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**Deformation Mechanisms and
Convection in Ice I**

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Ph.D.

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Title: Deformation Mechanisms and Convection in Ice I

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Abstract: In recent decades, icy moons have become the centre of attention because of the possible existence of conditions favourable to life, such as long-term stable subsurface oceans. Whether such conditions exist depends largely on the bodies' thermal evolution, linked to the heat transfer efficiency through the uppermost ice shell. To assess the heat transfer efficiency, it is necessary to characterise the mechanisms responsible for the deformation in the ice. Experimental studies suggest multiple mechanisms contributing to the deformation of the ice. We use a rheological model informed by these studies to identify the prevailing deformation mechanism and to explore how the ice deforms over a range of stress, temperature, and grain size. Using numerical modelling and community code ASPECT, we perform a parametric study with a model inspired by Europa to investigate how the thermal evolution of an ice shell depends on the ice shell thickness, grain size and gravitational field strength. We conclude that the thermal evolution is highly sensitive to these parameters, with relatively small changes significantly affecting the result.

Keywords: subsolidus convection, deformation mechanisms, ice I, numerical simulations

Název práce: Deformační mechanismy a konvekce v ledu I

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Abstrakt: Ledové měsíce se v posledních desetiletích dostaly do popředí zájmu díky možnosti existence podmínek vhodných k životu, jako je např. dlouhodobě stabilní podpovrchový oceán. Zda takové podmínky skutečně existují závisí na jejich termálním vývoji, který je určen efektivitou přenosu tepla ve svrchní ledové vrstvě. K posouzení účinnosti přenosu tepla je nutné charakterizovat mechanismy zodpovědné za deformaci ledu. Z experimentálních studií plyne, že na deformaci ledu se podílí několik mechanismů. Pomocí reologického modelu založeném na těchto studiích jsme určili převládající mechanismus deformace a prozkoumali deformaci ledu pro různá napětí, teploty a velikosti zrna. Pomocí metod numerického modelování a volně dostupného kódu ASPECT jsme provedli parametrickou studii s inspirovanou strukturou Europoy. Cílem bylo prozkoumání závislosti termálního vývoje ledové slupky na její tloušťce, velikosti zrna a gravitačním zrychlení. Výsledky ukazují, že termální vývoj je na tyto parametry velmi citlivý, kdy jejich relativně malé změny výrazně ovlivní výsledek.

Klíčová slova: konvekce, deformační mechanismy, led I, numerické modelování

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Introduction

Water ice is one of the most common frozen volatiles in our Solar System. Beyond the snow line, now roughly corresponding with the main asteroid belt, the solar radiation is weak and temperatures are low enough for the water vapour to condense into a solid. This allowed water ice to become one of the dominant components of smaller bodies in the outer parts of the Solar System, where it has been observed on the surface of almost all objects.

The number of known phases of ice currently exceeds 20 (Hansen, 2021). Among them, the low-pressure phase of ice Ih is the most prevalent under the near-surface conditions on these bodies. The surfaces of these bodies, with a wide range of surface features, imply their very diverse history and evolution. Some, like the Jovian moon Callisto, have old surfaces covered with craters without a sign of activity. Others, like Jovian Europa or Saturnian Enceladus, have young surfaces with, e.g., double ridges, faulting systems, ridged planes or chaos terrain, pointing towards current or recent activity.

Understanding the thermal evolution of these bodies is essential for determining their inner structure, as well as how our Solar System formed. Thermal evolution depends on the efficiency of heat transfer from the interior through the ice shell. That can happen either by heat conduction or convection. Transfer by conduction occurs anytime there is a temperature gradient present, whereas for heat to be transferred by the more efficient convection—a combination of conduction and matter advection—the Rayleigh number has to be larger than the critical value. The convection not only enhances the cooling of the satellite, but the convective layer, located in the warmer lower parts of the ice shell, puts stress on the upper parts, which can lead to material failure and the creation of surface geological features. Thermal evolution of icy moons also closely relates to the much debated possibility of existence of subsurface oceans on larger icy moons which may be able to harbour life. The three icy Galilean moons are currently the target of two upcoming missions—ESA’s Juice and NASA’s Europa Clipper—among whose main objectives is to determine whether such favourable conditions may exist on them.

Since the deformation is involved in the heat transfer, the understanding of its mechanisms is essential for determining how ice shell evolves and how heat is transferred through it.

1 Rheology

Ice in the shell is subjected to stress caused by buoyant force arising from the density anomalies compared to the background density profile, gravity, tidal forces, etc. How ice behaves under stress on geological timescales is important in determining the efficiency and mode of heat transport. Central to this are deformation mechanisms by which the ice deforms under stress—they affect the viscosity, which in turn affects the Rayleigh number that determines whether the heat will be transported by convection. There are several of them operating in ice, and each depends differently on stress as well as other conditions such as temperature or grain size.

The deformation of the ice shell is mostly strain-independent. In this regime, the rheological relationship can be described by (Goldsby and Kohlstedt, 2001)

$$\dot{\gamma} = \tilde{A} d^{-p} \sigma_d^n \exp\left(-\frac{Q + PV}{RT}\right), \quad (1.1)$$

where $\dot{\gamma}$ is the ordinary strain rate, σ_d is the differential stress, $\tilde{A}$ is a preexponential parameter, T is the temperature, d is the grain size, Q is the activation energy, V is the activation volume, P is the hydrostatic pressure, and R is the gas constant. The preexponential parameter can further depend on the state variables, such as temperature, i.e., $\tilde{A} = \tilde{A}(T)$. The relationship, therefore, expresses dependence on grain size for $p > 0$, Arrhenius dependence on temperature, and possible non-linear dependence on stress for $n > 1$.

In numerical modelling, the ordinary strain rate $\dot{\gamma}$ and the differential stress σ_d are usually replaced with different scalar representations of the respective tensor quantities. Instead of the ordinary strain rate, the second invariant of deviatoric strain rate is used, defined as $\dot{\epsilon}'_{\text{II}} = (1/2 \dot{\epsilon}'_{ij} \dot{\epsilon}'_{ij})^{1/2}$, where $\dot{\epsilon}'_{ij}$ are elements of the deviatoric strain rate tensor $\dot{\epsilon}'$. Similarly, instead of the differential stress tensor σ_d , the second invariant of deviatoric stress tensor is used, defined as $\sigma'_{\text{II}} = (1/2 \sigma'_{ij} \sigma'_{ij})^{1/2}$, where σ'_{ij} are elements of the deviatoric stress tensor σ' . The relationship between these two representations depends on the geometry of the experiment from which the ordinary strain rate and differential stress are derived. For the axial compression experiment, which is often used to determine the properties of ice and rock under stress, the relations between the two representations are $\sigma_d = \sqrt{3} \sigma'_{\text{II}}$ and $\dot{\gamma} = (2/\sqrt{3}) \dot{\epsilon}'_{\text{II}}$ (Gerya, 2010). If we also assume that the preexponential parameter $\tilde{A}$ depends on the l -th power of the temperature, the rheological relationship eq. (1.1) can be rewritten as

$$\dot{\epsilon}'_{\text{II}} = A T^{-l} d^{-p} \sigma'_{\text{II}}{}^n \exp\left(-\frac{Q + PV}{RT}\right), \quad (1.2)$$

where

$$A T^{-l} = \frac{3^{(n+1)/2}}{2} \tilde{A}. \quad (1.3)$$

For the purposes of numerical modelling, the rheological relationship needs to be reformulated in terms of effective viscosity, defined as

$$\eta_{\text{eff}} = \frac{\sigma'_{\text{II}}}{2 \dot{\epsilon}'_{\text{II}}}. \quad (1.4)$$

Using the eq. (1.2), the effective viscosity can be written as a function of either σ'_{II} or $\dot{\epsilon}'_{\text{II}}$. We have used the formulation in terms of the second invariant of strain rate. The effective viscosity is then

$$\eta_{\text{eff}} = \frac{1}{2} A^{-1/n} T^{l/p} d^{p/n} (\dot{\epsilon}'_{\text{II}})^{(1-n)/n} \exp\left(\frac{Q + PV}{nRT}\right). \quad (1.5)$$

1.1 Deformation mechanisms

A single ice crystal is highly anisotropic due to its hexagonal structure. Here, we consider polycrystalline ice, which consists of many individual ice crystals (grains). In an idealised case, which we consider here, the ice crystals are randomly oriented, so the anisotropy of individual crystals averages out on a macroscopic scale, making the polycrystalline ice behave isotropically.

During the ductile deformation, several mechanisms contribute to the total strain rate. Their relative importance depends on multiple factors such as applied stress, temperature, and grain size. This section describes the primary mechanisms responsible for the deformation of ice.

Dislocation creep

At high stress, the ice deforms primarily by the motion of dislocations. Dislocations can either glide along their glide planes, which can be either basal or non-basal planes, or climb by diffusion. Both processes are needed during the secondary creep regime (Durham et al., 2010).

Dislocation creep depends strongly on stress, with a stress exponent $n = 4.0$, and is independent of grain size. At temperatures close to the melting point, a so-called premelting occurs, which enhances the creep. The premelting happens when the ice melts at local pressure variations that can develop, e.g., at grain boundaries.

Grain-boundary sliding and basal slip

The activation of glide on non-basal planes requires high stress, which makes dislocation creep inefficient as the stress decreases. Instead, basal slip—gliding dislocations along the basal plane—together with grain boundary sliding (GBS), contribute the most to the total strain rate. The ice can deform in two different regimes, depending on which of the two processes limits the deformation.

When GBS is the slower mechanism, ice deforms primarily under GBS-accommodated basal slip regime characterised by $n = 1.8$ and grain size dependence $p = 1.4$. During the GBS-accommodated creep, the effect of premelting was also measured. Conversely, when the basal slip is less efficient, the deformation enters a basal slip-accommodated GBS regime. Here, the strain rate is governed primarily by the basal slip rate, which is relatively insensitive to grain size, and the stress exponent is approximately $n = 2.4$.

Diffusion creeps

At lower stress, diffusion creep is expected to be dominant. However, the transition to diffusion creep is highly sensitive to grain size. The diffusion can happen through either the interior of the grain or along the grain boundaries. Direct experimental observations of diffusion creep in ice are limited. Thus, the creep equations are derived from first principles, supported by experimental data from other materials. Calculations yield different results for different grain shapes and magnitudes of strain. For infinitesimal strains and spherical grains, the equations are as follows (Kohlstedt, 2007)

$$\dot{\epsilon}_{\text{II,v. diff}} = 21 \frac{V_m \sigma'_{\text{II}}}{RT d^2} D_{0v} \exp \left(-\frac{Q_v + PV}{RT} \right), \quad (1.6)$$

$$\dot{\epsilon}_{\text{II,gb. diff}} = 21 \pi \frac{\delta V_m \sigma'_{\text{II}}}{RT d^3} D_{0b} \exp \left(-\frac{Q_b + PV}{RT} \right), \quad (1.7)$$

for the volume diffusion and the grain-boundary diffusion, respectively. V_m is the molar volume, δ is the grain boundary width, and D_{0i} is a material-dependent preexponential parameter. Equations (1.6) and (1.7) can be rewritten in the manner of eq. (1.2) when all the constants are included in the preexponential factor A . For the volume diffusion $A_{\text{v. diff}} = 21 V_m D_{0v} / R$ and for the grain-boundary diffusion $A_{\text{gb. diff}} = 21 \pi \delta V_m D_{0b} / R$.

For larger strains, the processes involved in diffusional creep become more complicated and the strain rate increases. Ashby et al. (1978) derived lower and upper bounds for strain rates of both grain-boundary and volume diffusion creeps. When averaged, the lower and upper bounds are 1.7 and 3.4 times the strain rate at zero (infinitesimally small) strain, respectively, for the volume diffusion and 1.7 and 5.5 times the strain rate at zero strain, respectively, for the grain-boundary diffusion. Considering the stress, and thus the strain rate, during the deformation is very low, the infinitesimal derivation of the creep law is good even for large strains.

1.2 Composite rheology

The total strain rate produced by all mechanisms operating when stress is applied is described by (Goldsby and Kohlstedt, 2001)

$$\dot{\epsilon}_{\text{total}} = \dot{\epsilon}_{\text{v. diff}} + \dot{\epsilon}_{\text{gb. diff}} + \left(\dot{\epsilon}_{\text{GBS}}^{-1} + \dot{\epsilon}_{\text{bs}}^{-1} \right)^{-1} + \dot{\epsilon}_{\text{disl}}, \quad (1.8)$$

where the subscripts denote volume diffusion creep (v. diff), grain-boundary diffusion creep (gb. diff), GBS-accommodated basal slip (GBS), basal slip-accommodated GBS (bs), and dislocation creep (disl). Strain rates are added together if the mechanisms are independent. If the mechanisms are mutually dependent, the reciprocal values of the strain rates they produce are added. This is the case for GBS-accommodated basal slip and basal slip-accommodated GBS.

The creep parameters were taken from Goldsby and Kohlstedt (2001) and are summarised in table 1.1. The premelting activation energies of dislocation creep and GBS-accommodated basal slip were modified to ensure the continuity of the strain rate as a function of T and σ'_{II} . Goldsby and Kohlstedt (2001) do not list

uncertainties with their results. However, considering the difficulty of measurement near the melting temperature of ice, together with the used extrapolation and the uncertainties of creeps at lower temperatures as described by Durham and Stern (2001), we inferred that our slight modification of activation energies falls within the ranges introduced by uncertainties.

Using the definition of effective viscosity, the eq. (1.8) can be rewritten as

$$\eta_{\text{total}}^{-1} = \eta_{\text{v. diff}}^{-1} + \eta_{\text{gb. diff}}^{-1} + (\eta_{\text{GBS}} + \eta_{\text{bs}}) + \eta_{\text{disl}}^{-1}. \quad (1.9)$$

To ensure convergence of the solution of the model, the possible values of effective viscosity are often limited by the maximum allowed value η_{max} . To reflect that in the effective viscosity equation, the eq. (1.9) is modified to

$$\eta_{\text{total}}^{-1} = \eta_{\text{v. diff}}^{-1} + \eta_{\text{gb. diff}}^{-1} + (\eta_{\text{GBS}} + \eta_{\text{bs}}) + \eta_{\text{disl}}^{-1} + \eta_{\text{max}}^{-1}. \quad (1.10)$$

Table 1.1 Creep parameters adapted from Goldsby and Kohlstedt (2001), modified (see text).

	$A/\text{K}^l \text{m}^p \text{Pa}^{-n} \text{s}^{-1}$	l	p	n	$Q/\text{kJ mol}^{-1}$
volume diffusion creep	4.528×10^{-8}	1.0	2.0	1.0	59.4
grain-boundary diffusion creep	8.196×10^{-17}	1.0	3.0	1.0	49.0
GBS-accom. basal slip ($T < 255 \text{ K}$)	1.439×10^{-13}	0.0	1.4	1.8	49.0
GBS-accom. basal slip ($T > 255 \text{ K}$)	1.107×10^{16}	0.0	1.4	1.8	190.0
basal slip-accom. GBS	7.087×10^{-7}	0.0	0.0	2.4	60.0
dislocation creep ($T < 258 \text{ K}$)	3.118×10^{-18}	0.0	0.0	4.0	61.0
dislocation creep ($T > 258 \text{ K}$)	4.677×10^5	0.0	0.0	4.0	175.5

1.2.1 Deformation mechanism maps

Deformation mechanisms maps are constructed to visualise under what conditions different creep mechanisms dominate. Several of such maps for ice Ih are shown in Fig. 1.1.

The deformation mechanism maps were constructed by calculating the strain rate associated with each mechanism using eq. (1.2) with parameters from table 1.1 over a range of stress (10^{-5} MPa to 10^3 MPa) and temperature (100 K to 270 K) with grain size kept constant at 0.1 mm, 0.2 mm, 0.5 mm, 1.0 mm, 2.0 mm and 5.0 mm. The grey lines are selected isolines of the total strain rate obtained by plugging all computed strain rates into eq. (1.8). The thick black lines represent boundaries between the regions where different deformation mechanisms control the strain rate. The computed strain rates were compared at each point of the created temperature-stress grid to determine the threshold values, where the dominant deformation mechanism changes. The dominant mechanism is the one

contributing the most to the total strain rate. An independent mechanism is dominant if it has the highest strain rate of all the contributions. On the contrary, out of mutually dependent mechanisms, the one contributing the lowest strain rate of all dominates the deformation. Thus, among independent mechanisms, the dominant mechanism was chosen to be the one with the highest strain rate. In regions where combined GBS or basal slip creep was dominant, a distinction was made between areas dominated by GBS-accommodated basal slip and basal slip-accommodated GBS by comparing their respective strain rates and choosing the mechanism limiting the deformation.

The selected temperature interval roughly corresponds to the temperatures expected in the ice shells in the ice Ih stability region. The upper bound represents the melting temperature of the ice, while the lower bound approximately corresponds to the estimate of the average surface temperature for the Jupiter system. Thus, Fig. 1.1 indicates which mechanisms are likely to dominate under the conditions in the ice shell. The grain-boundary diffusion creep is dominant only in very fine-grained ice at low temperatures and stress. Thus, it will most likely not contribute to the deformation as a dominant mechanism under the conditions corresponding to icy satellites. The dislocation creep requires high stress to dominate, which makes it also unlikely to be dominant. This leaves the volume diffusion creep and the creeps involving GBS and basal slip.

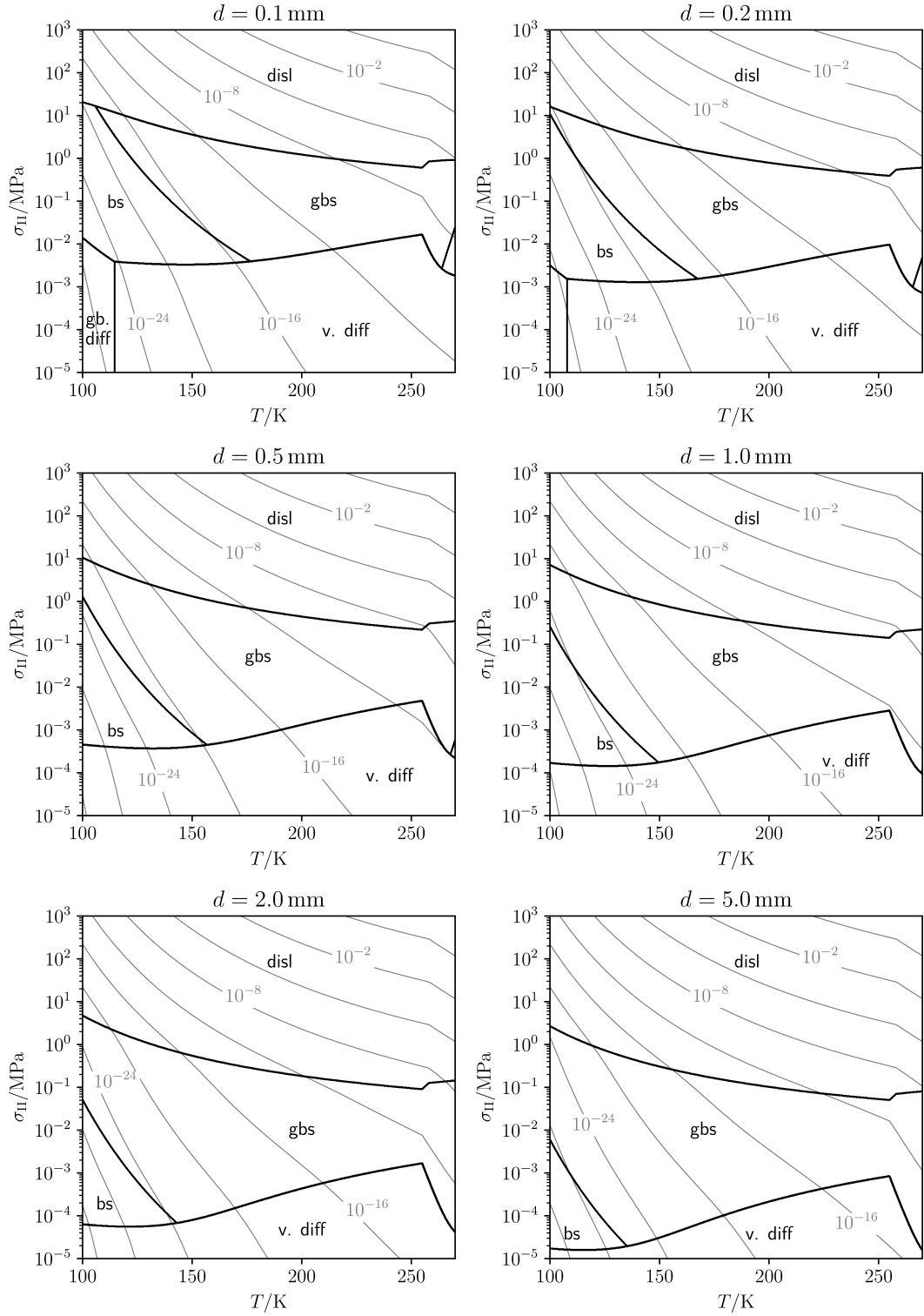


Figure 1.1 Deformation mechanisms maps for several grain sizes over a selection of the stress-temperature domain.

2 Governing equations and modelling approach

In this chapter, we describe the equations governing the convection, the software used for the numerical simulations, and the setup of the simulated models.

2.1 Approximation of the governing equations

To approximate the governing equations, we used modified incompressible extended Boussinesq approximation with no internal heating sources to consider full density everywhere (instead of only in the buoyancy term in momentum equation), temperature-dependent material parameters, and assume compressible material in viscous dissipation and adiabatic heating terms in energy conservation equation. Thus, the material derivative of the density is zero and the continuity equation takes the following form

$$\nabla \cdot \vec{v} = 0, \quad (2.1)$$

where $\vec{v}$ is the velocity. Assuming the only body force acting on the system is gravity characterised by $\vec{g}$ and neglecting the inertia term (on the right-hand side), the momentum equation is

$$-\nabla p + \nabla \cdot \boldsymbol{\sigma}' + \rho \vec{g} = 0, \quad (2.2)$$

where p is the pressure, $\boldsymbol{\sigma}'$ is the deviatoric stress, and ρ is the density. For the deviatoric stress tensor, we used the following rheological relationship

$$\boldsymbol{\sigma}' = 2\eta_{\text{eff}} \dot{\boldsymbol{\epsilon}}', \quad (2.3)$$

where η_{eff} is the effective viscosity defined by eq. (1.5). Finally, the energy conservation equation is

$$\rho c_p \left(\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T \right) - \nabla \cdot (k \nabla T) = \boldsymbol{\sigma}' : \dot{\boldsymbol{\epsilon}}' + \alpha T (\vec{v} \cdot \nabla p), \quad (2.4)$$

where c_p is the specific heat capacity at constant pressure, T is the temperature, k is the thermal conductivity, and α is the thermal expansivity. On the left-hand side, the second term in parentheses corresponds to the transport of heat due to advection. The last term on the left-hand side describes heat diffusion. On the right-hand side, the first term corresponds to viscous heating, while the second term corresponds to the heat generated by the adiabatic compression/decompression of the material.

To approximate the temperature dependence of the material parameters k , c_p , and α , we used the following approximations. Hobbs (1974)

$$k(T) = k_1 + \frac{k_2}{T}, \quad (2.5)$$

Feistel and Wagner (2006)

$$c_p(T) = c_1 + c_2 T, \quad (2.6)$$

and Röttger et al. (1994)

$$\alpha(T) = \frac{1}{V(T)} \frac{dV(T)}{dT}, \quad (2.7)$$

where

$$V(T) = A_0 + A_3 T^3 + A_4 T^4 + A_5 T^5 + A_6 T^6 + A_7 T^7. \quad (2.8)$$

The constants $A_0 - A_7$, c_1 , c_2 , k_1 , and k_2 are in table 2.1. The temperature dependence of the material parameters is also plotted in Fig. 2.1 for the relevant range of temperatures.

Table 2.1 Coefficients used in approximations of material parameters.

A_0	$128.217 \times 10^{-30} \text{ m}^3$
A_3	$-1.3152 \times 10^{-36} \text{ m}^3 \text{ K}^{-3}$
A_4	$2.4837 \times 10^{-38} \text{ m}^3 \text{ K}^{-4}$
A_5	$-1.6064 \times 10^{-40} \text{ m}^3 \text{ K}^{-5}$
A_6	$4.6097 \times 10^{-43} \text{ m}^3 \text{ K}^{-6}$
A_7	$-4.661 \times 10^{-46} \text{ m}^3 \text{ K}^{-7}$
c_1	$185.0 \text{ J K}^{-1} \text{ kg}^{-1}$
c_2	$7.037 \text{ J K}^{-2} \text{ kg}^{-1}$
k_1	$0.4685 \text{ W m}^{-1} \text{ K}^{-1}$
k_2	488.19 W m^{-1}

The governing equations can also be rewritten in a dimensionless form, where we can introduce several dimensionless numbers. One of them, important for the description of convection, is the Rayleigh number, defined as

$$\text{Ra} = \frac{\alpha \Delta T \rho c_p g D^3}{\eta k}, \quad (2.9)$$

where $\Delta T = T_{\text{bot}} - T_{\text{surf}}$ is the temperature difference between the bottom and the surface of the shell and D is the shell thickness.

2.2 ASPECT

For the numerical modelling, we have used ASPECT, a code for solving the governing equations, see Heister et al. (2017), Kronbichler et al. (2012), Bangerth et al. (2024a), and Bangerth et al. (2024b). It uses a finite element method with adaptive mesh refinement to solve the governing equations.

ASPECT allows users to create plugins to extend the functionality of the code, including new material models. To simulate the ice rheology, we have used a custom material model provided by our advisor.

2.2.1 Time discretisation

The timestep size is computed as (Bangerth et al., 2024b)

$$\Delta t = \text{CFL} \min_k \frac{h_k}{\|v\|_{\infty, k} p_T}, \quad (2.10)$$

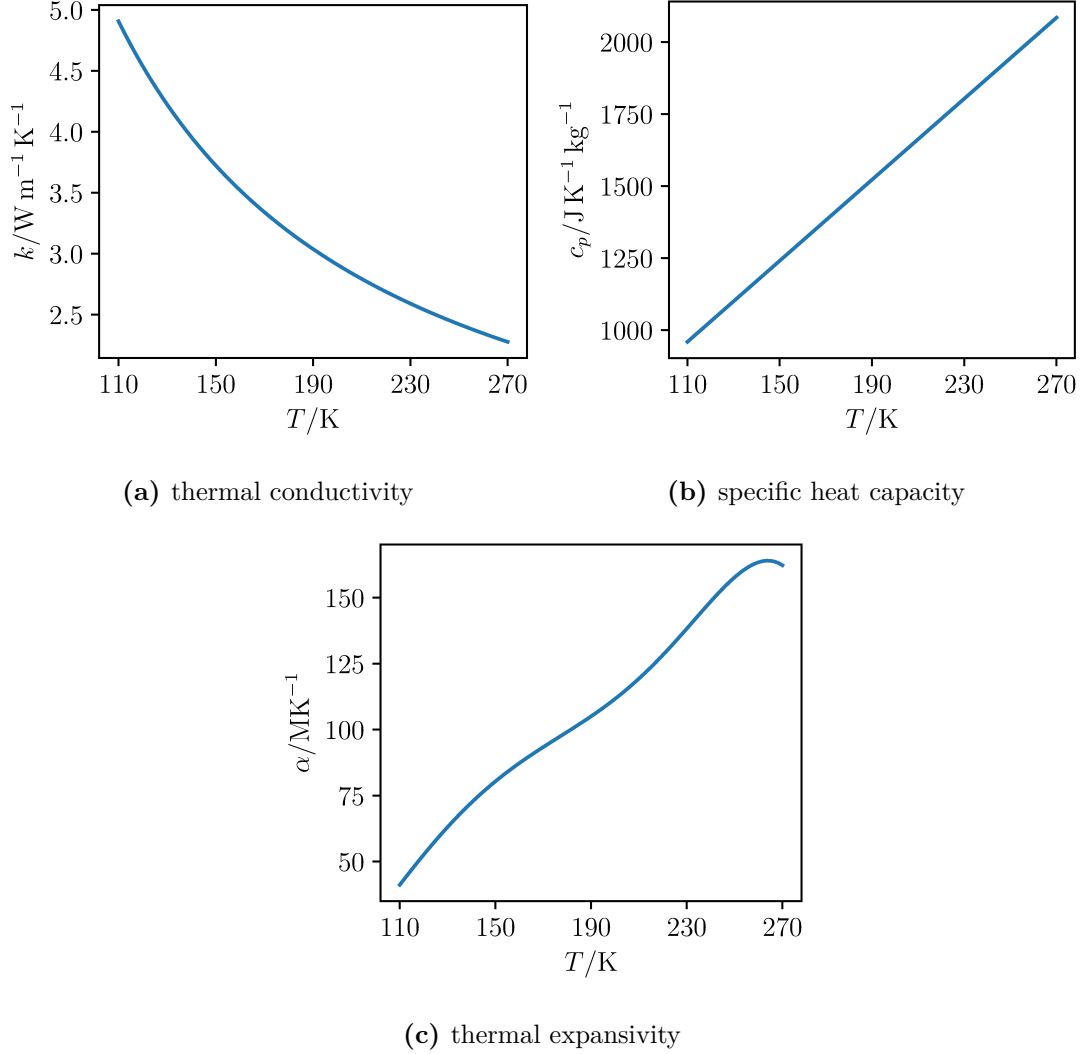


Figure 2.1 Temperature dependence of material parameters.

where k is the index of the cell, h_k is the diameter of the cell, $\|v\|_{\infty,k} = \max(|x^i|)_k$ is the maximal magnitude of the velocity in the cell and p_T is the polynomial degree of the temperature discretisation and CFL is a model parameter.

2.2.2 Spatial discretisation and mesh refinement

ASPECT uses quadrilateral (or hexahedral in 3D) elements on which it discretises and solves the governing equations. At the beginning of the simulation, ASPECT creates a coarse mesh by dividing the initial shape into a grid based on a specified number of subdivisions in each coordinate direction. From there, the mesh can be globally or adaptively refined before and during the simulation. There are many different strategies which can be used to determine which parts of the mesh should be further refined and which can be coarsened.

We have used three criteria. The first flags all cells at the bottom boundary, the second flags cells where there are large variations of strain rate and the third flags cells with large variations of temperature. Each of the criteria produces a refinement indicator for each cell. These are first normalised to the interval $[0, 1]$ and then multiplied by a criterion scaling factor. Finally, for each cell, the

indicators from all the criteria are summed. The cells with the highest refinement indicators are then refined. An example of an adaptively refined mesh using the described criteria is in Fig. 2.2.

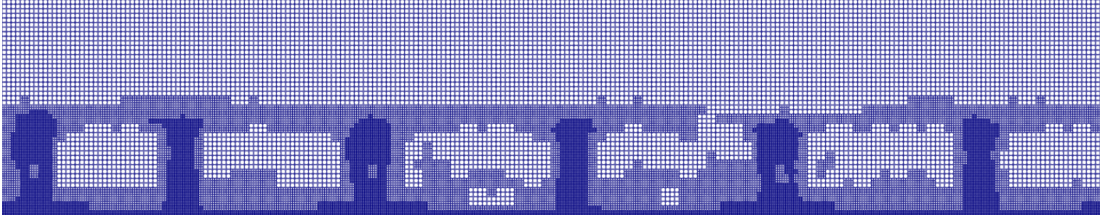


Figure 2.2 Example of an adaptively refined mesh.

The solution of the governing equations is approximated by continuous piecewise polynomial functions of degree n . The polynomial degree of the resultant fields of temperature and velocity can be specified; we used the default setting of $n = 2$ for both temperature and velocity fields. Also, by default, the pressure field solution has a polynomial degree one less than that of velocity; thus, $n = 1$ in our case.

2.3 Models setup

To represent the ice shell, we have used a two-dimensional box geometry of depth D and length $l = xD$, where x is the aspect ratio. At the top and the bottom boundaries, we prescribed free-slip velocity conditions and fixed temperatures—110 K at the surface and the melting temperature of ice, 270 K, at the bottom. The temperature at the top boundary corresponds to the estimate of the average surface temperature for Jupiter’s moon Europa calculated from the Stefan-Boltzmann law. The temperature at the bottom boundary is meant to represent the melting temperature of the ice. We decided to use a simplified approach with bottom temperature constant accross all our models, even though the melting temperature depends on pressure which changes with shell thickness. The error of this approach should be minimal, maybe with the exception of the model with the highest shell thickness, where the melting temperature is about 10 K lower.

At the left and right boundaries, we imposed periodic conditions on both velocity and temperature fields. Alongside these conditions, we also prescribed a zero net horizontal velocity constraint to avoid its non-uniqueness in the momentum equation due to the neglect of the inertia term. The maximum viscosity $\eta_{\max}$ was set to 10^{23} Pa s.

The initial temperature in the box was defined by a function to include temperature perturbations in the linear temperature profile,

$$T(x, y) = \max \left\{ \min \left\{ 10 \cdot \text{rand}() + 100 + \frac{D - y}{D}(270 - 100), 110 \right\}, 270 \right\}, \quad (2.11)$$

where $\text{rand}()$ generates a random number from the interval $(0, 1)$ and D is the thickness of the ice shell. The grain size was set in the beginning and did not vary throughout the simulation.

As the end time we chose the upper bound of the diffusion time,

$$\tau_{\text{diff}} = \frac{\rho_0 c_p D^2}{k}, \quad (2.12)$$

calculated for the specific heat capacity c_p and thermal conductivity k at the bottom of the shell ($T = 270$ K) and the reference density $\rho_0 = 920 \text{ kg m}^{-3}$. The diffusion times used are in table 2.2.

The general model setup is sketched in Fig. 2.3. Table 2.3 lists all models and their parameters. In section A.1 is an ASPECT configuration file for the reference model.

Table 2.2 Upper bounds for diffusion times τ_{diff} for different shell thicknesses D used as end times in simulations.

D/km	$\tau_{\text{diff}}/\text{Myr}$
10	2.6
20	10.5
21	11.8
22	12.8
25	16.5
30	24.0
50	66.0
100	264.0

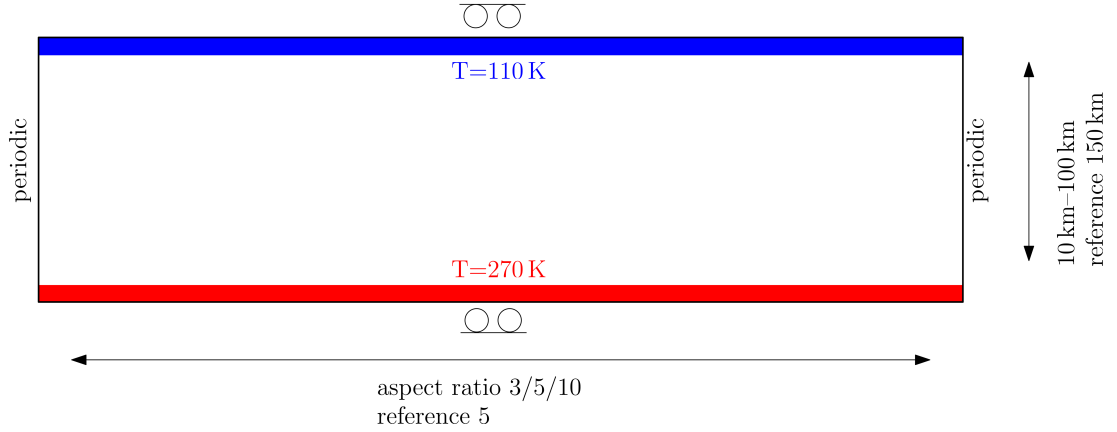


Figure 2.3 Model setup

Table 2.3 Models setup.

shell thickness/km	grain size/mm	aspect ratio	gravity	end time/yr	maximum refinement
10	0.5	1:5	1.3	2.6	64
20	0.5	1:5	1.3	10.5	64
21	0.5	1:5	1.3	11.8	64
22	0.5	1:5	1.3	12.8	64
25	0.5	1:5	1.3	16.5	64
30	0.5	1:5	1.3	24.0	96
50	0.5	1:5	1.3	66.0	96
100	0.5	1:5	1.3	264.0 †	192
30	0.3	1:5	1.3	24.0	96
30	0.8	1:5	1.3	24.0	64
30	0.9	1:5	1.3	24.0	64
30	1.0	1:5	1.3	24.0	64
30	0.5	1:3	1.3	24.0	96
30	0.5	1:10	1.3	24.0	96
30	0.5	1:5	2.6	24.0	96

† simulation was stopped sooner at 44.8 Myr due to time considerations (see section 3.4).

3 Results

This chapter focuses on the simulation results. First, we describe the parameters with which we describe the results. Then, we examine in detail the results of the reference model in section 3.2 and how the results are affected by the change of the aspect ratio. The following sections 3.3 to 3.5 focus on the sensitivity of results to changes in grain size, shell thickness, and gravity.

3.1 Results characterisation

Let us first define several terms we use throughout the chapter that relate to the evolution of the ice shell and processes therein. By lithosphere we mean the part of the shell characterised by an approximately linear temperature field depth-profile, which results in viscosity changing exponentially (see eq. (1.5)). Additionally, the lithosphere is characterised by low velocity and strain rate. In a purely conductive case, the lithosphere coincides with the whole shell. In the case of ongoing convection, the definition of lithosphere is tied to the definition of the convective zone and boundary layers.

A convective zone (or layer) is characterised by an almost constant temperature in the depth-averaged temperature profile. A convective zone is composed of a series of convective cells. To describe the placement of convective cells in the zone, we use their relative wavelength defined as x/n , where x denotes the relative length of the horizontal boundaries and n is the number of convective cells. Adjacent to the convective zone are boundary layers, separating the convective layer from the bottom boundary on one side and the lithosphere on the other. Temperature changes in boundary layers are strongly nonlinear and lead to large changes in viscosity.

Related to the concept of convective layer is convective temperature T_c , which we define as the average of the temperature depth profile over the convective zone. Analogously, we can also determine the logarithm of convective viscosity $\log \eta_c$ as the average of the logarithm of the viscosity depth profile over the convective zone. Knowing the convective temperature and convective viscosity, it is possible to calculate convective Rayleigh number Ra_c by plugging the reference density ρ_0 , convective viscosity η_c and material parameters at temperature T_c to eq. (2.9).

As a system develops, it tends towards an equilibrium. This equilibrium is defined by 1) an equality in heat influx (through the bottom boundary) and the heat outflux (through the top boundary), and 2) heat fluxes that are constant in time. A chaotic system usually does not arrive at a perfect equilibrium, but a statistical equilibrium, where time-averaged heat fluxes satisfy the definition above, but heat fluxes alone (especially the heat influx) may fluctuate.

Since our models are often chaotic, describing them in terms of instantaneous quantities, such as temperature or heat flux, would be unphysical. Instead, we shall use time-averaged quantities over approximately¹ the last 1 Myr. The affected quantities are heat influx, heat outflux, and all depth profiles. Thus,

¹The non-exactness arises from the time stepping, which means the exact time of 1 Myr before the end of the simulation is usually stepped over.

when referencing them, we will mean the time-averaged quantities by default, not the instantaneous ones.

Since the temperature profile was too coarse in most cases, we resampled² and smoothed³ it to make the detection of the convective layer more convenient. Then we refined the data again to a resolution high enough that linear interpolation between the points was not necessary when determining the extent of the upper boundary layer. The position of the convective layer and the convective temperature were determined together iteratively. First, an estimate of the convective zone position was made by assigning it the part of the temperature depth profile where

$$\frac{d\bar{T}}{dy} > -1 \text{ K km}^{-1}. \quad (3.1)$$

Then, the convective temperature was calculated as the average over this zone. If there were areas at convective temperature outside of the convective zone, the affected boundary was shifted accordingly. Then, a new value of convective temperature was calculated. This process was repeated as long as there were areas of convective temperature outside the convective zone.

With the extent of the convective layer determined, the logarithm of convective viscosity was calculated again as an average over the convective layer from a depth profile, which was obtained by the same process as for the temperature profile. Then, we denoted the lower boundary layer as the layer separating the convective zone from the bottom boundary. We defined the boundary layer at the top of the convective zone as the region where the temperature difference between its upper and lower boundaries matches that of the lower boundary layer. Finally, the lithosphere width of a convective model was calculated as

$$D_{\text{lith}} = D - (D_{\text{conv.}} + D_{\text{bound.top}} + D_{\text{bound.bot.}}), \quad (3.2)$$

where $D_{\text{conv.}}$ is the thickness of the convective layer and $D_{\text{bound.top}}$ and $D_{\text{bound.bot.}}$ are the thicknesses of the upper and lower boundary layers, respectively.

3.2 Reference model

For a reference model, we chose a grain size of 0.5 mm, shell thickness of 30 km and aspect ratio of 1:5. We chose such parameters to ensure the reference model exhibits convection while maintaining physically reasonable parameters.

The simulation started from a perturbed linear field, as was described in section 2.3. A perturbed temperature field also implies perturbed density field, which lead to the initiation of convection and a subsequent development of convective cells, as shown in Fig. 3.1. Since convection is much more effective in heat transport than conduction, there is an excess heat in the developing convective layer, which leads to its growth, as can be seen also in Fig. 3.2 This imbalance of heat influx and outflux is shown in Fig. 3.3—the heat influx through the bottom boundary shortly after the start of the simulation is much larger

²For resampling we used the function `interp1d` from the Python's SciPy module `interpolate`.

³For smoothing we used the function `savgol_filter` from the Python's SciPy module `signal` with the window length of 5 and polynomial order of 1 in the mode `'nearest'`.

than the heat outflux through the top boundary. The convective cells and the convective zone grow until a (statistical) steady state is reached. As the convective zone grows in thickness, the convective cells gradually merge. This change of configuration is accompanied by rapid changes in heat flux through the bottom boundary, as shown in Fig. 3.3, where the configuration change corresponds to spikes in the heat flux function. Simultaneously, on top of the convective layer, a cold, rigid lithosphere develops, where heat is transported by conduction only. Velocity and strain rate in the lithosphere are very low, as is the surface velocity, corresponding to the stagnant-lid convection.

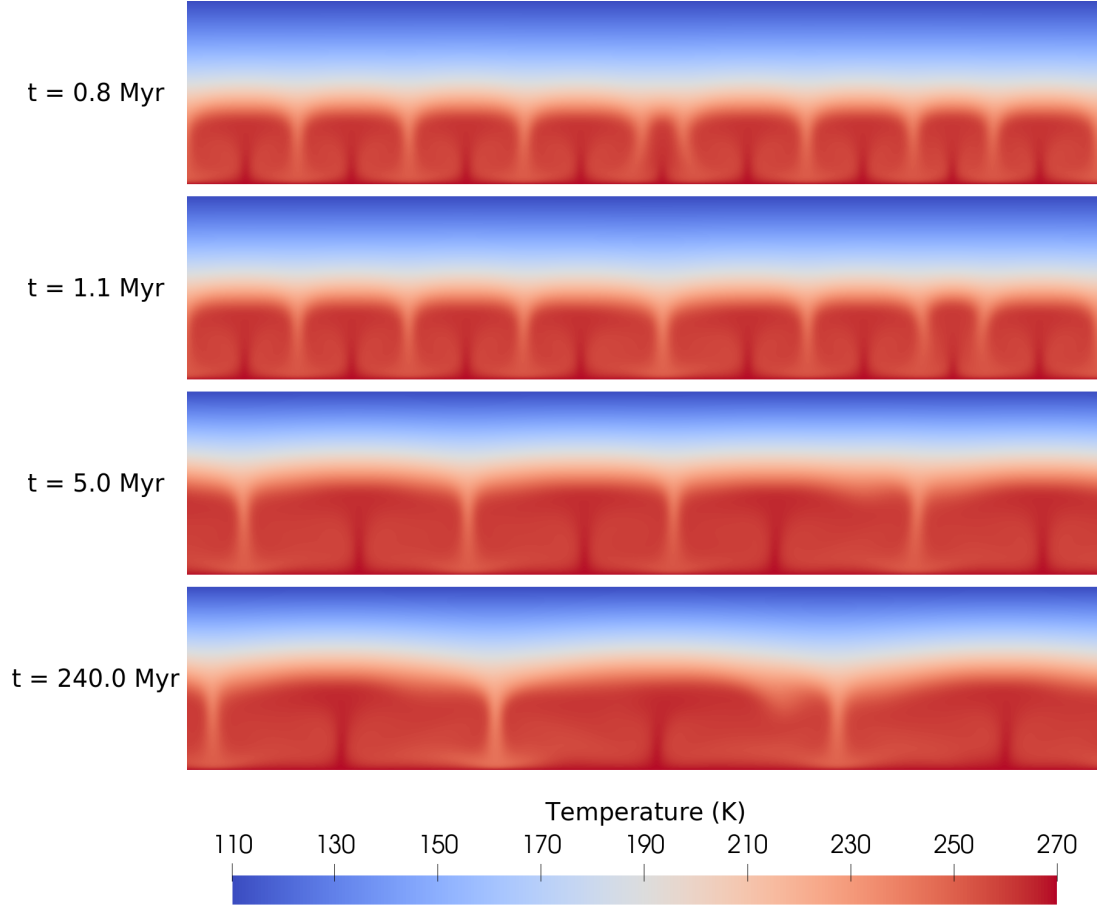


Figure 3.1 Evolution of the temperature field for the reference model.

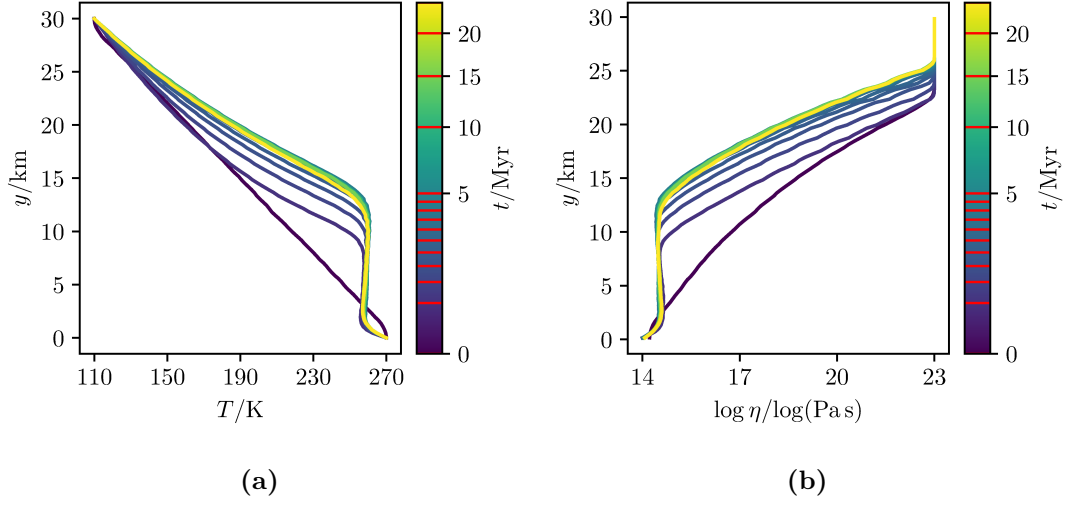


Figure 3.2 Temperature (a) and viscosity (b) depth profiles at selected times.

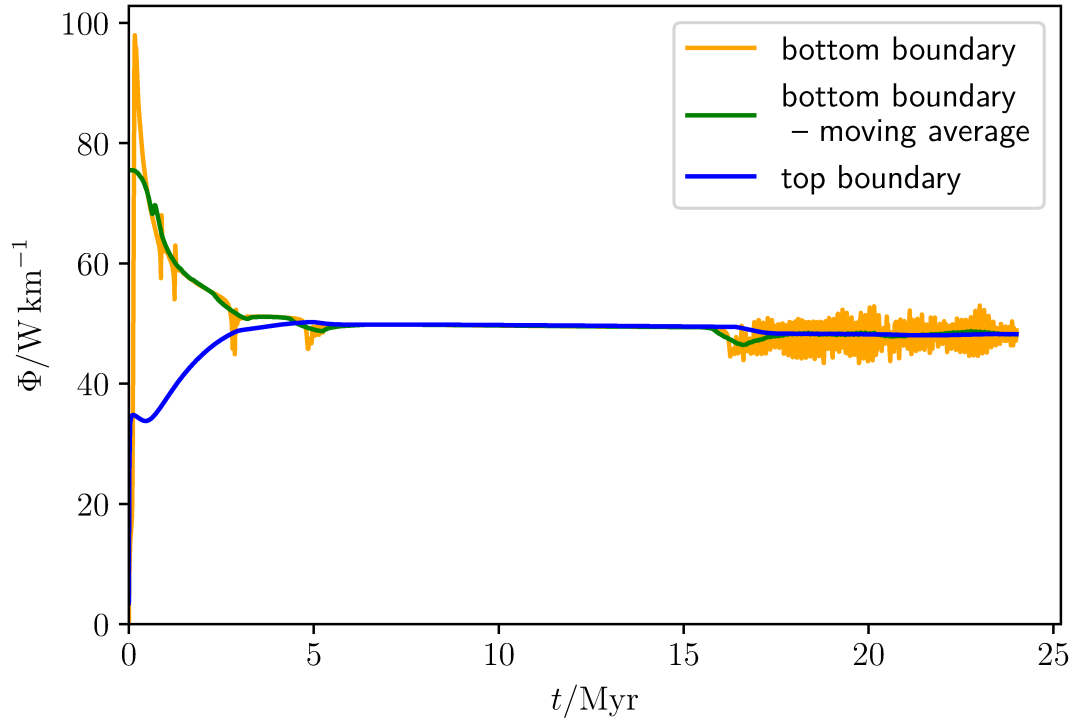


Figure 3.3 Time evolution of heat influx (through the bottom boundary) and outflux (through the top boundary). The moving average of heat influx was calculated using the function `uniform_filter1d` from Python's SciPy module `ndimage`.

3.2.1 End state

The simulation was stopped after 24 Myr. As Fig. 3.3 shows, the system converged into a statistical steady state. The convection occurs in three convective cells, making the relative convective cell wavelength 1.7. The convective zone thickness is 9.85 km, and the lithosphere thickness is 16.56 km, or 0.33 and 0.55, relative to the shell thickness, respectively. The convective Rayleigh number is 351.67. The end state fields of chosen quantities are shown in Fig. 3.4.

In most places in the convective zone, the dominant deformation mechanism is volume diffusion creep due to a low strain rate. However, in areas with high velocity gradients, the strain rate is high enough for GBS (and occasionally basal slip at the hottest places with high strain rate) to become dominant. This happens mainly in upwellings, where the warm, soft ice rises, and downwellings, where the cold, rigid ice sinks. The volume diffusion again dominates the deformation in the innermost parts of both upwellings and downwellings, because the gradient of velocity there, and thus the strain rate, is low. The deformation in the lithosphere is once again dominated by the GBS. There, both the temperature and velocity are low, but the stress in the rigid ice is higher. At the very top, roughly 5 km below the surface, a maximum viscosity is reached.

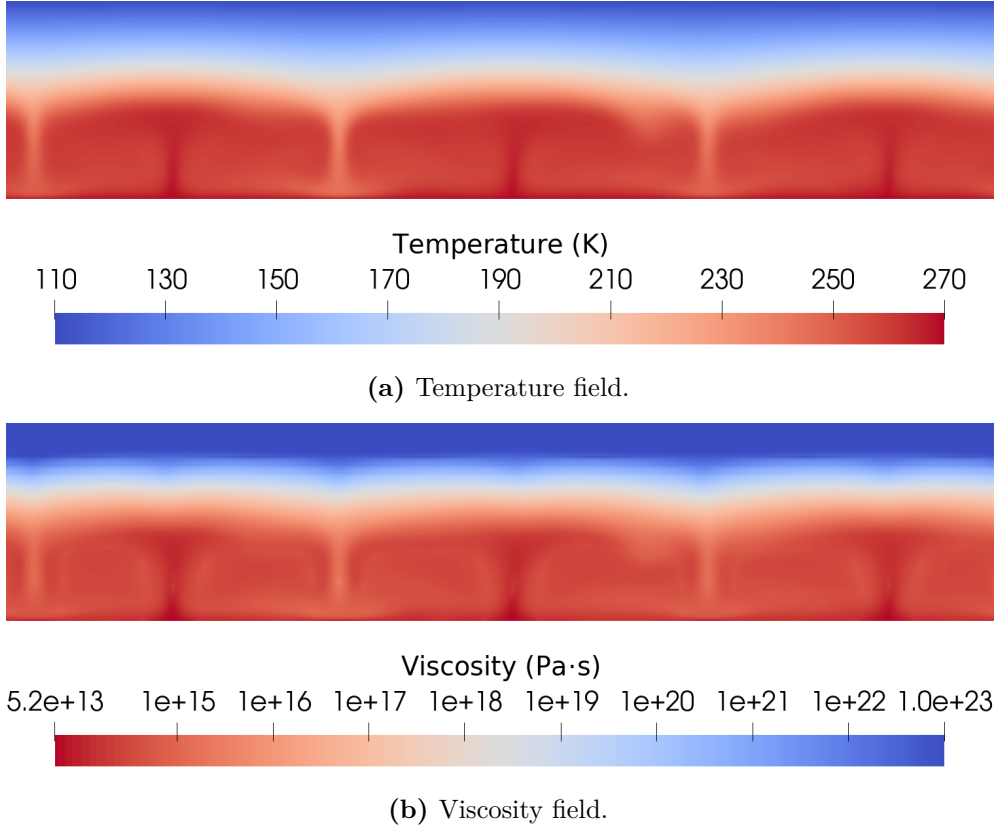
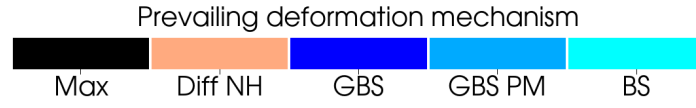
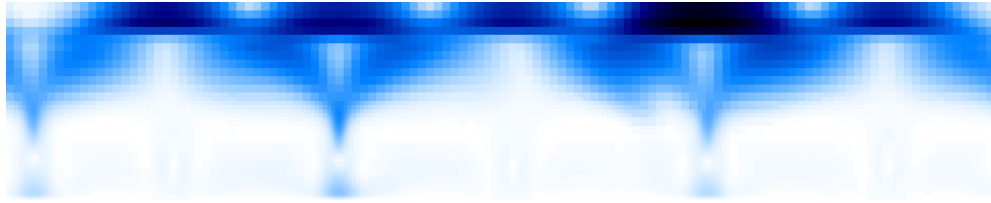


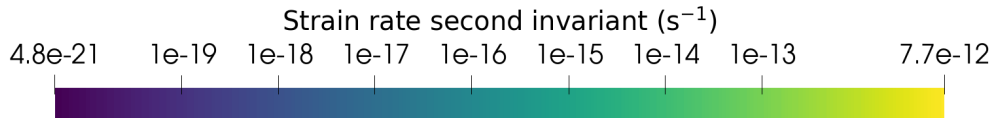
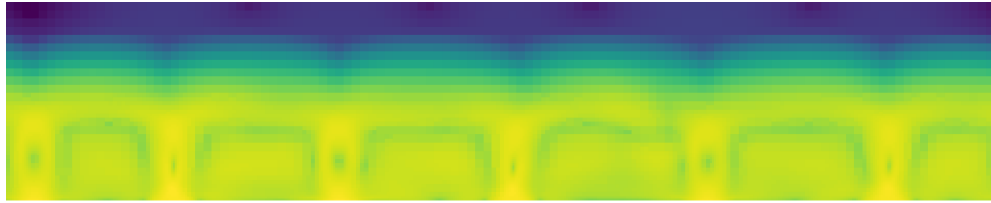
Figure 3.4 Fields of selected quantities for the reference model at end state at diffusion time 24 Myr.



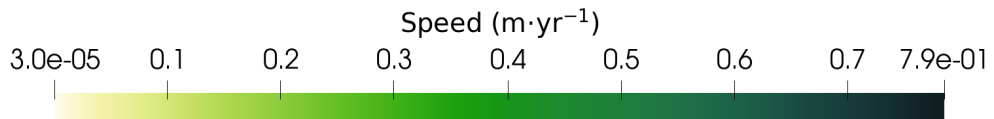
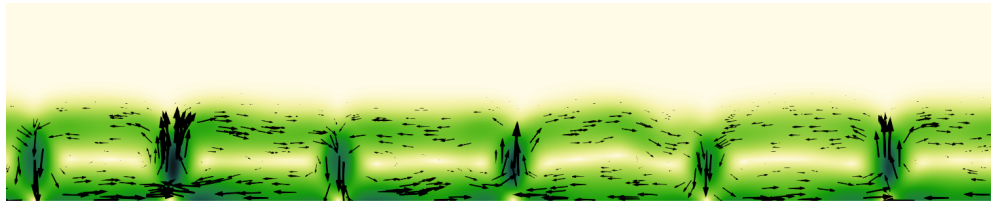
(c) Prevailing deformation mechanism. Max denotes areas of maximum viscosity, Diff NH is volume diffusion creep, GBS is GBS-accommodated basal slip, GBS PM is GBS-accommodated basal slip in the premelting regime, and BS is basal slip-accommodated GBS.



(d) Second invariant of stress tensor field.



(e) Second invariant of strain rate tensor field.



(f) Velocity field.

Figure 3.4 Fields of selected quantities for the reference model at end state at diffusion time 24 Myr (cont.).

3.2.2 Different aspect ratios

There is a finite, integer number of convective cells that can fit a model of a given aspect ratio. As such, the chosen aspect ratio may affect the simulated convection. Generally, the higher the aspect ratio, the better. However, a higher aspect ratio implies more degrees of freedom, which means higher computation times. Thus, it is necessary to choose an aspect ratio so that it does not significantly affect the simulated processes, while computation is reasonably long.

Here, we try to determine how our selection affects the aspect ratio of 1:5 affect our results by comparing it with two models—one with lower and the other with higher aspect ratio. The end states of the three models are shown in Figure 3.5, table 3.1 shows the relative convective cell wavelengths calculated for them, and Fig. 3.6 compares the end-state heat fluxes. We see that lower aspect ratio somewhat affects the convective cell wavelength and heat fluxes. However, when comparing heat fluxes for the models with the aspect ratio of 1:5 and 1:10, we see that the difference is less than 3 %.

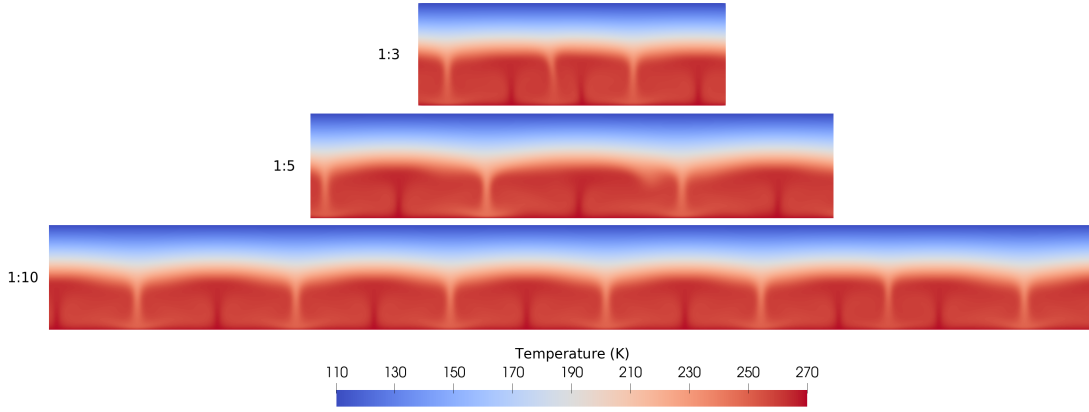


Figure 3.5 Comparison of end-state temperature fields of models with various aspect ratios at 24 Myr.

Table 3.1 Comparison of relative wavelengths of convective cells for different aspect ratio.

aspect ratio	relative wavelength
1:3	1.0
1:5	1.7
1:10	1.4

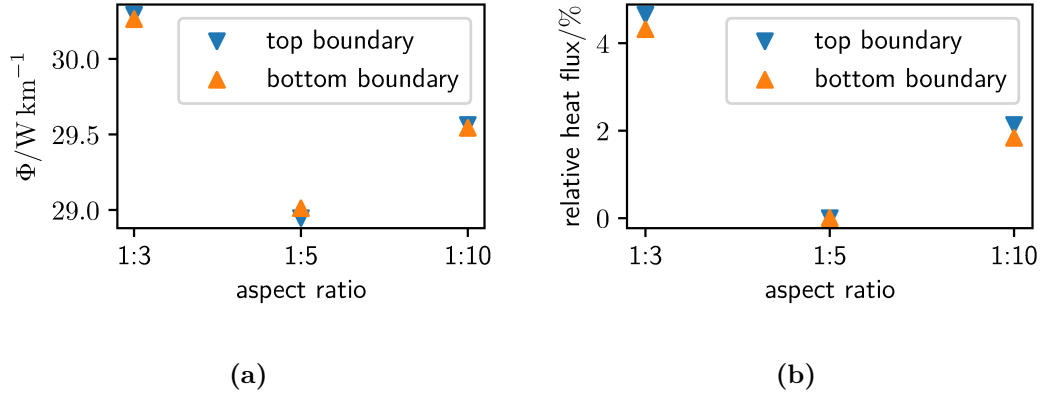


Figure 3.6 Comparison of end-state (a) heat fluxes, (b) heat fluxes relative to the reference model heat fluxes (aspect ratio 1:5) at 24 Myr.

3.3 Grain size

Grain size is one of the parameters controlling the viscosity of the ice and, through that, the dynamics of the ice shell. To investigate the influence of grain size on the thermal evolution of the ice shell, we modified the reference model to explore a range of grain sizes. The key parameter deciding whether a system will be convective is the Rayleigh number, which depends on the grain size through its dependence on viscosity. For the deformation mechanisms active in most of the reference model, the viscosity increases with increasing grain size. This implies decreasing Rayleigh number with increasing grain size, which means less vigorous or even no convection at higher grain sizes.

The comparison of end-state temperature fields and dominant deformation mechanisms of different models is in Figs. 3.7 and 3.8, respectively. For larger grains, the models are purely conductive with almost linear temperature depth profiles, where the slight nonlinearity is introduced by the temperature dependence of the thermal conductivity k . The change from a purely conductive to convective regime occurs between 0.9 mm and 0.8 mm. The models are convective for grain sizes 0.8 mm and lower, with the convection becoming more vigorous as the grain size decreases. Consequently, the convective zone becomes thicker with decreasing grain size, while the lithosphere becomes thinner (see Fig. 3.9). With a thinner lithosphere, more heat can be transported through it, which results in higher heat flux in the steady state. Figure 3.10 shows a change in the heat flux together with a change in the convective Rayleigh number, which also increases with decreasing grain size. Rayleigh number is closely related to the Nusselt number, a ratio of total heat transfer to conductive heat transfer; the Nusselt number depends on the Rayleigh number as $\text{Nu} \sim \text{Ra}^{1/3}$ (Solomatov, 1995).

Figure 3.8 shows which mechanisms are dominant for different grain sizes. In a model with a grain size of 0.8 mm, the GBS is dominant in most of the shell. That is caused by 1) smaller convective cells, which result in high strain rate not only in downwellings and upwellings but also along the areas where velocity is horizontal, and 2) GBS being available for lower strain rates at lower stresses (see Fig. 1.1), which is due to GBS being less sensitive to grain size than the volume diffusion creep. With decreasing grain size, convective cells become larger,

which reduces the area of high velocity gradients/strain rate. This, together with GBS becoming the dominant deformation mechanism at higher strain rate/stress, results in GBS being gradually replaced by volume diffusion creep in the convective cells, with the exception of the bases of the upwellings, where high strain rate and temperature favour basal slip as the dominant mechanism. Neither dislocation nor grain-boundary diffusion creep is a rate-controlling mechanism for any of the models. The dislocation-dominated region is unreachable due to the low stress involved in the deformation. The grain-boundary diffusion-dominated region could not be reached due to the grain sizes being too large.

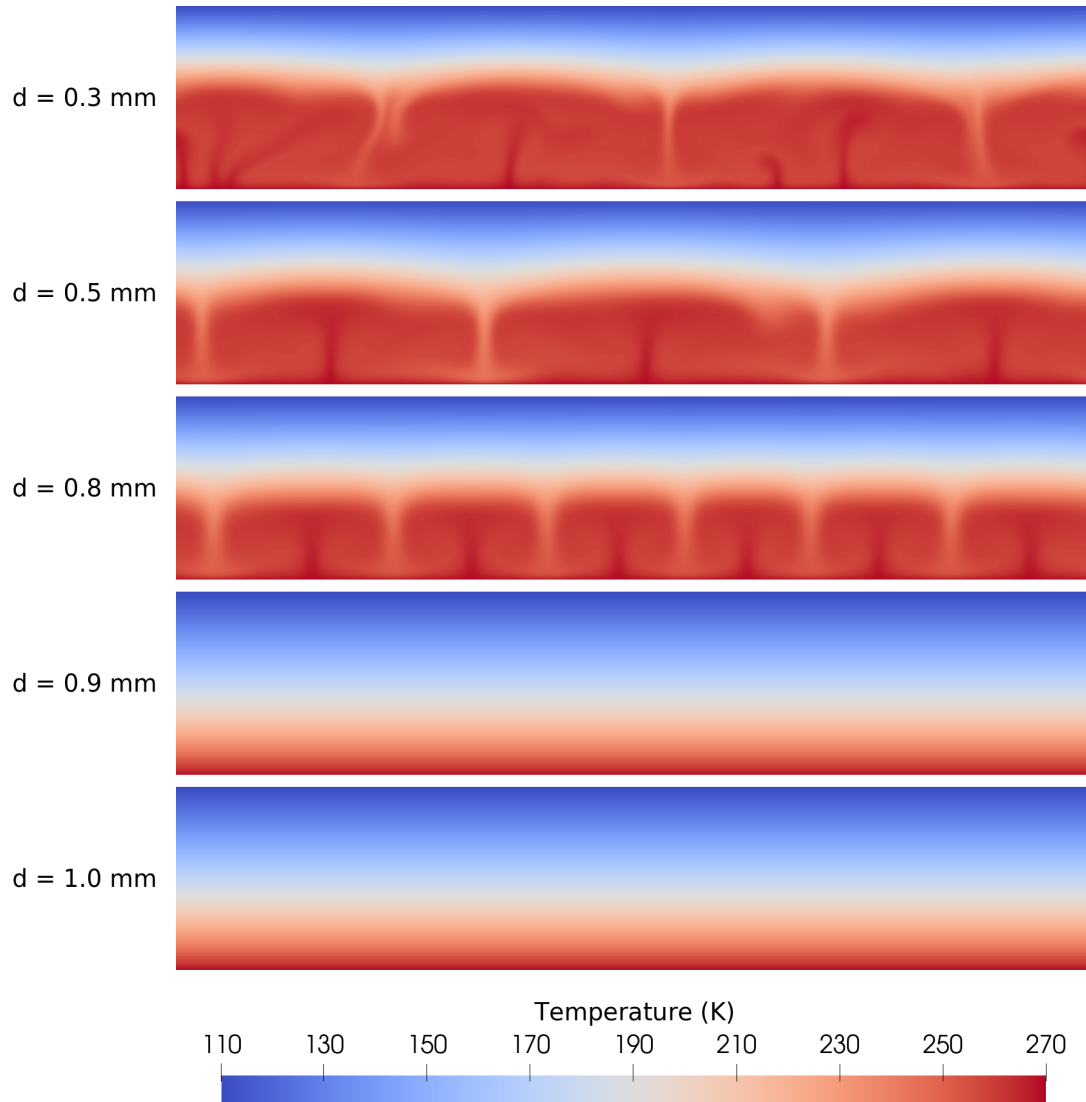


Figure 3.7 Comparison of end-state temperature profiles of models with varying grain sizes at 24 Myr.

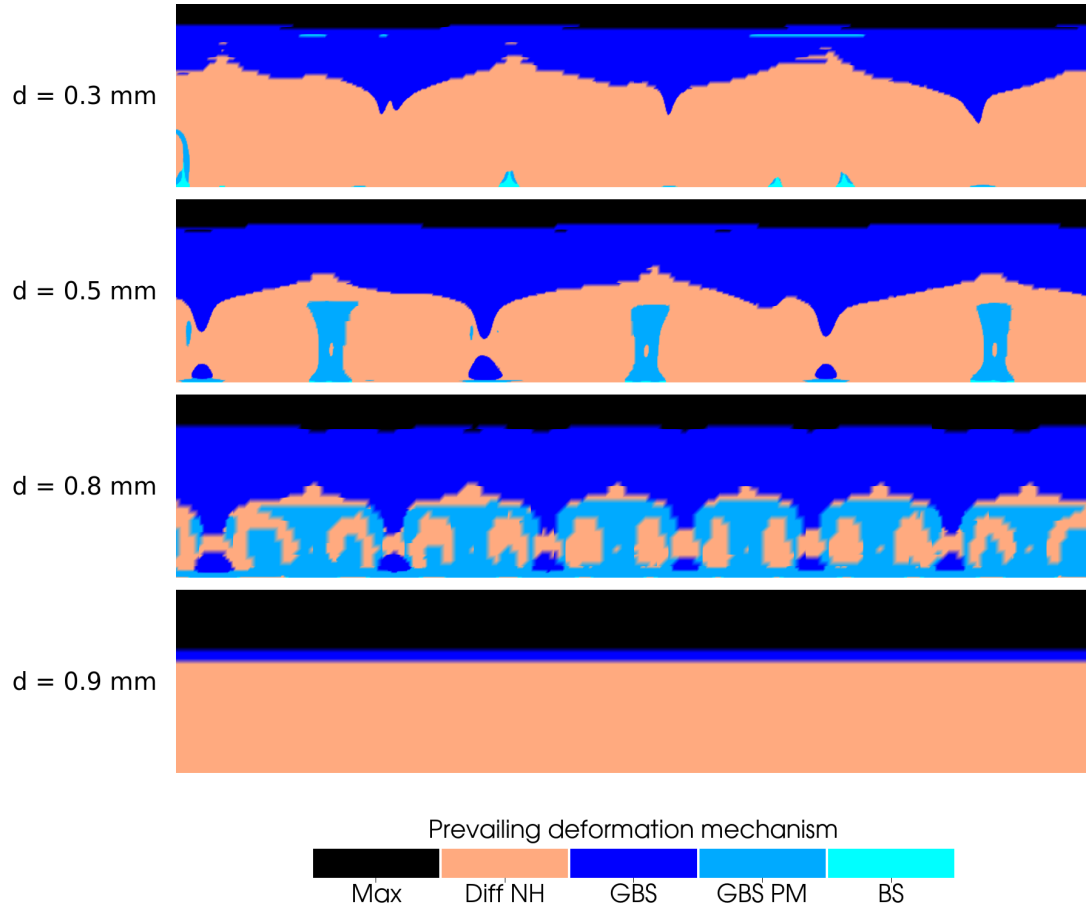


Figure 3.8 Comparison of dominant deformation mechanisms in models with varying grain sizes at 24 Myr.

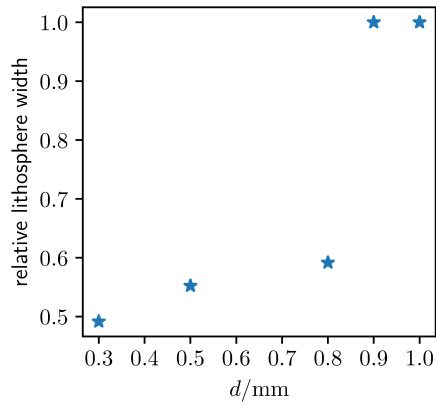


Figure 3.9 Relative lithosphere thickness.

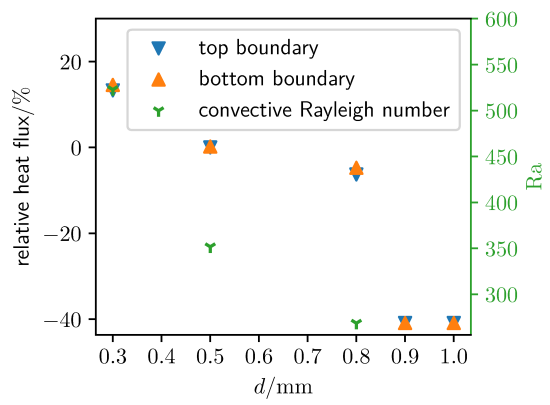


Figure 3.10 Relative heat flux with convective Rayleigh number.

3.4 Shell thickness

Shell thickness influences the thermal evolution of the ice shell, since the Rayleigh number depends on it as $Ra \sim D^3$. To investigate the influence of shell thickness on the thermal evolution, we again modified the reference model, this time to explore a range of shell thicknesses. Since the Rayleigh number depends strongly on shell thickness, it will increase when the shell thickens, which implies more vigorous convection.

The model with the shell thickness of 100 km did not run until the diffusion time as listed in table 2.2, but only to 44.8 Myr, due to time consideration. As can be seen in Fig. 3.11, structures in the shell are very thin, and thus the mesh needed to have high resolution at these places, which significantly increased the computation time. Although ending earlier than planned, the model still reached statistical equilibrium well before the simulation ended.

Figures 3.11 and 3.12 compare the end-state temperature fields and dominant deformation mechanisms of the selected models. For thinner shells, the models are purely conductive with almost linear temperature depth profiles. A change from a purely conductive to convective regime occurs between 21 km and 22 km. For shells thicker than 22 km, the convection is more vigorous with increasing thickness—the relative convective layer thickness increases while the relative lithosphere thickness decreases (see Fig. 3.13).

Since models with various shell thicknesses are involved, interpreting changes in end-state heat fluxes is not as straightforward as before, because heat flux directly depends on the shell thickness. Thus, instead of heat fluxes, we use the Nusselt number, defined as the ratio of the actual solution to the conductive solution of the same shell thickness and boundary conditions. Naturally, for purely conductive solutions, the Nusselt number is equal to one, as can be seen in Fig. 3.14. For convective solutions, the Nusselt number increases with shell thickness, meaning the higher the shell thickness, the more efficient the heat transfer. Since the efficiency of heat transfer is linked to the vigour of convection ($Nu \sim Ra^{1/3}$), the Rayleigh number also increases with increasing shell thickness, as shown again in Fig. 3.14.

Figure 3.12 shows how dominant mechanisms change with changing shell thickness. In the lithosphere, the stress is always high enough for the GBS to be dominant. In the convective layer, three mechanisms can dominate. For lower shell thicknesses, basal slip can occur as a dominant mechanism at the very bottom of upwellings, where temperature and strain rate are high. As the shell thickness increases, basal-slip dominated areas of deformation disappear in favour of GBS in the premelting regime, because stress is too high. GBS dominates the rest of the regions with high strain rate, along downwellings and upwellings, where the velocity gradient is high. Whether GBS appears in the low-temperature or the premelting regime depends on the temperature of the ice. The third mechanism which controls the deformation in the convective layer is the volume diffusion creep. The volume diffusion creep dominates in the regions of lower strain rate, which roughly corresponds to the areas between upwellings and downwellings, where GBS or basal slip are rate-controlling mechanisms. As the shell thickness increases, the volume diffusion creep-dominated regions become larger due to lower strain rates involved in the deformation in the majority of the convective layer. Neither

dislocation nor grain-boundary diffusion creep are rate-controlling mechanisms for any of the models. The grain-boundary diffusion, at the selected grain size, cannot be a dominant mechanism due to high temperature in the entirety of the ice shell, as is apparent from Fig. 1.1. For the dislocation creep to be dominant, the stress would need to reach values at least two orders of magnitude higher than the highest stress reached in any of the models.

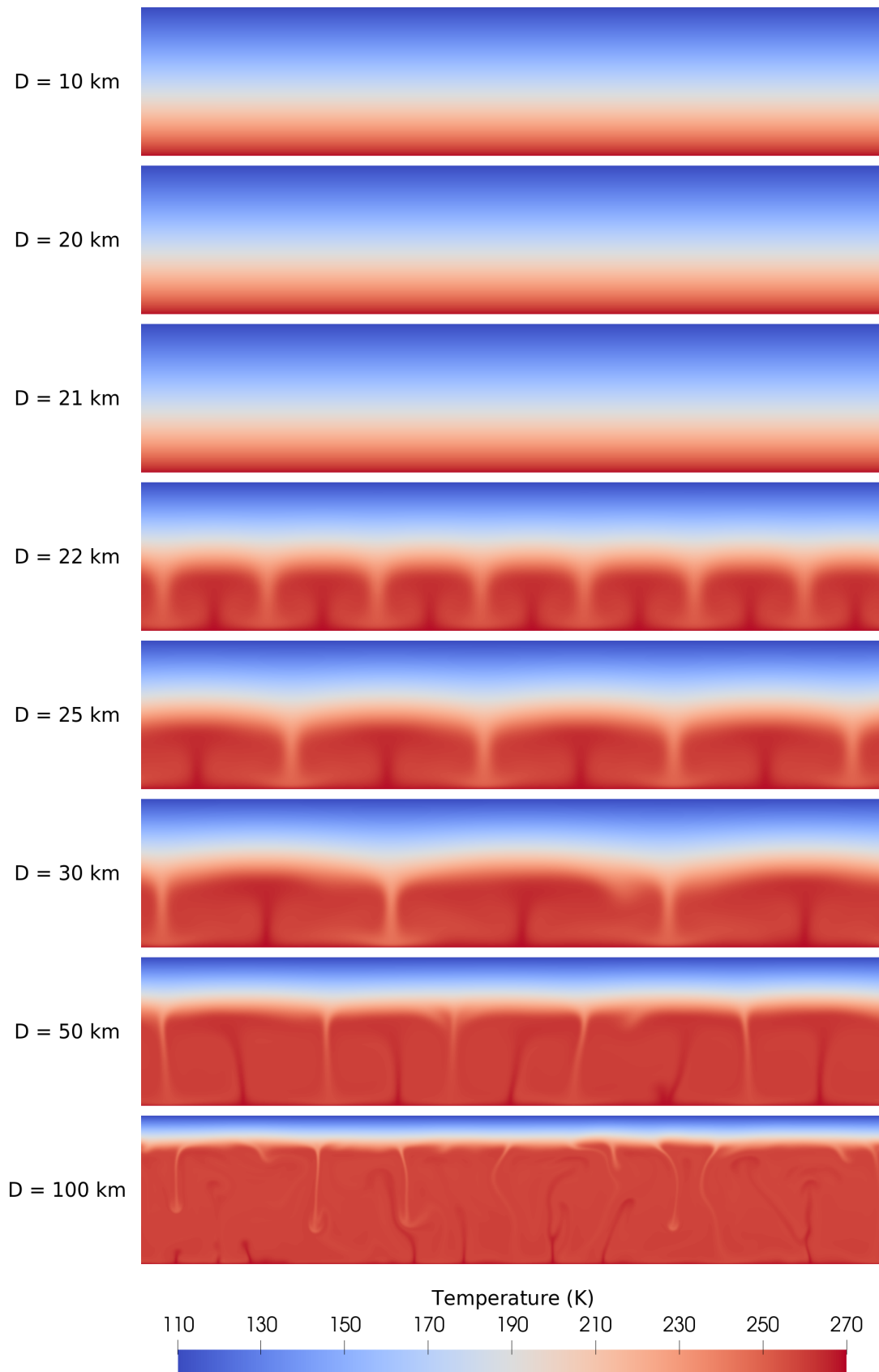


Figure 3.11 Comparison of end-state temperature profiles of models with varying grain sizes.

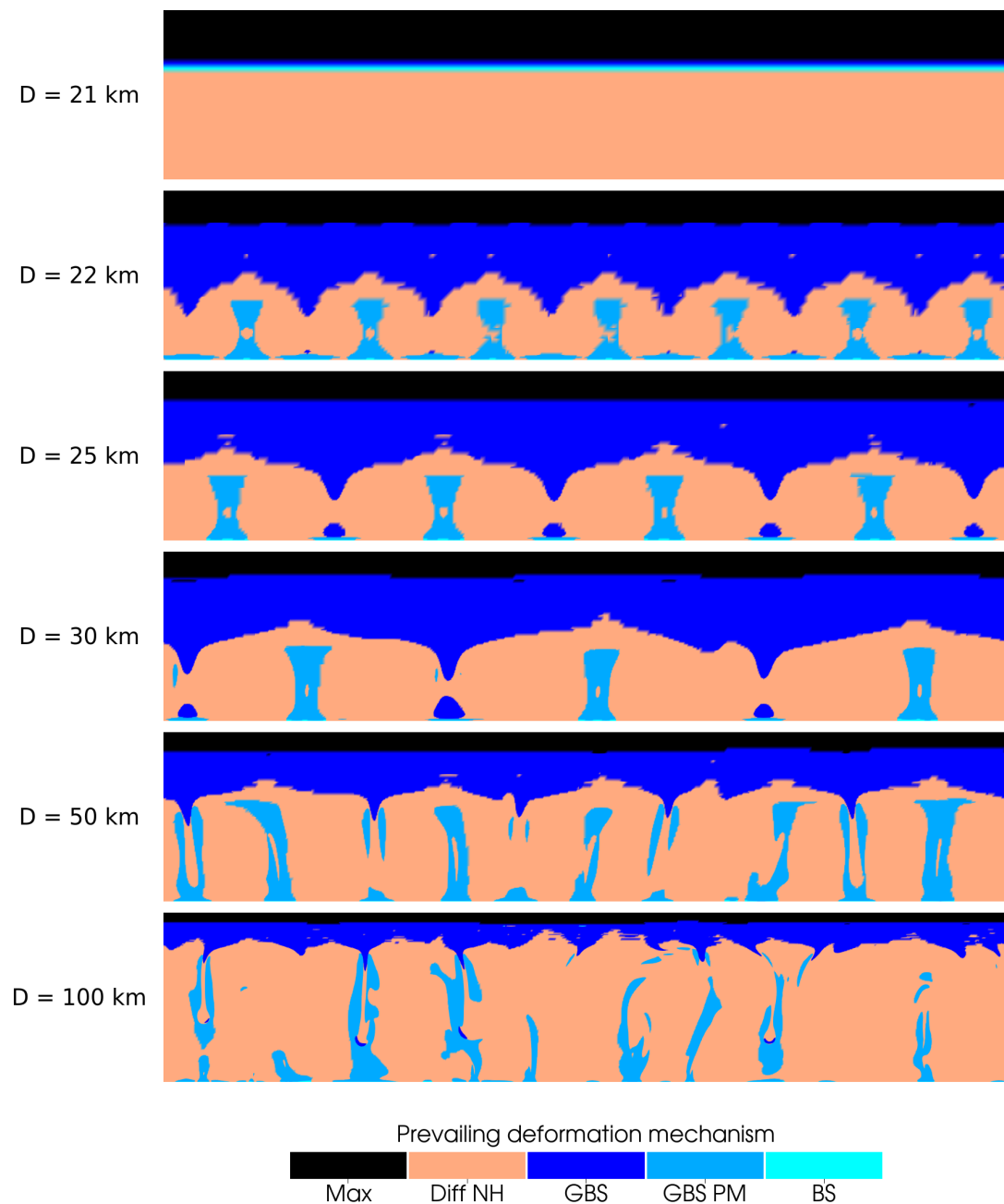


Figure 3.12 Comparison of dominant deformation mechanisms in models with varying grain sizes.

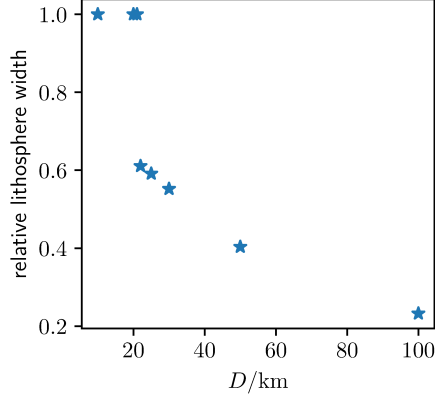


Figure 3.13 Relative lithosphere thickness.

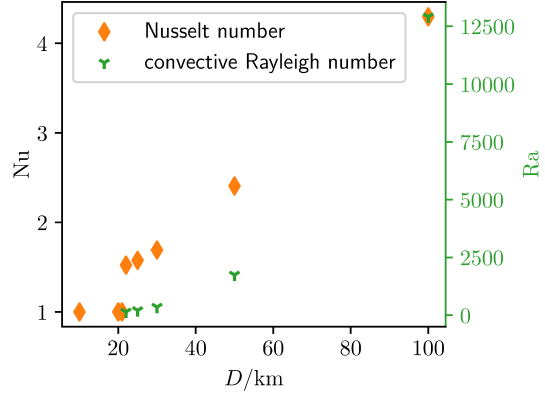


Figure 3.14 Nusselt number and convective Rayleigh number.

3.5 Gravity

Another parameter influencing thermal evolution is the gravitational field strength. The gravitational field strength contributes to both the dissipation number, which controls the contribution of the adiabatic and viscous heating, and the Rayleigh number. However, since the dissipation number for the ice shell is low, the effect of gravity field strength on convection can be estimated by Rayleigh number scaling. Thus, the investigation of the influence of gravity can be done by changing the shell thickness instead of the gravity field strength since the Rayleigh number has a dependence of $Ra \sim gD^3$, which means that increasing the gravity n times corresponds to increasing the shell $\sqrt[3]{n}$ times.

We ran one model with different gravitational field strength by modifying the reference model to have the gravitational field with twice the magnitude of the reference one, i.e., $g = 2.6 \text{ m s}^{-1}$. That corresponds to the shell thickness of approximately 37.8 km. The comparison with the reference model is in Figs. 3.15 and 3.16.

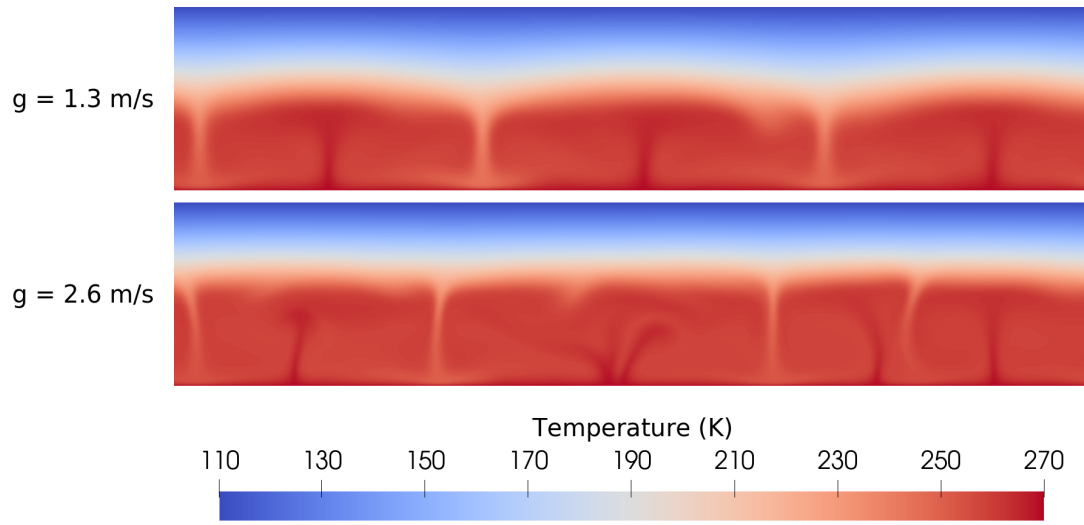


Figure 3.15 Comparison of temperature field for different gravity magnitude at diffusion time 24 Myr.

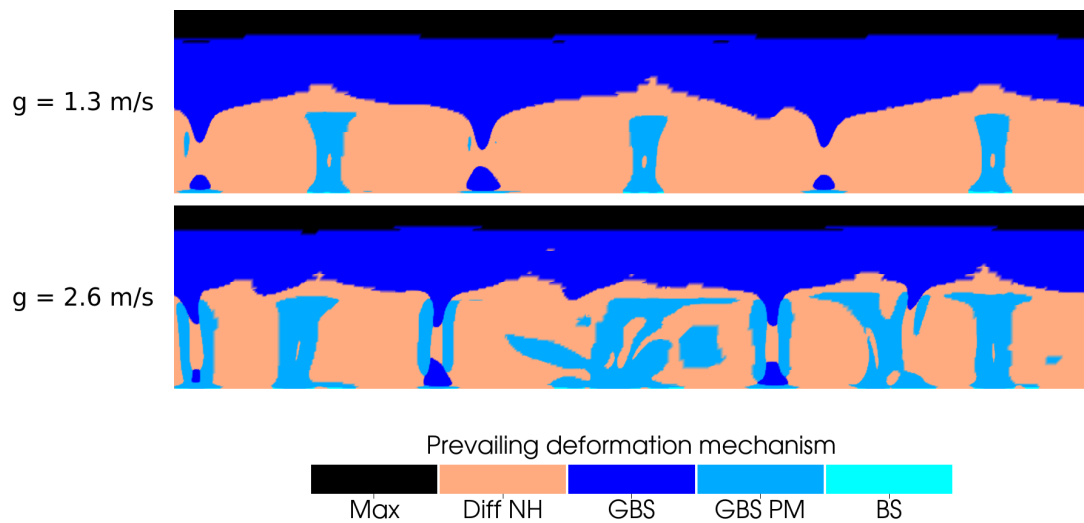


Figure 3.16 Comparison of dominant deformation mechanism field for different gravity magnitude at diffusion time 24 Myr.

Conclusion

We used an existing rheological model of ice Ih based on experimental studies and first-principle derivations to investigate dominant deformation mechanisms over a range of conditions. Using this model, we constructed deformation mechanism maps to visualise how the dominant mechanism varies with stress, temperature and grain size.

Numerical simulations were conducted to explore thermal convection in an ice shell over a range of shell thicknesses and grain sizes in a model inspired by Jupiter’s moon Europa. The results show that for a reference grain size of 0.5 mm, convection is present when the shell is at least 22 km thick, whereas for a reference shell thickness of 30 km, convection occurs only for grain sizes less than or equal to 0.8 mm. We have also concluded that grain-boundary diffusion and dislocation creeps are unlikely to dominate the deformation in a convecting ice shell.

These results highlight the strong dependence of shell dynamics of an ice shell on the microstructural properties of ice and show that small changes in grain size or shell thickness can significantly affect its thermal evolution.

Our models were simplified and neglected many phenomena that may occur in reality. For example, our simulations did not consider grain size evolution due to dynamic recrystallisation and grain growth. Also, we considered the unrealistic case of both the shell and the underlying ocean being composed only of pure water without any impurities, which affects their physical properties, especially the melting temperature of ice at the interface of the ice shell and the water ocean.

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A Attachments

A.1 Sample ASPECT parameter file

```
set Dimension = 2
set Use years in output instead of seconds = true
set Output directory = europa_output-05mm-5
set Additional shared libraries = ./plugins/libvisco_plastic_ice.so

set CFL number = 0.8
set Maximum relative increase in time step = 20
set End time = 24e6
set Resume computation = false

set Nonlinear solver scheme = single Advection, iterated Newton Stokes

set Nonlinear solver tolerance = 1e-5
set Max nonlinear iterations = 20

subsection Solver parameters
  subsection Stokes solver parameters
    set Number of cheap Stokes solver steps = 20
  end
  subsection Newton solver parameters
    set Max pre-Newton nonlinear iterations = 10
    set Use Newton residual scaling method = true
  end
end

subsection Termination criteria
  set Termination criteria = steady state heat flux
  subsection Steady state heat flux
    set Boundary indicators = bottom, top
    set Maximum relative deviation = 0.01
    set Time in steady state = 1e7
  end
end

subsection Geometry model
  set Model name = box
  subsection Box
    set Box origin X coordinate = 0.
    set X extent = 150.0e3
    set X repetitions = 15
    set X periodic = true
    set Box origin Y coordinate = 0.
    set Y extent = 30.e3
    set Y repetitions = 3
  end

subsection Initial temperature model
  set List of model names = function
  subsection Function
    set Coordinate system = cartesian
    set Variable names = x,y
    set Function constants = p=10., pi=3.1415926536, y0=0e3, y1=30e3
    set Function expression = max(min(p * rand() + 100.+(y1-y)/(y1-y0)*(270.-100.),270),110)
  end
end

subsection Boundary temperature model
  set Fixed temperature boundary indicators = bottom, top
  set List of model names = box
  subsection Box
    set Bottom temperature = 270.
    set Top temperature = 110.
  end
end
```

```

subsection Boundary velocity model
  set Tangential velocity boundary indicators = bottom, top
end

subsection Nullspace removal
set Remove nullspace = net x translation
end

subsection Gravity model
  set Model name = vertical
  subsection Vertical
    set Magnitude = 1.3
  end
end

subsection Heating model
  set List of model names = shear heating, adiabatic heating
end

subsection Material model
  set Model name = visco plastic ice
  subsection Visco Plastic Ice
    set Densities = 920.
    set Reference temperature = 200.
    set Thermal conductivities = 0.4685
    set Thermal conductivities 1_T = 488.19

    set Viscous flow law = composite

    set Prefactors for diffusion NH creep = 4.53e-8
    set Activation energies for diffusion NH creep = 59e3
    set Activation volumes for diffusion NH creep = 0e0
    set Grain size exponents for diffusion NH creep = 2

    set Prefactors for diffusion C creep = 8.20e-17
    set Activation energies for diffusion C creep = 49e3
    set Activation volumes for diffusion C creep = 0e0
    set Grain size exponents for diffusion C creep = 3

    set Prefactors for dislocation creep = 3.12e-18
    set Activation energies for dislocation creep = 60e3
    set Activation volumes for dislocation creep = 0e0
    set Stress exponents for dislocation creep = 4.0
    set Prefactors for dislocation PM creep = 4.68e5
    set Activation energies for dislocation PM creep = 175.5e3
    set Activation volumes for dislocation PM creep = 0e0
    set Stress exponents for dislocation PM creep = 4.0
    set Transition temperature dislocation = 258.

    set Prefactors for GBS creep = 1.44e-13
    set Activation energies for GBS creep = 49e3
    set Activation volumes for GBS creep = 0e0
    set Stress exponents for GBS creep = 1.8
    set Grain size exponents for GBS creep = 1.4
    set Prefactors for GBS PM creep = 1.11e+16
    set Activation energies for GBS PM creep = 190e3
    set Activation volumes for GBS PM creep = 0e0
    set Stress exponents for GBS PM creep = 1.8
    set Grain size exponents for GBS PM creep = 1.4
    set Transition temperature GBS = 255.
    set Prefactors for BS creep = 7.09e-07
    set Activation energies for BS creep = 60e3
    set Activation volumes for BS creep = 0e0
    set Stress exponents for BS creep = 2.4
    set Grain size exponents for BS creep = 0

    set Grain size = 0.5e-3

    set Reference strain rate = 1e-10
    set Maximum viscosity = 1e23
    set Minimum viscosity = 1
  end
end

```

```

end

subsection Mesh refinement
  set Initial global refinement      = 3
  set Initial adaptive refinement   = 2
  set Minimum refinement level      = 3
  set Time steps between mesh refinement = 5
  set Strategy                      = boundary, strain rate, temperature
  set Refinement criteria merge operation = plus
  set Refinement criteria scaling factors = 0.95, 1, 1.
  subsection Boundary
    set Boundary refinement indicators = bottom
  end
end

subsection Postprocess
  set List of postprocessors = velocity statistics, temperature statistics,\
    heat flux statistics, heating statistics, visualization, depth average,\
    velocity boundary statistics
  subsection Visualization
    set Time between graphical output = 0.1e6
    set List of output variables = material properties, strain rate,\
      stress second invariant, temperature anomaly, gravity, heating,\
      named additional outputs
    set Output format = hdf5
    set Write in background thread = true
  subsection Material properties
    set List of material properties = thermal conductivity,\
      thermal diffusivity, specific heat, viscosity, density,\
      thermal expansivity
  end
end
subsection Depth average
  set Number of zones = 100
  set Output format = gnuplot, txt
  set Time between graphical output = 0.01e6
  set List of output variables = viscosity, temperature, velocity magnitude,\
    vertical heat flux, vertical mass flux, sinking velocity,\
    rising velocity, log viscosity
end
end

subsection Checkpointing
  set Steps between checkpoint = 500
end

set Timing output frequency = 1

```