earth Planet. Sci.*



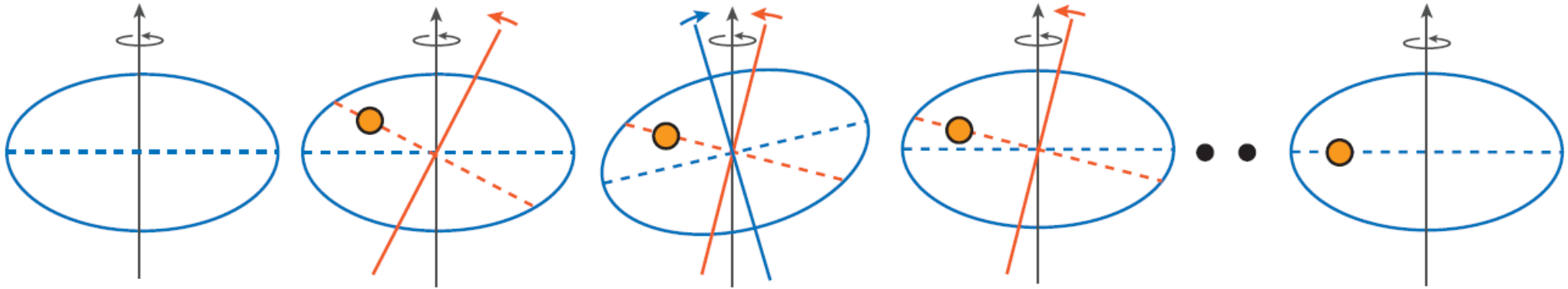
Mechanism: bulge readjustment



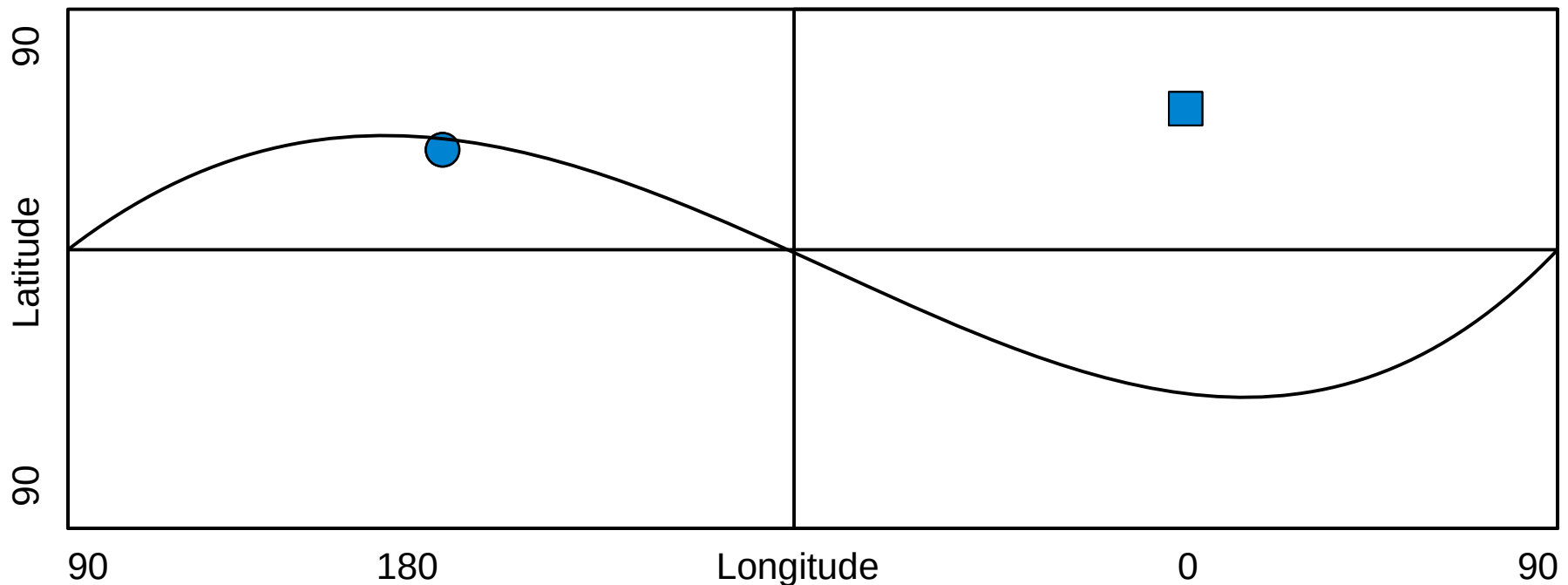
Matsuyama I., Nimmo F., and Mitrovica J.X. (2014): Planetary reorientation. *Ann. Rev. Earth Planet. Sci.*



Mechanism: load to equator scenario



Matsuyama I., Nimmo F., and Mitrovica J.X. (2014): Planetary reorientation. *Ann. Rev. Earth Planet. Sci.*

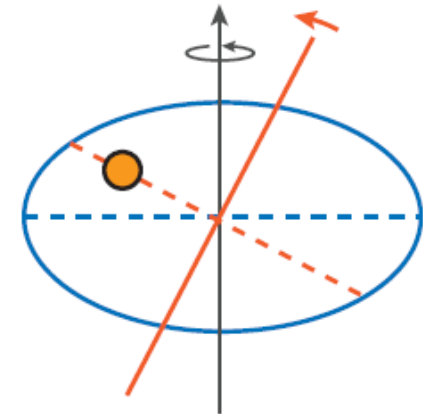


Mechanism: rotation vs. tides

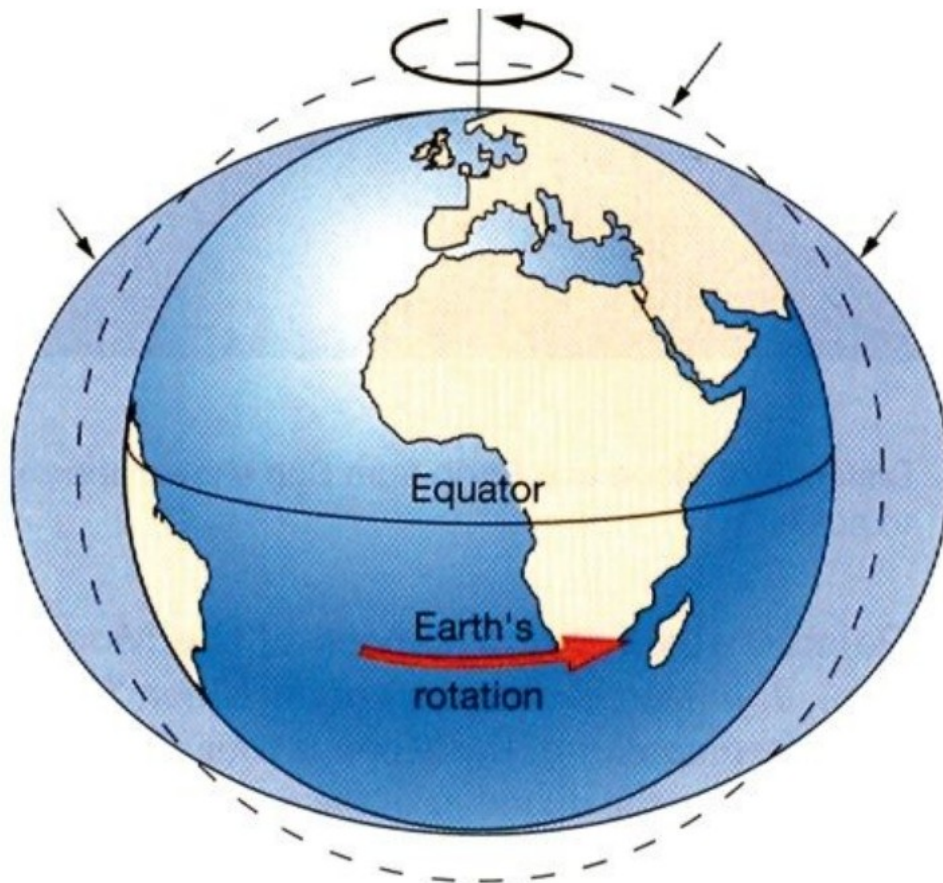
Rotation: bulge perpendicular to ω axis

Tides: bulge along the tidal axis

rotational axis



rotational axis



tidal axis

S

when tidally locked:

$$\omega = \sqrt{\frac{G(M_h + M)}{a^3}} = \omega \sqrt{\frac{M_h + M}{3M_h}}$$

$$0 = \frac{d}{dt}(\mathbf{I} \cdot \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega})$$

TPW: methods (Earth)

Peltier, W. R. (1974). The impulse response of a Maxwell Earth.

Willemann, R. (1984). Reorientation of Planets with Elastic Lithospheres.

Ricard, Y., Spada, G., & Sabadini, R. (1993, MAY). Polar wandering of a dynamic

Lefftz, M., Legros, H., & Hinderer, J. (1991, MAR). Non-linear equations for the rotation of a viscoelastic planet taking into account the influence of a liquid

Vermeersen, L., & Sabadini, R. (1996, NOV). Significance of the fundamental mantle rotational relaxation mode in polar wander simulations.

Recently:

Hu, H., van der Wal, W., & Vermeersen, L. L. A. (2017a, JAN). A numerical method for reorientation of rotating tidally deformed viscoelastic bodies.

Hu, H., van der Wal, W., & Vermeersen, L. L. A. (2017b, DEC). A Full-Maxwell Approach for Large-Angle Polar Wander of Viscoelastic Bodies.

Hu, H., van der Wal, W., & Vermeersen, L. L. A. (2019, MAR 15). Rotational dynamics of tidally deformed planetary bodies and validity of fluid limit and quasi-fluid approximation.

$$\frac{d}{dt}(\mathbf{I} \cdot \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) = \mathbf{o} \times (\mathbf{I} \cdot \mathbf{o}).$$

TPW: methods (moons)

Willemann, R., 1984. Reorientation of Planets with Elastic Lithospheres.

Ojakangas, G.W., Stevenson, D.J., 1989. Polar wander of an ice shell on Europa. *Icarus*

The glory:

Matsuyama, I., Nimmo, F., & Mitrovica, J. X. (2014)

Planetary Reorientation. *Annual Review of Earth and Planetary Sciences*

Recently:

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Hu, H., van der Wal, W., & Vermeersen, L. L. A. (2019, MAR 15). Rotational dynamics of tidally deformed planetary bodies and validity of fluid limit and quasi-fluid approximation.

Dynamics of reorientation

Ojakangas, G.W., Stevenson, D.J., 1989. Polar wander of an ice shell on Europa. *Icarus*

4. POLAR WANDER OF A RIGID SYNCHRONOUSLY ROTATING BODY

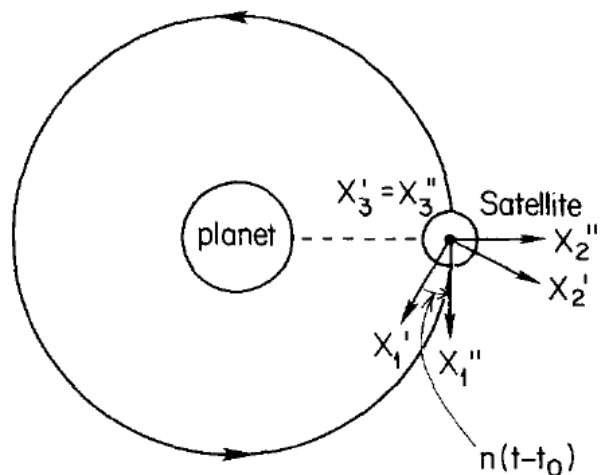


FIG. 2. Rotation from the nonrotating x'_i coordinate system to the x''_i coordinate system that rotates synchronously with the mean motion, n , about the x'_3 -axis. The x''_1 - and x''_2 -axes are directed opposite to the orbital velocity and directly away from the planet, respectively.

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} (n + \dot{\varphi}) \sin \psi \cos \theta - \dot{\theta} \cos \psi \\ (n + \dot{\varphi}) \cos \psi \cos \theta + \dot{\theta} \sin \psi \\ (n + \dot{\varphi}) \sin \theta + \dot{\psi} \end{bmatrix}. \quad (4.5)$$

ligible. We conclude that if any large-scale polar wander of the shell occurs on a time scale long compared to the rotation period, the components of the inertia tensor due to the hydrostatic tidal and rotational bulges as well as the relative angular momentum due to the motion of those bulges over the shell may be ignored.

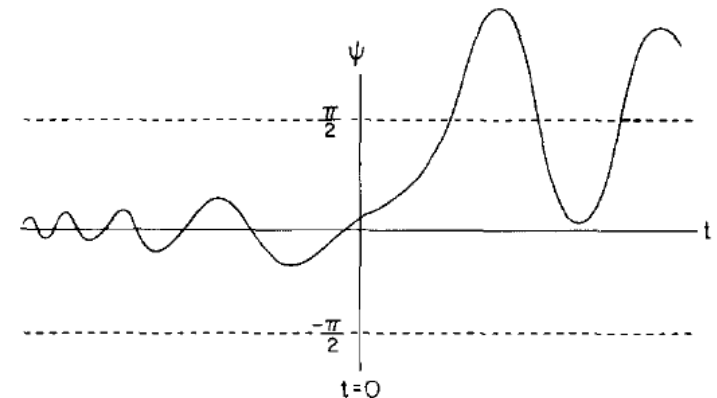
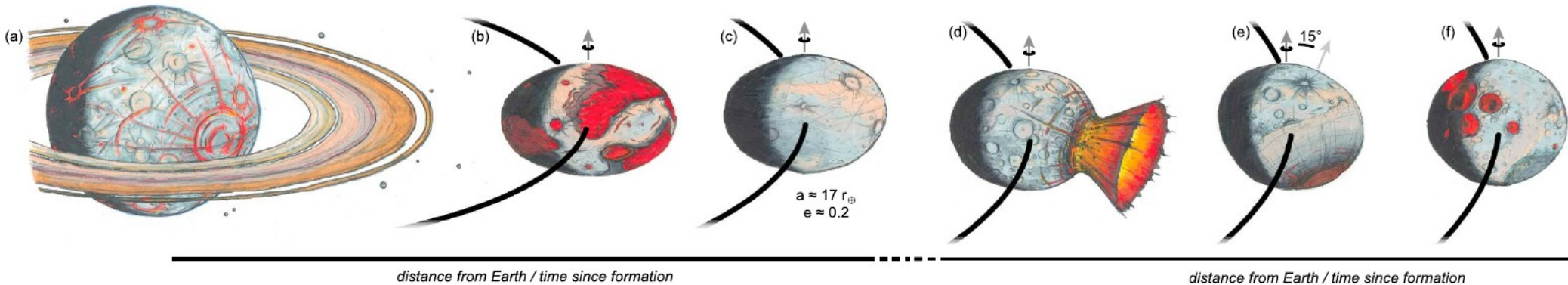
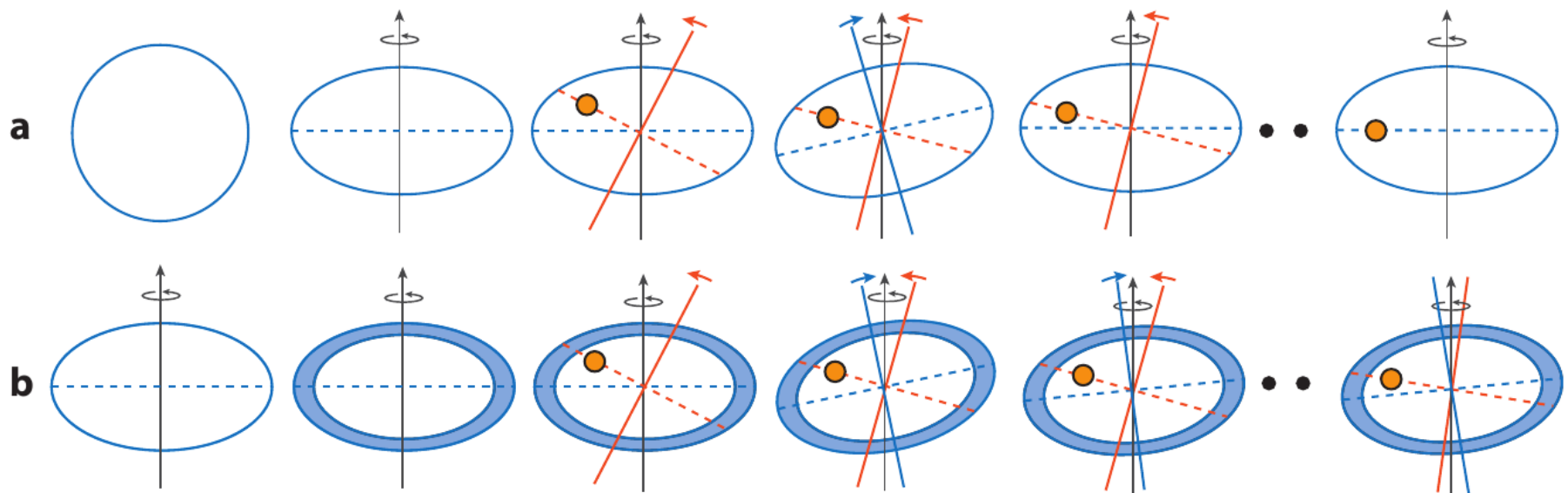


FIG. 3. The form of the solution to Eq. (4.17) with no damping. ψ may oscillate about $\pm\pi/2$ when time $t > 0$, depending on the values of ψ and $\dot{\psi}$ at $t = 0$. Circulation of ψ is also possible, but is unlikely for any real body because of the presence of dissipation.

Fossil (or remnant) bulge

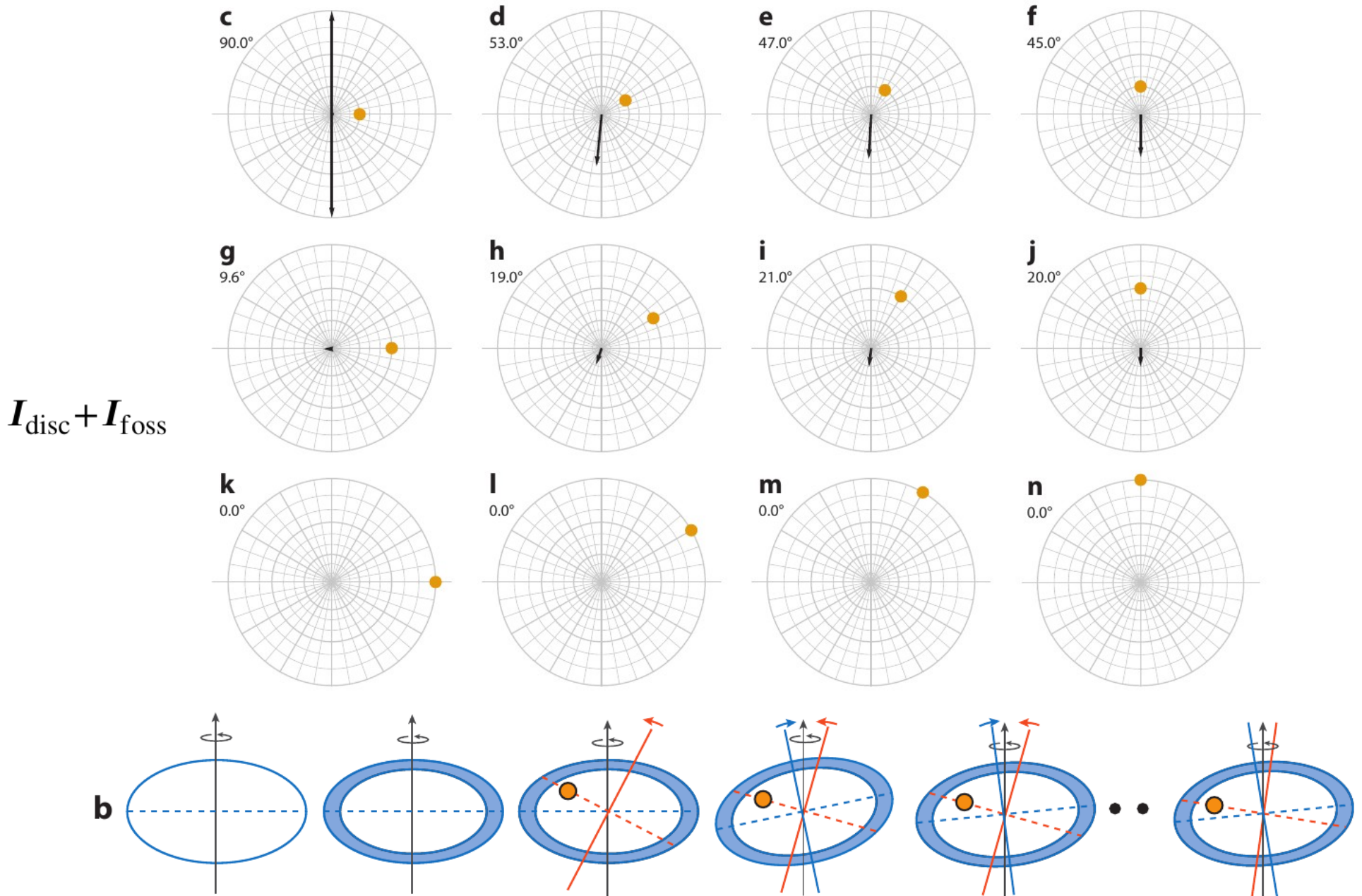


Keane and Matsuyama (2014): Evidence for lunar true polar wander and past low-eccentricity,...



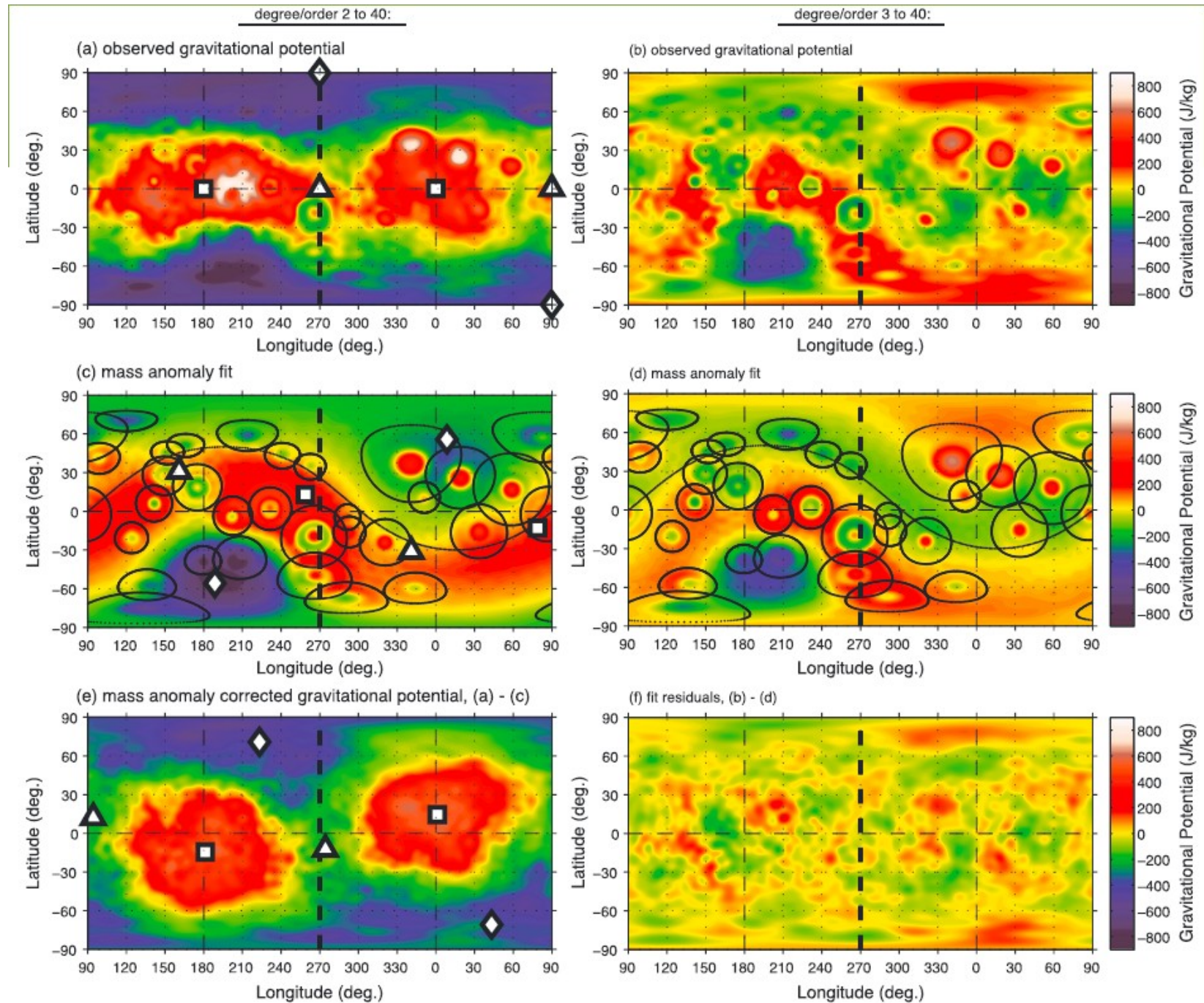
Matsuyama I., Nimmo F., and Mitrovica J.X. (2014): Planetary reorientation. *Ann. Rev. Earth Planet. Sci.*

Equilibrium reorientation



The Moon

Keane and Matsuyama, 2014: Evidence for lunar true polar wander...



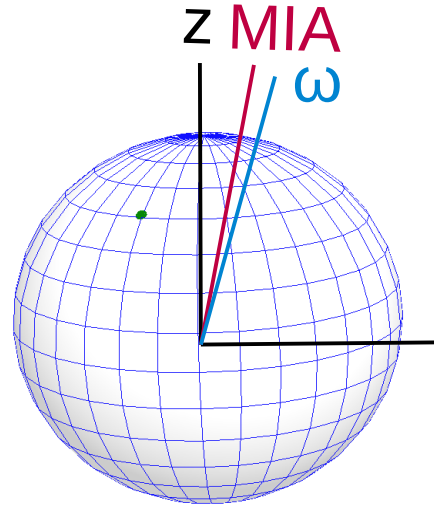
Dynamics: PWL approach of Hu et al.

Linearized Liouville equation:

$$m_1(t) = \frac{\Delta I_{13}(t)}{C - A} + \frac{C \Delta \dot{I}_{23}(t)}{\Omega(C - A)(C - B)}$$

$$m_2(t) = \frac{\Delta I_{23}(t)}{C - B} - \frac{C \Delta \dot{I}_{13}(t)}{\Omega(C - A)(C - B)}$$

$$m_3(t) = -\frac{\Delta I_{33}}{C}$$



Transformation of coordinates:

$$\mathbf{Q} = \begin{pmatrix} \omega_3 + \frac{\omega_2^2}{1+\omega_3} & -\frac{\omega_1\omega_2}{1+\omega_3} & \omega_1 \\ -\frac{\omega_1\omega_2}{1+\omega_3} & 1 - \frac{\omega_2^2}{1+\omega_3} & \omega_2 \\ -\omega_1 & -\omega_2 & \omega_3 \end{pmatrix}$$

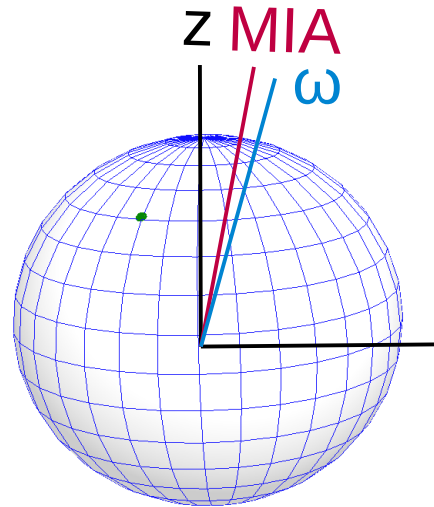
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$$m_2(t) = \frac{\Delta I_{23}(t)}{C - B} - \frac{C \Delta \dot{I}_{13}(t)}{\Omega(C - A)(C - B)}$$

$$m_3(t) = -\frac{\Delta I_{33}}{C}$$



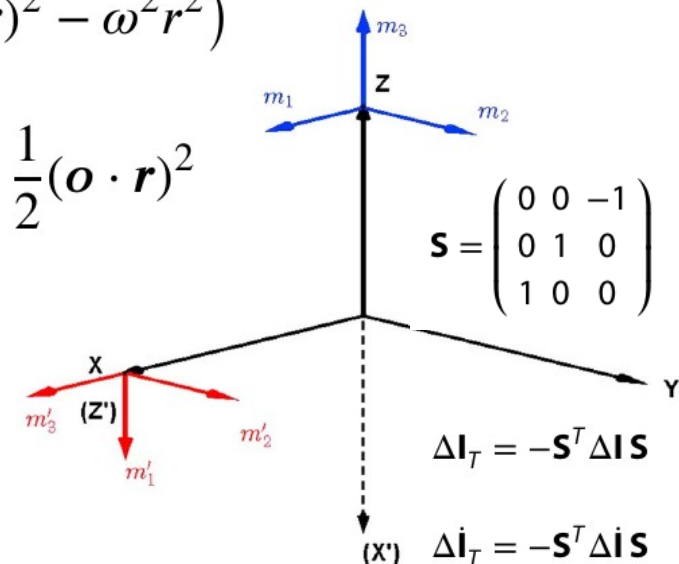
Transformation of coordinates:

$$\mathbf{Q} = \begin{pmatrix} \omega_3 + \frac{\omega_2^2}{1+\omega_3} & -\frac{\omega_1\omega_2}{1+\omega_3} & \omega_1 \\ -\frac{\omega_1\omega_2}{1+\omega_3} & 1 - \frac{\omega_2^2}{1+\omega_3} & \omega_2 \\ -\omega_1 & -\omega_2 & \omega_3 \end{pmatrix}$$

Tidal axis wander: iterations

$$\Psi = \frac{1}{2} ((\boldsymbol{\omega} \cdot \mathbf{r})^2 - \omega^2 r^2)$$

$$\Theta = \frac{1}{6} o^2 r^2 - \frac{1}{2} (\mathbf{o} \cdot \mathbf{r})^2$$



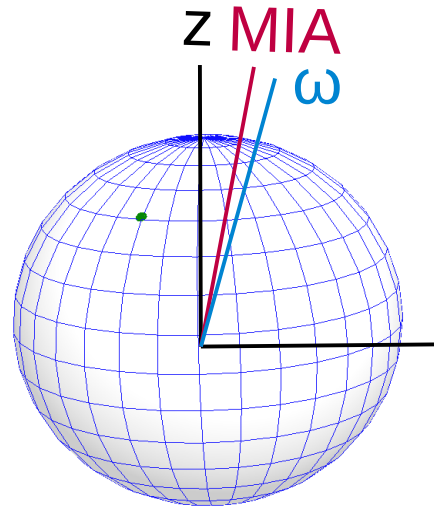
Dynamics: PWL approach of Hu et al.

Linearized Liouville equation:

$$m_1(t) = \frac{\Delta I_{13}(t)}{C - A} + \frac{C \Delta \dot{I}_{23}(t)}{\Omega(C - A)(C - B)}$$

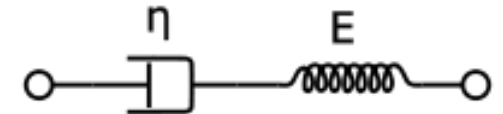
$$m_2(t) = \frac{\Delta I_{23}(t)}{C - B} - \frac{C \Delta \dot{I}_{13}(t)}{\Omega(C - A)(C - B)}$$

$$m_3(t) = -\frac{\Delta I_{33}}{C}$$



$$\nabla \cdot \tau + f = 0,$$

$$\nabla \cdot u = 0,$$



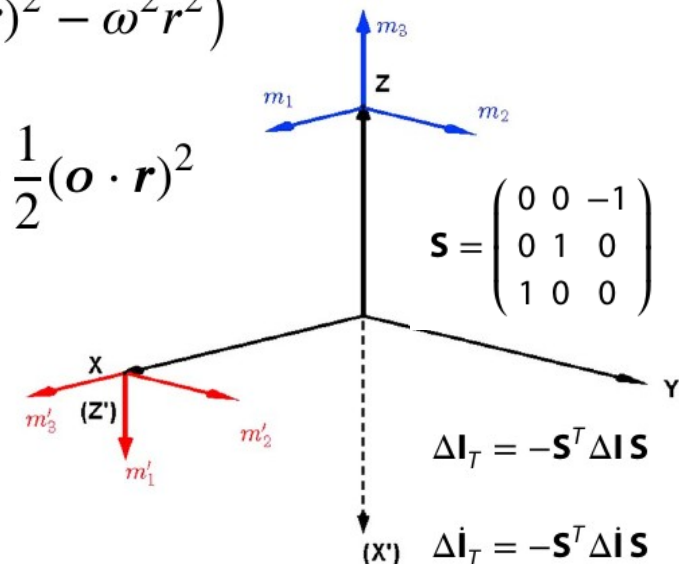
Transformation of coordinates:

$$\mathbf{Q} = \begin{pmatrix} \omega_3 + \frac{\omega_2^2}{1+\omega_3} & -\frac{\omega_1\omega_2}{1+\omega_3} & \omega_1 \\ -\frac{\omega_1\omega_2}{1+\omega_3} & 1 - \frac{\omega_2^2}{1+\omega_3} & \omega_2 \\ -\omega_1 & -\omega_2 & \omega_3 \end{pmatrix}$$

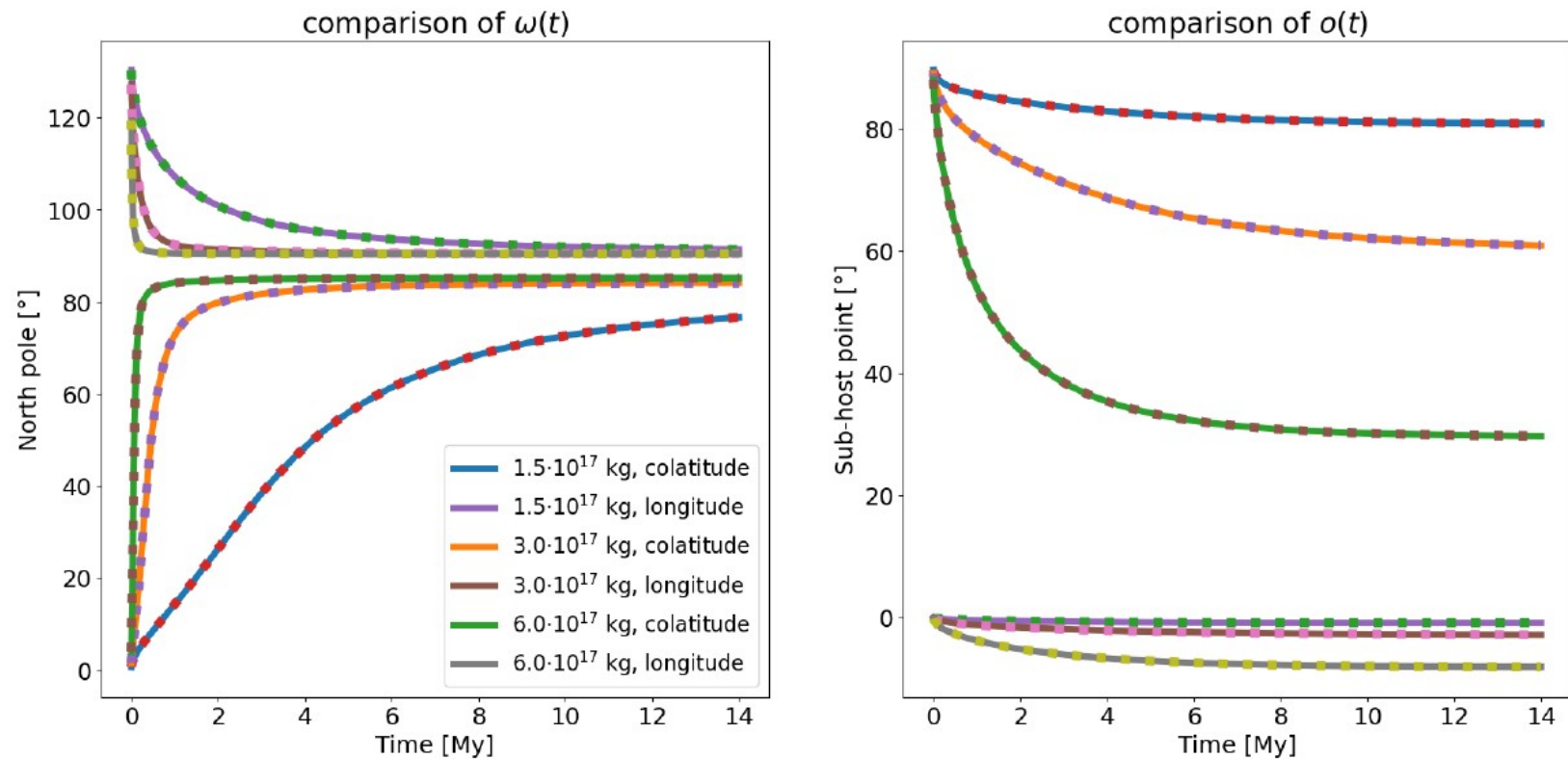
Tidal axis wander: iterations

$$\Psi = \frac{1}{2} ((\omega \cdot r)^2 - \omega^2 r^2)$$

$$\Theta = \frac{1}{6} o^2 r^2 - \frac{1}{2} (o \cdot r)^2$$

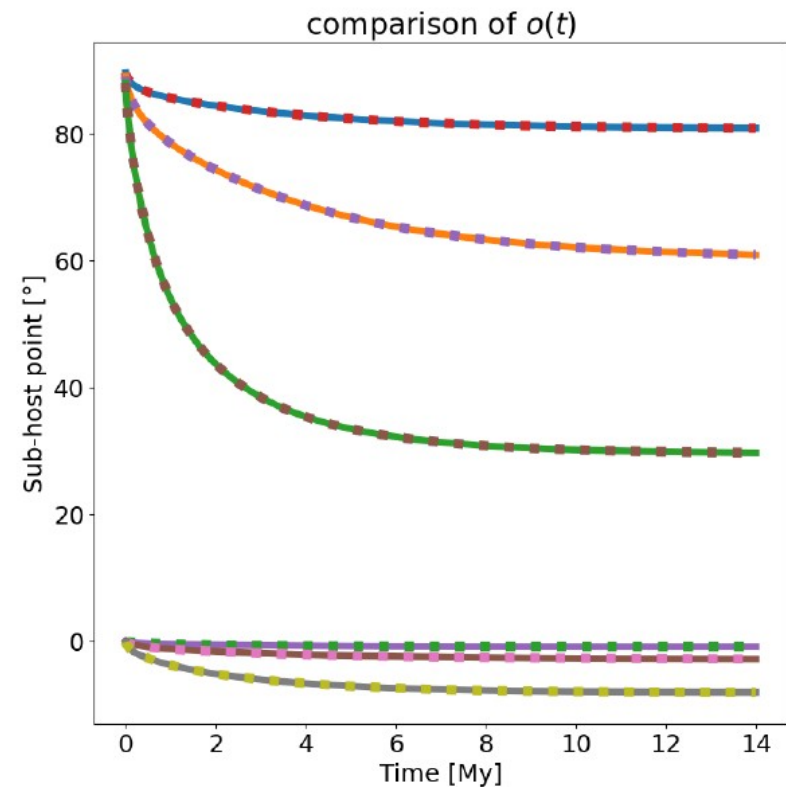
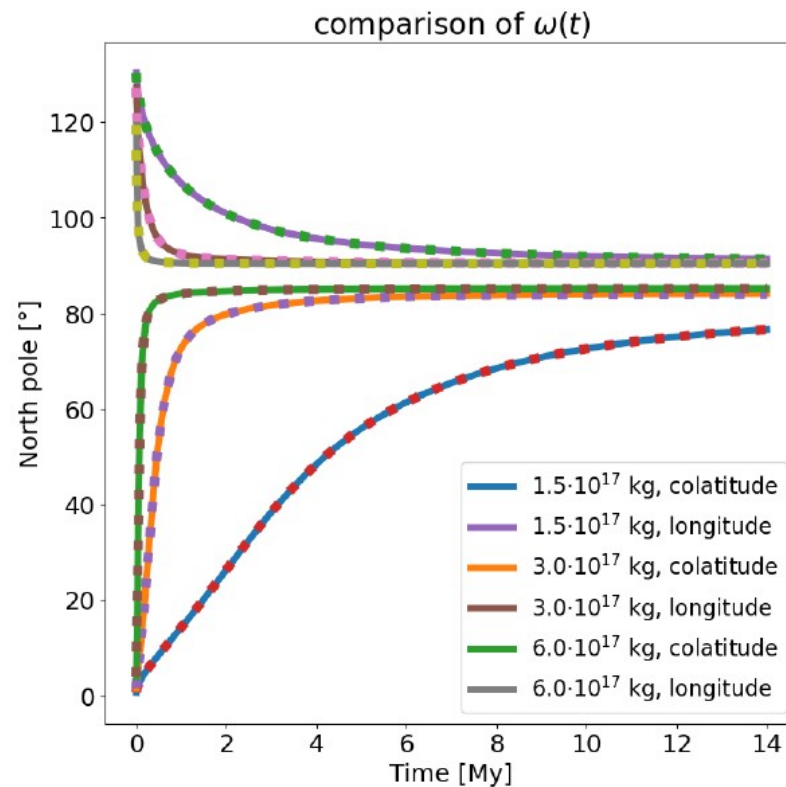


Dynamics: PWL approach of Hu et al.



Test case (Triton): positive load placed at 75°N, 15°E

Dynamics: PWL approach of Hu et al.



The $o\omega||m$ MIA Approximation

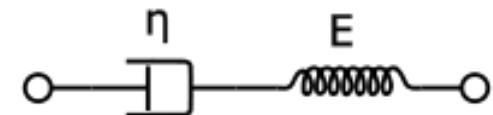
$$\omega \times (I \cdot \omega) = 0,$$

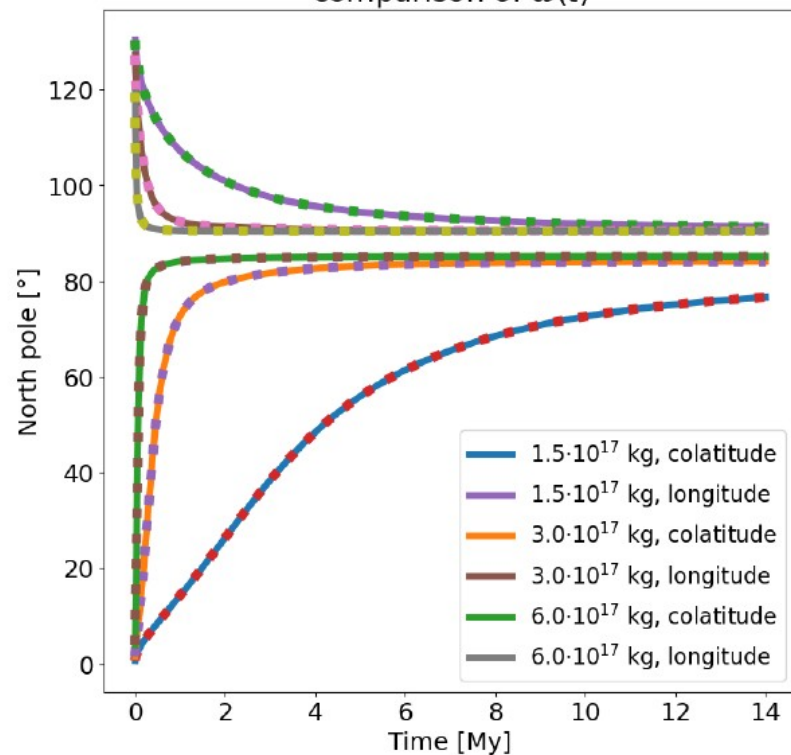
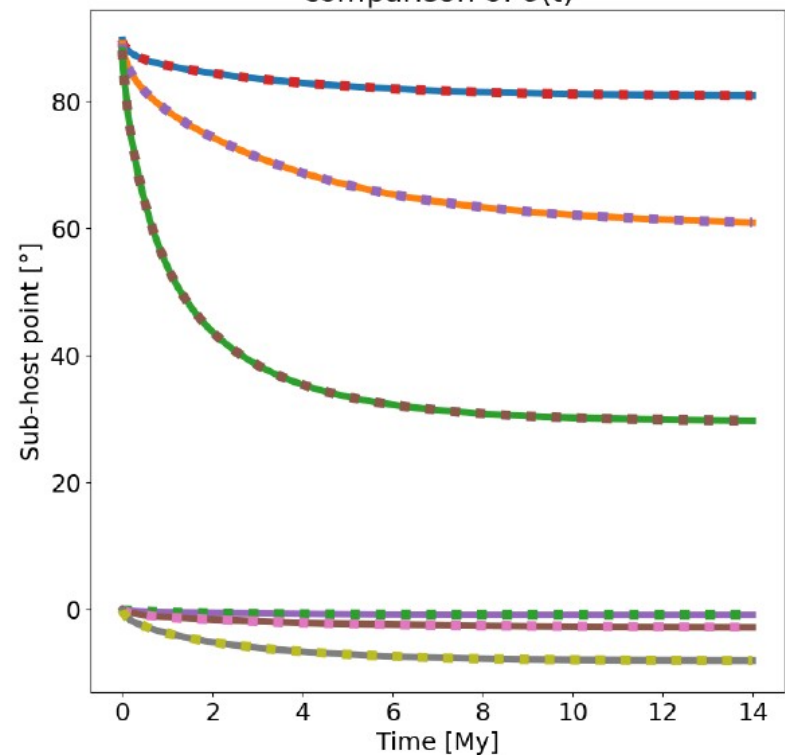
$$o \times (I \cdot o) = 0,$$

$$\frac{d}{dt}(I \cdot \omega) = 0.$$

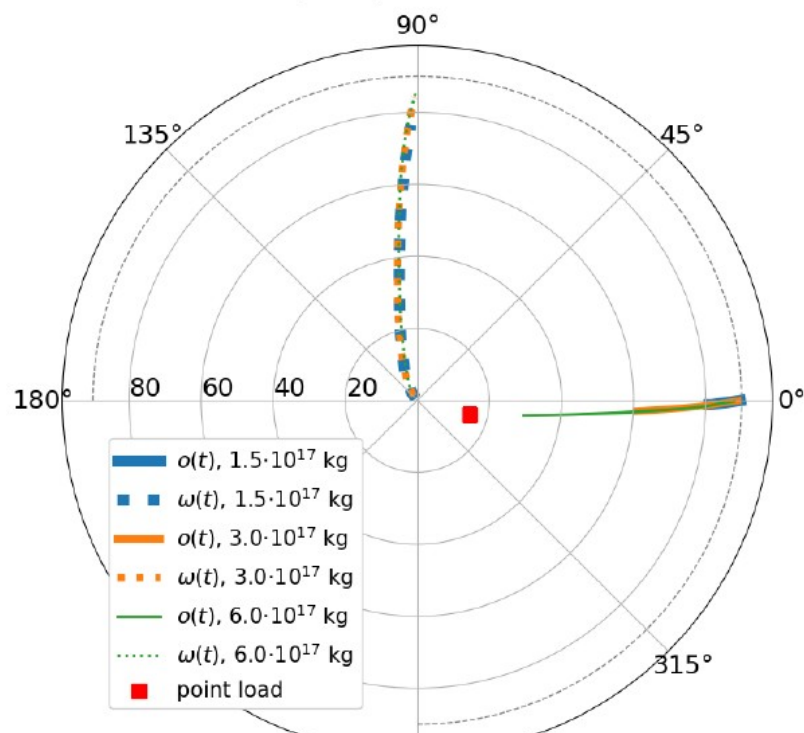
$$\nabla \cdot \tau + f = 0,$$

$$\nabla \cdot u = 0,$$

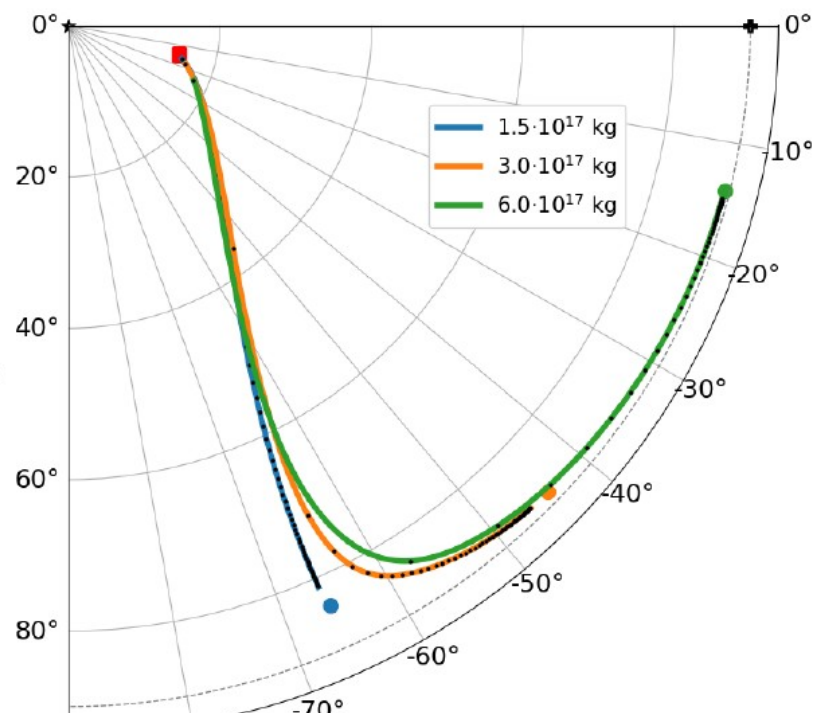


comparison of $\omega(t)$ comparison of $\alpha(t)$ 

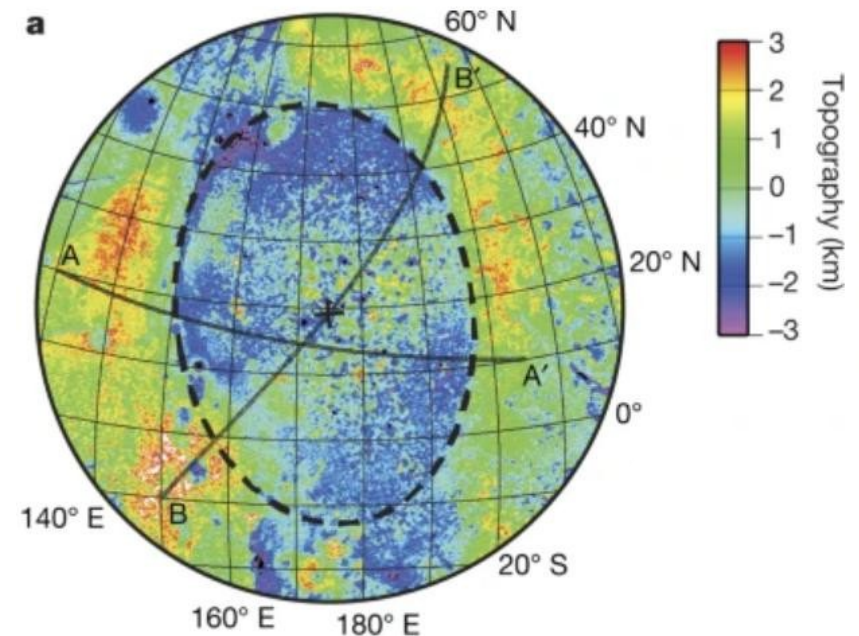
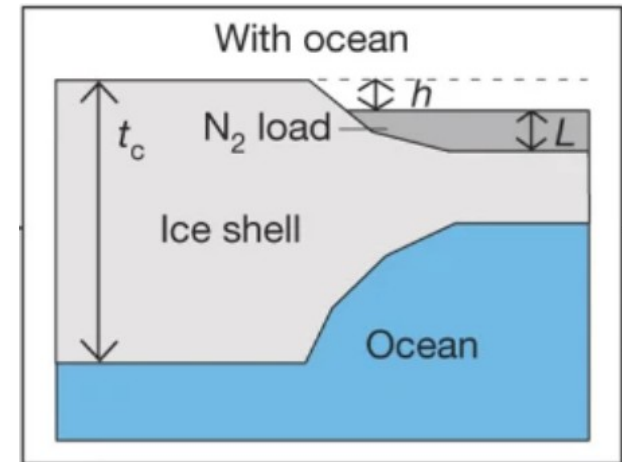
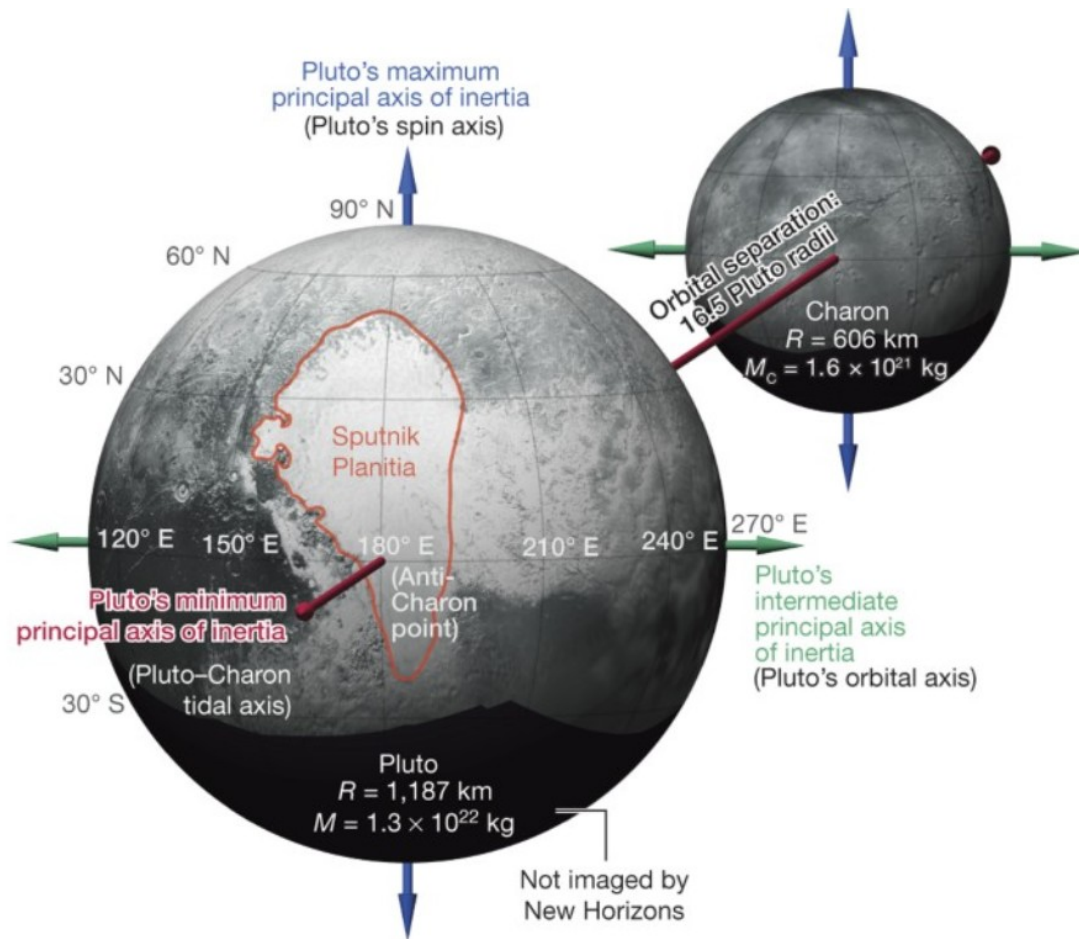
c) body-fixed view



d) bulge-frame view



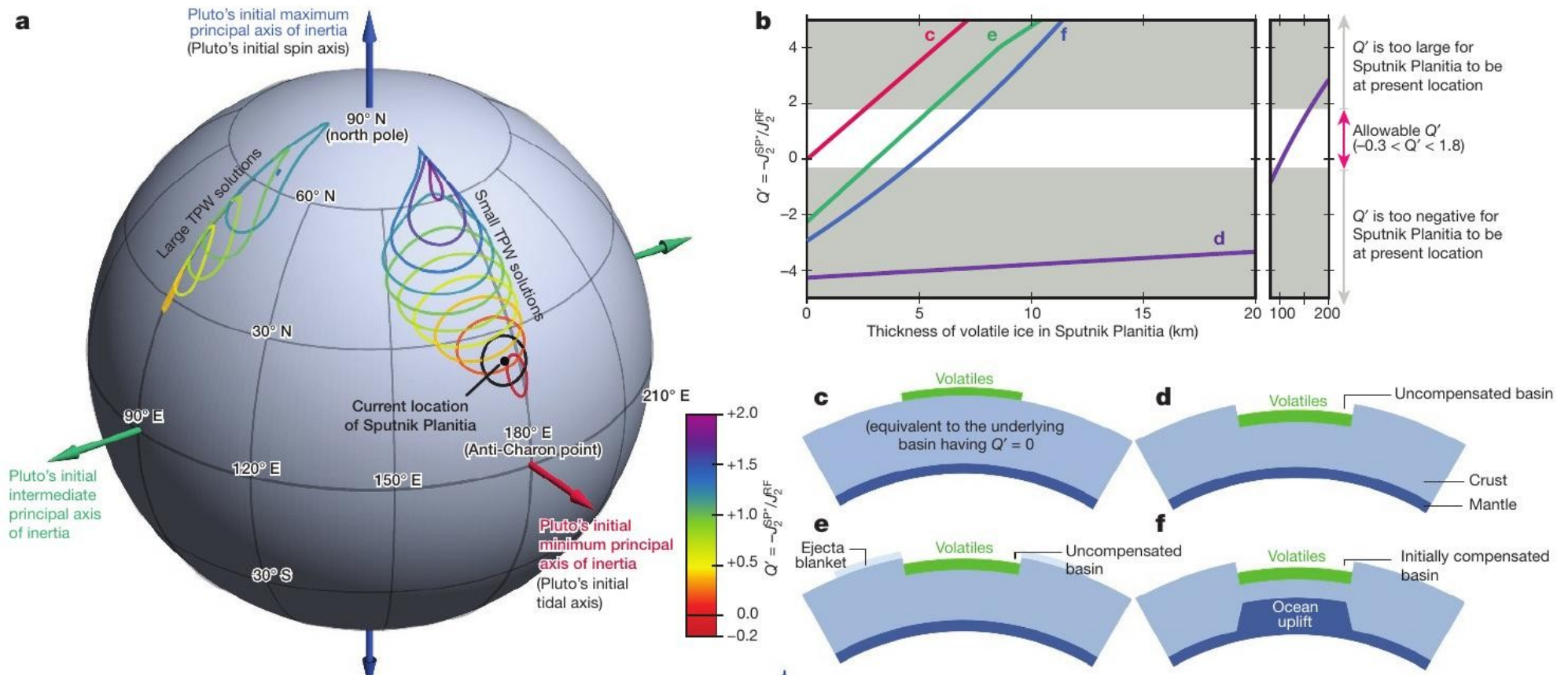
Pluto: Sputnik Planitia basin



Keane et al., 2016: Reorientation and faulting of Pluto due to volatile loading within Sputnik Planitia, *Nature*

Nimmo et al., 2016: Reorientation of Sputnik Planitia implies a subsurface ocean on Pluto, *Nature*

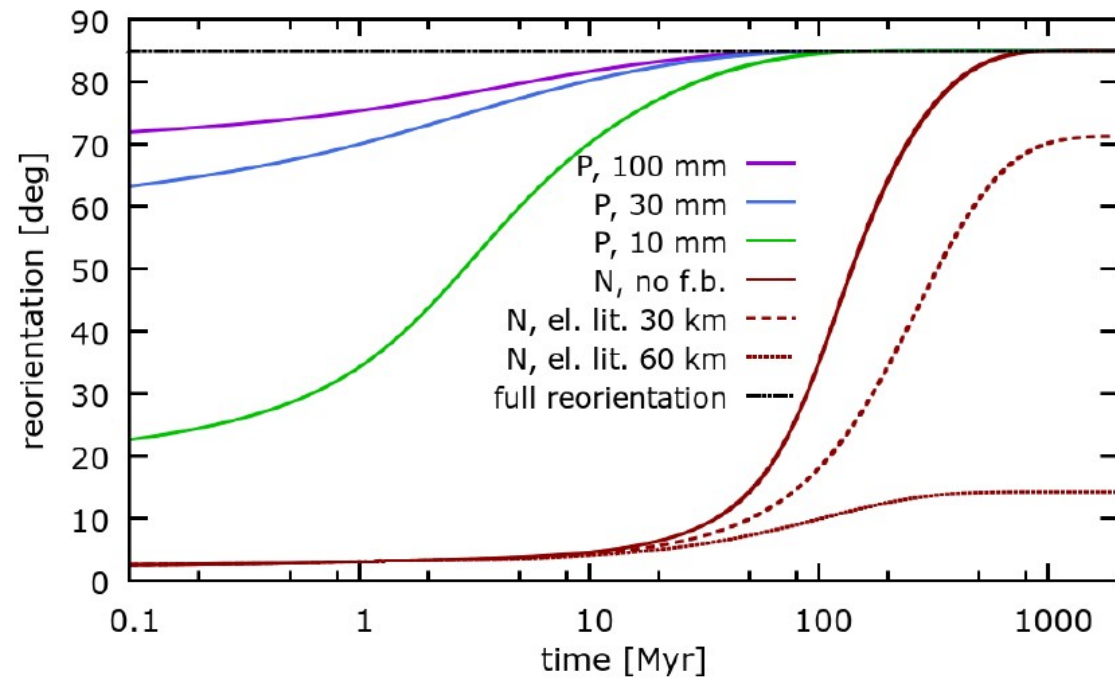
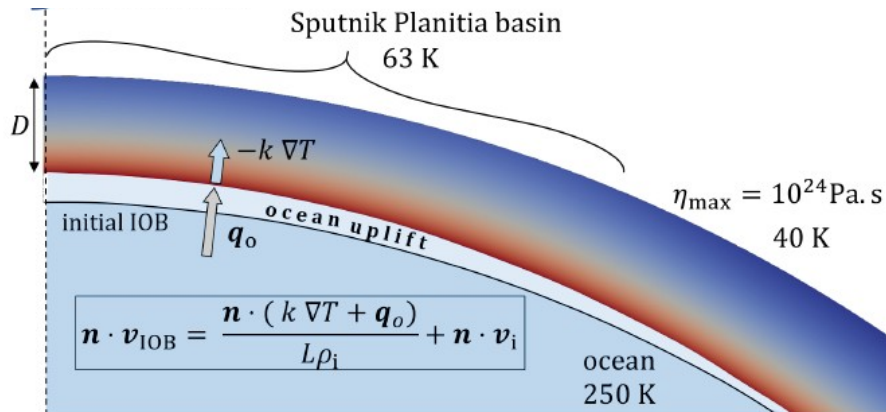
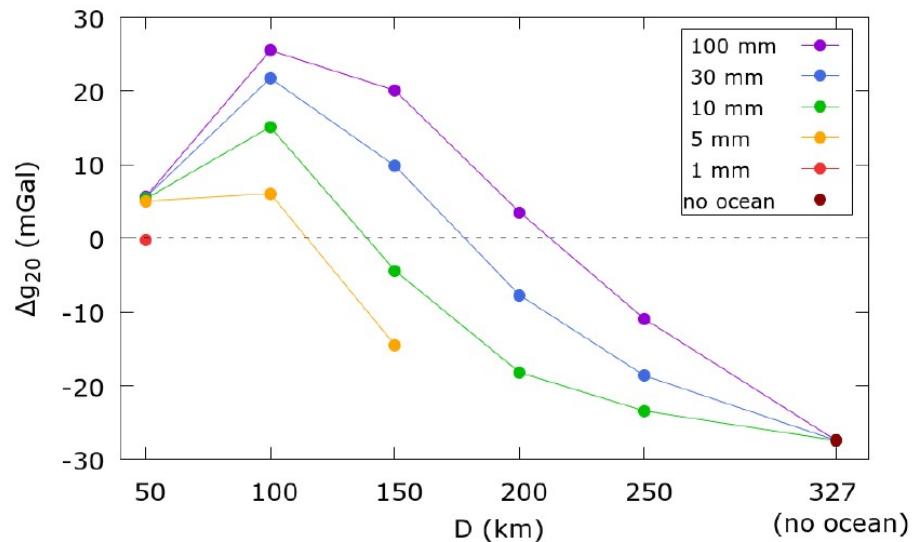
Pluto: Sputnik Planitia basin



Keane et al., 2016: Reorientation and faulting of Pluto due to volatile loading within Sputnik Planitia, *Nature*

Pluto: Sputnik Planitia basin

SPUTNIK PLANITIA BASIN AS A TRIGGER FOR MELTING AND REORIENTATION OF PLUTO'S ICE SHELL. M. Kihoulou¹ and V. Patočka¹. ¹ Department of Geophysics, Faculty of Mathematics and Physics, Charles University, V Holesovickach 2, 18000 Prague, Czech Republic



Dynamics of reorientation: Straight or not?

Equilibrium orientation

- only the final state
- “quasi-steady” dynamics

Small satellites

- M_h/M large

Are load paths in the bulge-fixed frame straight or not?

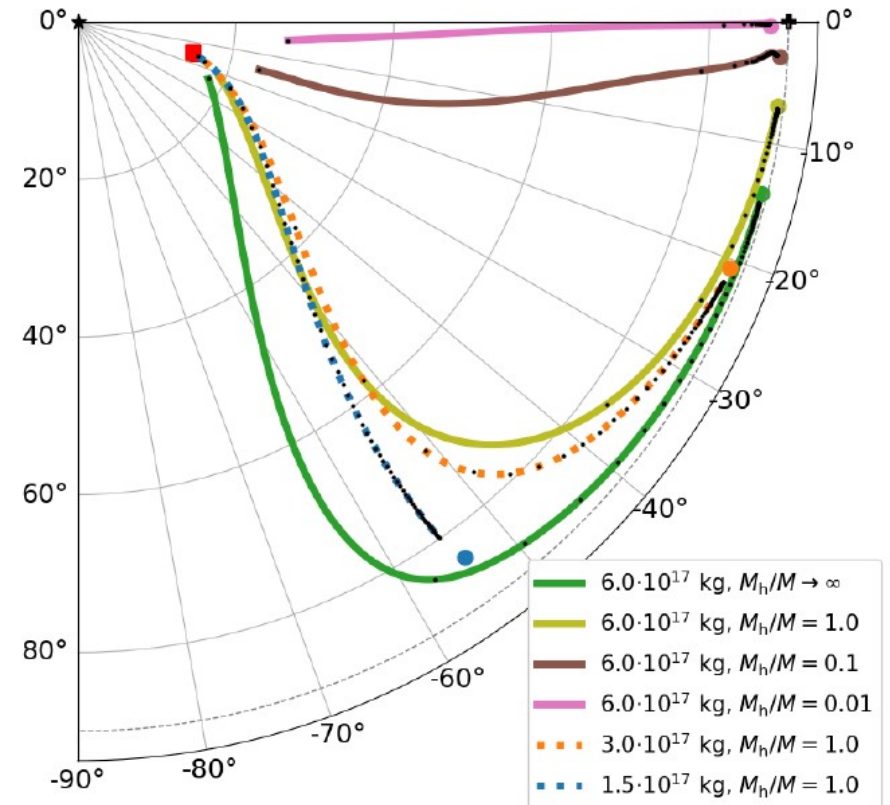
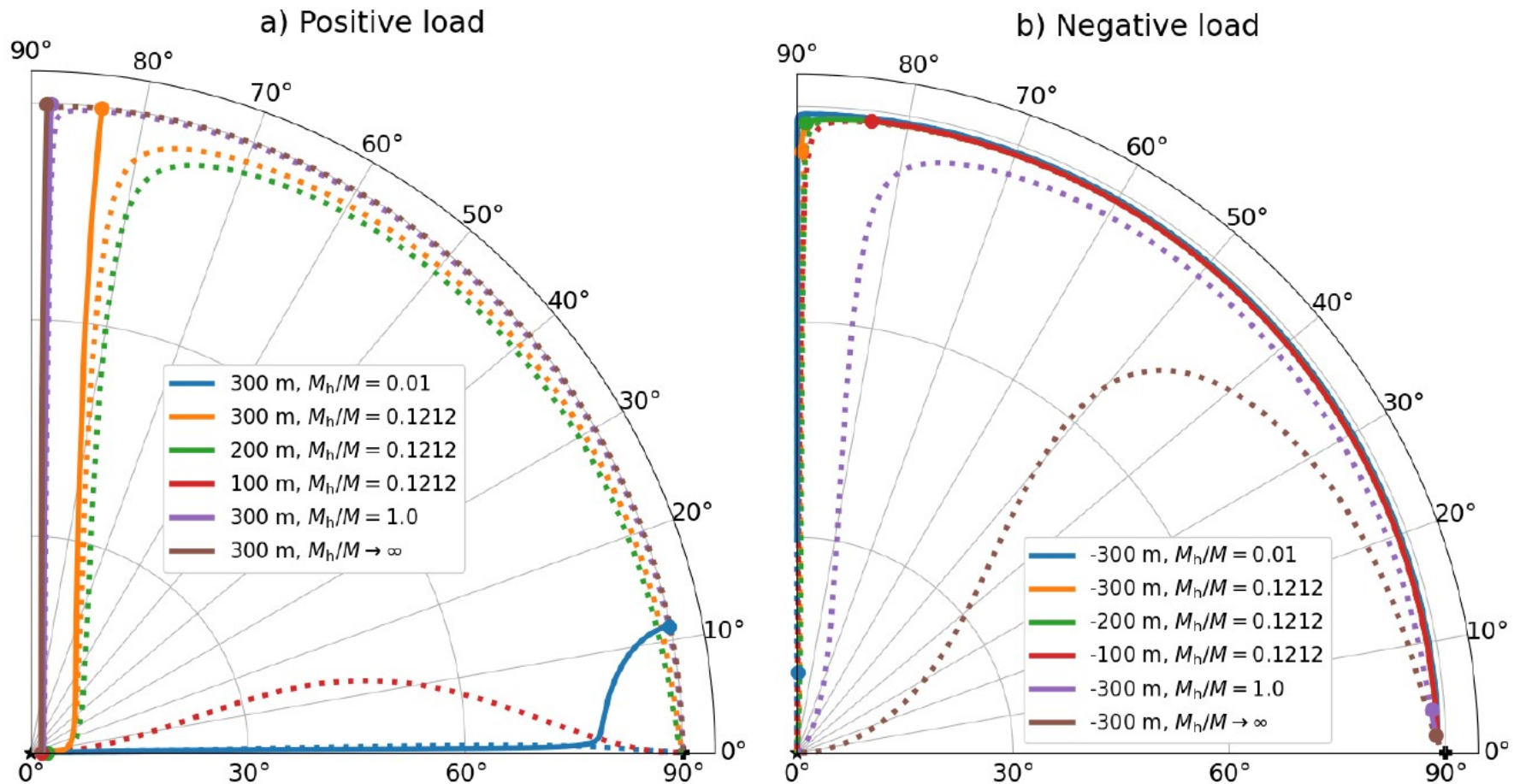


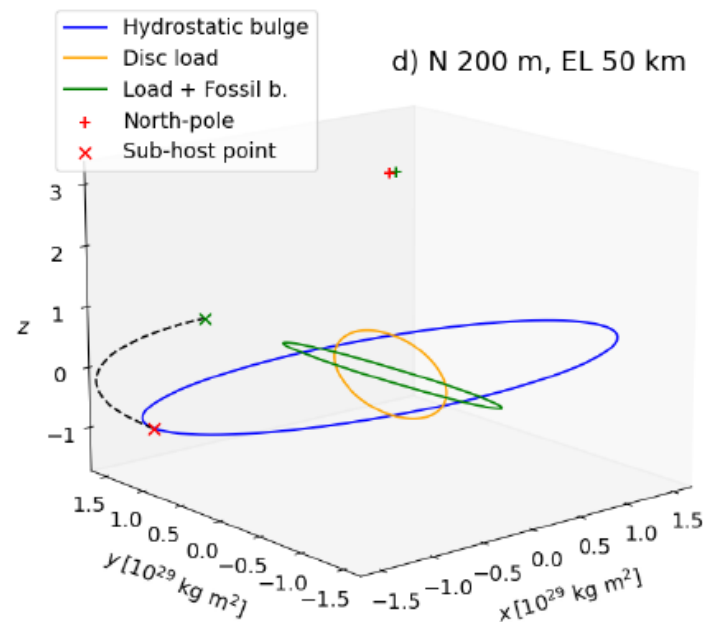
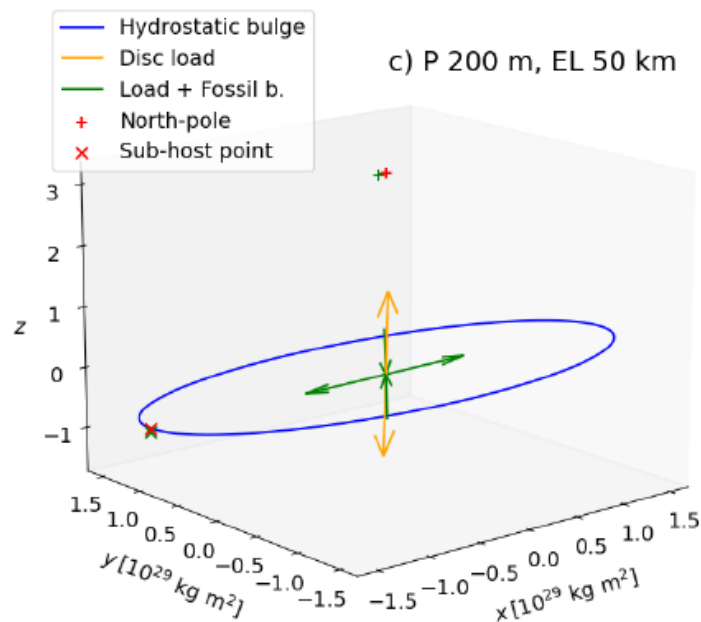
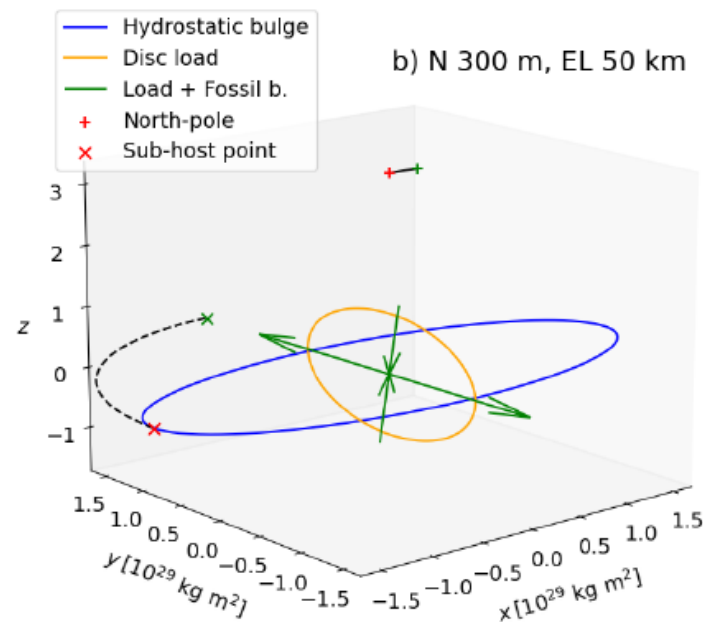
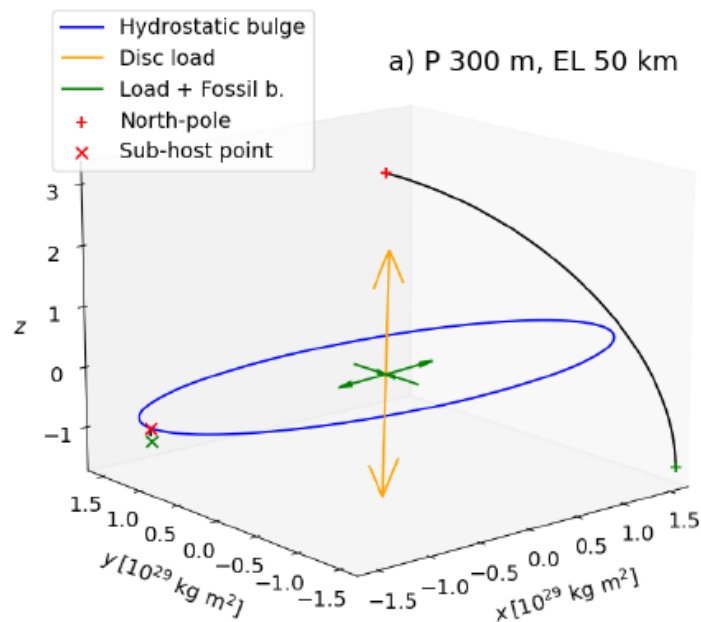
Figure 2: The influence of M_h/M on the path of a positive load, depicted in the bulge-fixed frame (cf. Fig. 1d). Temporal evolution is illustrated by the small black dots that are evenly sampled in time at intervals of 250 ky. The red square marks the initial position of the load.

Dynamics of reorientation: Straight or not?

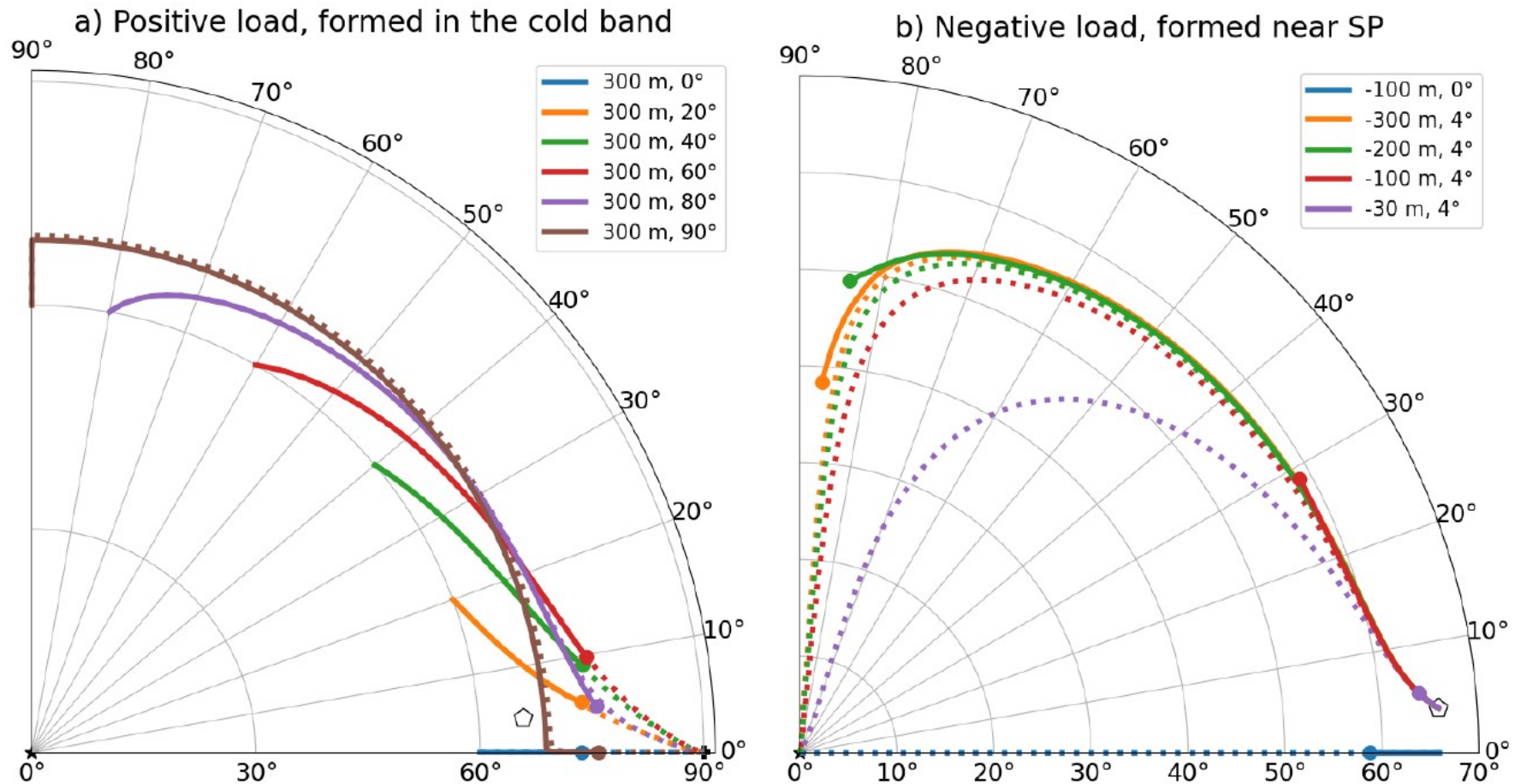


Patočka et al.: Dynamic reorientation of tidally locked bodies: Application to Pluto, *in prep*

Equilibrium reorientation

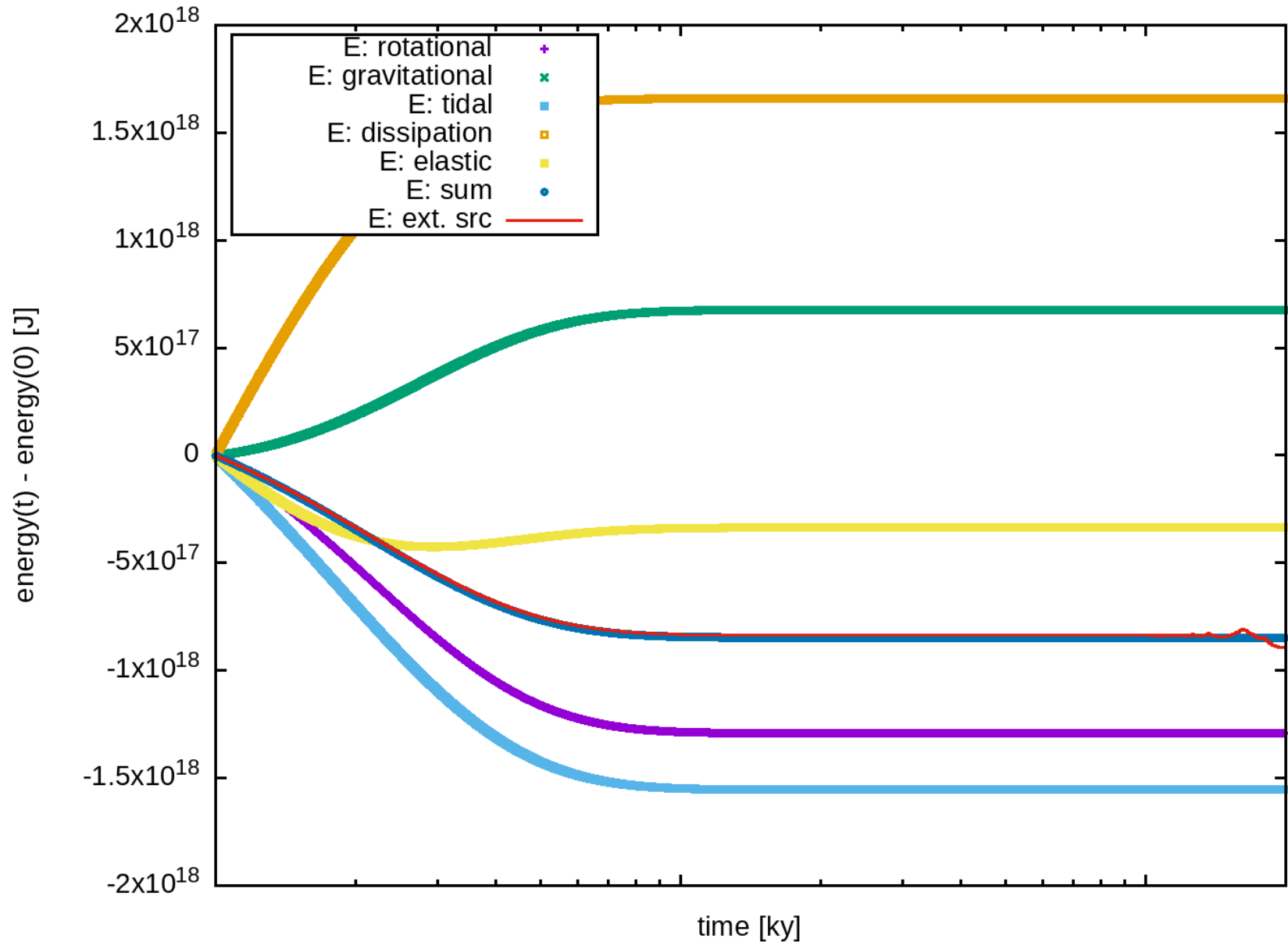


Dynamics of reorientation: Straight or not?



Patočka et al.: Dynamic reorientation of tidally locked bodies: Application to Pluto, *in prep*

General appeal: use energy balance



Future perspectives: stress

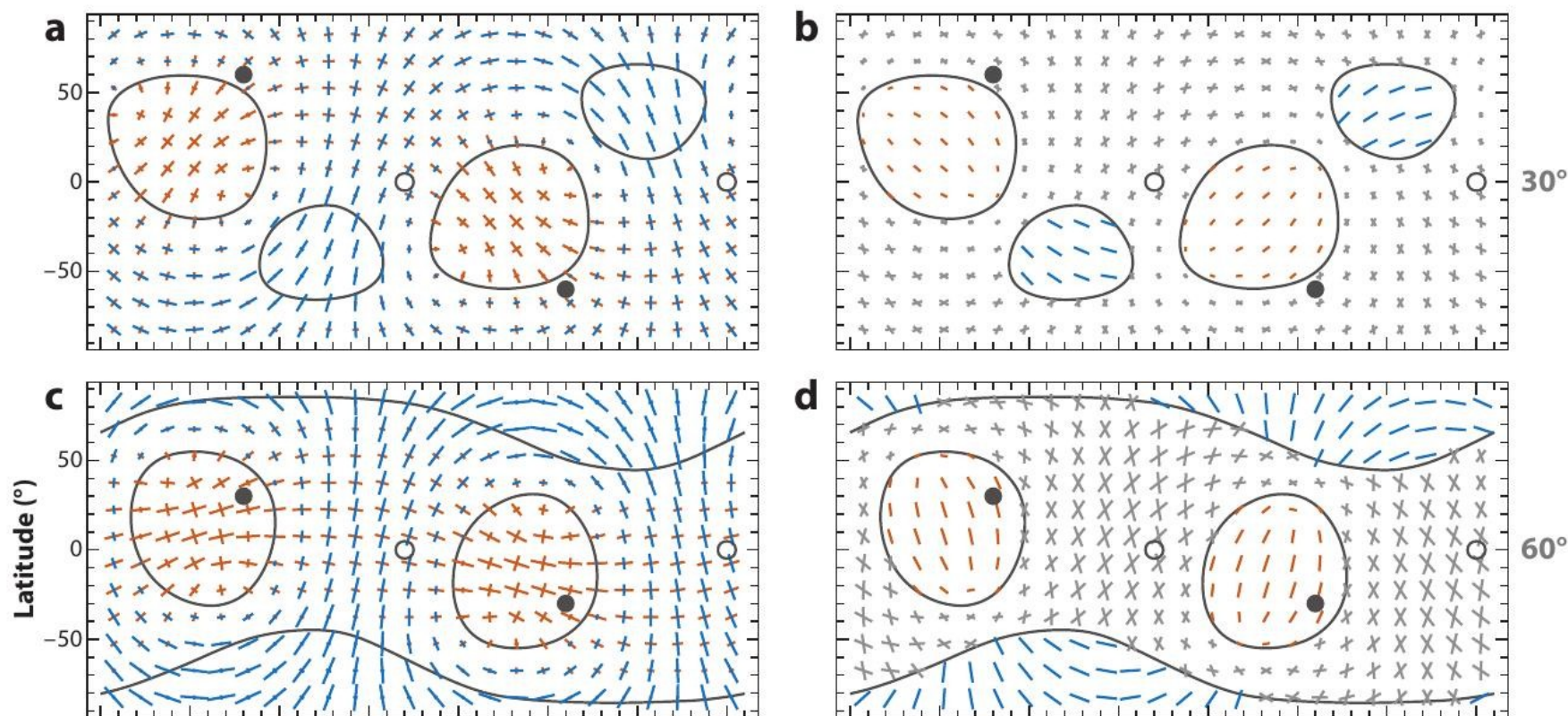
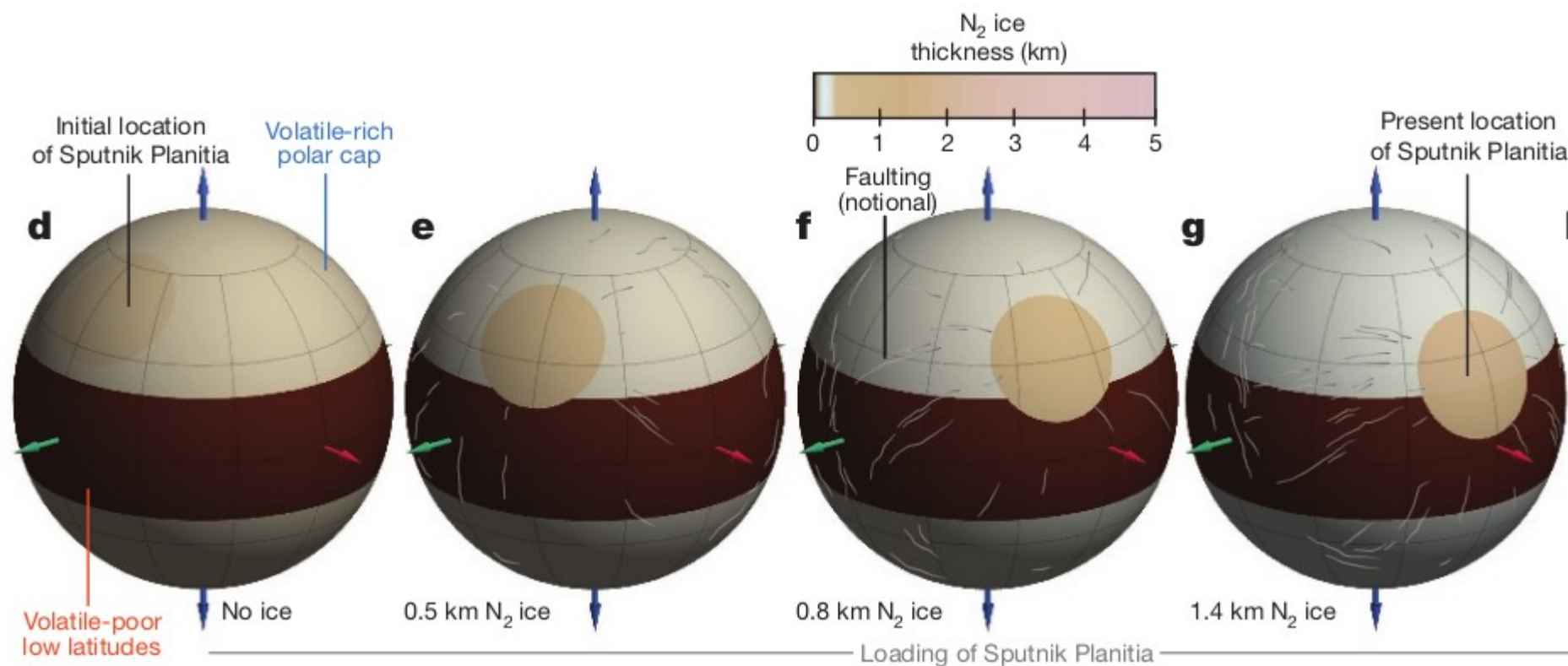


Figure 6

Predicted global-scale stress and tectonic patterns due to polar wander for a rotationally and tidally deformed planetary body. We assume different amounts of polar wander, as labeled, along the 80° meridian. Solid circles indicate the locations of the initial rotation poles, and open circles indicate the locations of the initial tidal poles (the poles associated with the initial tidal axis). In the stress pattern panels (*left column*), crosses indicate directions and relative magnitudes of principal stresses, and orange and blue lines correspond to extensional and compressive stresses, respectively. In the tectonic pattern panels (*right column*), the orange, blue, and light gray lines indicate the strike of the expected normal, thrust, and strike-slip faults, respectively. The length of the lines is proportional to the maximum shear stress. Solid dark gray lines mark the boundaries between different tectonic regions.

Future perspectives: stress



Keane et al., 2016: Reorientation and faulting of Pluto due to volatile loading within Sputnik Planitia, *Nature*

Equilibrium orientation

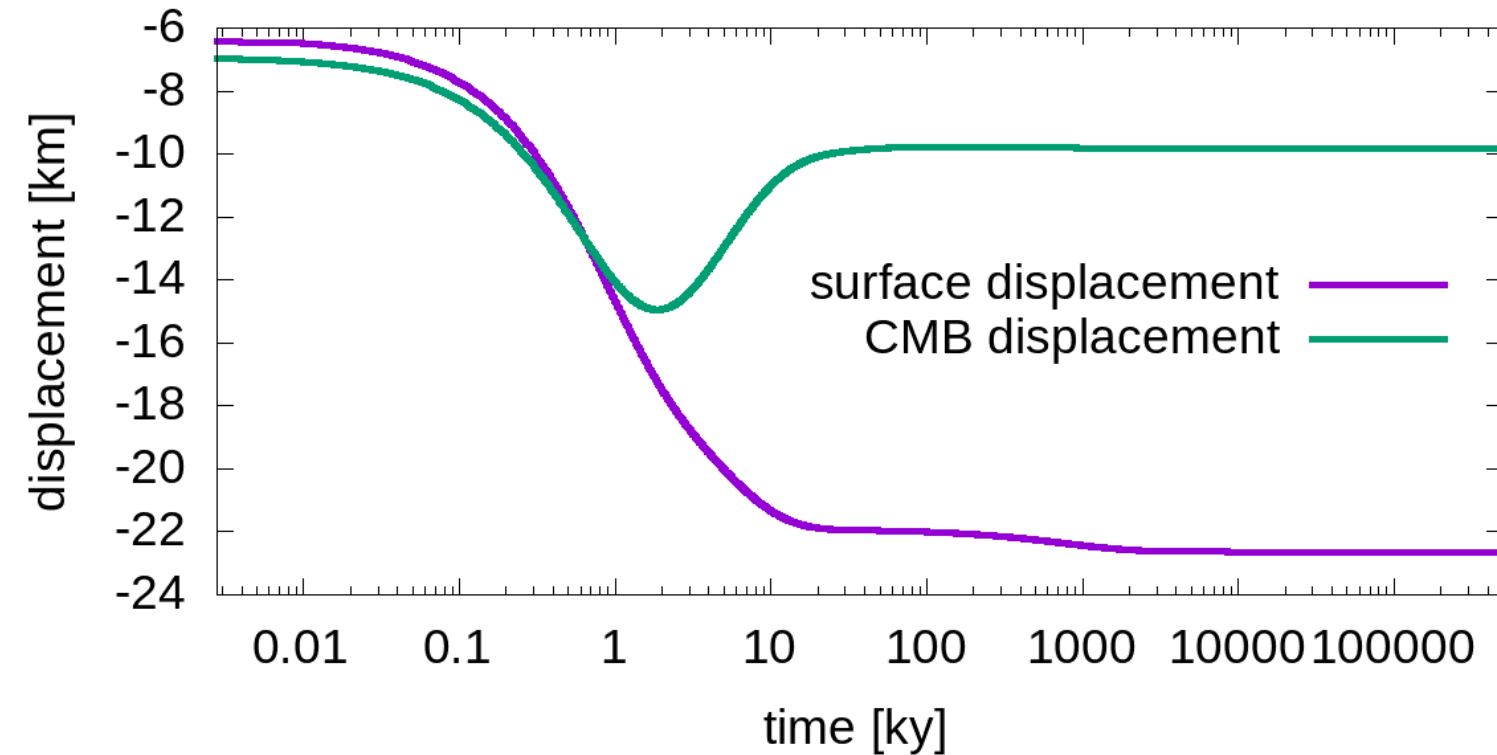
- “quasi-steady” dynamics: $t_{\text{relax}} \ll t_{\text{load}} \ll t_{\text{stress}}$

Time? Quiz?

Conclusions

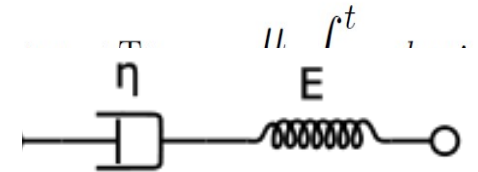
- The “ow||mMIA” method – at your command
- TPW rates can vary by orders of magnitude and can be checked
- Positive and negative loads act differently as M_h/M decreases

TPW: methods overview



$$\nabla \cdot \tau + f = 0,$$

$$\nabla \cdot u = 0,$$



$$k^T(s) = k_e^T + \sum_{i=1}^M \frac{k_i^T}{s - s_i}$$

$$0 = \frac{d}{dt}(\mathbf{I} \cdot \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega})$$

Lefftz et al. (1991)

$$\approx k_e^T - \sum_{i=1}^M \frac{k_i^T}{s_i} \approx k_e^T - \sum_{i=1}^M \frac{k_i^T}{s_i} = k_f^T (1 - T_1 s)$$

$$\boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) = 0$$