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**Exploring Internal Structure of Icy
Bodies: A Statistical Perspective**

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Ph.D.

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Abstract: Understanding the internal structure and composition of icy bodies in the Solar System is one of the key questions, providing crucial insights into their formation, evolution, and potential habitability. In our work, we have focused on modelling the structure of the Galilean moons Europa and Ganymede using known satellite characteristics, thermodynamic parameters, equations of state, and the physical properties of likely building materials, such as different types of ice. We used the Markov chain Monte Carlo method to estimate the posterior probability of the different structures, based on known available data. These structures include the thickness and properties of the hydrosphere (including salinity of the ocean), the rocky mantle, and the iron core. Our results revealed several trade-offs and sensitivities between different characteristics of the moons, depending on the assumptions made.

Keywords: icy satellites, structure, ice properties, inverse problems

Název práce: Vnitřní struktura ledových měsíců a planet: Statistický pohled

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Abstrakt: Poznanie vnútornej štruktúry a zloženia ľadových telies v Slniečnej sústave je jednou z kľúčových otázok, ktorá poskytuje dôležité informácie o ich vzniku, vývoji a potenciálnej obývateľnosti telies. V našej práci sme sa zamerali na modelovanie štruktúry Galileových mesiacov Europa a Ganymede s využitím známych charakteristík satelitov, termodynamických parametrov, stavových rovníc a fyzikálnych vlastností pravdepodobných stavebných materiálov, ako sú napr. rôzne druhy ľadu. Na odhad pravdepodobnosti rôznych štruktúr sme použili metódu Markov chain Monte Carlo na základe známych dostupných meraní. Tieto štruktúry zahŕňajú hrúbku a vlastnosti hydrosféry (vrátane salinity oceánu), horninový plášť a železné jadro. Naše výsledky odhalili niekoľko vzájomných vzťahov a citlivostí medzi rôznymi charakteristikami mesiacov v závislosti od použitých predpokladov.

Klíčová slova: ľadové mesiace, štruktúra, vlastnosti ľadu, obrátené úlohy

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Introduction

Outer planet systems illustrate the diversity of the solar system, featuring a wide range of moons (Figure 1) as well as varied rings and magnetic environments. Considerable attention has been paid to the icy moons due to their unique surface features, revealing their geological history and evolution. Despite missions like Voyager, Galileo, and the ongoing Juno mission that have significantly advanced our understanding of icy moons, many key questions, such as the exact internal structure of icy moons or the precise nature of subsurface oceans, remain unanswered. Characterisation of the icy moons’ interiors provides important clues about their formation, evolution, and habitability. However, this area of planetary science suffers from a lack of data and measurements, primarily caused by the great distance of the satellites from Earth, production and operational costs of dedicated missions, and the dependence on remote observations from orbit.



Figure 1 Outer solar system satellites, compared to Earth’s Moon. Adapted from Nimmo (2018).

Currently, two new missions are heading to Galilean icy moons: JUICE, launched in April 2023, and Europa Clipper, launched in October 2024.

Among the primary scientific objectives of the JUICE mission are the characterisation of the ocean layers and detection of subsurface oceans; topographical, geological, and compositional mapping of the surface; and study of the physical properties of the icy shells (Science Objectives). The mission will conduct multiple flybys of Europa and Callisto and will ultimately enter orbit around Ganymede, making it the first spacecraft to orbit a moon in the outer solar system.

Additionally, for Ganymede, JUICE aims to examine its intrinsic magnetic field and its interactions with the magnetosphere of Jupiter. Another important objective of the JUICE mission is to provide the subsurface radar sounding of the moon, which will determine the minimum thickness of the outer icy layer.

JUICE also includes multispectral imaging and spectroscopy equipment, which maps surface chemistry with better resolution than previous missions. Tidal deformation in Ganymede's outer ice shell will be measured by a laser altimeter (GALA).

For Europa, the JUICE mission will also focus on studying the conditions necessary for the origin of life, as well as the study of the formation of surface features and the composition of non-water materials deposited on the surface. However, JUICE will perform only two flybys of Europa due to radiation exposure, reaching up to 400 km of the moon's surface.

Europa will be the primary focus of NASA's mission Europa Clipper. Its main objectives include determining the composition of Europa's subsurface ocean; the interaction of the subsurface ocean with Europa's surface; and the geological exploration of Europa (Science Objectives). Europa Clipper also includes a radar sounding (REASON) that will help determine the thickness of Europa's icy shell. It also consists of the Europa Imaging System (EIS), which will map ~95% of Europa, which is a dramatic improvement over previous missions. Europa Clipper also features mass spectrometers, which will sample surface particles and atmospheric gases to detect organic and inorganic materials. Additionally, it includes magnetic and plasma sensors that will evaluate tidal responses, ocean depth, and salinity with high accuracy.

The JUICE and Europa Clipper missions are expected to provide new data (including tidal deformation, topography, and gravity field) that will reduce data uncertainty that will help refine our understanding of subsurface oceans and internal structures.

1 Icy moons

The determination of the icy moons’ internal structure relies on measurable satellite parameters, such as the radius of the body R_{surf} , its total mass M , and the polar moment of inertia factor MOI

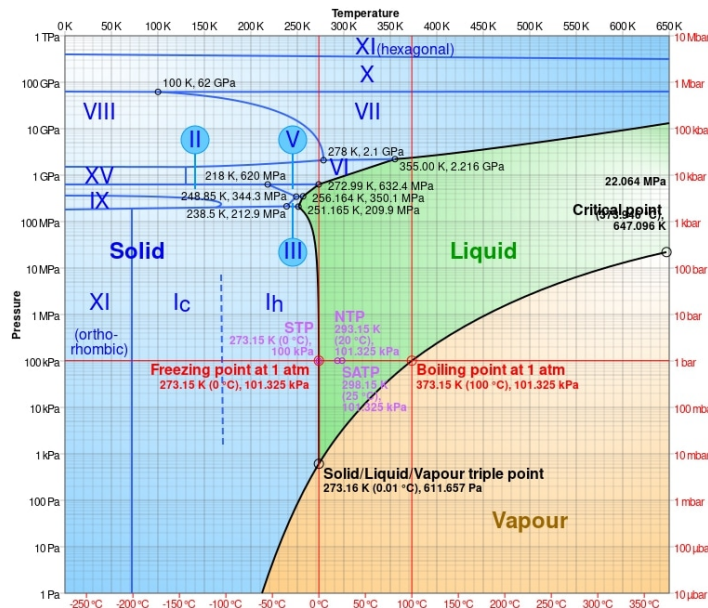
$$\text{MOI} = \frac{C}{MR_{\text{surf}}^2}, \quad (1.1)$$

where C is the polar moment of inertia. In addition to these measurements for individual bodies, the key piece of information comes from the knowledge of the properties of the building materials available in the solar system, including ice/water, silicates, and iron-rich materials.

A crucial insight into the differentiation state of the planetary bodies is encoded in the value of MOI, a parameter sensitive to the radial distribution of the mass. The closer the MOI value for the body studied is to the theoretical MOI value for a homogeneous body ($\text{MOI}=0.4$), the lower is the degree of differentiation of the body. For mid- to large-sized bodies, the structure is typically expected to consist of an outer hydrosphere and an inner, deeper layer composed of silicate-iron material. The latter can be further differentiated into a silicate-rich mantle and an iron-rich core.

The hydrosphere of icy moons consists of an outer ice shell and, potentially, a subsurface ocean of liquid water. The surface layer of ice is composed of hexagonal ice, Ih, which also commonly occurs on the Earth’s surface. It is also sometimes referred to as low-pressure ice because of its formation under low-pressure conditions. In bodies with a larger radius, such as Ganymede, where the internal pressures in the hydrosphere reach higher values (up to 1.8 GPa), another layer of ice is formed, known as high-pressure ice (HP ice). Currently, about 20 crystalline polymorphs have been experimentally confirmed (Komatsu, 2022), and several dozen other polymorphs have been theoretically predicted (Engel et al., 2018). In the icy satellites with sufficient pressure, attention is focused on stable high-pressure ice polymorphs II, III, V, and VI (Journaux et al., 2020a), see the phase diagram of water in Figure 1.1.

For the Galilean satellites, the existence of a subsurface liquid layer in the ocean was first deduced from the induced magnetic field measurements by the Galileo mission. The detected induced field was consistent with an electrically conductive layer interpreted as a salty ocean (Sohl et al., 2002). Although in the case of Ganymede, the situation is more complicated due to its internally generated magnetic field (Section 1.2). The size and properties of a possible subsurface ocean depend on several factors: pressure, temperature, and composition, including dissolved salts and other chemical compounds. When examining the hydrospheres of icy moons, several possible ocean compositions are usually considered. These range from pure water to salt-rich mixtures. The most commonly mentioned salts are MgSO_4 or NaCl . In satellites of Saturn and beyond, the presence of ammonia is also considered (Nimmo, 2018). The presence of these dissolved compounds in the ocean is a crucial factor in evaluating the internal structures of hydrospheres. In particular, their presence lowers the melting temperature of ice and thus increases the P - T interval at which the liquid subsurface ocean can occur, effectively increasing its possible size.



The state of the art of internal structure and composition is limited due to the lack of data and measurements. Many studies have used the thermodynamic properties of building materials, gained over the last decade, to model the internal structures of icy moons. Vance et al. (2014) modelled the internal structure of Ganymede’s hydrosphere for an ocean composed of MgSO_4 at various concentrations and subsequently used a similar approach to study Europa’s hydrosphere (Vance et al., 2018). Styczinski et al. (2023) published freely available software *PlanetProfile*, which models the internal structures of bodies by integrating known parameters and using established EOS and knowledge about materials that are assumed to be the building materials of the given bodies.

Howell (2021) focused his work on determining the probability density distribution of the thickness of Europa’s ice shell using heat flow balance. The distribution was then evaluated using the code they developed, utilising the MCMC approach in Python.

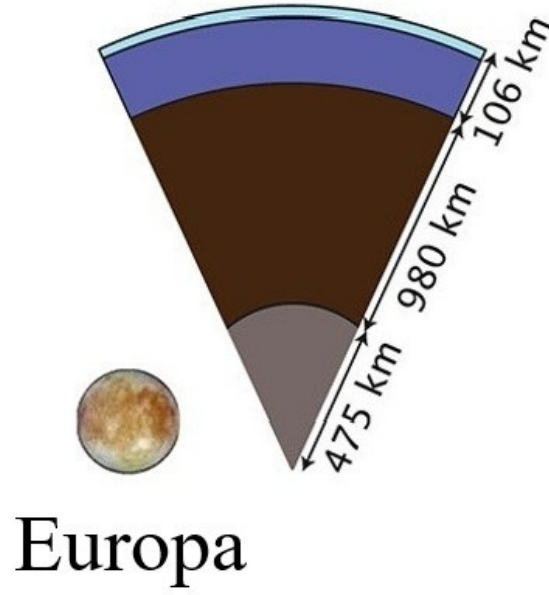


Figure 1.2 Internal structure of Europa, adapted from Vance et al., 2018

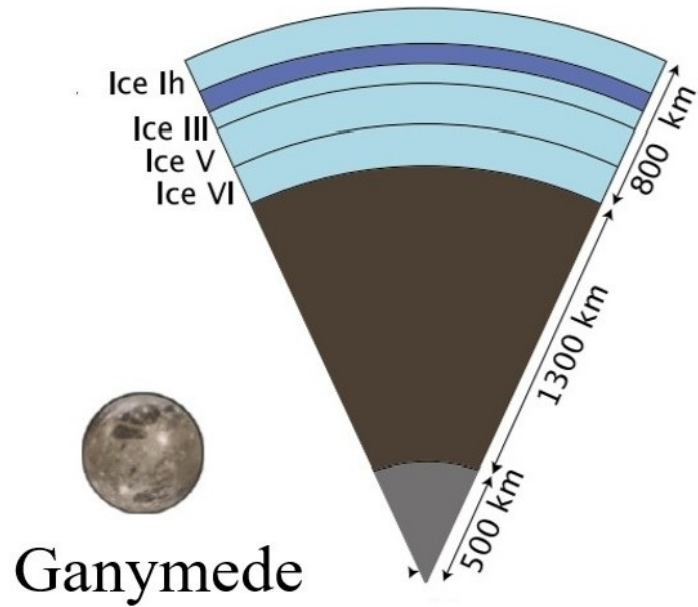


Figure 1.3 Internal structure of Ganymede, adapted from Journaux et al., 2020b

In our work, we focused on studying the internal structures of two icy moons, Europa and Ganymede. Ganymede, as the largest moon in our solar system, reaches pressures in its hydrosphere that correspond to the conditions for the formation of high-pressure (HP) ice phases. Their presence would prevent the contact between water and silicates necessary for the existence of life. Europa,

which has a smaller radius, represents smaller icy bodies that should not reach such pressures in their hydrosphere.

For both bodies, we assumed they are differentiated into three layers: hydrosphere, silicate mantle, and metallic core. The hydrosphere is further divided into water and ice phases, depending on internal pressures. Reference structures are depicted in Figures 1.2 and 1.3. However, in the case of Europa, this assumption is uncertain. We briefly elaborate on this issue in the following section.

1.1 Europa

Europa is the second moon in terms of distance from Jupiter, belonging to the Galilean moons, along with Io, Ganymede, and Callisto, with a radius $R_{\text{surf}} = 1560.8 \text{ km}$ (JPL). Its average density is 3018 kg m^{-3} . These values indicate that Europa’s interior is located between bodies composed mainly of ice, such as Ganymede or Callisto, and rock satellites, such as Io or Earth’s Moon (Hussmann et al., 2010). It is, therefore, expected that Europa will have a higher proportion of silicates in its structure and thus a proportionally smaller hydrosphere than, for example, Ganymede.

The level of differentiation of the internal structure of Europa is an open question. Anderson et al., 1998 determined the value of MOI from the mission Galileo to be $\text{MOI} = 0.346 \pm 0.005$, which suggests that Europa might be fully differentiated into hydrosphere, silicate mantle, and metallic core. However, the Juno mission provided new additional measurements, based on which a new value for Europa’s MOI was calculated: $\text{MOI} = 0.3547 \pm 0.0024$ (Gomez Casajus et al., 2021), which led to an increase in the value of MOI, challenging the full differentiation. In our calculations, we worked with both the Juno-based MOI and the Galileo MOI for Europa and compared them. In both cases, we assumed a three-layer model. However, our work can also be applied to models without silicate-iron layer differentiation in future studies.

Internal pressures in Europa’s hydrosphere are not high enough to create high-pressure ice phases; therefore, their occurrence in the hydrosphere is highly unlikely. This means that the layer of liquid ocean is below the layer of surface ice Ih and lies directly in contact with silicates, resulting in the water-rock interaction, one of the processes considered important for the emergence of extraterrestrial life.

1.2 Ganymede

Ganymede is the third Galilean moon in terms of distance from Jupiter. With a radius of $R_{\text{surf}} = 2631.2 \text{ km}$ (JPL), it is the largest moon in the solar system. The value of $\text{MOI} = 0.3159 \pm 0.0052$ (Gomez Casajus et al., 2022) makes Ganymede the body with the lowest MOI among the solid satellites and terrestrial planets. This value is modified from the long-used value $\text{MOI} = 0.3115 \pm 0.0028$ (Schubert et al., 2004); however, the difference is within 1-sigma. In our work, we used only the new value. Ganymede’s mean density is 1940 kg m^{-3} (Lissauer and de Pater, 2013), which suggests that it is a body composed largely of ice.

As the only satellite in the solar system, Ganymede has its own magnetic field. It is most likely generated by a dynamo in its liquid, possibly an Fe-FeS, core (Schubert et al., 1996). However, the measured magnetic field of Ganymede cannot be fully explained by an internally generated dipole alone. Kivelson et al. (2002), therefore, proposed two models: one combining dipole and quadrupole components of Ganymede’s dynamo, and another consisting of a dynamo-generated dipole together with an induced field due to the presence of a conductive subsurface layer. While both fit the observations, the simpler induced-field model was preferred.

Saur et al. (2015) proposed a new technique for detecting the subsurface ocean in Ganymede, using observations and modelling of auroral ovals with the Hubble Space Telescope. The position of auroral ovals oscillates due to the time-varying magnetic field of Jupiter, in which Ganymede is located. If there is a subsurface liquid conductive layer in its structure, an induced magnetic field is created, which reduces the time-dependent magnetic field, and thus the oscillation amplitude of the auroral ovals decreases.

The low value of MOI and the existence of an internally generated magnetic field strongly suggest that the structure is fully differentiated into a hydrosphere, a silicate mantle, and an iron-rich metallic core (Journaux et al., 2020b). Since Ganymede is the largest moon in the solar system, a high-pressure layer of ice forms in its hydrosphere beneath the subsurface ocean layer, preventing water from coming into contact with the silicate mantle. However, if heat transfer within the hydrosphere is sufficiently efficient, the high-pressure ice layers may melt, allowing the transfer of substances between the silicate mantle and liquid water. The efficiency of this heat transfer depends on the thermodynamic and rheological properties of the materials involved (Vance et al., 2014).

2 Method

In our work, we developed an algorithm and a code for calculating the internal structures of the icy satellites Europa and Ganymede, which are also applicable or easily adaptable to other bodies, such as the moons of Saturn and Uranus, and terrestrial exoplanets.

Our goal is to numerically determine the internal structures of the icy bodies, using known properties of the satellites such as mass (M), moment of inertia factor (MOI) and body radius (R_{surf}). The code is also designed to accommodate additional data expected from ongoing missions, such as the precise value of the tidal potential Love number k_2 , the possible value of the Love number h_2 , or the minimum thickness of the surface ice layer. Such constraints help to determine the internal structure, including the thickness of individual chemically distinct layers.

We structure the problem into two parts: a forward problem and an inverse problem. The forward problem (Section 2.1) evaluates the observable quantities (data space, $\mathbf{d}$) and the radial profile of the density based on the assumed structural and compositional parameters (model space, $\mathbf{m}$). The inverse problem (Section 2.2) then assesses the posterior probabilities of the model space parameters using the affine invariant Markov chain Monte Carlo (MCMC) ensemble sampler (Goodman and Weare, 2010; Foreman-Mackey et al., 2013). Although the present internal structure model builds on knowledge gained during the bachelor’s thesis (Košíková, 2023), the current approach allows for a more precise assessment of the internal structure and includes posterior probability estimation through inverse modelling, an aspect not addressed in previous work. Furthermore, the methodology and code presented here have been developed from scratch.

2.1 Forward problem

The scheme of the forward problem is depicted in Figure 2.1. In the forward problem, we predict the observable data $\mathbf{d}$ from given model parameters $\mathbf{m}$. To do so, we take into account several assumptions. We assume that the structure is primarily pressure-dependent and the bodies are close to their hydrostatic equilibrium. Radially symmetric bodies are, therefore, considered and lateral variations are neglected. The output (data) of the forward problem are measurable properties of bodies: radius, mass, MOI and k_2 Love number necessary to calculate the inverse problem (Section 2.2).

Governing equations

Given the input structural parameters, we assess the observables, interior structure, and density profile. Since density depends on pressure and temperature (expressed by the equation of state EOS, $\rho(p, T) = \rho(p(r), T(r)) = \rho(r)$) we must evaluate the following variables as a function of radius that allows us to assess the density profile $\rho(r)$:

- $m(r)$ – all mass below r : $4\pi \int_0^r \rho(r')(r')^2 dr'$,

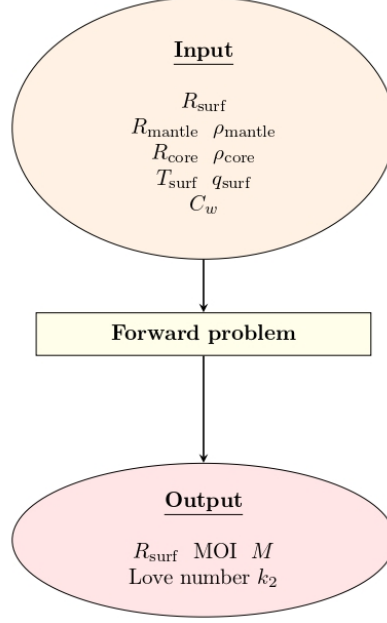


Figure 2.1 Flowchart depicting forward problem.

- $g(r)$ – gravitational acceleration at radius r : $g(r) = \frac{Gm(r)}{r^2}$,
- $P(r)$ – pressure at radius r : $P(r) = -\int_{R_{\text{surf}}}^r g(r')\rho(r')dr'$,
- $T(r)$ – temperature at radius r .

For convenience, we also integrate the polar moment of inertia $c(r)$, assuming $c(r = R_{\text{surf}}) = 0$ and $c(r = 0) = C$, i.e., the total polar moment of inertia. We therefore integrate the following system of equations:

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r) \quad (2.1)$$

$$\frac{dg(r)}{dr} = 4\pi G \rho(r) - \frac{2Gm(r)}{r^3} \quad (2.2)$$

$$\frac{dP(r)}{dr} = -\rho(r)g(r) \quad (2.3)$$

$$\frac{dc(r)}{dr} = -\frac{8}{3}\pi\rho(r)r^4 \quad (2.4)$$

In order to calculate the temperature as a function of depth, several assumptions must be made. As in Vance et al. (2014), we assume that the heat flow in the outer ice shell is purely conductive and stationary. Neglecting lateral changes in temperature, we therefore solve the following equations:

$$\begin{aligned} \frac{dT}{dr} &= -\frac{q(r)}{k_{\text{therm}}(r)} \\ \frac{dq}{dr} &= \rho(r)H - \frac{2}{r}q(r), \end{aligned} \quad (2.5)$$

where k_{therm} is the thermal conductivity and $q(r)$ is the heat flux at radius r .

In the liquid ocean, we assume convective heat transport and describe the temperature using the adiabatic profile:

$$\frac{dT}{dr} = -\frac{\alpha(r)g(r)T(r)}{c_p(r)}, \quad (2.6)$$

where α is the thermal expansion coefficient and c_p is the specific heat capacity at constant pressure.

In the high-pressure phases of ice, we assume that the temperature profile follows the melting curve to approximate a two-phase flow (Šrámek et al., 2007).

The above equations allows us to assess the total mass $M = m(R_{\text{surf}})$, $\text{MOI} = \frac{C}{MR_{\text{surf}}^2} = \frac{c(r=0)}{MR_{\text{surf}}^2}$ for the known surface radius R_{surf} . In addition, we calculate the tidal response and the corresponding Love numbers (k_2 , h_2) using the library Walterová et al. (2023). We consider a purely elastic tidal response depending on the shear modulus $\mu(r)$. The value of μ is calculated using the thermodynamic properties of the building material, depending on the temperature and pressure. For numerical reasons, the value of the shear modulus in the liquid ocean and core is prescribed as $\mu = 1$ Pa. The value in the mantle is set as $\mu = 70 \times 10^9$ Pa.

Equation of state and thermodynamic properties

Equations (2.1–2.6), along with the calculation of Love numbers, depend on several key parameters: density ρ , thermal conductivity k_{therm} , thermal expansion coefficient α , specific heat capacity at constant pressure c_p , and shear modulus μ . In addition, the melting temperature T_m is required to model the thermal behaviour of high-pressure (HP) ices and ice-ocean interfaces. These parameters are based on the published properties of the relevant building materials and their dependencies on temperature and pressure.

For the hydrosphere material, we consider three possible ocean compositions: pure water, a magnesium sulphate MgSO_4 solution with a concentration between 1-10 wt%, and a sodium chloride NaCl solution with a concentration of 0-15 wt%. Beyond this range, no thermodynamic data are available without extrapolation. The EOS and the thermodynamic data required in the calculations were obtained from the open-source library *PlanetProfile* (MgSO_4 Styczinski et al., 2023) and *SeaFreeze* (NaCl , pure water Journaux et al., 2020a; Bollengier et al., 2019; Andersson and Inaba, 2005). The case of NaCl was applied only to Europa, as *SeaFreeze* does not currently support the higher pressure in the presence of salt required to model HP phases present in Ganymede. Phase diagrams for concentration 5wt% of MgSO_4 and NaCl solutions, are depicted in Figure 2.2 and Figure 2.3. The phase diagram for pure water is in Figure 2.4. In Attachment A.1, we have also shown a density diagram for 5 wt% of MgSO_4 , NaCl and pure water for illustration purposes.

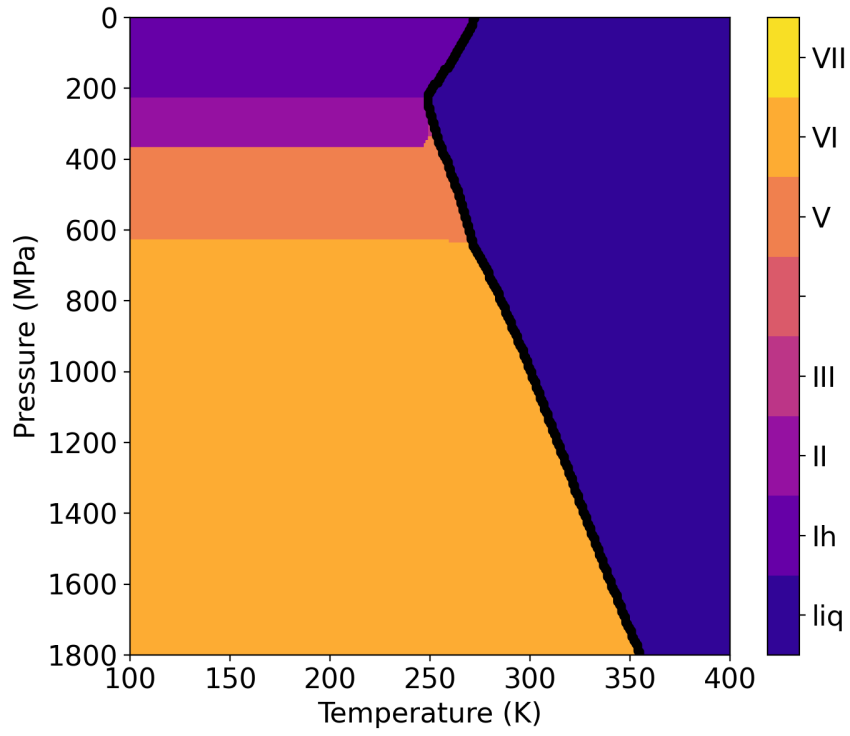


Figure 2.2 Phase diagram for salt MgSO_4 solution, concentration wt = 5%.

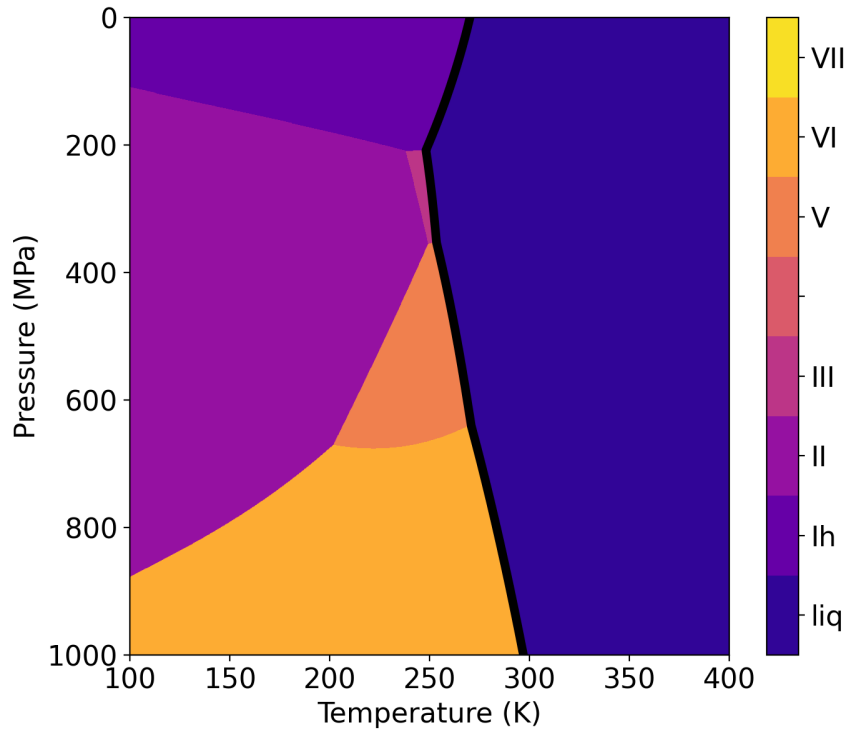


Figure 2.3 Phase diagram for salt NaCl solution, concentration wt = 5%.

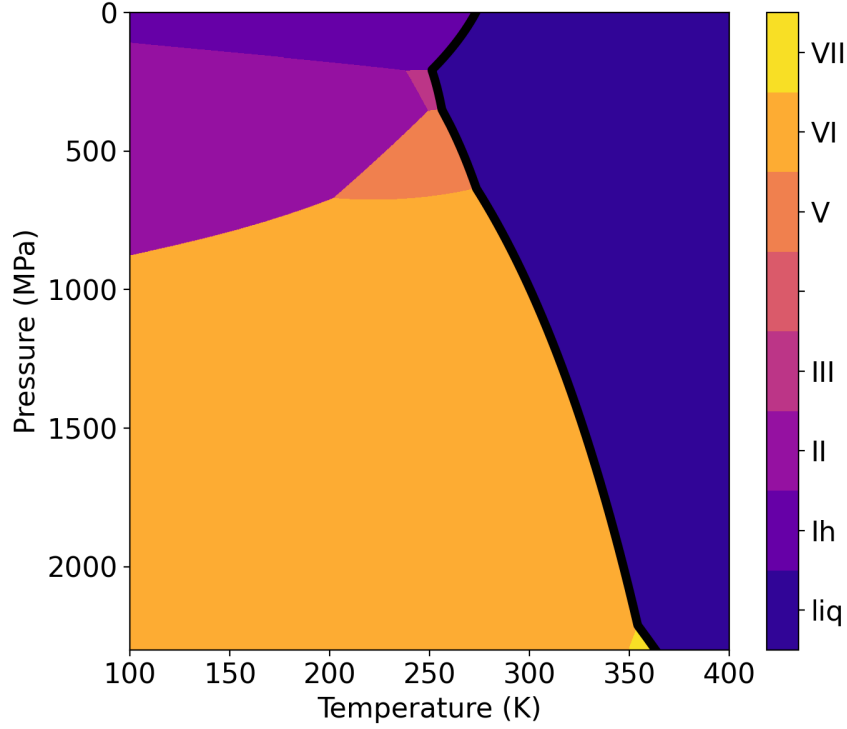


Figure 2.4 Phase diagram for pure water. No salt present.

Using *PlanetProfile* and *SeaFreeze*, we generate lookup tables of material properties (phase diagram, ρ , α , k_{therm} , c_p , μ) as functions of pressure, temperature, and salt concentration (P, T, C_w). Similarly, we construct tables for melting curves depending on pressure and salt concentration, which are used to evaluate the ocean-ice boundary and to assess temperature in HP ices. Material properties at any given point (P, T, C_w) or (P, C_w) are obtained through linear interpolation using a custom-made **Fortran** library for **Python**.

The properties of the inner layers of the body are simplified. We consider constant densities (independent of P - T) for both the mantle and the core materials. However, a range of density values is explored to enable comparison across different internal structure models. The a priori values adapted in our model are discussed in Section 2.2.

Modelling workflow

In this subsection, we explain the procedure used in our code to calculate the internal structures of icy bodies. The procedure is divided into individual layers of the body, i.e., integration in the hydrosphere, the mantle, and the core (Figure 2.5). For the individual layers: hydrosphere, mantle, and core, the integration schemes are depicted in Figure 2.6 and Figure 2.7, respectively. We identified the following parameters in our setting that are needed to evaluate the internal structure: radii of the surface R_{surf} , mantle R_{mantle} , and core R_{core} ; salt concentration in the ocean C_w ; mantle ρ_{mantle} and core ρ_{core} densities; surface heat flux q_{surf} .

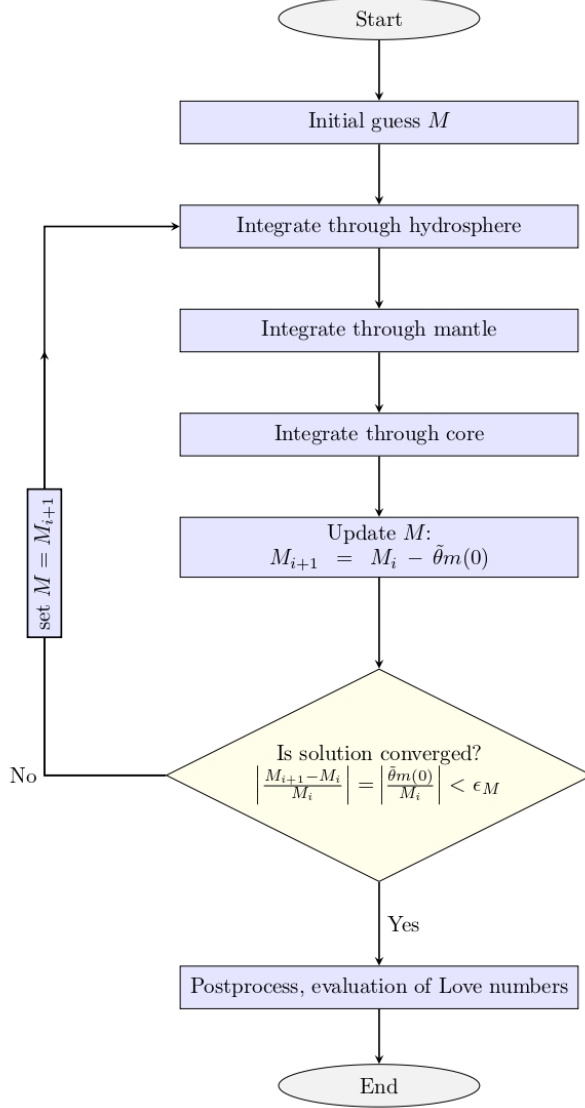


Figure 2.5 Flowchart showing scheme of algorithm iteration in the code.

Assessing internal structure: iteration scheme

The iteration scheme used in the code is shown in Figure 2.5. Our computational scheme begins by assigning the initial estimate for the total mass of the body $M_{i=0}$ (typically based on the mean mass value). For i -th iteration, the following initial conditions at the surface $r = R_{\text{surf}}$ are applied:

$$\begin{aligned}
 m(r = R_{\text{surf}}) &= M_i \\
 g(r = R_{\text{surf}}) &= \frac{GM_i}{R_{\text{surf}}^2} \\
 P(r = R_{\text{surf}}) &= 0 \text{ Pa} \\
 c(r = R_{\text{surf}}) &= 0 \text{ kg m}^2 \\
 T(r = R_{\text{surf}}) &= 110 \text{ K} \\
 q(r = R_{\text{surf}}) &= q_{\text{surf}}
 \end{aligned} \tag{2.7}$$

We then integrate Equations 2.1-2.4, along with the appropriate equations for temperature (2.5, 2.6), through each layer from the surface to the centre. For integration, we use the Runge-Kutta integration RK45 from the `scipy` library. It is based on the Runge-Kutta method of 5th and 4th order, which uses the 5th order Runge-Kutta formula in the individual integration steps, but 4th order accuracy to calculate the error. We assume continuity in mass, pressure, gravitational acceleration, and temperature at material/phase interfaces.

When the integration reaches the centre of the body, the mass at the centre, $m(r = 0)$, is obtained. A new estimate of the total mass is computed $M_{i+1} = M_i - \tilde{\theta}m(r = 0)$. The parameter $\tilde{\theta}$ (Table 2.1) is a numerical parameter that stabilises convergence and avoids oscillatory behaviour. If the relative difference between the iterations is less than ϵ_M (Table 2.1), i.e., $\left| \frac{M_{i+1} - M_i}{M_i} \right| < \epsilon_M$, the model is accepted, and the corresponding calculations are saved at discretisation points. Note that this condition leads naturally to $m(r = 0) \leq \epsilon_m / \tilde{\theta} M_i$. If the relative difference is greater than ϵ_M , the code iterates with the new total mass until the convergence condition is met, or we reach the maximum number of iterations $n = n_{\max}$ (Table 2.1).

Integration through hydrosphere

As we integrate from the surface to the centre, we first integrate the equations in the hydrosphere. A flowchart, depicting integration in the hydrosphere, is shown in Figure 2.6.

The code starts the integration from the surface, $r = R_{\text{surf}}$, using the boundary conditions (Equations 2.7). First, the size of the discretisation step in the outer ice shell (Δr_{ice}) is determined: ($\Delta r_{\text{ice}} = \min(d_{\text{ice}}/n_{\text{ice}}, \Delta r_{\text{ice}}^{\min})$), d_{ice} is the thickness of the outer ice shell, estimated from the previous iteration). A discretisation step determines the radii at which relevant parameters are stored. However, between two discretisation steps, the equations (2.1 – 2.5) are integrated using `scipy` RK45 utilising adaptive steps to ensure local relative accuracy ϵ_I (Table 2.1). Numerical integration proceeds between each pair of discretisation radii, from the current radius r_d to the lower bound $r_{d+1} = \max(r_d - \Delta r_{\text{ice}}, R_{\text{mantle}})$. This approach ensures that the integration remains confined to the hydrosphere. After this integration, the code determines whether at the next discretisation step r_{d+1} , pressure and temperature conditions for i) the presence of ice are reached, ii) the ocean are reached, or iii) the mantle are reached.

- i) When the P - T conditions for ice are reached, we move the integration scheme to the next discretisation step.
- ii) The ocean is reached. The ice-ocean boundary is determined using the bisection method, with a relative error ϵ_B (Table 2.1). The current value of the radius is then set as the ice-ocean boundary, and integration continues in the ocean.
- iii) If the mantle is reached, it simply means that there is no ocean present, and the outer shell may consist of Ih ice and possibly also HP ices and integration within the hydrosphere ended.

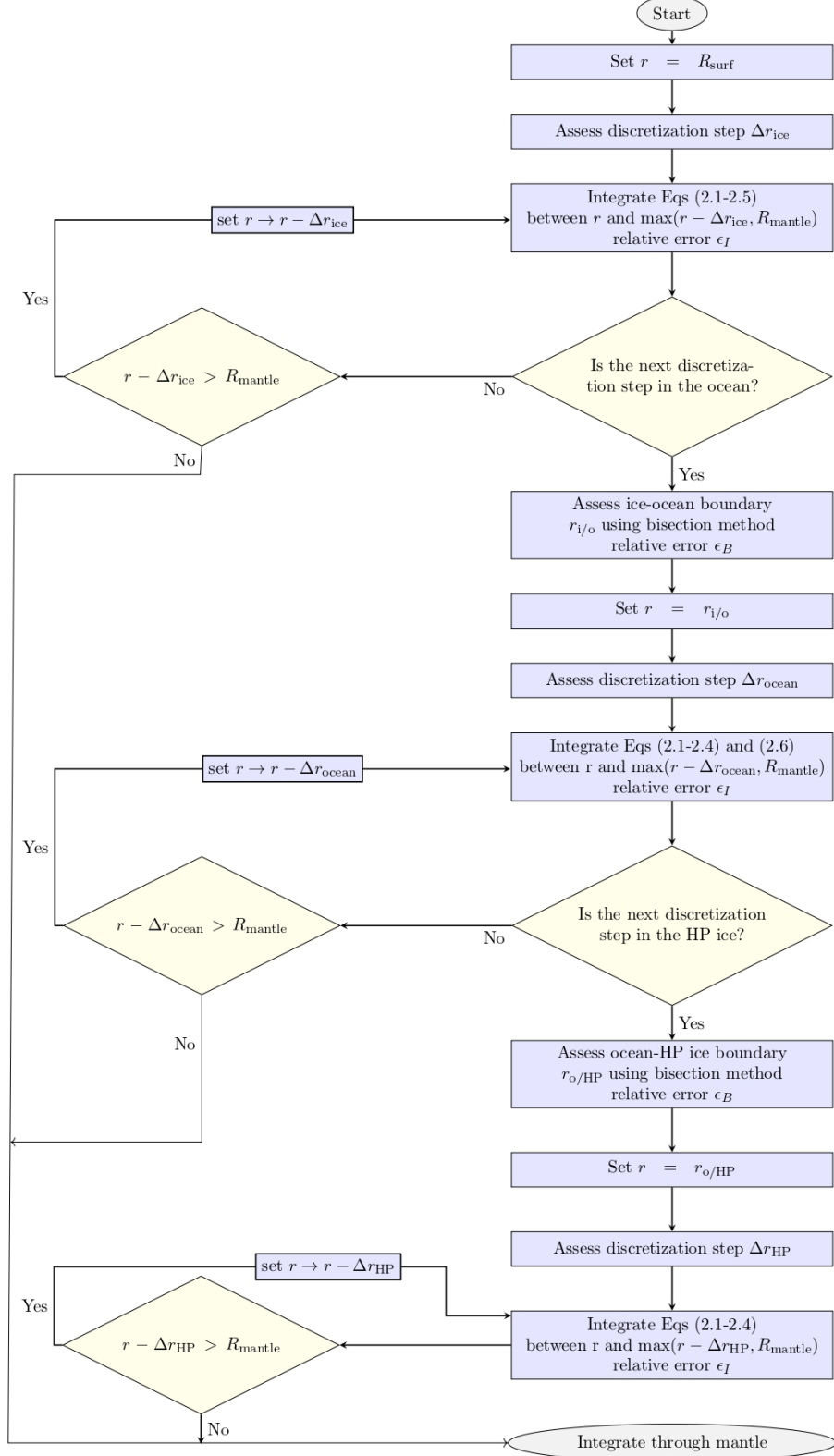


Figure 2.6 Flowchart showing scheme of integration in the hydrosphere. Equations used in the integrations are (2.1)-(2.4), including equation calculating thermal profile (2.5) or (2.6), depending on the layer. Values or all relative errors are provided in Table 2.1. Notation: $r_{i/o}$ is the low pressure ice (Ih ice) - ocean interface radius, $r_{o/HP}$ is the ocean - high pressure ices (HP ices) interface radius, Δr_{ice} is discretization step in the Ih ice, Δr_{ocean} is discretization step in the ocean and Δr_{HP} is discretization step in the HP ices.

The integration procedure in the ocean is similar to that for integration through Ih ice. The discretisation step (Δr_{ocean}) is determined ($\Delta r_{\text{ocean}} = [i = 0 : 1 \text{ km}, i > 0 : \Delta r_{\text{ocean}}^{\text{min}} = d_{\text{ocean}}^{i-1}/n_{\text{ocean}}]$), d_{ocean} is the thickness of the ocean layer, and numerical integration of the equations (2.1 – 2.4, 2.6) proceeds using `scipy` RK45 with local relative accuracy ϵ_I (Table 2.1) between two discretization steps. Again, after integration, the code checks P - T conditions of the next discretisation step for i) the ocean, ii) HP ices, or iii) the mantle:

- i) The ocean is reached, and we move the integration scheme to the next discretisation step.
- ii) The next discretisation step is located in HP ice; the value of the ocean-HP ice boundary is again determined using the bisection method with respect to the relative error ϵ_B (Table 2.1), and integration continues in the HP ice layer. The current value of the radius is set as the radius of the ocean-HP ice boundary.
- iii) If the mantle is reached, there are no HP ices present, and the integration in the hydrosphere comes to an end.

Again, in the HP ice layer, the principle of integration is the same as in the previous layers of the hydrosphere. The discretization step (Δr_{HP}) is determined as $\Delta r_{\text{HP}} = d_{\text{HP}}/n_{\text{HP}}$, d_{HP} is the thickness of the HP ices, and the equations (2.1 – 2.4) are numerically integrated (local relative accuracy ϵ_I , Table 2.1) between two discretization steps. The procedure continues until the mantle is reached.

Parameter	Value
ϵ_M	1×10^{-6}
ϵ_I	1×10^{-8}
ϵ_B	1×10^{-8}
$\tilde{\theta}$	0.75
n_{max}	10
n_{ice}	10(E) or 30(G)
n_{ocean}	10 (E) or 30(G)
n_{HP}	10(E) or 30(G)
Δr_{min}	$5 \times 10^4 \text{ m}$

Table 2.1 Table containing numerical parameters. Value used for Europa is noted as (E), value used for Ganymede as (G).

An important note is that because of the phase transitions and the difference between the equations describing the temperature, the boundary between each phase (Ih ice-ocean, ocean-HP ice) needs to be determined with higher accuracy than a discretisation step. That is why our code uses the standard bisection method.

Not all layers of the hydrosphere may be present in every model, depending on the conditions prevailing in the body. Therefore, our code must be adapted to all possibilities in which individual layers of the hydrosphere could be present.

If the conditions for the presence of the ocean are not met, the outer ice shell consists of LP ice and HP ice. In this special case, we assume a conductive profile also in the HP ices, and the whole integration scheme is simplified.

Integration through mantle and core

When integration through the hydrosphere is complete, the code continues into the deeper inner layers of the body. Since the properties of both layers are simplified, the principle of integration in both is very similar.

After reaching the boundary of the mantle, the discretisation step in the mantle ($\Delta r_{\text{mantle}} = (R_{\text{mantle}} - R_{\text{core}})/n_{\text{mantle}}$) is assessed. The integration of equations (2.1 - 2.4) proceeds discretisation step by discretisation step through the mantle until the core boundary is reached and integration is complete.

In the last layer, the core, the code proceeded the same way as in the case of the mantle. The discretisation step ($\Delta r_{\text{core}} = R_{\text{core}}/n_{\text{core}}$) is determined, and numerical integration of equations (2.1 - 2.4) occurs, the same as in the mantle. Note that due to a singularity for $r \rightarrow 0^+$, we use a regularisation to ensure a finite solution.

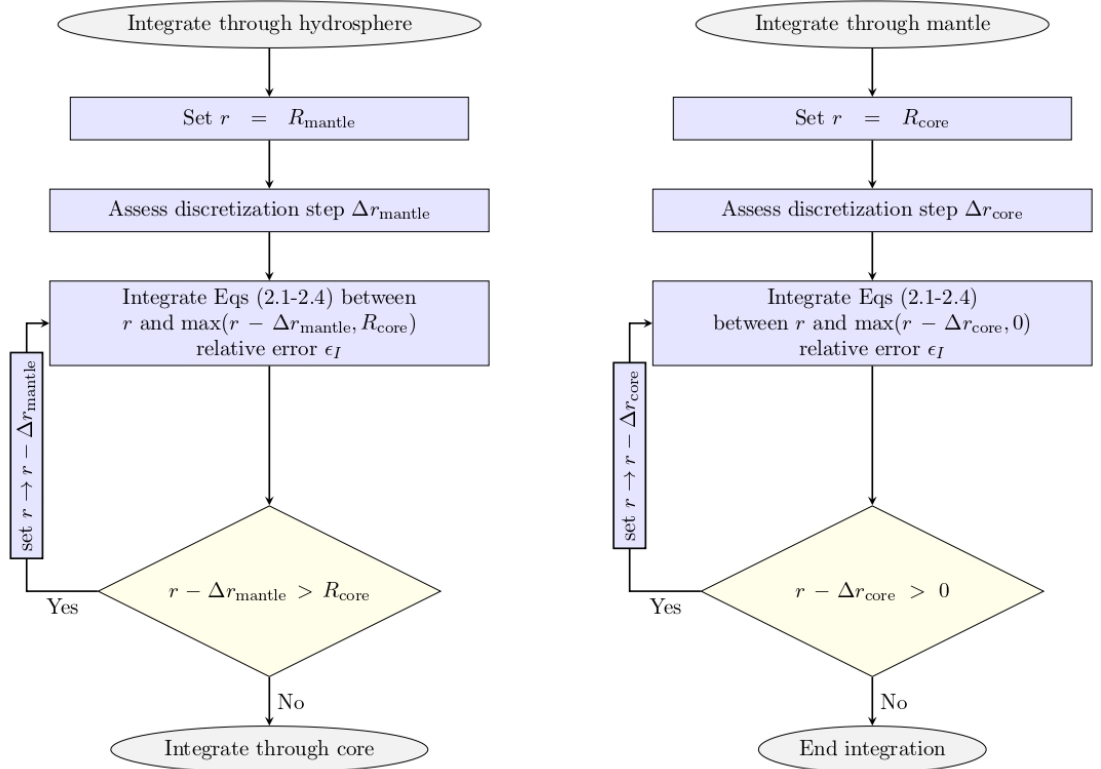


Figure 2.7 Flowchart showing scheme of integration in the mantle (left) and core (right). Equations used in the integrations are (2.1)-(2.4). EOS and material properties are simplified in the case of mantle and core, only taking into account constant values. Values and all relative errors are mentioned in the text.

2.2 Inverse problem

Modelling the internal structures of icy moons presents significant challenges due to the limited availability of data. As a result, the problem is strongly underdetermined, with trade-offs between the structural parameters leading to degeneracies. However, many parameters can be constrained using assumptions based on material properties and other scientific knowledge, providing a priori information.

To formulate the inverse problem, the logarithm of the likelihood function is defined as

$$\log p(\mathbf{d}|\mathbf{m}) = -\frac{1}{2} \sum_{i \in \mathcal{D}} \left(\left(\frac{d_i - f_i(\mathbf{m})}{\sigma_i} \right)^2 + \log \sigma_i^2 \right), \quad (2.8)$$

where $\mathcal{D} = \{M, \text{MOI}, R_{\text{surf}}\}$ or $\mathcal{D} = \{M, \text{MOI}, R_{\text{surf}}, k_2\}$, $\mathbf{m}$ represents the model and $\mathbf{d}$ corresponds to the data. For model parameters, we further consider a priori information that limits the allowed interval:

$$\log p(m_j) = \begin{cases} 0 & m_j \in [m_j^{\text{lower}}, m_j^{\text{upper}}] \\ -\infty & \text{otherwise} \end{cases} \quad (2.9)$$

or we consider no a priori information

$$\log p(m_j) = 0. \quad (2.10)$$

The a priori probability therefore follows

$$\log p(\mathbf{m}) = \sum_j^m \log p(m_j). \quad (2.11)$$

To explore the posterior distribution defined by the log-likelihood and the prior constraints, we adopt a Bayesian sampling approach based on Markov Chain Monte Carlo (MCMC) methods. The goal is to generate representative samples of the model parameters $\mathbf{m}$ from the posterior distribution $p(\mathbf{m}|\mathbf{d})$, which is proportional to the product of the likelihood and the prior constraints.

$$p(\mathbf{m}|\mathbf{d}) \propto p(\mathbf{d}|\mathbf{m})p(\mathbf{m}).$$

emcee

To sample the posterior probability, we use the freely available **Python** library **emcee** (Foreman-Mackey et al., 2013), which uses the Markov Chain Monte Carlo (MCMC) Affine Invariant method for evaluating the likelihood of the parameters. **emcee** samples posterior distributions, especially those that are non-Gaussian and strongly correlated. The **emcee** library is based on the stretch move algorithm (Attachement A.2) created by Goodman and Weare (2010).

The **emcee** sampler is initialised with multiple walkers K (typically $K = 96$), each starting from random positions drawn from the a priori space. The sampler is then run for a sufficient number of iterations to ensure convergence. To suppress dependence on initial conditions, an initial burn-in phase (defined as more than 50 times the autocorrelation time) is discarded. The remaining ensemble, comprising at least 50 times the autocorrelation time, is used to estimate credible intervals, marginal distributions, and parameter correlations.

2.3 Models

We employ `emcee` library in our code with the forward problem. The diagram describing the inverse problem is shown in Figure 2.8. Data **d** are depicted as an orange ellipse. For chosen models, we also include the Love number k_2 , whose possible value should be measured by the ongoing missions JUICE and Europa Clipper, together with its uncertainties. Both JUICE and Europa Clipper should also measure the Love number h_2 , but it has not yet been included in the inversion for Europa. One reason for this is the significant expected uncertainty, which for Europa is estimated to be 0.04-0.17 (Steinbrügge et al., 2018a). In the case of Ganymede, the reason was simply due to time requirements. For Ganymede, the measurement error h_2 is assumed to be only 2% (Steinbrügge et al., 2015).

The parameters **m** are represented by the blue ellipse (Figure 2.8). The model parameters comprises of body (R_{surf}), mantle (R_{mantle}), and core (R_{core}) radii; densities of mantle (ρ_{mantle}) and core (ρ_{core}); surface heat flux (q_{surf}) and for some models also salt concentration within the ocean (C_w). Note that we include the body radius R_{surf} into both the data and into the model space for convenience (the mapping function Equation 2.8: $f_{R_{\text{surf}}}(\mathbf{m}) = R_{\text{surf}}$). A priori information in the form of the allowed interval is prescribed for R_{mantle} , R_{core} , ρ_{mantle} , ρ_{core} , q_{surf} , and C_w . We limit the radii of the core and mantle to arbitrary but wide intervals that satisfy the results or assumptions from previous studies, e.g., Vance et al. (2014). We limit mantle densities to those corresponding to serpentinite $\text{Mg}_3\text{Si}_2\text{O}_5\text{OH}_5$ and values corresponding to fayalite Fe_2SiO_4 ; the interval for core densities is set between the eutectic value of FeS and pure iron. The heat flux interval is chosen in order to cover heat flux for the case of no ocean present in the hydrosphere, as well as the case with a large ocean present.

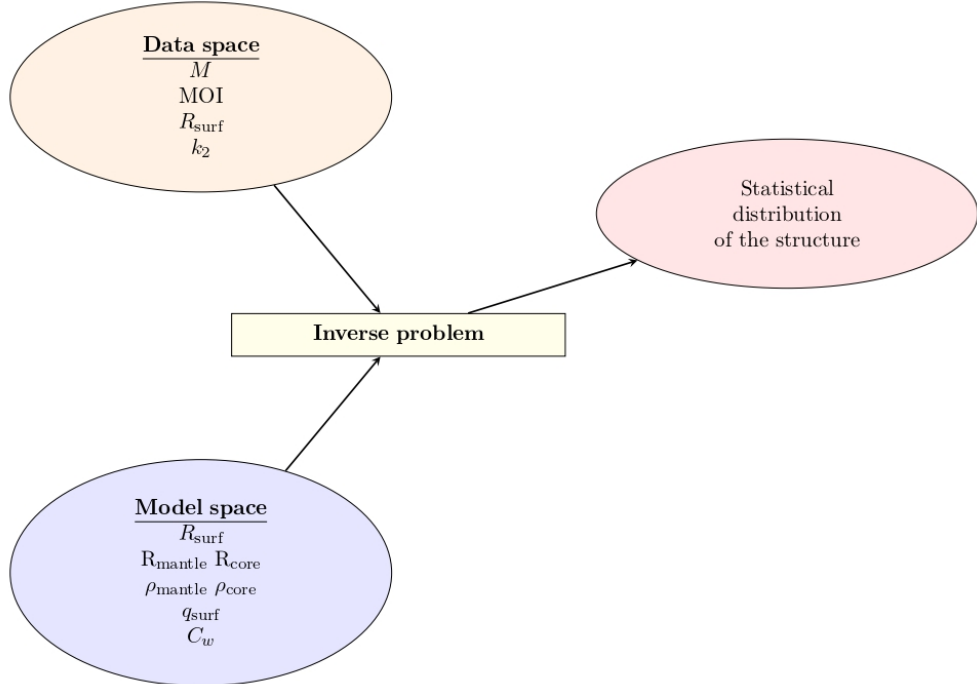


Figure 2.8 Flowchart showing scheme of inverse problem

For statistical analysis, we solve the inverse problem for several combinations of data and model parameters:

Reference model

Observed data: $\mathbf{d} = (M, \text{MOI}, R_{\text{surf}})$.

Model parameters: $\mathbf{m} = (R_{\text{surf}}, R_{\text{mantle}}, R_{\text{core}}, \rho_{\text{mantle}}, \rho_{\text{core}}, q_{\text{surf}})$.

Ocean composition: magnesium sulphate (MgSO_4) solution with concentration $C_w = 5 \text{ wt}\%$.

k_2 model:

Reference model with observed data $\mathbf{d} = (M, \text{MOI}, R_{\text{surf}}, k_2)$.

This model incorporates tidal Love number k_2 as an additional observational constraint.

Concentration model:

Reference model with $\mathbf{m} = (R_{\text{surf}}, R_{\text{mantle}}, R_{\text{core}}, \rho_{\text{mantle}}, \rho_{\text{core}}, q_{\text{surf}}, C_w)$. The salt concentration in the range $C_w = 1 \text{ wt}\%$ to $10 \text{ wt}\%$.

This model therefore incorporates an additional compositional parameter.

NaCl model:

Parameters and observed data as in the reference model. Ocean composition: sodium chloride (NaCl) with concentration $C_w = 5 \text{ wt}\%$.

PureWater model:

Parameters and observed data as in the reference model. No salt.

2.4 Post-processing and statistical analysis

Our depicted parameters, along with data and model parameters, are **thickness of the hydrosphere** (D_{hydro}), **thickness of the Ih ice** (D_{Ih}), **thickness of the ocean** (D_{ocean}), **thickness of the HP ice** (D_{HP}) (if present), **temperature at the Ih ice/ocean interface** (T_{b}), **tidal Love numbers** k_2 ($\text{Re } k_2$) and h_2 ($\text{Re } h_2$), **diminishing factor** $(1 + k_2 - h_2)$ (Delta or Δ) and **ratio of tidal Love numbers** h_2 and k_2 ($\text{Re}(h_2/k_2)$). In the concentration model, **concentration** (C_w) is also depicted. The value of $\text{Re}(h_2/k_2)$ is included among the parameters inspired by Steinbrügge et al. (2018b). According to their findings, $\text{Re}(h_2/k_2)$ suppresses the influence of the upper layers of the structure (hydrosphere) and can increase the correlation of the deeper ones, i.e., the mantle and the core. They used this principle to determine the size of Mercury's inner core and claimed that the measurement of $\text{Re}(h_2/k_2)$ for Mercury could provide at least an upper bound on the radius of the inner core (Steinbrügge et al., 2018b). Note that our algorithm can be easily adapted to include an a priori condition (Equation 2.9) on any of these additional parameters.

The statistical distributions are then plotted using the `corner` library in `Python`, and we refer to them as corner graphs. A corner graph is commonly used to visualise relationships between multiple variables in a dataset, particularly in statistical modelling and Bayesian analysis, for displaying joint distributions and individual parameter histograms. On the off-diagonal cells, the graph displays 2D contour plots or scatter plots, illustrating the relationship between various quantities, and 1D histograms along the diagonal that show the marginal posterior distributions for individual parameters.

To better visualise the dependencies between the individual parameters, we also plot their pairwise correlations using the `seaborn` library in `Python`.

3 Results

The purpose of this chapter is to present statistical results of internal structures for various inversion models (2.3), using the code presented in Chapter 2. As mentioned, our work focuses on exploring the internal structures of the Galilean moons Europa and Ganymede, assuming that both bodies are fully differentiated. For each body, different runs were performed with varying inversion parameters, and, in the case of Europa, also with different salt concentrations and varying values of MOI.

3.1 Europa

First, we focus on Europa. The physical parameters of Europa are summarised in Table 3.1. Table 3.2 provides an overview of the a priori information used in this study.

Parameter	Value
R_{surf} [km]	1560.8 ± 0.8
M [kg]	$4.7986 \times 10^{22} \pm (\text{value} \times 1 \times 10^{-4})$
MOI Gomez Casajus et al. (2021), GC21	0.3547 ± 0.0024
MOI Anderson et al. (1998), A98	0.346 ± 0.005

Table 3.1 Physical parameters of Europa

Parameter	Value
T_{surf} [K]	110
P_{surf} [MPa]	0.0
$C_{w,\text{range}}$ (MgSO ₄)	0.01-0.1
$q_{\text{surf,lower}}$ [W/m ²]	0.001
$q_{\text{surf,upper}}$ [W/m ²]	0.03
Mantle	
$\rho_{\text{mantle,lower}}$ [kg/m ³]	2500 (serpentine Mg ₃ Si ₂ O ₅ OH ₅)
$\rho_{\text{mantle,upper}}$ [kg/m ³]	4400 (fayalite Fe ₂ SiO ₄)
$R_{\text{mantle,lower}}$ [km]	1300
$R_{\text{mantle,upper}}$ [km]	1550
Core	
$\rho_{\text{core,lower}}$ [kg/m ³]	5150 (eutectic Fe:FeS)
$\rho_{\text{core,upper}}$ [kg/m ³]	8000 (pure iron)
$R_{\text{core,lower}}$ [km]	20
$R_{\text{core,upper}}$ [km]	800

Table 3.2 A priori information for Europa.

For Europa, six models are explored. First, we focus on a *reference model*: data M , R_{surf} , and MOI. We then examine how the results change when additional data or model parameters are considered. In the k_2 *model*, the potential Love number k_2 is included as additional data, while in the *concentration model*, the ocean salt concentration C_w is treated as a model parameter. All models in this initial set, referred to as the sensitivity model set, assume a MgSO₄ solution as the ocean composition and the GC21 value of MOI (Gomez Casajus et al., 2021).

In a second set of models, we compare the resulting structures for two different values of MOI for the same MgSO_4 salt concentration of 5 wt%: The GC21 value used in the *reference model* ($\text{MOI} = 0.3547 \pm 0.0024$) and the earlier estimate of $\text{MOI} = 0.346 \pm 0.005$ (A98 MOI, Anderson et al., 1998, *A98 reference model*). We label this set as MOI model set.

Finally, we investigate the effect of ocean composition by comparing the *reference model* for MgSO_4 salt with a concentration of 5 wt%, the model using NaCl salt with a concentration of 5 wt% (*NaCl model*), and the model where we consider an ocean for pure water (*Pure Water model*). When comparing different ocean compositions, the value $\text{MOI} = 0.3547 \pm 0.0024$ from Gomez Casajus et al. (2021) is used. This set of models is labelled the ocean composition model set.

3.1.1 Sensitivity model set

Reference model

We begin our analysis by modelling the *reference model*, which serves as the baseline for all subsequent models. The data space for this model is listed in Table 3.1, and a priori constraints are provided in Table 3.2. This reference model allows us to examine the system’s behaviour and explore the trade-offs and degeneracies inherent in the inverse problem.

The corner plot for the Europa *reference model* is shown in Figure 3.1. It presents both the statistical distributions and the pairwise relationships between model parameters. To further highlight the correlations among these quantities, a correlation matrix is shown in Figure 3.2.

Several trade-offs are evident between individual parameters. As expected, we observe trade-offs among the mantle radius (R_{mantle}), the mantle density (ρ_{mantle}), and the core radius (R_{core}), which collectively balance the constraints on the total mass and MOI. The value of the density of the core (ρ_{core}) is poorly constrained, due to the fact that it is not strongly correlated with any parameter, except for a slight anti-correlation with the R_{core} (Figure 3.2).

Table 3.2 lists the a priori values of bounds for q_{surf} , R_{mantle} , ρ_{mantle} , R_{core} and ρ_{core} . Comparison of these with the a posteriori distributions in Figure 3.1 reveals several trends. The mantle radius R_{mantle} does not reach values below ~ 1400 km, despite the a priori allowing values down to 1300 km. The core radius R_{core} samples the entire a priori interval; structures with cores larger than ~ 600 km are relatively uncommon.

For mantle density ρ_{mantle} , the posterior distribution prefers higher values, but these values do not exceed $\sim 3750 \text{ kg m}^{-3}$, which is less than the maximum value given in Table 3.2. The marginal distributions suggest that there are structures where ρ_{mantle} is around the minimum allowed value, but they were rare. For ρ_{core} , we notice an almost uniform distribution, i.e., the entire a priori interval is almost uniformly covered. Similarly, the marginal posterior distribution of q_{surf} is nearly uniform throughout the allowed range ($3\text{--}30 \text{ mW m}^{-2}$), suggesting that the available data are insufficient to narrow its a priori values meaningfully.

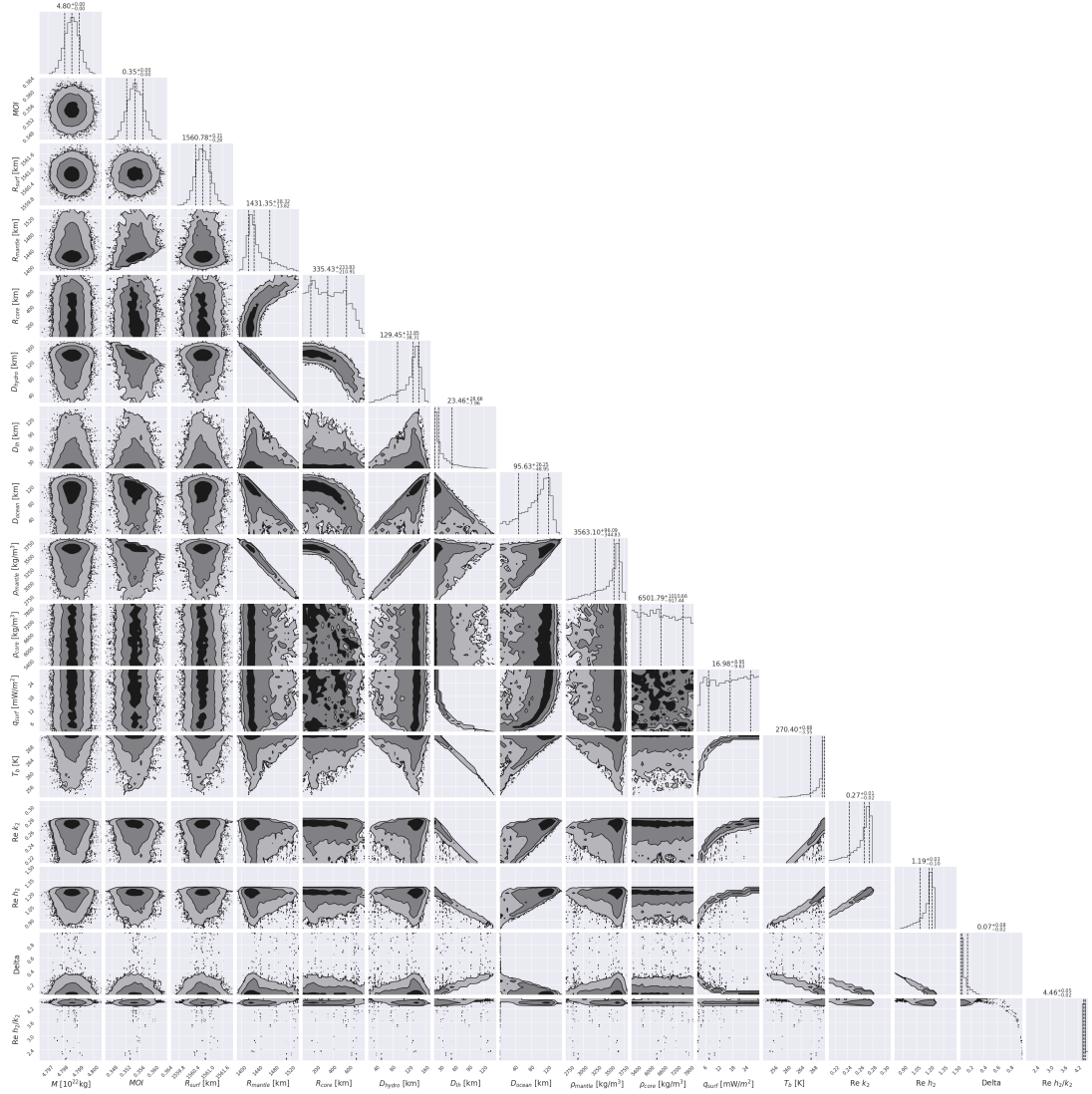


Figure 3.1 *Europa reference model*: Corner plot showing posterior distributions of model parameters and selected derived quantities. Vertical lines in the diagonal panels indicate the 16th, 50th, and 84th percentiles, representing the central credible interval. Coloured contours in the off-diagonal panels correspond to the 1- σ (black), 2- σ (dark grey), and 3- σ (light grey) confidence levels; outliers are marked by scattered points.

The surface heat flux (q_{surf}) plays a key role in determining the thermal and structural properties of Europa's outer shell. In particular, the surface heat flux (q_{surf}) controls both the temperature at the ice Ih–ocean interface (T_b) and the ice shell thickness D_{Ih} (Figure 3.1). This sensitivity is also supported by the correlation plot (Figure 3.2), where q_{surf} and T_b are strongly correlated while q_{surf} and D_{Ih} are strongly anti-correlated. The anti-correlation between D_{Ih} and q_{surf} is a direct consequence of the conductive heat transfer profile assumed. In contrast, the anti-correlation between T_b and D_{Ih} reflects the influence of the melting curve and the unique thermodynamic properties of ice Ih.

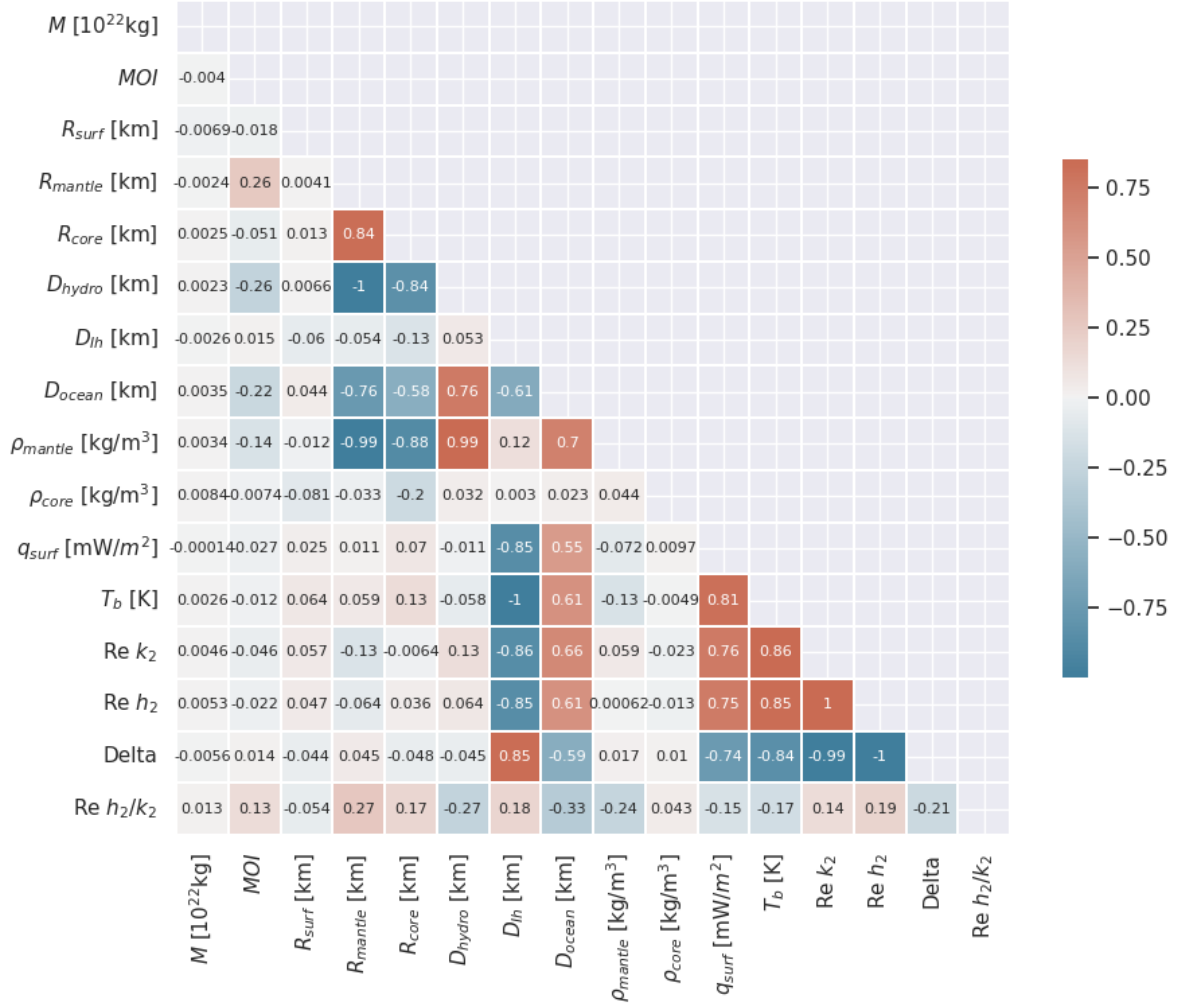


Figure 3.2 Europa reference model: Correlation plot depicting pairwise relationships between model parameters and derived quantities.

However, since q_{surf} remains poorly constrained, T_b and D_{lh} could not be reliably inferred from the current constraints. This highlights the need for additional observational constraints, such as improved estimates of surface heat flow or Love numbers, to better resolve the thermal structure of the hydrosphere.

To visualise the diversity of possible internal structures, we plot the density profiles of 100 randomly selected resulting models as a function of radius (Figure 3.3). The figure highlights the individual layers that occur in the structure of Europa: $\rho \approx 900 \text{ kg m}^{-3}$ corresponds to the density of ice Ih, $\rho \approx 1000 \text{ kg m}^{-3}$ to the density of the liquid ocean, $\rho \approx (2500 - 3500) \text{ kg m}^{-3}$ to the density of the silicate mantle, and $\rho \approx (5000 - 8000) \text{ kg m}^{-3}$ to the density of the metallic core.

As can be seen in the plot, the value of the core density is spread across the entire a priori interval. The thicknesses of the individual layers vary significantly, and in some cases, the structure lacks a liquid ocean entirely, illustrating the possible structural diversity.

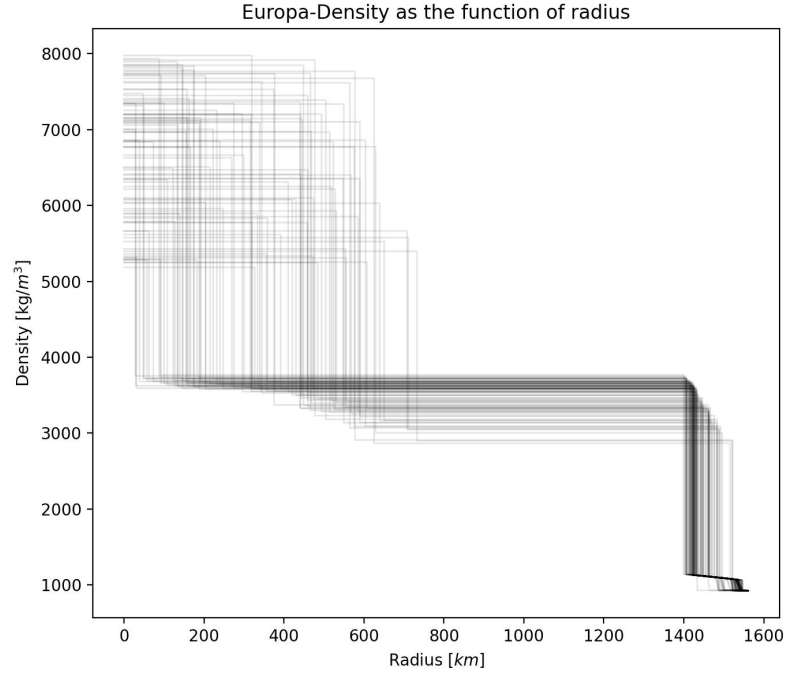


Figure 3.3 Europa *reference model*: Density profile for 100 randomly selected internal structures.

k_2 model

In the next run, we add the tidal Love number k_2 in our data space for the inverse problem. Its value is arbitrarily taken from the 1- σ interval in the posterior distribution of the *reference model*:

$$k_2 = (0.25 \pm 0.015)$$

The uncertainty is based on Mazarico et al. (2023) and corresponds to the value achievable with dozens of flybys of Europa. Model space is the same as in the reference model.

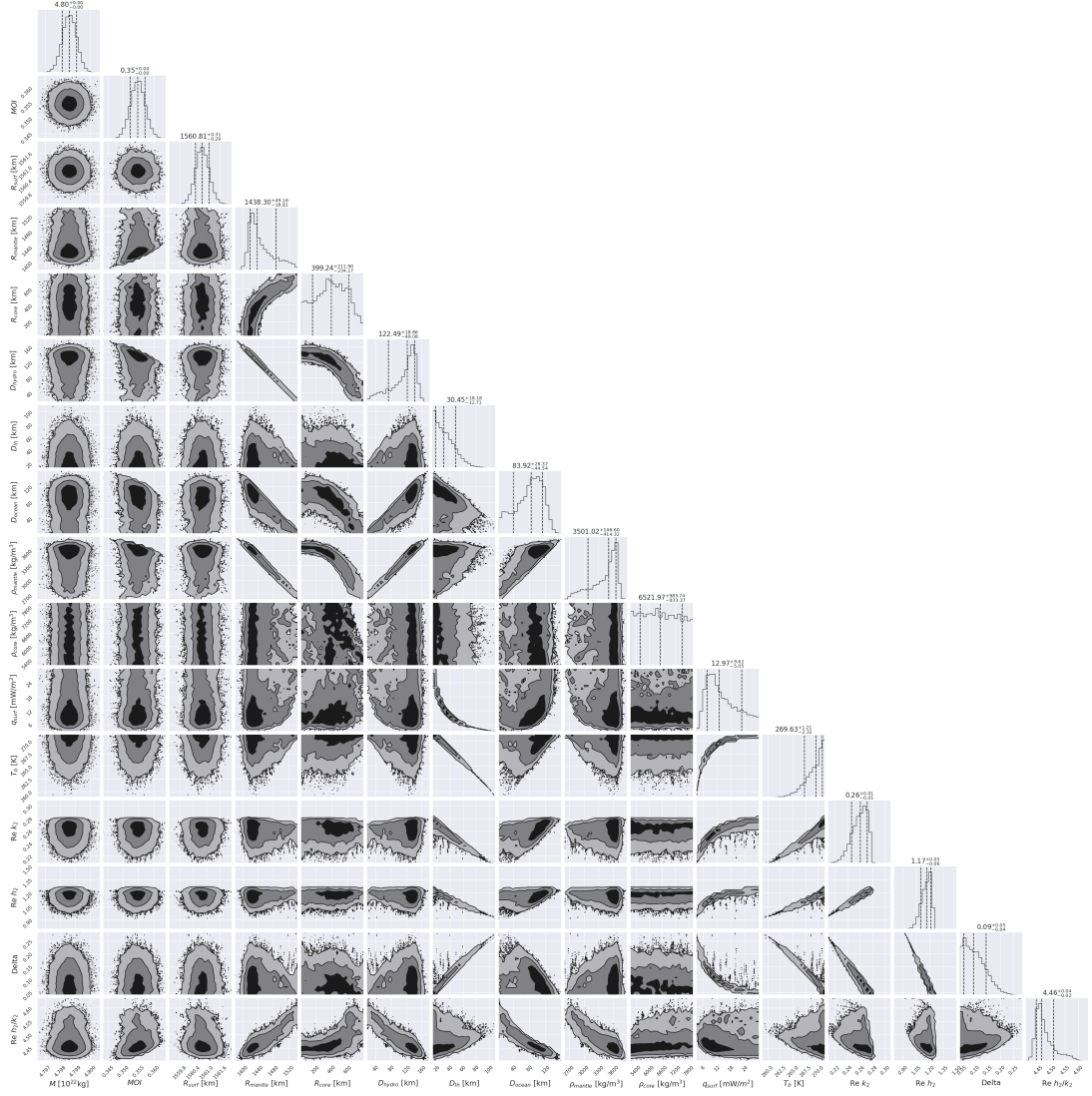


Figure 3.4 Europa k_2 model: Corner plot showing posterior distributions of model parameters and selected derived quantities.

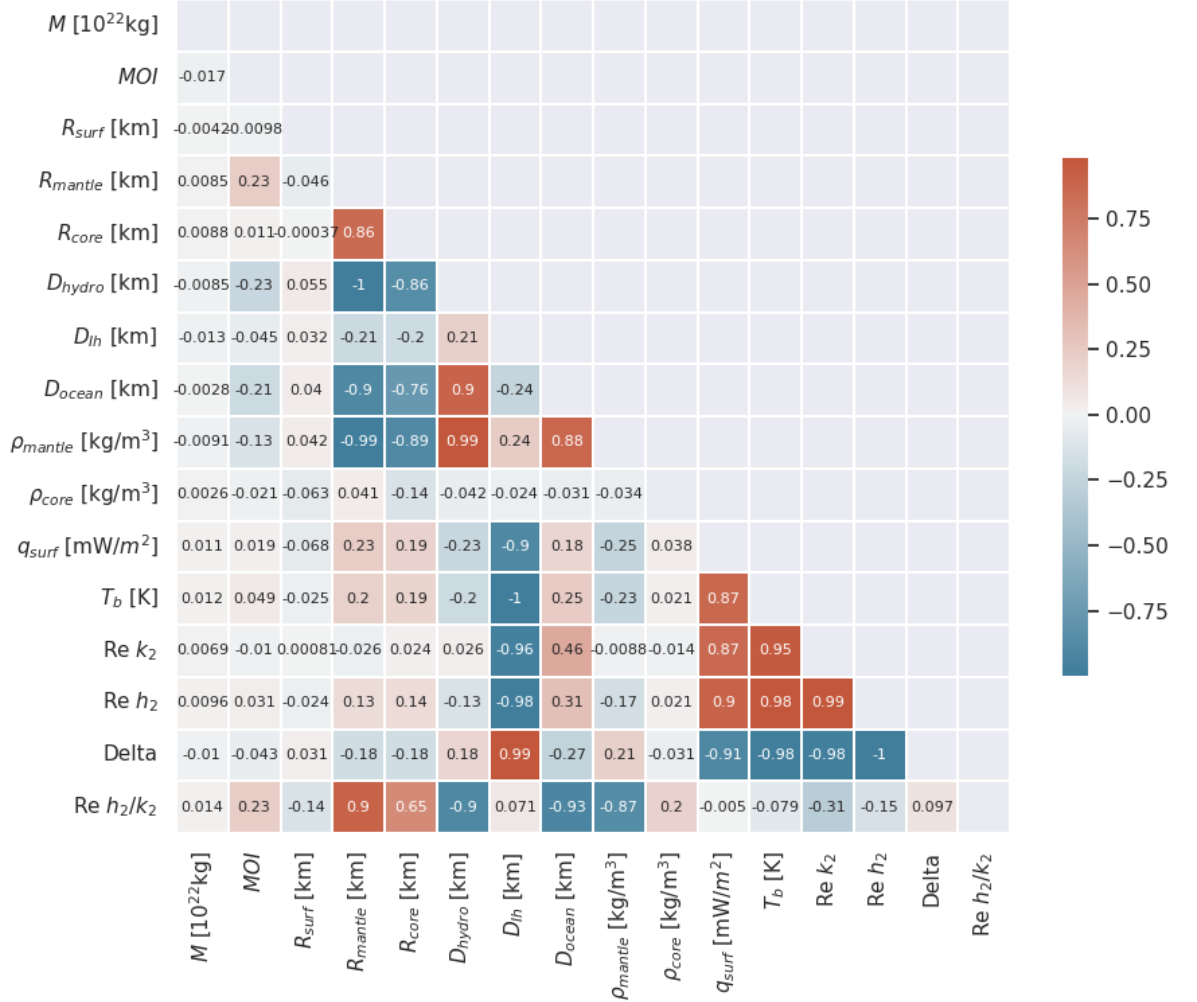


Figure 3.5 Europa k_2 model: Correlation plot depicting pairwise relationships between model parameters and derived quantities.

As in the reference model, we plot the corner plot (Figure 3.4) and the correlation matrix (Figure 3.5).

The observed trade-offs and sensitivities are broadly similar to those in the *reference model*. However, notable changes appear in the marginal posterior distributions. The most evident difference, compared to the *reference model* (Figure 3.1), is a more restricted range for the surface heat flux q_{surf} and consequently for D_{lh} , T_b , h_2 , Δ , and h_2/k_2 . This means that knowledge of the tidal Love number k_2 , i.e., adding another a priori information to the data space, would limit

the size of the uncertainties.

A comparison of the correlation plots of the *reference model* 3.2 and the k_2 *model* 3.5 reveals that many of the resulting correlations or anti-correlations have been strengthened. The most significant is the correlation between $\text{Re}(h_2/k_2)$ and the radii of the deeper layers, the mantle and the core, and the anti-correlation with D_{hydro} . Notably, the relationship between $\text{Re}(h_2/k_2)$ and R_{mantle} is almost linear, and the correlation with R_{core} is higher than in the *reference model*. This suggests that if we knew the value of the tidal Love number k_2 and $\text{Re}(h_2/k_2)$, we could estimate the thickness of the mantle and, consequently, the core with reduced uncertainties.

We also see changes in the values of the maximum a posteriori estimates. Compared with the *reference model*, larger mean values of R_{mantle} and R_{core} are present in the k_2 *model*, and therefore a lower value of D_{hydro} .

Concentration model

In this next model, we include MgSO_4 salt concentration in the ocean (C_w) in the model space. We prescribe the following a priori range:

$$C_w = [1 - 10]\%$$

The data space stays the same as in the case of the *reference model*. We plot the statistical distributions in Figure 3.6 and the correlation between the parameters in Figure 3.7.

We again observe similar trade-offs and sensitivities as in the *reference model*. A notable aspect of this model is the relationship between the salt concentration C_w and other parameters describing the internal structure. The concentration C_w shows a weak linear dependence on k_2 , but it appears uncorrelated with h_2 . This is also reflected in the correlation matrix, where k_2 and C_w exhibit a slight positive correlation while the correlation between C_w and h_2 is negligible. Interestingly, C_w is relatively strongly anti-correlated with the ratio $\text{Re}(h_2/k_2)$.

As expected, introducing C_w as an additional free parameter results in broader posterior distributions for some structural parameters, consistent with the idea that increasing model complexity generally increases uncertainty in the inverse problem.

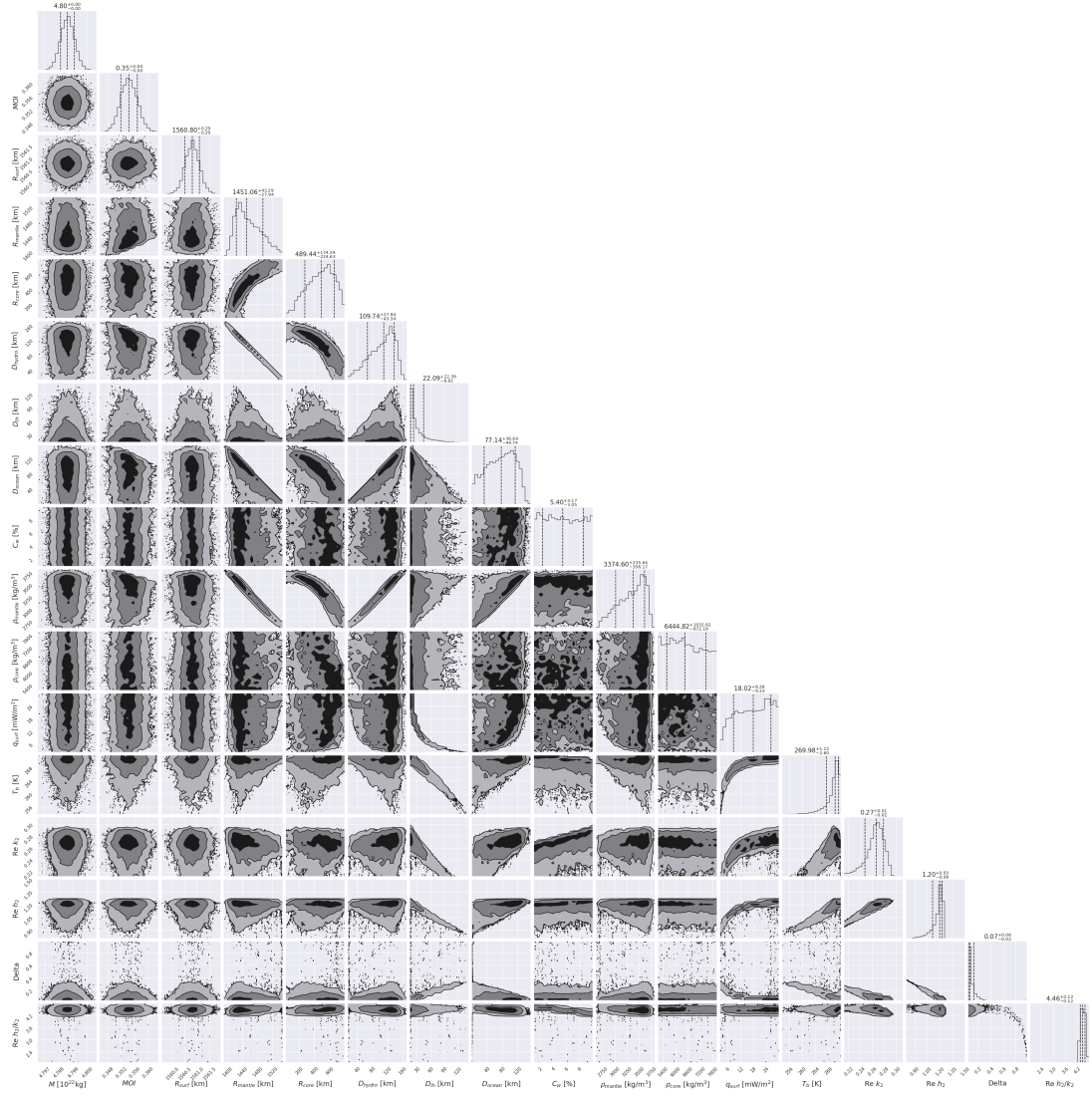


Figure 3.6 Europa *concentration model*: Corner plot showing posterior distributions of model parameters and selected derived quantities.

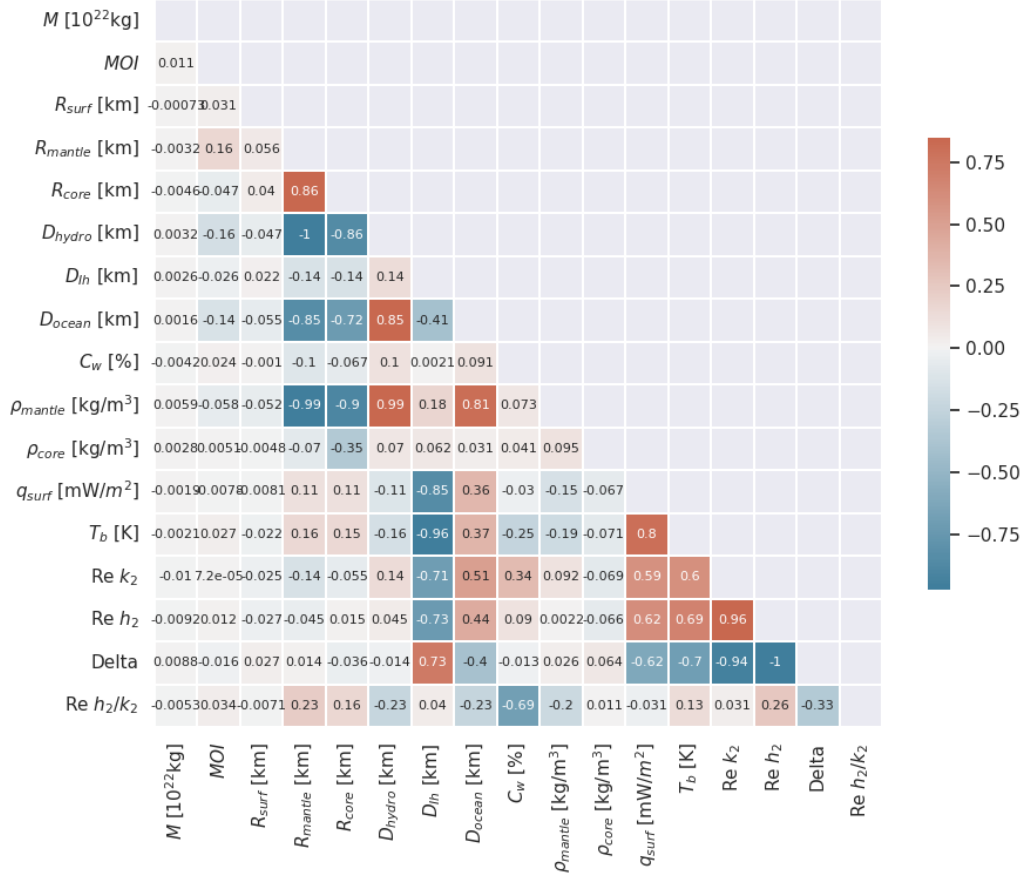


Figure 3.7 Europa *concentration model*: Correlation plot depicting pairwise relationships between model parameters and derived quantities.

Comparison of sensitivity model set

To better compare the differently constrained models, we depict the histograms (plots reflecting the marginal posterior distribution) of three parameters: q_{surf} , R_{mantle} , and R_{core} . This allows us to assess how the resulting posterior intervals change between the different model setups.

First, we display the histogram of q_{surf} for all three models (Figure 3.8). Comparing the three models, the *concentration model*, which introduces an additional model parameter C_w , still yields an almost uniform distribution for q_{surf} . In contrast, the k_2 *model*, which incorporates an additional observation constraint, significantly narrowed the posterior distribution interval. We observe that lower values of q_{surf} are preferred, between 7-12 mW m⁻². It should be noted that this result depends on our choice of the k_2 value used in the inversion.

Figure 3.9 shows the histograms of R_{mantle} . Introducing an additional model parameter in the inversion (*concentration model*) broadens the range of plausible values compared to the *reference case*. In the k_2 *model*, the posterior distribution is also slightly wider compared to the *reference model*, but a preference for lower R_{mantle} values is still evident. This shows that k_2 mainly determines the hydrosphere structure of the body.

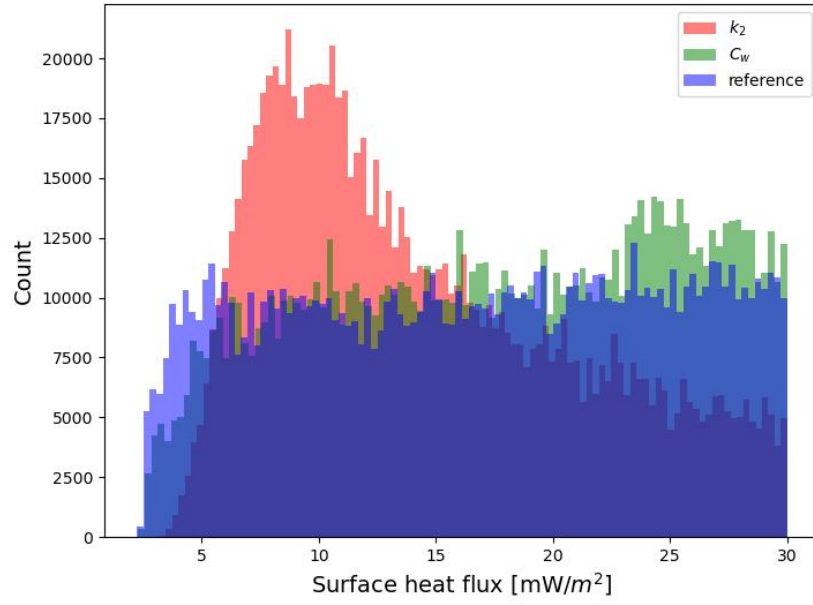


Figure 3.8 Histogram of the surface heat flux for Europa: the *reference model* (blue), the k_2 *model* (red), and the *concentration model* (green).

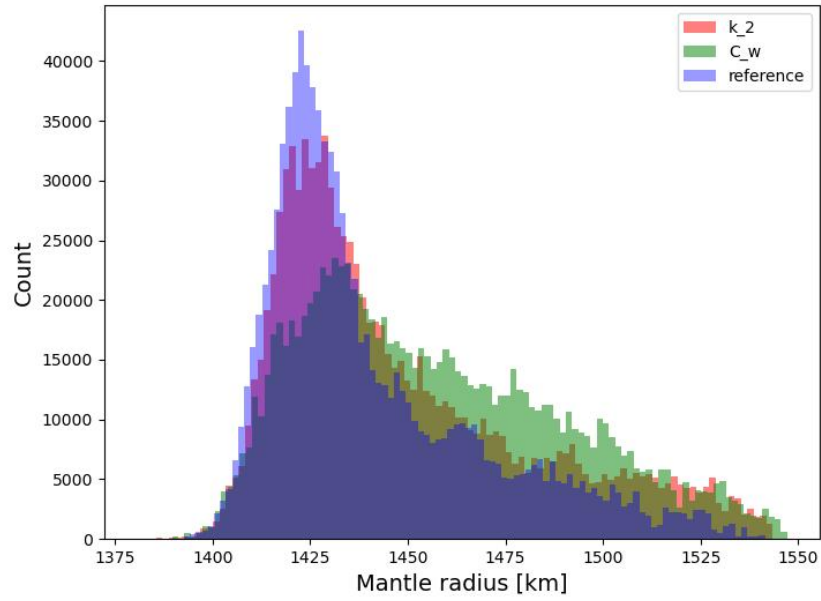


Figure 3.9 Histogram of the mantle radius for Europa: the *reference model* (blue), the k_2 *model* (red), and the *concentration model* (green).

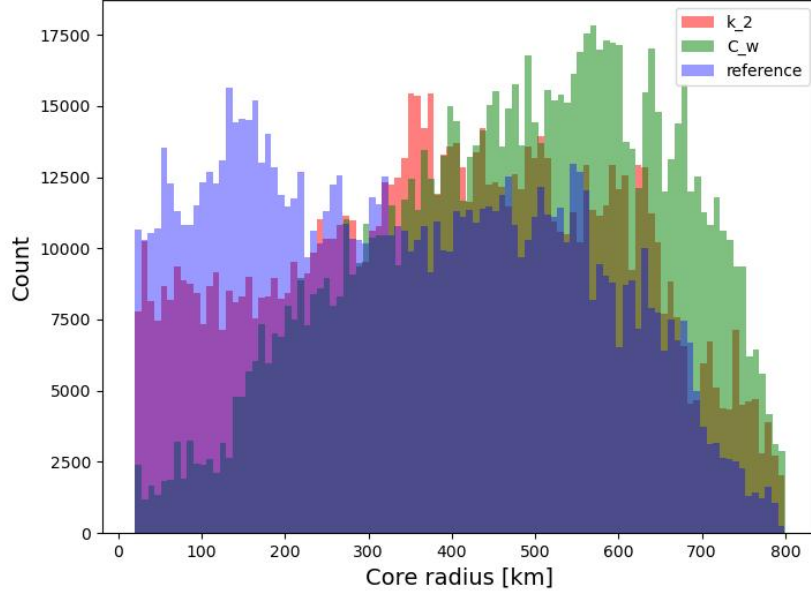


Figure 3.10 Histogram of the core radius for Europa: the *reference model* (blue), the k_2 model (red) and the *concentration model* (green).

The last Figure 3.10 depicts histograms of the R_{core} for the three models. The histograms show a significant difference. While the *reference model* prefers smaller values, the k_2 model and the *concentration model* prefer higher values of R_{core} .

3.1.2 MOI model set

A98 reference model

Up to this point, our models differ in various parameters that we add to the data or model space, but the values of the measured quantities stay the same. In this as well as the following sections, however, the model and data space are kept identical to those in the *reference model*. Here, we instead vary the input value of MOI. Specifically, we use a previously published value of MOI for Europa: $\text{MOI } 0.346 \pm 0.005$, derived from close flybys of the Galileo mission (Anderson et al., 1998). This value differs from the GC21 model estimate by more than $1\text{-}\sigma$. The salt composition and concentration remain the same as in the previous cases, i.e., 5 wt% of MgSO_4 . In this section, the *reference model* is labelled as the *GC21 model*, referring to the MOI used.

The statistical distributions are shown in Figure 3.11 and the correlation diagram in Figure 3.12.

Again, the trade-offs and sensitivities between both the *GC21 model* and the *A98 model* are similar. Since the value A98 MOI 0.346 ± 0.005 is smaller than the GC21 value, the mass in the structure of Europa should be concentrated more towards the centre of the body than in the *GC21 model*. Consequently, we focus on the parameters that describe the deep layers: the mantle and the core.

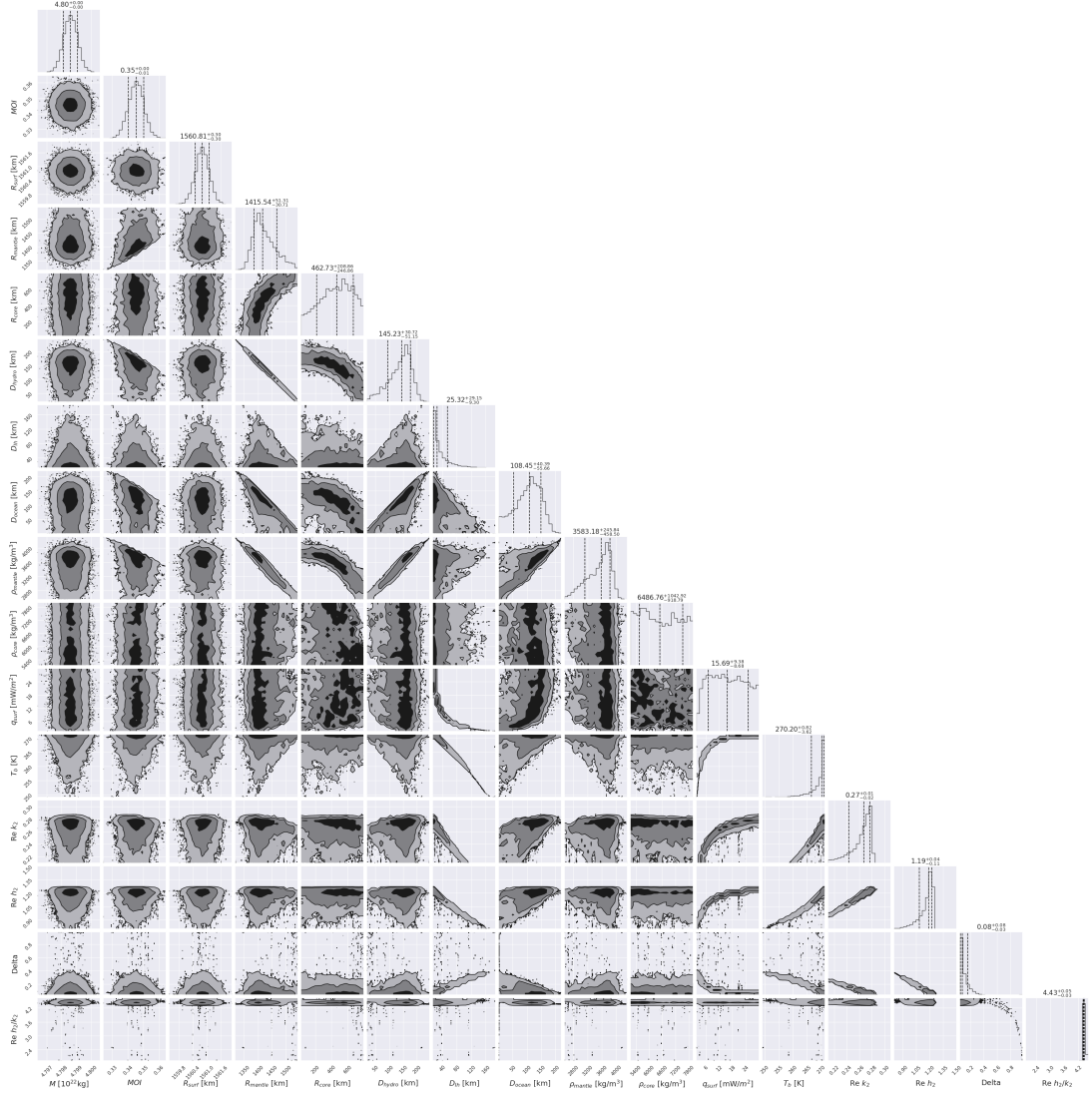


Figure 3.11 Europa *A98 reference model*: Corner plot showing posterior distributions of model parameters and selected derived quantities.

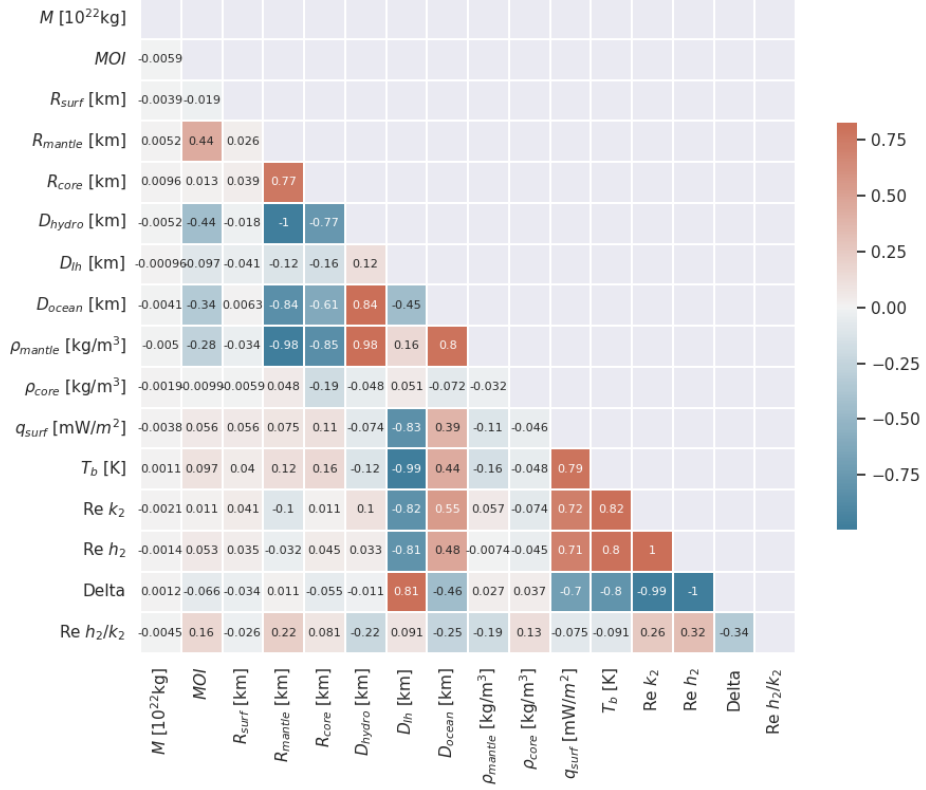


Figure 3.12 Europa *A98* reference model: Correlation plot depicting pairwise relationships between model parameters and derived quantities.

We observe that R_{core} has a higher mean value than in the *GC21* model. Vice versa, the value of R_{mantle} is lower, which is consistent with theoretical assumptions.

Comparison of MOI model set

For better visualisation, we compare histograms for R_{mantle} (Figure 3.13) and R_{core} (Figure 3.14) for the *A98* model and the *GC21* model.

Comparing the Figures 3.13 and 3.14, we see distinct differences between the two models. The mantle radius R_{mantle} yields a broader posterior distribution, encompassing smaller radii that are absent in the *GC21* model. R_{mantle} marginal distribution, in the case of the *GC21* model, is notably narrower, with the most values falling between approximately 1400 – 1450km. For the core radius R_{core} , the *A98* model shows a clear shift towards larger values, compared to *GC21* model, consistent with expectations based on the lower moment of inertia. *GC21* model may be consistent with a very small or non-existent core size, questioning Europa’s differentiation.

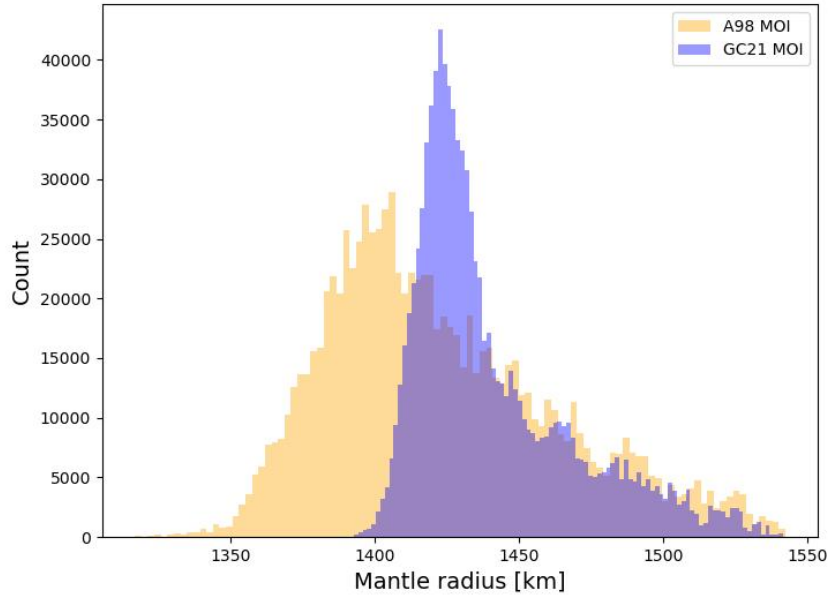


Figure 3.13 Histogram for mantle radius of Europa for two values of MOI: GC21 $\text{MOI} = 0.3547 \pm 0.0024$ (blue) and for A98 $\text{MOI} = 0.346 \pm 0.005$ (orange).

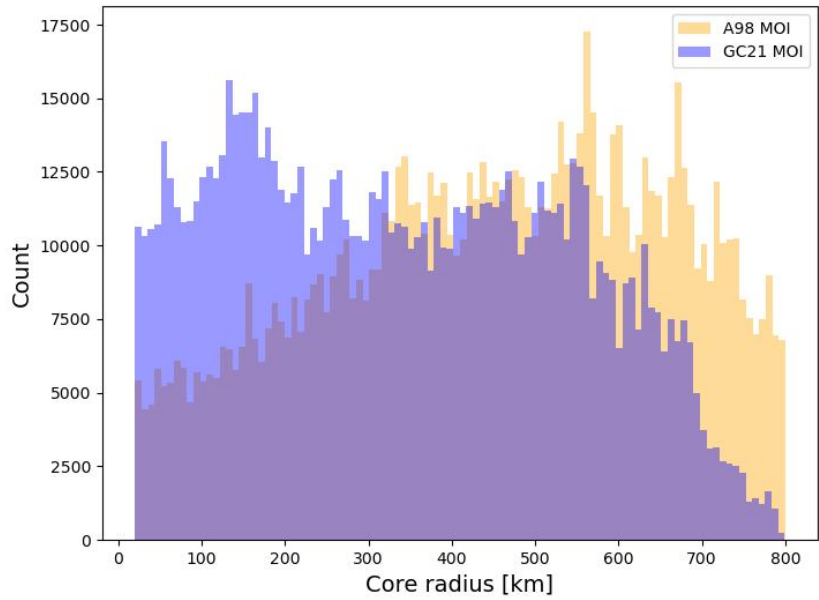


Figure 3.14 Histogram for core radius of Europa for two values of MOI: GC21 $\text{MOI} = 0.3547 \pm 0.0024$ (blue) and for A98 $\text{MOI} = 0.346 \pm 0.005$ (orange).

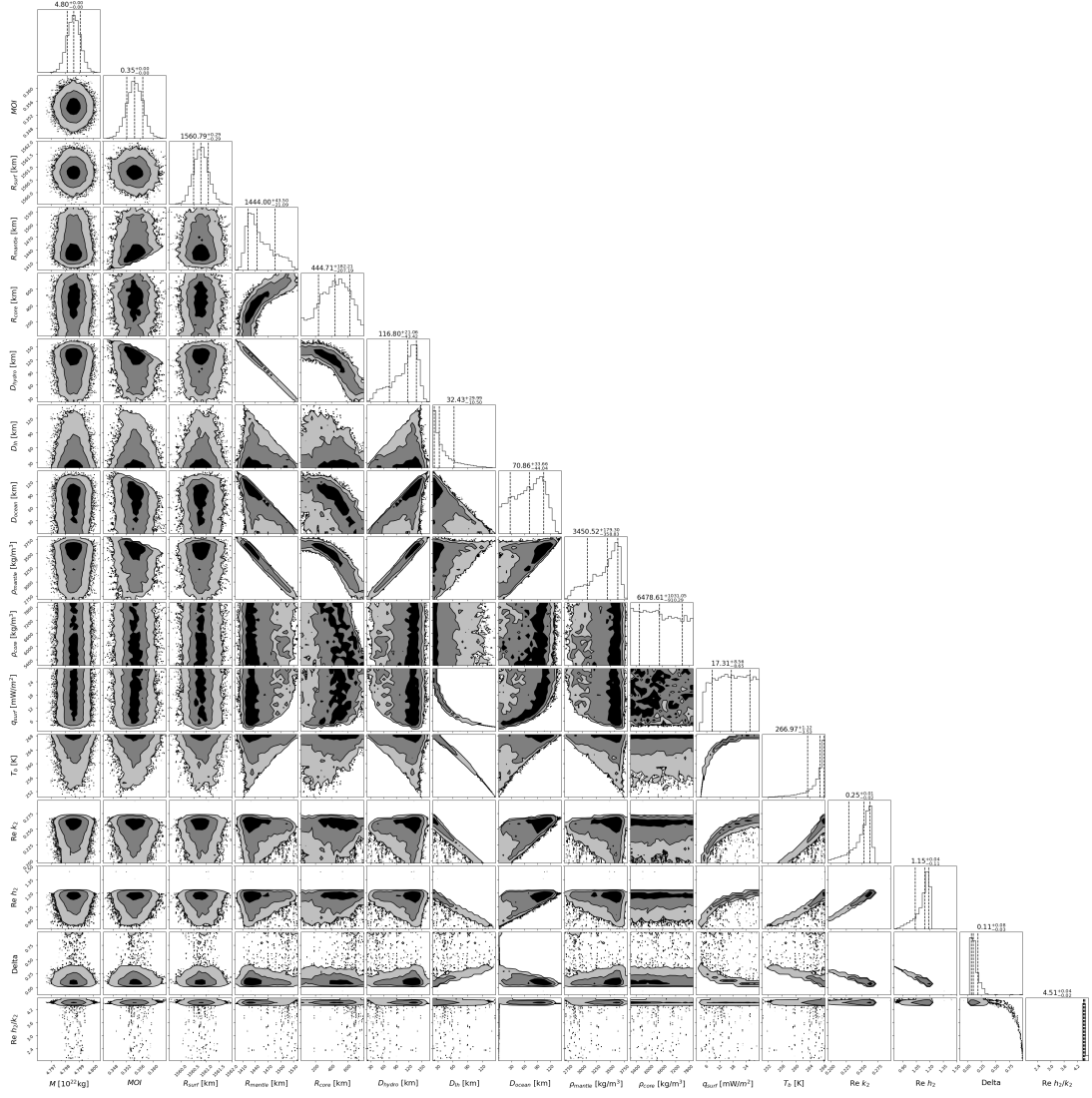


Figure 3.15 Europa *NaCl* model: Corner plot showing posterior distributions of model parameters and selected derived quantities.

3.1.3 Ocean composition model set

NaCl model

In this section, we investigate how the ocean's salt composition affects Europa's internal structures by considering NaCl instead of MgSO_4 . We use the same setup as in the *reference model*, i.e., the data and model parameters remain the same. The resulting posterior distributions are shown in the corner plot in Figure 3.15, and the corresponding correlation matrix is in Figure 3.16.

As expected, the trade-offs, sensitivities and correlation relationships are similar to those in the *reference model*. One notable difference lies in the maximum a posteriori estimate of the thickness of the ocean (D_{ocean}). The *NaCl model* allows for a smaller maximum a posteriori estimate of D_{ocean} than the *reference model*. Since D_{ocean} is smaller, the thickness of the ice shell D_{ih} is larger for the *NaCl model* than for the *reference model*. This affects T_{b} , which is inversely proportional to D_{ih} . It follows that when considering NaCl salt in the ocean, our results suggest that the maximum a posteriori estimate of T_{b} is smaller than when considering MgSO_4 . We also notice in Figure 3.16 a slightly enhanced correlation between T_{b} , q_{surf} with R_{mantle} and R_{core} .

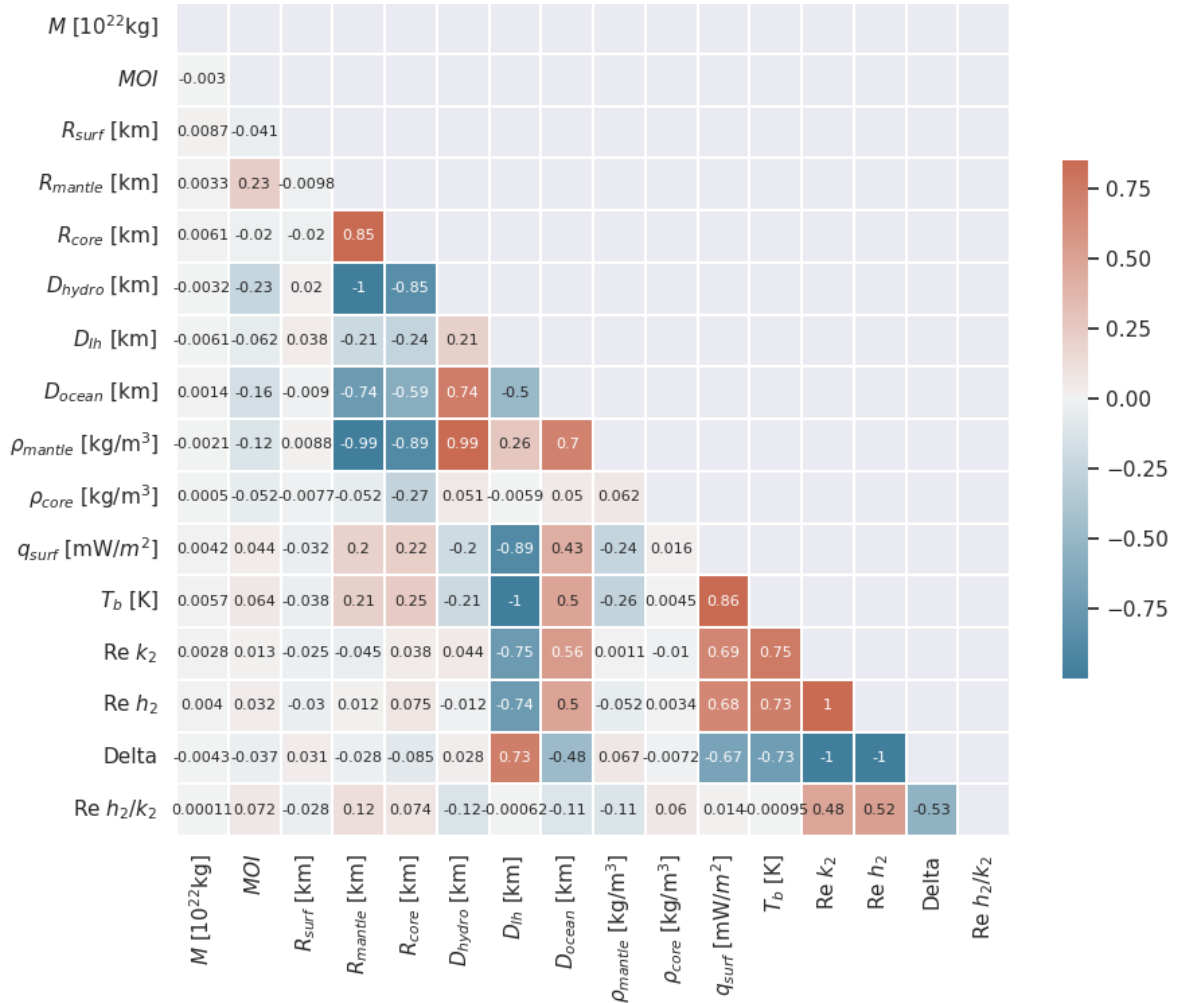


Figure 3.16 Europa *NaCl model*: Correlation plot depicting pairwise relationships between model parameters and derived quantities.

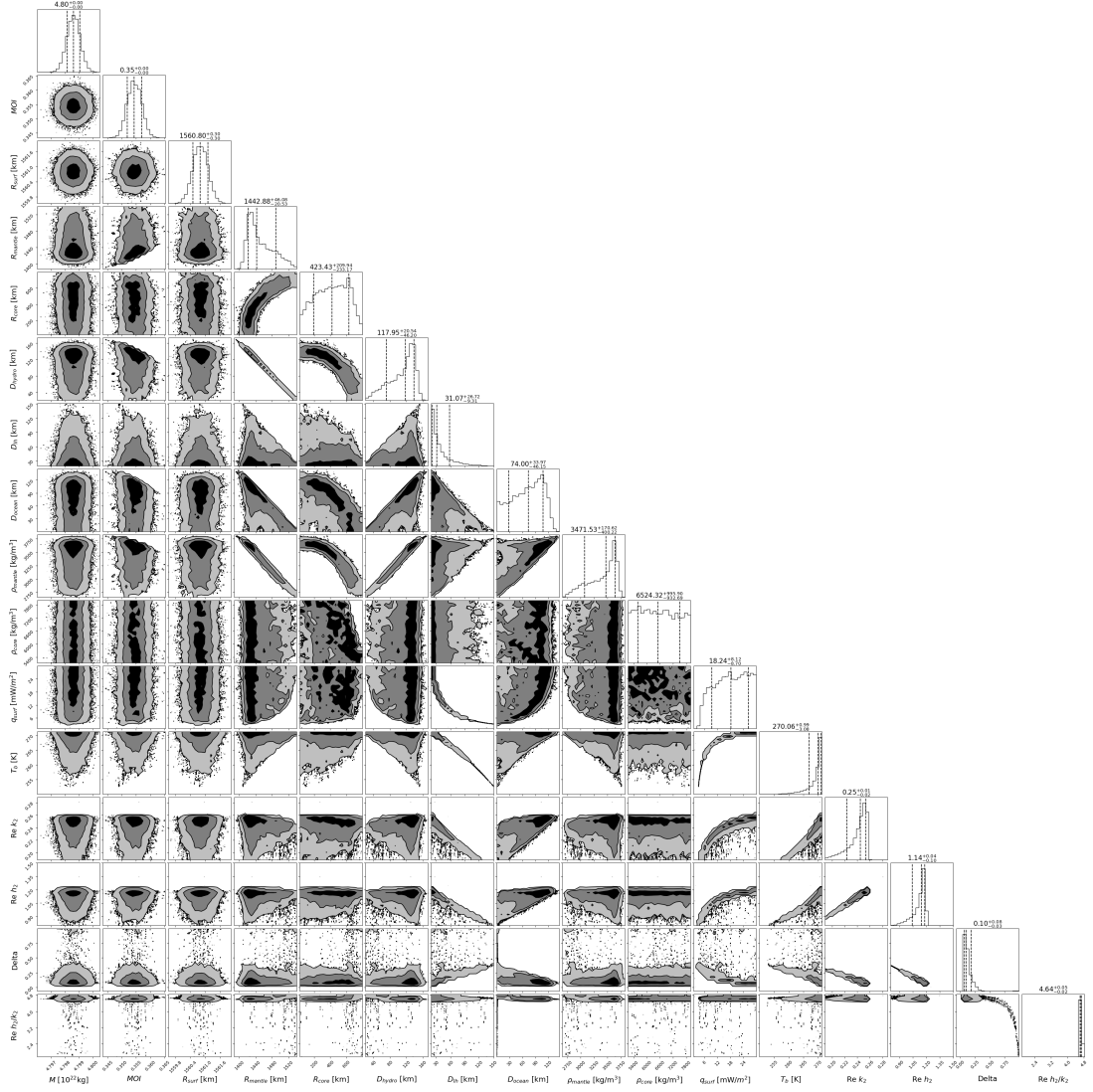


Figure 3.17 Europa *Pure Water* model: Corner plot showing posterior distributions of model parameters and selected derived quantities.

Pure Water model

As the final model for Europa, we investigate how the structure changes when assuming a pure water ocean, i.e., no dissolved salt. As before, we use the same setup as the *reference model*, with model and data space, while modifying the ocean composition to pure water. The results are displayed in Figure 3.17 and correlation matrix in Figure 3.18.

By comparing the results from Figure 3.1 and Figure 3.2, we notice that the results do not differ significantly. The most noticeable difference appears in the posterior marginal distribution of the surface heat flux q_{surf} . For *Pure Water model*, the distribution of q_{surf} declines for values below the midpoint of the prior interval. This suggests that models with pure water oceans are more likely to favour higher surface heat fluxes. However, the posterior distribution remains nearly uniform across the upper half of the range, making it difficult to constrain a precise value for q_{surf} .

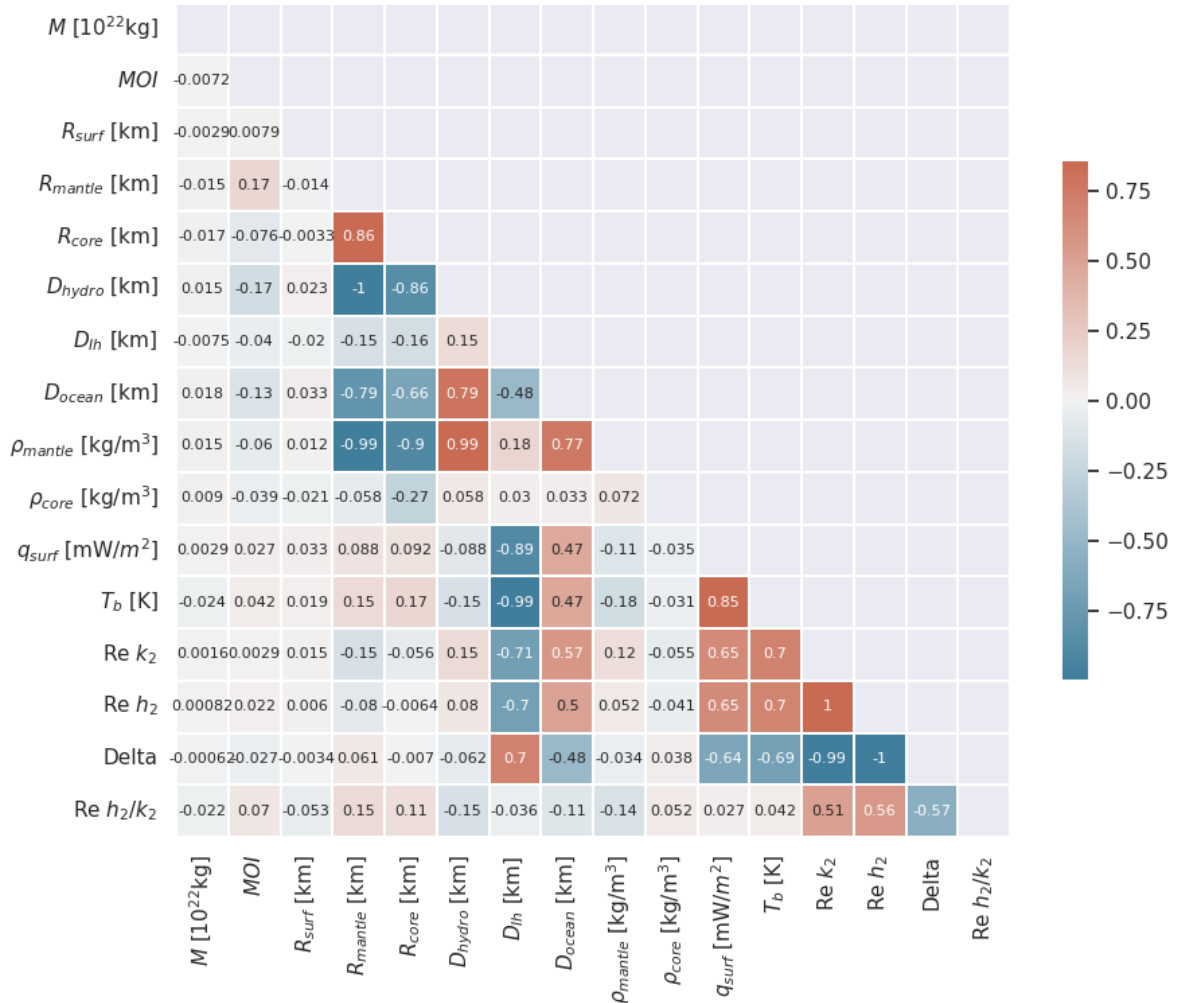


Figure 3.18 Europa *Pure Water model*: Correlation plot depicting pairwise relationships between model parameters and derived quantities.

Comparison of ocean composition model set

To investigate how ocean composition influences the inferred internal structure of Europa, we compare histograms representing the marginal posterior distributions of the mantle radius and core radius, respectively, for models with different ocean salinities. In Figure 3.19, we depict histograms of R_{mantle} for the *reference model*, *NaCl model*, and *Pure Water model*. For the *reference model*, the values of R_{mantle} are narrowly distributed around the value 1420 km. In contrast, when considering *NaCl model* and *Pure Water model*, the distribution of R_{mantle} is broader, although it still tends to favour smaller values.

Figure 3.20 shows the histograms of R_{core} for the *reference model*, *NaCl model*, and *Pure Water model*. While the reference model favours smaller values of R_{core} , both the NaCl and Pure Water models prefer larger R_{core} values.

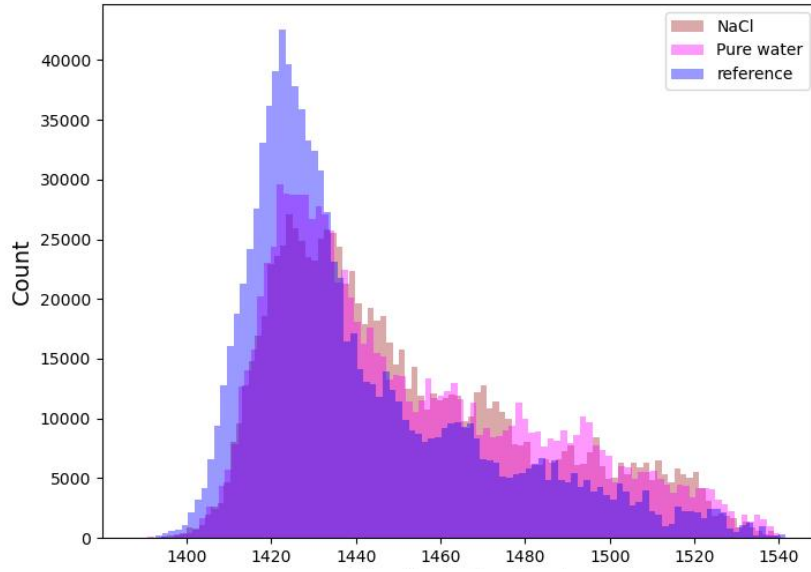


Figure 3.19 Histogram for mantle radius of Europa for the *reference model* (blue), the *NaCl model* (brown) and the *Pure Water model* (pink).

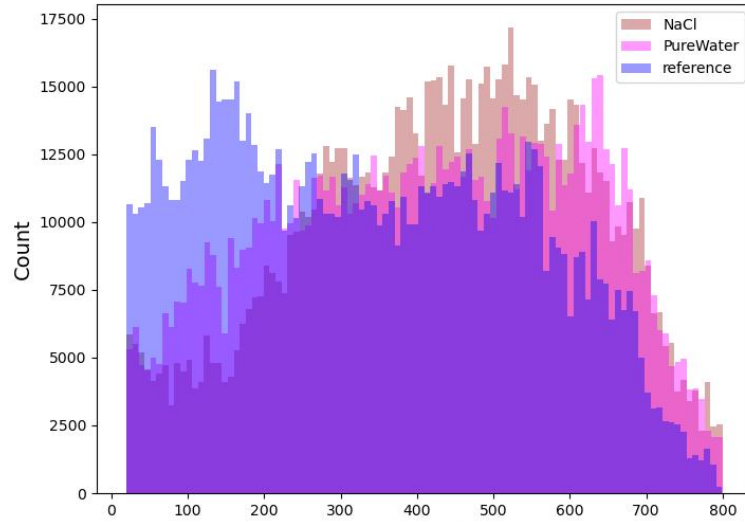


Figure 3.20 Histogram for core radius of Europa for the *reference model* (blue), the *NaCl model* (brown) and the *Pure Water model* (pink).

3.2 Ganymede

We selected Ganymede as the second target of our study. The physical parameters of Ganymede used for modelling are listed in Table 3.3. A priori information for the inverse problem is listed in Table 3.4. The allowed density intervals for the mantle and core are adopted from the Europa model.

Parameter	Value
R_{surf} [km]	2631.2 ± 1.7
M [kg]	$14.814\,79 \times 10^{22} \pm (\text{value} \times 1 \times 10^{-4})$
MOI Gomez Casajus et al. (2022)	0.3115 ± 0.0028

Table 3.3 Physical and initial parameters of Ganymede

Parameter	Value
T_{surf} [K]	110
P_{surf} [MPa]	0.0
$C_{w,\text{reference}}$	0.05
$q_{\text{surf,lower}}$ [W m^{-2}]	0.001
$q_{\text{surf,upper}}$ [W m^{-2}]	0.005
Mantle	
$\rho_{\text{mantle,lower}}$ [kg m^{-3}]	2500 (serpentine $\text{Mg}_3\text{Si}_2\text{O}_5\text{OH}_5$)
$\rho_{\text{mantle,upper}}$ [kg m^{-3}]	4400 (fayalite Fe_2SiO_4)
$R_{\text{mantle,lower}}$ [km]	1400
$R_{\text{mantle,upper}}$ [km]	2200
Core	
$\rho_{\text{core,lower}}$ [kg m^{-3}]	5150 (eutectic Fe:FeS)
$\rho_{\text{core,upper}}$ [kg m^{-3}]	8000 (pure iron)
$R_{\text{core,lower}}$ [km]	100
$R_{\text{core,upper}}$ [km]	1100

Table 3.4 A priori information for Ganymede.

Reference model

Due to the time requirements associated with modelling Ganymede, only the *reference model* is calculated. The data space for the *reference model* includes the moment of inertia (MOI), mass (M), and surface radius (R_{surf}) of Ganymede (Table 3.3). The model space is again defined by the radii and densities of the mantle and core, as well as the surface heat flux (Table 3.4). The resulting statistical distribution is shown in Figure 3.21, and the correlation relationships are depicted in Figure 3.22.

Compared to the *reference model* for Europa (Figure 3.1), we observe similar trade-offs in the case of Ganymede. As expected, Ganymede’s hydrosphere is larger than that of Europa. Internal pressure within Ganymede’s hydrosphere reaches high values due to its larger size and thicker hydrosphere, and our models consistently predict the presence of the HP ices.

When we compare the correlation patterns for Ganymede (Figure 3.22) with those of Europa *reference model* (Figure 3.2), they are broadly similar, although some correlations appear to change. This is expected because Ganymede’s hydrosphere includes an additional layer of HP ices.

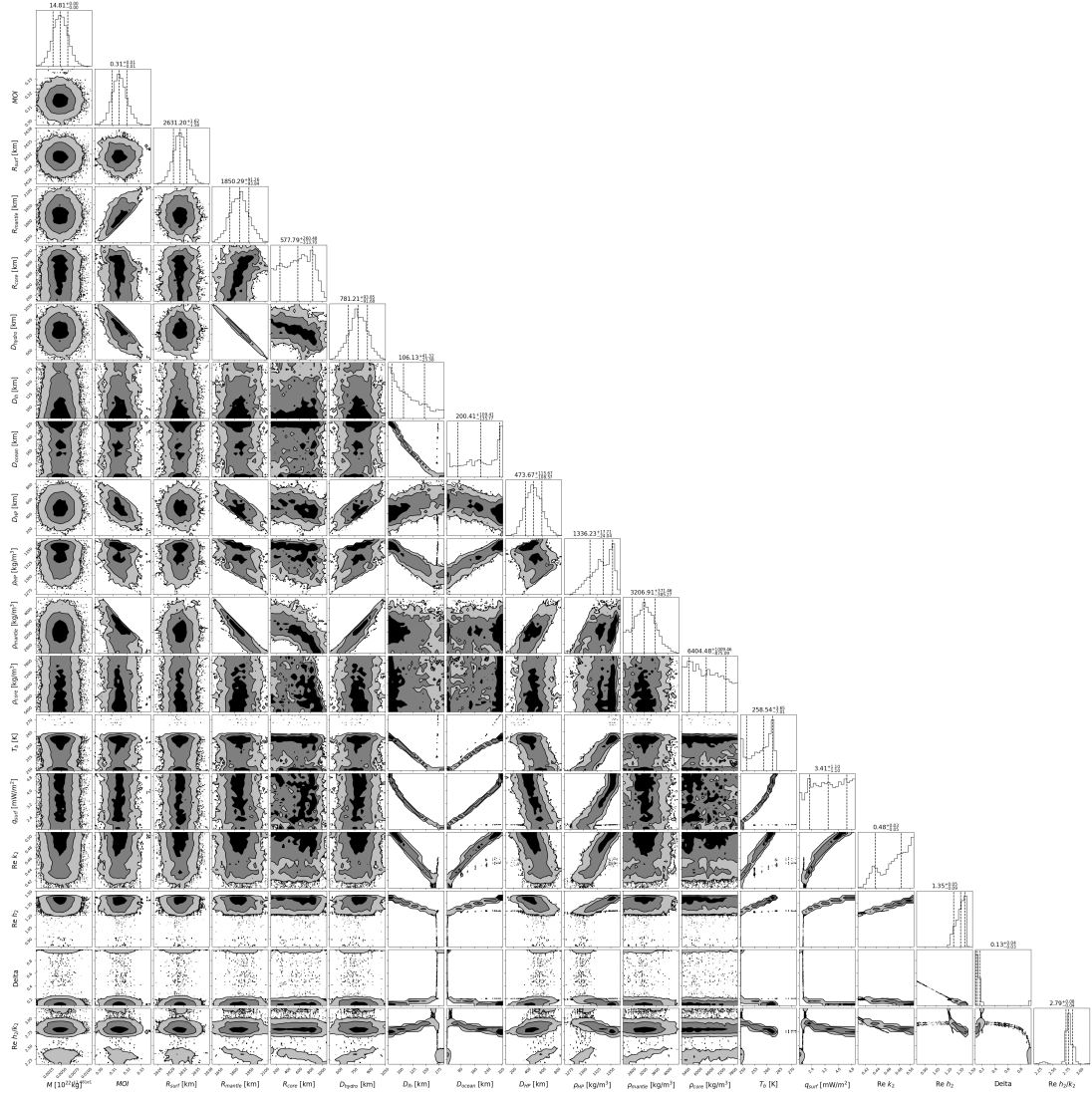


Figure 3.21 *Ganymede reference model*: Corner plot showing posterior distributions of model parameters and selected derived quantities.

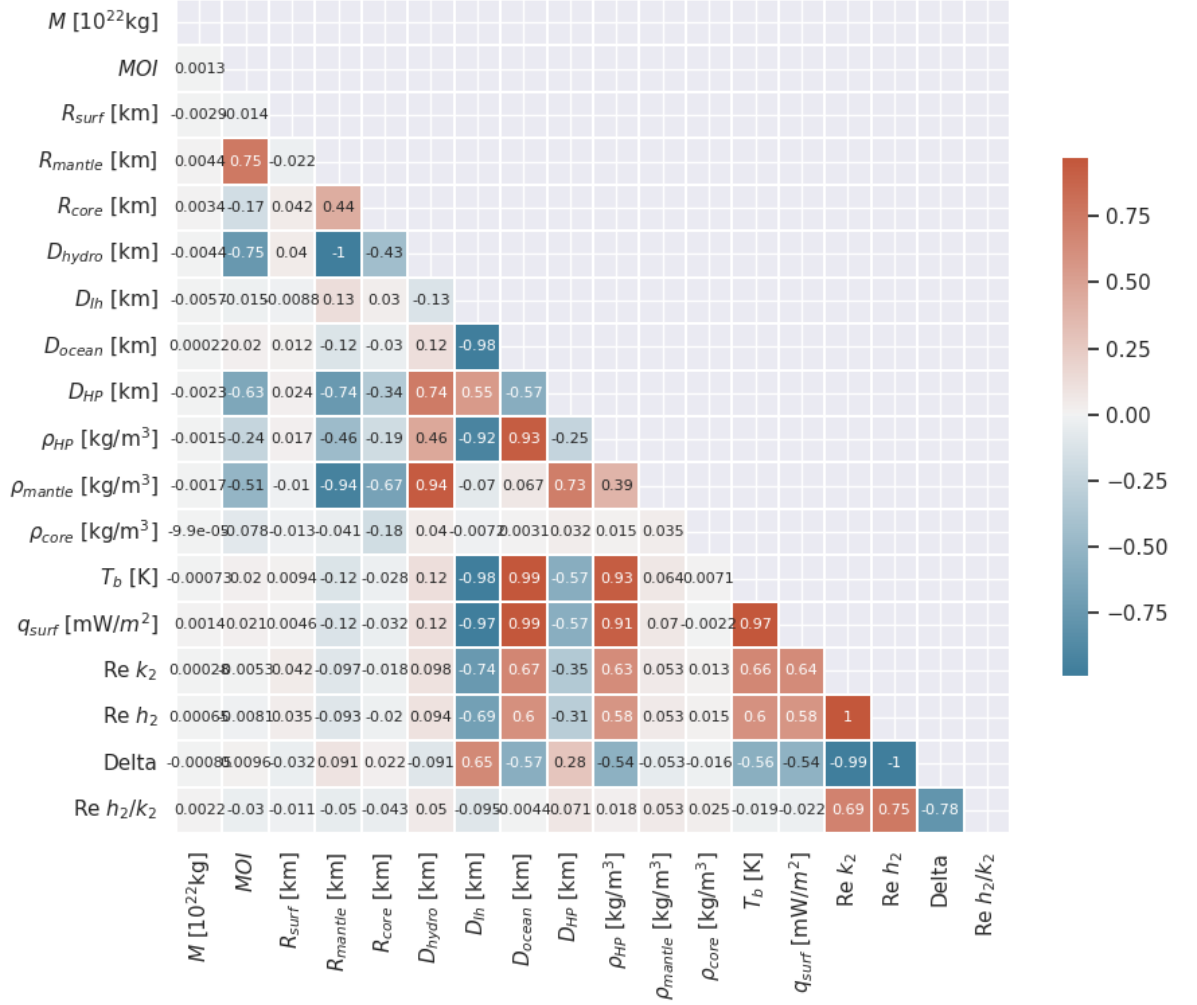


Figure 3.22 Ganymede *reference model*: Correlation plot depicting pairwise relationships between model parameters and derived quantities.

As a result, correlations that previously involved ocean thickness D_{ocean} and deeper interior layers (mainly the mantle, but also slightly the core) are now shifted to D_{HP} .

However, compared to Europa, we notice stronger correlations/anti-correlations between the surface heat flux q_{surf} and the temperature at the Ih ice-ocean interface T_b with the thicknesses of individual layers of the hydrosphere. In particular, D_{lh} shows a strong anti-correlation with T_b and also q_{surf} , consistent with the theoretical expectation. For the ocean layer, we see an almost perfect correlation between D_{ocean} and both T_b and q_{surf} .

In contrast, the HP ice layer D_{HP} exhibits noticeably weaker anti-correlations with T_{b} and q_{surf} , although these correlations remain moderately strong. On the other hand, the correlations of T_{b} and q_{surf} with tidal Love numbers $\text{Re } k_2$ and $\text{Re } h_2$ are weaker than those observed in the Europa *reference model*.

When inspecting the statistical distributions of $\text{Re}(h_2/k_2)$ with other parameters in Figure 3.21, we observe three clearly distinct solution clusters. To better understand these solutions, we plot a scatter plot of $\text{Re}(h_2/k_2)$ and R_{mantle} with the ocean thickness indicated by colour. This visualisation allows us to reveal how variations in the ocean thickness influence the observed clustering (Figure 3.23).

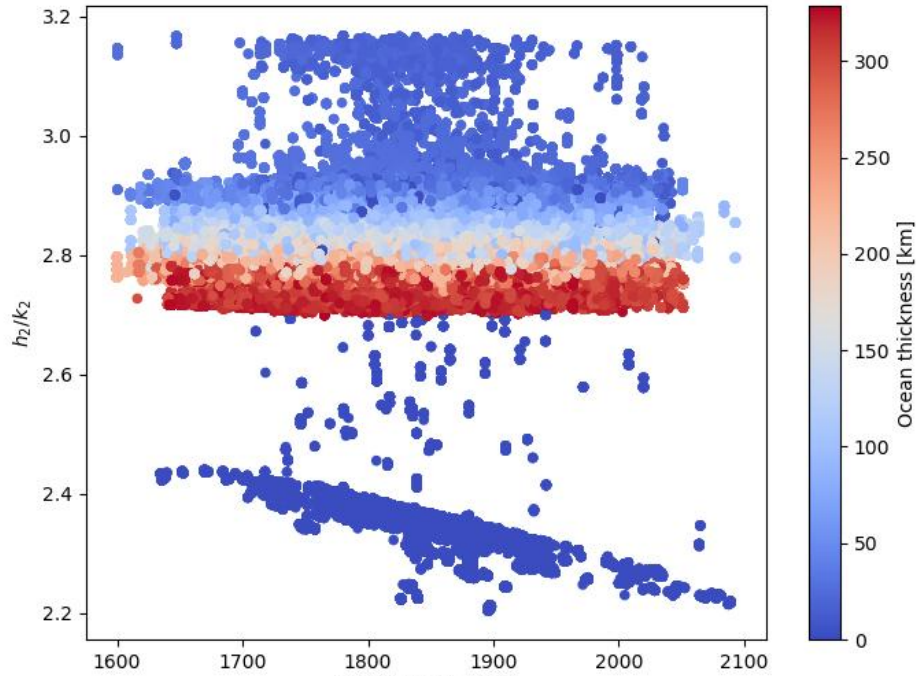


Figure 3.23 Ganymede *reference model*: Scatter plot of $\text{Re}(h_2/k_2)$ as a function of mantle radius, with colours indicating the ocean thickness.

The figure demonstrates that the ocean thickness indeed governs the three solution clusters. They corresponded to (1) internal structures with large oceans, (2) those with moderate to small oceans, and (3) structures lacking a liquid ocean layer. While the majority of models contain a liquid ocean of some size, the number of models with no liquid layer is non-negligible. The figure also shows that cases without the ocean present a weak linear dependence on R_{mantle} , while models with an ocean are practically independent of R_{mantle} . This suggests that the correlations for $\text{Re}(h_2/k_2)$ with other properties warrant a more detailed analysis for individual clusters in the future.

We also depict a scatter plot of k_2 and R_{mantle} again coloured by the ocean thickness D_{ocean} (Figure 3.24). The graph shows that structures without a liquid ocean present in the hydrosphere correspond to very low values of the Love number k_2 . This result suggests that if and when the JUICE mission measures the value of k_2 , we could predict whether a liquid ocean is actually present in Ganymede's structure or not.

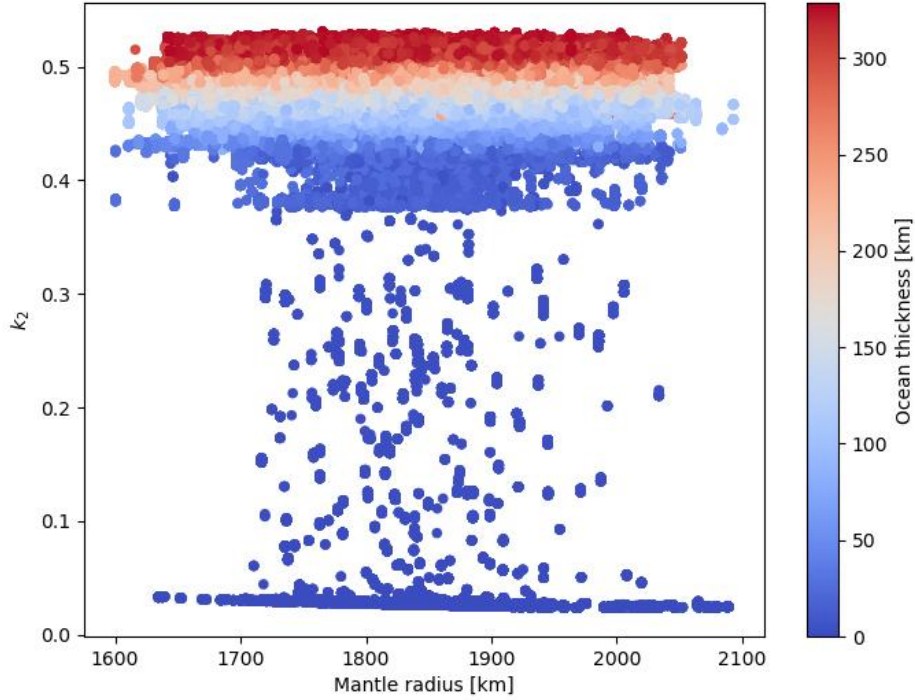


Figure 3.24 *Ganymede reference model*: Scatter plot of $\text{Re } k_2$ as a function of mantle radius, with colours indicating the ocean thickness.

As in the case of Europa, we plot the density profiles of 100 randomly selected resulting structures as a function of radius (Figure 3.25). The figure highlights the individual layers that occur in the structure of Ganymede: $\rho \approx 900 \text{ kg m}^{-3}$ corresponds to the density of ice Ih, $\rho \approx 1000 \text{ kg m}^{-3}$ to the density of the liquid ocean, $\rho \approx 1300 \text{ kg m}^{-3}$ to the density of the HP ices, $\rho \approx (2500 - 3500) \text{ kg m}^{-3}$ to the density of the silicate mantle, and $\rho \approx (5000 - 8000) \text{ kg m}^{-3}$ to the density of the metallic core.

Similarly to Europa (Figure 3.3), the density of the core spans the entire range of the a priori interval. A wide range of internal structures is predicted, characterised by varying thicknesses of individual layers. Some structures showed a complete absence of a liquid ocean.

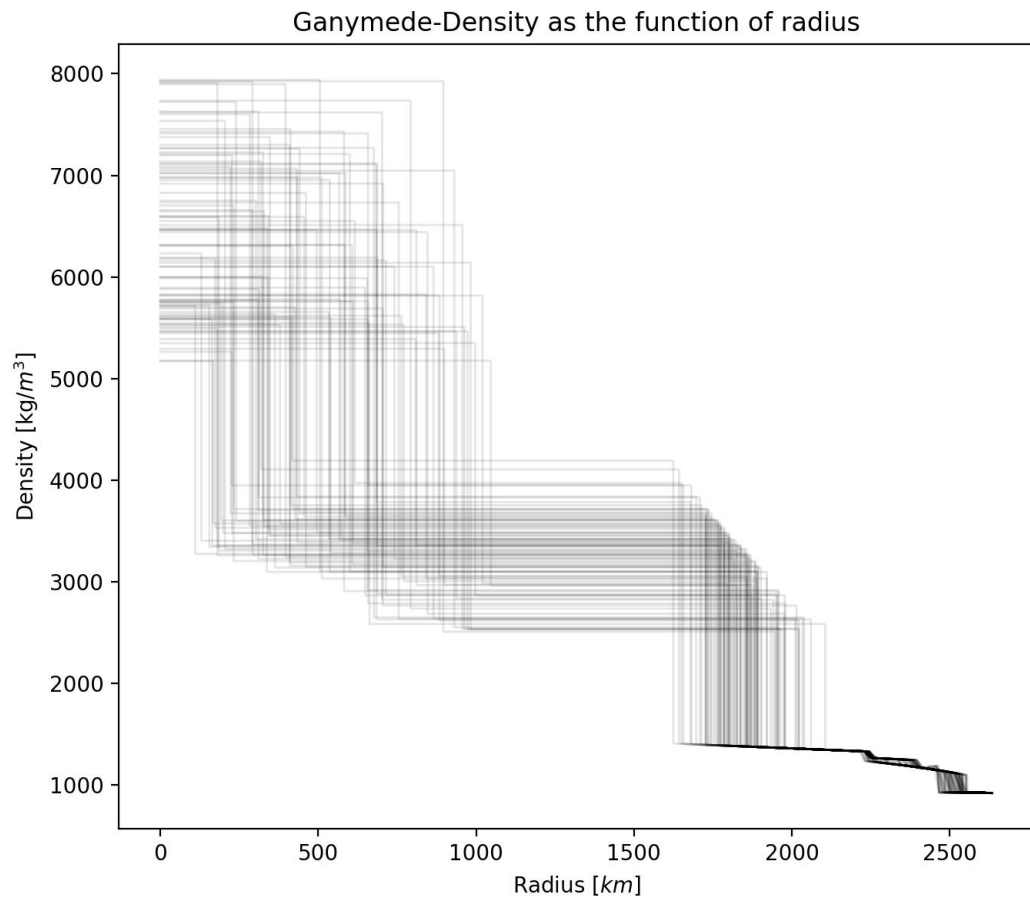


Figure 3.25 Ganymede *reference model*: Density profile for 100 randomly selected internal structures.

Conclusions

In this work, we investigated the internal structures of icy moons by developing a numerical algorithm, currently applied to Europa and Ganymede but easily adaptable to other planetary bodies. The code computed internal structure profiles based on observed planetary properties, using EOS and thermodynamic data sourced from Styczinski et al. (2023) and Journaux et al. (2020a). To explore the parameter space and identify models consistent with the observational constraints and uncertainties, we employed an ensemble sampler implemented in the Python library `emcee`.

For Europa, we calculated several models in which we changed the parameters of the data space, model space, value MOI, or ocean composition. We divided the models into three groups in which we compared them with each other: sensitivity model set, where we changed the model and data space, MOI model set where we investigated the impact of the two measured values of MOI, and ocean composition model set, where we compared models with different salt compositions in the ocean.

We found that for all models, we did not predict the presence of high-pressure ice phases (HP ices) in their structure. As a result, there is direct contact between the liquid ocean layer and silicates – one of the conditions required for the emergence of life.

For all models, the core density remained mostly unconstrained, and although the core radius spanned the allowed range, the reference model tended to favour a relatively small core. This suggests the possibility of only partial differentiation. For the mantle, the preferred density values were relatively high, but were associated with large uncertainties. The mantle radius was better constrained, with most models favouring values between approximately 1350 and 1450 km. In contrast, the surface heat flux typically showed an almost uniform distribution, indicating limited constraint from the available data.

However, when we added the tidal Love number k_2 to the data parameters, the range of possible values for the surface flux constraint, with the most likely value, occurring in the range $7 - 12 \text{ mW m}^{-2}$. The incorporation of k_2 also significantly limited the intervals of other internal structure characteristics, determined by the surface heat flux, namely, the thickness of the Ih ice, temperature at the ocean/Ih ice interface, and parameters depicting tidal Love numbers. Measuring k_2 , as well as h_2 , is one of the objectives of both the JUICE and Europa Clipper missions, and our results showed that knowing it should improve our determinations of internal structures.

Conversely, when we included salt concentration among the model inversion parameters, we observed a slight widening of the range of possible solutions for the mantle radius, which is consistent with theory.

For the second set of models, the varying MOI values influenced the resulting structure. We found that when using the value $\text{MOI} = 0.346 \pm 0.005$ from Anderson et al. (1998), the maximum a posteriori estimates of the core radius shifted to larger values, compared to using the value $\text{MOI} = 0.3547 \pm 0.0024$ (Gomez Casajus et al., 2021). This corresponds to theory, since the value MOI is closely related to the distribution of mass in the body, and thus to its differentiation state (the closer

the value is to the value MOI for a homogeneous body, the less differentiated it is). We also observed a widening of the range of possible values for the mantle radius when using MOI by Anderson et al. (1998).

The last set of models consisted of models with different salt compositions (MgSO_4 , NaCl , or pure water) in the ocean. When considering MgSO_4 salt composition, the maximum a posteriori estimate of the mantle radius narrowed (to a value of ~ 1420 km) compared to using other salts in the ocean.

Due to the time consideration, we used only one model for Ganymede, the reference model. Unlike Europa, Ganymede always had HP ices present in its structure. There is therefore a barrier between direct contact between silicates and the liquid layer, if it is present in the structure. Plotting the dependence of the ratio of Love numbers h_2 and k_2 and the radius of the mantle, relative to the thickness of the ocean, showed that in our model, there are structures that do not have a liquid ocean layer. These structures correspond to low values of k_2 , and therefore, future measurements could indicate the existence of an ocean in Ganymede’s hydrosphere. However, this result was highly dependent on the a priori model of allowed values for surface heat flux. It should also be noted that in the case where only a layer of Ih ice and a layer of HP ice are present in the hydrosphere, we assumed a purely conductive heat transfer profile, which may not necessarily be the case for Ganymede.

In future work, we plan to generalise the temperature profile in ice Ih and silicate layers to account for convection in the stagnant lid regime. The current assumption on the conductive profile is a significant limitation of the model, particularly in the case of Ganymede’s hydrosphere structure. We also want to include viscoelastic response for tidal deformation. We also plan to incorporate realistic mantle and core properties, using, for example, the *PerpleX* library.

Additionally, we aim to implement magnetic induction calculations using one of the existing Python libraries, such as the *MagMoon* library (Styczinski et al., 2022) or published scaling (Khurana et al., 2009; Jia et al., 2024). This would allow us to incorporate the magnitude of the induced magnetic field into our dataset in the inversion.

Given the time-consuming nature of our current code, we would like to incorporate machine learning techniques, specifically system learning, into our code. To create our neural networks, we want to use existing Python libraries such as TensorFlow, MXNet and Keras.

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List of Abbreviations

EOS Equation of state

HP High-pressure

ice Ih ice I crystallizing in the hexagonal lattice

i/o ice Ih - ocean interface

JUICE Jupiter Icy Moons Explorer

LP Low-pressure

MOI Moment of inertia factor

A Attachments

A.1 Density diagrams

To illustrate the EOS used in the code, we display density diagrams for a salt concentration of 5wt% for salt MgSO_4 (Figure A.1), salt NaCl (Figure A.2), and for pure water in the ocean (Figure A.3). The look-up tables are created using the open-source library *PlanetProfile* (MgSO_4 Styczinski et al., 2023) and *SeaFreeze* (NaCl , pure water Journaux et al., 2020a; Bollengier et al., 2019; Andersson and Inaba, 2005).

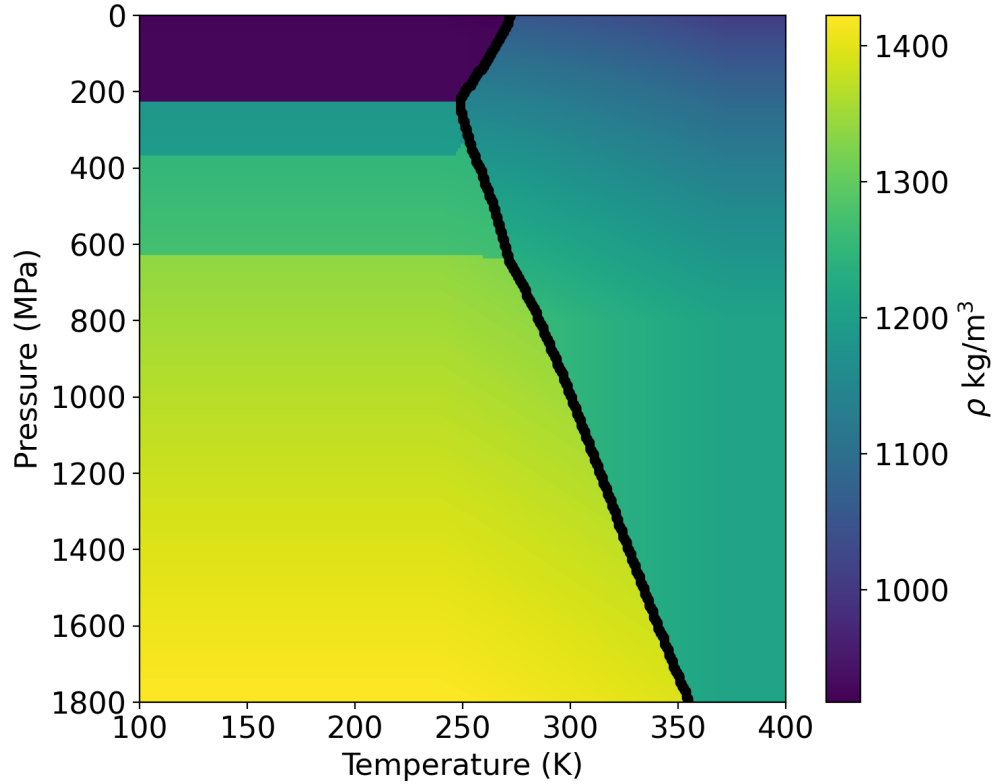


Figure A.1 Density diagram for salt MgSO_4 solution, concentration $\text{wt} = 5\%$.

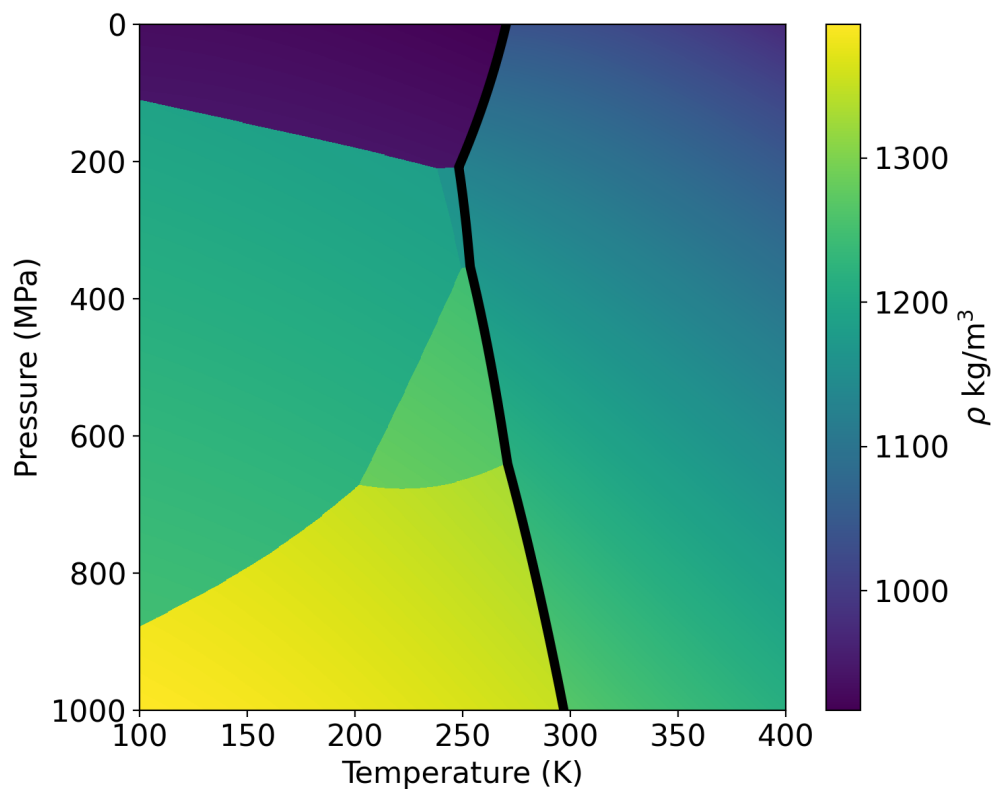


Figure A.2 Density diagram for salt NaCl solution, concentration wt = 5%.

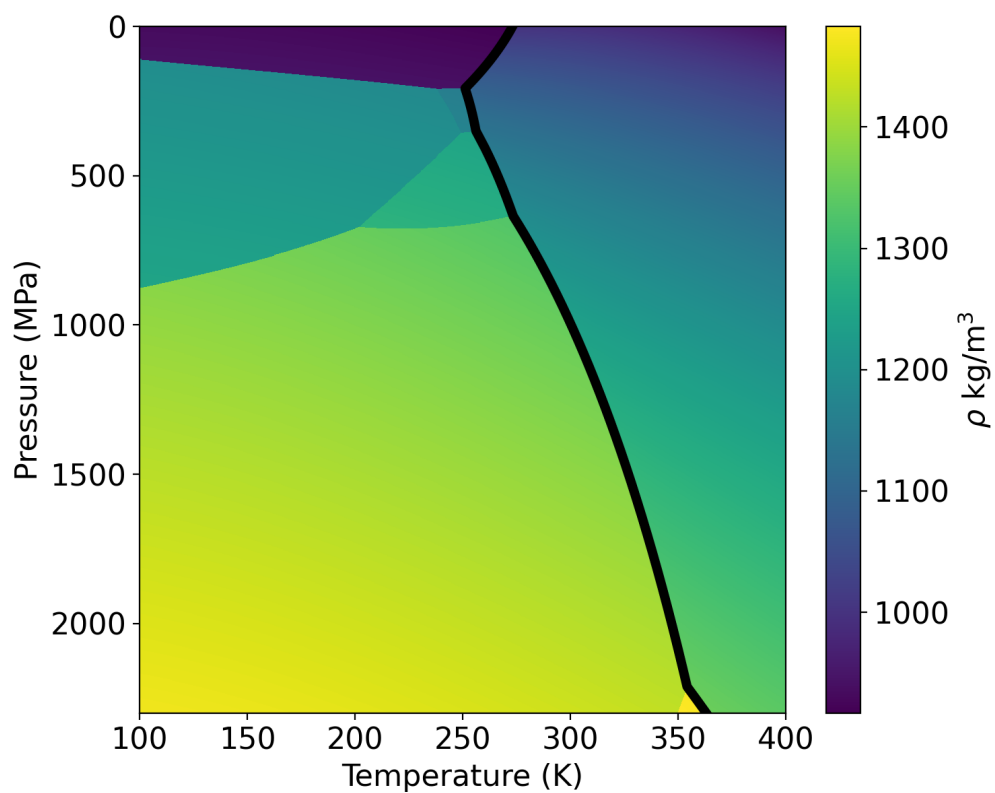


Figure A.3 Density diagram for pure water. No salt present.

A.2 Stretch Move Algorithm

This section is based on Goodman and Weare (2010) and Foreman-Mackey et al. (2013). The algorithm uses an ensemble of chains, so-called walkers, to explore the parameter space. At the beginning of the inversion, the number of walkers is initialised to satisfy the following condition:

$$K \geq 2 \times (N \text{ dim})$$

Each of the given K walkers starts at a different position in the parameter space. However, what makes this algorithm significantly different from classical MCMC methods is that each walker updates its position based on information about the positions of other walkers in the space. This improves the convergence of the inversion, and the algorithm is thus better able to explore the space formed by correlated or even anisotropic parameters.

The main mechanism of the **emcee** algorithm is the so-called stretch move proposal, i.e. a proposal for a new step that is affine-invariant (preserves the geometry of the distribution after a linear transformation). Each walker X_k randomly selects another walker X_j ($j \neq k$) and proposes a new position:

$$Y = X_j + Z(X_k - X_j)$$

where Z is a randomly selected scaling number from a special distribution $g(z)$, which must satisfy the condition:

$$g(z^{-1}) = zg(z).$$

in order to preserve affine invariance. Goodman and Weare (2010) recommend setting $g(z)$ as

$$g(z) \propto \frac{1}{\sqrt{z}}$$

$$z \in \left[\frac{1}{a}, a \right],$$

where the value of a is typically set to 2. This scaling allows walkers to move efficiently along the boundary between them and adapt to the shape of the target distribution.

The acceptance rule determines whether the proposed step is accepted. The new position Y is accepted with probability

$$\min \left(1, Z^{n-1} \cdot \frac{p(Y)}{p(X_k)} \right),$$

where $p(x)$ is the posterior probability at point x and n is the number of dimensions of the parameter space. If the acceptance rule does not accept the proposed step, the walker remains at its original position. The combination of ensemble updates and affine-invariant stretch moves makes the **emcee** algorithm a very effective tool for sampling from complex, highly correlated, and non-Gaussian distributions.