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DOCTORAL THESIS

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**Thermal convection in the subsurface
ocean of an icy moon**

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In date

Author's signature

Dedicated to prof. Čadek, ibuprofen and my colleagues from the room T035.
Thanks for your support. Also, special thanks goes to my wife and son, who
made it as hard as possible to finish this thesis. It really sped up my character
development.

*The intentions of this work were beyond my knowledge.
While filling the gaps, I started to worry: are they not
beyond my intellect?*

coffee break thoughts

Title: Thermal convection in the subsurface ocean of an icy moon

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Abstract: This thesis investigates thermal convection in the subsurface oceans of icy moons, focusing on Titan, Ganymede, and Europa. Using pseudospectral numerical simulations, it explores the dynamical flow regimes arising from the interplay between buoyant forcing and planetary rotation. While the behavior of systems dominated by the Coriolis force is well studied, particularly regarding flow speed and heat transfer efficiency, less attention has been paid to regimes where buoyancy and rotational effects are of comparable magnitude. Building on recent advances, this work categorizes observed flow patterns and introduces a set of empirical parameters that quantify the transitions between them. These parameters offer a systematic framework for classifying flow behaviors and predicting the magnetic signatures generated by ocean dynamics. The study further examines how these convective patterns affect heat transport across the ice-ocean interface and assesses their implications for surface topography of Titan and Ganymede.

Keywords: icy moons, subsurface ocean dynamics, topography, induced magnetic field

Název práce: Termální konvekce v podpovrchovém oceánu ledového měsíce

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Abstrakt: Tato disertační práce se zabývá tepelnou konvekcí v podpovrchových oceánech ledových měsíců, se zaměřením na Titan, Ganymed a Europu. Pomocí pseudospektrálních numerických simulací zkoumá dynamické proudové režimy, které vznikají v důsledku interakce vztlačového buzení a planetární rotace. Zatímco chování systémů dominovaných Coriolisovou silou je dobře prozkoumáno, zejména z hlediska rychlosti proudění a účinnosti přenosu tepla, méně pozornosti bylo dosud věnováno režimům, v nichž jsou vztlačové a rotační efekty srovnatelného řádu. Na základě nedávného pokroku práce kategorizuje pozorované proudové struktury a zavádí soubor empirických parametrů, které kvantifikují přechody mezi jednotlivými režimy. Tyto parametry poskytují systematický rámec pro klasifikaci proudových charakteristik a predikci magnetických signatur generovaných oceánskou dynamikou. Studie dále zkoumá, jak různé konvektivní vzory ovlivňují přenos tepla na rozhraní mezi ledem a oceánem, a hodnotí jejich dopad na povrchovou topografii Titanu a Ganymedu.

Klíčová slova: ledové měsíce, dynamika podpovrchového oceánu, topografie, indukované magnetické pole

Contents

Introduction	7
1 Evidence for water oceans in Jovian and Saturnian system	7
2 Modeling of subsurface oceans	12
3 Dynamical regime of subsurface oceans	15
3.1 Implications for motionally induced magnetic field	20
3.2 Implications for shape and topography	21
 1 A numerical model of convective heat transfer in Titan’s subsurface ocean	 24
1.1 Introduction	24
1.2 Control and diagnostic parameters	27
1.3 Results of numerical simulations	29
1.4 Role of boundary conditions	37
1.5 Comparison with Titan’s topography	39
1.6 Conclusions	42
1.A Governing equation and numerical method	43
1.B Control parameters relevant to Titan	45
1.C List of simulations	48
 2 The role of subsurface ocean dynamics and phase transitions in forming the topography of icy moons	 51
2.1 Introduction	51
2.2 Method	56
2.2.1 Ice shell	57
2.2.2 Evolution of the phase boundary	59
2.2.3 Water ocean	61
2.2.4 Computational strategy	63
2.3 Model parameters	65
2.4 Results	67
2.5 Comparison with observations	72
2.6 Conclusions	74
2.A The ratio of horizontal to vertical forcing	77
2.B List of simulations	78
 3 Magnetic field induced by convective flow in Europa’s subsurface ocean	 79
3.1 Introduction	79
3.2 Methodology	82
3.2.1 Ocean modeling	82
3.2.2 EM modeling	86
3.3 Results	90
3.3.1 EOIMF symmetries	90
3.3.2 Comparison of EOIMF generated by Modes I and II	92
3.3.3 EOIMF sensitivity to ice and ocean thicknesses	93
3.3.4 EOIMF sensitivity to mantle, ocean and ice conductivity	93

3.4	Conclusions	94
3.A	Ocean model	96
3.A.1	Determining the Rayleigh number	96
3.A.2	Velocity of ocean flow	98
3.B	EOIMF	99
3.B.1	The Lorentz force for Configurations 1–4	99
3.B.2	Comparison with the model of Vance et al. (2021)	100
3.C	Table of ocean simulations	102
4	Convective flow in Ganymede’s subsurface ocean: Implications for the induced magnetic field and topography	103
4.1	Introduction	103
4.2	Modeling the dynamics of subsurface oceans	104
4.3	Model of Ganymede’s ocean	108
4.3.1	Governing equations	108
4.3.2	Diagnostic parameters	108
4.3.3	Flow characteristics	112
4.3.4	Heat transfer characteristics	114
4.4	Implications for topography and magnetic field	117
4.4.1	Topography	117
4.4.2	Motionally induced magnetic field	120
4.5	Conclusions	121
4.A	Extrapolation technique	123
4.B	Results for $r_i/r_o = 0.80$ and 0.90	125
4.C	List of simulations	127
	Conclusion	130
	Bibliography	132
	List of Figures	155
	List of Tables	157
	List of Publications	158

Introduction

Prelude

Thirty years ago, our understanding of the Outer Solar System’s satellites relied entirely on data from two spacecraft. Voyager 1 flew by Jupiter in 1979 and Saturn in 1980, while Voyager 2 made similar flybys in the same years and continued on to encounter Uranus in 1986 and Neptune in 1989. The information gathered during these missions led scientists to hypothesize the potential presence of subsurface liquid reservoirs on Europa (Ojakangas and Stevenson, 1989) and Ganymede (Kirk and Stevenson, 1987). However, due to the absence of direct observations, these hypotheses were based solely on theoretical considerations such as radioactive decay rates (Lewis, 1971) and tidal heating mechanisms (Cassen et al., 1979).

The arrival of the Galileo mission at Jupiter in 1995 and Cassini at Saturn in 2004 dramatically expanded our understanding of icy moons. The traditional view of these bodies as cold, rigid, and inert was transformed by discoveries such as water plumes erupting from Enceladus’s south pole and strong indirect evidence for subsurface oceans. In this introductory chapter, we review the key evidence supporting the existence of these hidden oceans. The second part presents a simplified mathematical model, the numerical methods employed, and a summary of findings related to topography and magnetic fields.

1 Evidence for water oceans in Jovian and Saturnian system

Europa, one of Jupiter’s moons, presents a compelling case for the existence of a subsurface ocean. Data from the Galileo mission, particularly gravity and shape measurements, indicate that Europa’s interior is differentiated, with a hydrosphere approximately 100–200 km thick overlaying a silicate mantle (Anderson et al., 1998; Sohl et al., 2002; Vance et al., 2018). A more recent reanalysis of Galileo data refined this estimate to 125–150 km (Casajus et al., 2021). Evidence supporting the presence of liquid water within this hydrosphere can be grouped into three independent categories: (1) surface morphology, (2) tidal heating, and (3) induced magnetic field measurements.

Europa’s terrain features, including ridges, double ridges, and chaotic blocks, were found consistent with subsurface ocean dynamics (e.g., Carr et al., 1998; Pappalardo et al., 1999; Prockter et al., 2002). Their concurrence with salt deposits within 40° of the equator (Figueredo and Greeley, 2004; Brown and Hand, 2013) can be explained by the circulation of a global ocean beneath an active ice shell (Soderlund et al., 2014). Moreover, tidal flexing caused by Europa’s eccentric orbit generates internal heating sufficient to sustain a liquid ocean beneath the icy shell (Ojakangas and Stevenson, 1989; Hussmann et al., 2002). It should be noted, that the orbital eccentricity is assumed to vary in time due to the orbital resonance with Ganymede and Io, thus directly affecting the thickness of the ocean. Based on models of orbital evolution, Hussmann and Spohn (2004) proposed that the ice

shell thickness varies between 5 and 60 km on a gigayear time-scale due to global melting and freezing caused by variations in the tidal heating.

The thickness of Europa’s ice shell remains uncertain and is the focus of ongoing research (see Marusiak et al., 2021, for a review). Estimates based on induced magnetic field measurements suggest a shell roughly 10–30 km thick above a saline ocean (Khurana et al., 1998; Kivelson et al., 2000; Zimmer et al., 2000). A recent joint inversion of gravity and magnetic data by Petricca et al. (2023) expanded this range to 1–30 km. Thermal evolution modeling by Howell (2021) yielded an estimate of approximately 24 km, while Lemasquerier et al. (2023), using a model of heat transfer within a subsurface ocean, predicted an ice shell thickness of 12–24 km depending on whether tidal or radiogenic heating dominates.

Although theoretical models (e.g., Kargel et al., 2000; Zolotov and Shock, 2001) and analyses of observational data (e.g., Hand and Chyba, 2007; Petricca et al., 2023) consistently suggest the likely presence of salts in Europa’s ocean, the salinity remains poorly constrained, with estimates ranging from nearly fresh water to highly saline conditions (e.g., Zolotov and Kargel, 2009). As a result, the inclusion of salinity effects in ocean flow models varies study to study. For instance, Vance and Brown (2005) suggested a model of a stratified ocean, where a layer of cold and salty water is overlaid by warmer and less salty layer. Ashkenazy and Tziperman (2021) explored a range of scenarios from freshwater to saline oceans and concluded that if the salt concentration exceeds approximately 10 g/kg_{H₂O}, ocean dynamics are likely dominated by salinity gradients. Conversely, other studies adopt a freshwater assumption to investigate convective heat transport (e.g., Soderlund, 2019; Bire et al., 2022) or the generation of magnetically induced fields (Vance et al., 2021; Šachl et al., 2025). Conclusive observations constraining Europa’s ocean chemistry are expected to be provided by the Europa Clipper Mission after the year 2030 (e.g., Howell and Pappalardo, 2020).

Ganymede, the largest moon in the Solar System, is also considered a strong candidate for hosting a subsurface ocean. Gravity data indicate that Ganymede is a fully differentiated body, composed of a liquid metallic core, a silicate mantle, and a hydrosphere between 600 and 900 km thick (Anderson et al., 1996; Sohl et al., 2002; Schubert et al., 2004). Measurements of the magnetic field by the Galileo spacecraft further support the presence of a conductive layer (likely a saline ocean) beneath its surface (Kivelson et al., 1996, 1997a). Additional evidence comes from Hubble Space Telescope observations of oscillating auroral bands, which are consistent with the influence of a subsurface ocean beneath Ganymede’s ice shell (Saur et al., 2015) and presence of hydrated salt minerals at Ganymede’s surface (McCord et al., 2001).

The thicknesses of Ganymede’s ice shell and underlying ocean remain poorly constrained. Thermodynamic models allow for a wide range of possibilities, with the ice shell estimated to be between 5 and 157 km thick and the ocean between 24 and 518 km deep (Vance et al., 2018). These estimates are interdependent: a thinner ice shell implies a thicker ocean, and vice versa. Given the high pressures at the base of the ocean, it is likely situated between the outer ice shell and an internal layer of high-pressure ice, which prevents direct contact with the silicate mantle (Sohl et al., 2002; Sotin and Tobie, 2004; Vance et al., 2014, 2018). Assuming a conductivity similar to that of terrestrial water, a synthesis of thermodynamic modeling and magnetic field data is consistent with Galileo observations if the

ocean is approximately 200–300 km thick and lies beneath an ice shell at a depth of about 130 km (Khurana et al., 2002).

As with Europa, the composition of Ganymede’s ocean remains uncertain. Spectral analysis of reflected light suggests the presence of magnesium sulfate (MgSO_4) and possibly sodium sulfate (Na_2SO_4) on the surface (McCord et al., 2001). While this method offers some insight into the ocean’s chemical constituents, it does not provide information about salt concentration within the ocean itself. A more promising approach, based on estimating electrical conductivity, was employed by Jia et al. (2025). By combining magnetic field measurements from the Galileo and Juno missions, they estimated the ocean’s conductivity across different internal structure scenarios. Their results suggest that a thick ocean (≥ 300 km) beneath a thin ice shell (≤ 100 km) would have a conductivity about 0.1 S/m, indicative of a nearly freshwater composition. In contrast, a thin ocean (≤ 100 km) overlain by a thick ice shell (≥ 200 km) would require a conductivity greater than 1 S/m, implying a more saline environment.

To date, models of Ganymede’s ocean have focused on heat transfer and flow dynamics under the assumption of a freshwater composition, which is consistent with the presence of a thick ocean (e.g., Soderlund, 2019; Terra-Nova et al., 2023; Cabanes et al., 2024). This assumption will also be adopted in subsequent chapters of this thesis when discussing surface topography and magnetically induced fields. The extent to which such models remain valid without accounting for the influence of salts may soon be clarified, as two upcoming space missions are set to investigate Ganymede. NASA’s Europa Clipper (e.g., Howell and Pappalardo, 2020) will perform multiple close flybys of Ganymede starting in 2030, followed shortly by ESA’s Jupiter Icy Moons Explorer (JUICE) (Grasset et al., 2013), which will enter orbit around the moon for approximately one year. These missions are expected to deliver high-resolution data on Ganymede’s shape, as well as its gravitational and magnetic fields - key parameters that are closely tied to the ocean’s composition.

Turning to the Saturnian system, Enceladus has emerged as one of the most geologically active candidates for an ocean world. In 2005, NASA’s Cassini spacecraft discovered plumes of water vapor, ice grains, and organic molecules erupting from fractures near the moon’s south pole, commonly known as the “Tiger stripes” (e.g., Porco et al., 2006; Waite et al., 2006). The high dissipation rate in this region was explained by the presence of a subsurface liquid water reservoir beneath the ice shell (e.g., Collins and Goodman, 2007; Tobie et al., 2008). The gravity data suggested that Enceladus is differentiated into hydrosphere and low density core (Iess et al., 2014), and libration measurements later confirmed that the ice shell is decoupled from the rocky core by a liquid layer (Thomas et al., 2016). Additional support for a global subsurface ocean came from analyses of gravity and topography data (e.g., McKinnon, 2015; Beuthe et al., 2016; Čadek et al., 2016).

Refined analyses of Enceladus’s global shape and libration suggest that the ice shell has an average thickness of 27–33 km, overlying a subsurface ocean with an average depth of 21–26 km (Park et al., 2024). Topographic data from the Cassini mission revealed pronounced pole to equator variations in ice shell thickness, ranging from only a few kilometers at the south pole to nearly 35 km at the equator. A notable hemispheric asymmetry is present, with the ice shell at the north pole estimated to be around 10 km thick (e.g., Tajeddine et al.,

2017; Hemingway and Mittal, 2019; Čadek et al., 2019). The presence of various non-water compounds in ejected icy grains (Postberg et al., 2009, 2011; Waite et al., 2017), along with the detection of silicon-rich particles (Hsu et al., 2015), suggests ongoing hydrothermal activity at the seafloor of Enceladus. Such activity could contribute to the irregular shape of the ice shell, as localized melting and freezing above hydrothermal vents may drive shell deformation over time.

Hemingway et al. (2019) proposed a mechanism for the breaking the symmetry of Enceladus’s ice shell, showing that ocean overpressure arising from phase changes is sufficient to fracture the shell, most likely at its thinnest point (where stress accumulates). The resulting pressure relief would inhibit similar fracturing elsewhere, while enhanced heat dissipation in the fractured region (e.g., Tobie et al., 2008) would promote continued thinning. The polar regions are especially vulnerable to such thinning due to the spatial distribution of tidal heating, which peaks at high latitudes (e.g., Beuthe, 2013). An additional source of polar heating may stem from the seafloor, as heat flux from a porous, water-saturated core has been shown to be concentrated at high latitudes (Choblet et al., 2017b). Such a core composition is supported by recent magnetic field measurements indicating high electrical conductivity within the core (Saur et al., 2024).

The potential influence of hydrothermal activity on ocean circulation and ice shell deformation was investigated by Bouffard et al. (2025). Their findings suggest that, while the pattern of heat released from the core is largely homogenized at low latitudes, it is efficiently transferred to the base of the ocean in the polar regions. This preferential polar heat flux offers an explanation for the observed high-latitude ice shell thinning, fracturing, and hemispheric asymmetry as discussed above.

An alternative perspective was proposed by Kang et al. (2022a), who suggested that the observed hemispheric asymmetry may have originated from a random initial anomaly. Their study showed that, owing to the anomalous expansion behavior of water near its freezing point, ocean circulation can amplify such an anomaly, provided the ocean has low salinity and only moderate convective activity (see also Kang et al., 2022c; Kang, 2022). The initial anomaly could have been triggered by thinning caused by a giant impact (Roberts and Stickle, 2021), subsequently shifted to the south pole through true polar wander (e.g., Tajeddine et al., 2017), or may result from an inherently monopolar pattern of geological activity (Běhounková et al., 2012).

Titan, the largest moon of Saturn, is a notable candidate among the Solar System’s ocean worlds. Measurements of its mass and moment of inertia indicate a differentiated interior, comprising a silicate core overlain by a hydrosphere roughly 500 km thick (e.g., Néri et al., 2020). Two independent lines of evidence point to the existence of a liquid ocean within this hydrosphere: Titan’s unexpectedly large obliquity (Baland et al., 2011) and its gravitational response to tidal deformation (Iess et al., 2012).

The thickness of Titan’s ice shell remains poorly constrained, with estimates ranging from a few tens of kilometers up to 200 km (Hemingway et al., 2013; Baland et al., 2014). Consequently, the underlying ocean is thought to be between 130 and 420 km thick. Unlike Europa and Enceladus, Titan’s ocean is likely not in direct contact with the silicate mantle. Instead, a layer of high-pressure (HP) ices is expected to form under the extreme pressures, similar to the case of Ganymede (e.g., Vance et al., 2018). The ocean’s chemical composition remains uncertain.

Earlier interpretations of a high tidal response, based on initial estimates of Titan’s Love number (Durante et al., 2019), suggested a dense ocean ($\geq 1300 \text{ kg/m}^3$) (Iess et al., 2012; Mitri et al., 2014; Baland et al., 2014), potentially rich in ammonia sulfates (Fortes et al., 2007; Fortes and Choukroun, 2010) and/or magnesium sulfate (Vance and Brown, 2013). However, a recent revision halved the Love number estimate (Goossens et al., 2024), pointing instead to a lower-density ocean, possibly composed of freshwater or ammonia-rich water (Vance et al., 2018), as suggested by earlier studies (e.g., Tobie et al., 2005).

Cassini measurements revealed long-wavelength topographic variations on Titan, defined as deviations of the physical surface from an equipotential surface. These variations range from approximately -300 m in the south polar depression to $+400 \text{ m}$ in some midlatitude highlands (Durante et al., 2019). The weak correlation between gravity and topography indicates a high degree of compensation, prompting the application of isostatic models to explain the surface features. Nimmo and Bills (2010) assumed an Airy isostasy model, proposing that surface topography is supported by variations in the elevation of the ice-ocean interface. Subsequent studies by Lefèvre et al. (2014) and Kvorka et al. (2018) found that such basal topography would be dynamically unstable unless the viscosity at the base of the ice shell exceeds 10^{16} Pa s , which implies either a large grain size or a very cold ocean. The presence of ammonia in the ocean would fulfill this condition, as its low melting temperature allows the ocean to remain cold enough to ensure high basal viscosity. More recently, Amit et al. (2020) and Kvorka and Čadek (2022) proposed that rotationally constrained heat transfer within the ocean could generate and maintain the observed ice-ocean interface topography, supporting earlier theoretical predictions (Kvorka et al., 2018).

Alternatively, chemical processes in Titan’s nitrogen–methane atmosphere (e.g., Tobie et al., 2006; Bézard et al., 2014; Vuitton et al., 2014) have been proposed as a potential origin of the polar depressions. Choukroun and Sotin (2012) hypothesized that these features could result from the accumulation of ethane clathrate, a substance roughly 8% denser than water ice. This model relies on Pratt isostasy and is supported by observations of ethane precipitation at the poles (Griffith et al., 2006). Building on this framework, Čadek et al. (2021) used shape and gravity data inversion to constrain density variations within Titan’s ice shell. Their results indicate that, at least in some regions, the inferred density structure may reflect multiple contributing factors, such as ethane deposition at the poles, large-scale temperature variations due to the insulating effects of methane clathrate (Kalousová and Sotin, 2020), and heterogeneity in heat flux from the underlying ocean (e.g., Amit et al., 2020).

Other potential candidates for hosting subsurface water oceans beneath their icy crusts include Callisto (Cochrane et al., 2025), Mimas (Lainey et al., 2024), and Dione (Beuthe et al., 2016). However, as these bodies fall outside the scope of this thesis, they are not discussed further. To summarize the preceding discussion, although the study of convective heat transfer and flow dynamics within subsurface oceans is a relatively young field, it has already produced promising hypotheses that help explain observed surface shapes and topographic features on icy moons. The significance of ocean modeling is expected to grow with the arrival of magnetic field data from upcoming missions. In particular, the high precision of magnetometers onboard the Europa Clipper and JUICE missions (Kivelson

et al., 2023; Amtmann et al., 2024; Ellmeier et al., 2024) appears sufficient to detect magnetic fields induced by ocean flows (Vance et al., 2021; Šachl et al., 2025, chapter 4 of this thesis). This could enable direct comparisons between magnetic signatures predicted by convection models and actual observations, offering valuable insights into the validity of current numerical approaches.

2 Modeling of subsurface oceans

With growing evidence supporting the presence of subsurface oceans within icy bodies, the possibility of primitive life beyond Earth has garnered increasing interest. Initial studies drew parallels with ecosystems existing near the hydrothermal systems found in Earth’s deep oceans, where conditions such as high pressure, absence of sunlight, and chemically active rock-water interfaces may resemble those at the base of buried extraterrestrial oceans. In these extraterrestrial settings, hydrothermal activity may be driven by tidal heating within the silicate mantle (Reynolds et al., 1983) and chemical reactions occurring within a porous mantle could provide a viable energy source for primitive life forms (e.g., Gaidos et al., 1999; Chyba and Phillips, 2001; Vance et al., 2007). Evidence for such systems would suggest that environments hypothesized to be the cradle of life on Earth (Corliss et al., 1981) may also exist on other planetary bodies.

One hypothesis proposed to explain the origin of Europa’s chaotic terrains involved a hydrothermal activity. In this model, surface features were thought to result from localized melt-through events, potentially triggered by hot plumes rising from the base of the subsurface ocean (Greenberg et al., 1999). Building on this idea, Collins et al. (2000) and Thomson and Delaney (2001) applied scaling laws derived from studies of plume dynamics in Earth’s oceans (e.g., Speer and Marshall, 1995), suggesting that rotationally constrained plumes originating from heated regions of the silicate core could deliver sufficient heat to initiate such melting. This hypothesis has since been explored under a variety of conditions, drawing on analogies with processes observed in Earth’s oceanic and stratified environments.

Vance and Brown (2005) investigated plume ascent in a stratified ocean environment analogous to the Red Sea, identifying a range of conditions, including ice shell thickness, ocean temperature, and salinity, that could allow a plume to reach the ice shell and trigger a melt-through event on Europa. Taking a complementary approach, Goodman and Lenferink (2012) investigated plume dynamics in a non-stratified ocean using the MIT general circulation model (MITgcm, Marshall et al., 1997), showing that thermally driven plumes could attain spatial scales comparable to those of Europa’s chaotic terrains. Their simulations assumed a quiescent ocean with mean background flow speeds below a few millimeters per second. To support this assumption, they proposed that Europa’s ocean circulation might resemble that of Earth’s abyssal oceans, where typical mean velocities are around 1 mm/s (Vallis, 2012). However, the relevance of this comparison (not only) to Europa’s subsurface ocean dynamics has since been called into question.

A different approach was adopted by Soderlund et al. (2014), who moved beyond analogies with Earth’s oceans and instead focused on theoretical frameworks that relate the dynamical regime of rotating convection to the interplay between

Coriolis and buoyancy forces (Julien et al., 2012; King et al., 2009, 2012). While the Coriolis force arises from planetary rotation, which is an observable parameter, buoyancy, driven by temperature (and compositional) variations, is not directly measurable. To overcome this limitation, they employed a scaling relationship that links buoyant forcing to the total heat flux through the ocean (e.g., Christensen and Aubert, 2006). By estimating this heat flux from tidal dissipation and radiogenic heating within the mantle (e.g., Hussmann et al., 2002), their analysis indicated that heat is primarily transported by quasi-three-dimensional turbulence, rather than by organized columnar convection. Notable differences include a heat transfer pattern that peaks at the equator rather than being directed toward high latitudes, and the dominance of Ice Giant-like broad longitudinal circulations (fig. 1 in Soderlund et al., 2014), replacing the previously hypothesized Jovian-like structure of multiple narrow, alternating jets (fig. 10 in Goodman and Lenferink, 2012).

In addition to a major conceptual shift from earlier studies, a further distinction lies in the numerical methods employed. Goodman and Lenferink (2012) employed finite-volume simulations within a small Cartesian domain to model ascent of a single plume, enabling the use of realistic hydrothermal heat fluxes (0.1–10 GW for Europa, Lowell and DuBose, 2005; Vance and Goodman, 2009) and an effective viscosity on the order of $O(10^{-2})\text{ m}^2/\text{s}$ to represent mixing from background flows (Wunsch and Ferrari, 2004). In contrast, Soderlund et al. (2014) used the pseudo-spectral code MagIC (Wicht, 2002) to solve rotating thermal convection in a full spherical shell. While this approach is better suited for capturing global flow structures and heat transfer, it comes at a cost. Resolving the full range of spatial and temporal scales in oceanic flow is numerically impossible. As a result, 3D pseudospectral models must employ artificially high viscosities and thermal diffusivities (typically $1\text{--}100\text{ m}^2/\text{s}$) to suppress small-scale features that lie beyond the model’s resolution (e.g., Christensen and Wicht, 2007). To see how the gap between simulation and reality can be addressed, it is helpful to examine the development of the pseudospectral method and its various applications.

The pseudospectral method originated in early stellar dynamo simulations (Glatzmaier, 1984) and was later adapted for geodynamo modeling by Glatzmaier and Roberts (1995), soon becoming widely adopted (see, e.g., Roberts and Glatzmaier, 2000; Matsui et al., 2016). In this approach, solution variables are expanded in complete sets of basis functions—typically spherical harmonics for angular directions (Varshalovich et al., 1988), and either Chebyshev polynomials (e.g., Wicht, 2002) or finite differences (e.g., Kvorka and Čadež, 2022) for the radial dimension. While we do not dig into the technical aspects of variable decomposition (e.g., Ivers and Phillips, 2008; Golle et al., 2012), it is important to note that these expansions are truncated at finite degrees due to computational limitations. The maximum spherical harmonic degree sets the smallest resolved lateral scale, while radial resolution is determined either by the polynomial cutoff or the number of grid points in finite-difference schemes.

Since numerical simulations cannot achieve the resolution required for true dynamic similarity with the physical system, a statistical approach based on scaling analysis has been developed to extrapolate simulation results towards molecular values of viscosity and thermal diffusivity (see, e.g., Roberts and King, 2013, for a review). The aim of this analysis is to calibrate power-law relationships that link

key output variables, such as mean flow speed or heat transfer efficiency, to control parameters representing the dominant forces in the system (e.g., Christensen and Aubert, 2006). Calibration of these scaling laws typically relies on extensive datasets composed of numerous low-resolution simulations with high viscosities and thermal diffusivities, supplemented by a limited number of high-resolution runs at the lowest feasible diffusivity values, which are significantly more computationally demanding.

Advancements in computational algorithms (Schaeffer, 2013; Ishioka, 2018; Schaeffer, 2021) have significantly expanded the range of achievable resolutions, enabling clearer differentiation between various heat transfer regimes (Gastine et al., 2016; Cheng et al., 2018) as well as improved characterization of spatial patterns in flow and heat flux (e.g., Yadav et al., 2016; Gastine and Aurnou, 2023). In the context of subsurface ocean dynamics, the pseudospectral method has been applied to examine spatial variations in heat flux at the ocean’s upper boundary (Soderlund, 2019; Amit et al., 2020; Kvorka and Čadek, 2022), the effects of non-uniform bottom heating on heat transport (Lemasquerier et al., 2023; Terra-Nova et al., 2023; Bouffard et al., 2025), the structure of the mean flow (Soderlund, 2019; Cabanes et al., 2024; Šachl et al., 2025), and the influence of ice shell thickness on ocean circulation (Kvorka and Čadek, 2024; Gastine and Favier, 2025).

The finite volume approach introduced by Goodman and Lenferink (2012) has not been abandoned but has instead continued to evolve along a distinct path. Similar to the departure from Earth-like assumptions observed in pseudospectral modeling, recent applications of the finite volume method to subsurface ocean studies share conceptual similarities with investigations of ocean circulation during “Snowball Earth” episodes. During these periods, Earth’s oceans were entirely or nearly entirely covered by thick sea ice. A key distinction between present-day oceans and those of Snowball Earth lies in the dominant source of mechanical energy: wind-driven surface stress is absent under ice cover and is instead replaced by geothermal heating from the ocean floor (see, e.g., Ashkenazy et al., 2013, 2014; Jansen, 2016, and references therein). This configuration closely mirrors that of subsurface oceans on icy moons, making the modeling techniques developed for Snowball Earth scenarios directly applicable (Ashkenazy and Tziperman, 2021).

A key distinction from the pseudospectral method lies in how computational constraints are addressed. Local methods, such as finite volume or finite element approaches, are well-suited for simulations involving anisotropic viscosity and thermal diffusivity. This flexibility enables the parametrization of subgrid-scale processes—such as advection and diffusion occurring at scales smaller than the model resolution—and their influence on the resolved flow and heat transport. While a comprehensive review of parametrization techniques is beyond the scope of this thesis, examples of rotationally constrained parametrizations developed for Earth’s ocean can be found in Visbeck et al. (1997) and Jansen et al. (2015). For comparisons between fully resolved and subgrid-parametrized simulations in the context of convection-driven dynamos, see Matsui and Buffett (2005) and Matsushima (2006).

The parametrization of subgrid-scale processes in subsurface oceans is often simplified by assuming anisotropy between radial and lateral diffusivities (e.g., Ashkenazy and Tziperman, 2021; Kang and Jansen, 2022). Radial diffusivities

are generally inferred from flows driven by tidal forces and libration, with typical values on the order of 10^{-5} to 10^{-2} m²/s (e.g., Zeng and Jansen, 2021; Kang et al., 2022c). In contrast, lateral diffusivities are chosen as the minimum values that maintain numerical stability for a given computational grid and are usually several orders of magnitude greater than the radial values.

More recently, a novel finite-volume-based approach has been introduced (Ramadhan et al., 2020), enabling the use of significantly lower lateral viscosity and thermal diffusivity. This method models a localized section of the ocean using a Cartesian box, which substantially reduces computational costs. Its GPU-accelerated implementation permits simulations with diffusivity and viscosity values on the order of 10^{-2} m²/s (Bire et al., 2022, 2023). Although the model employs a Cartesian domain geometry, rotational effects are accurately captured through the “deep beta-plane approximation” (Dellar, 2011), ensuring physically consistent dynamics.

Studies employing finite-volume methods have highlighted several key aspects of subsurface ocean dynamics. These include the critical role of salinity forcing in driving ocean circulation (Ashkenazy and Tziperman, 2021), the potential influence of ocean stratification, resulting from the anomalous expansion of water (Melosh et al., 2004), on the thermal evolution of Enceladus (e.g., Zeng and Jansen, 2021; Kang et al., 2022a; Lobo et al., 2021), and the possibility of non-synchronous rotation of icy shells due to torques exerted by mean ocean currents (Ashkenazy et al., 2023; Kang, 2024).

3 Dynamical regime of subsurface oceans

Before proceeding with a more detailed analysis of the results, it is important to broaden the preceding discussion by addressing certain technical aspects related to the interpretation of idealized or unrealistic simulations. In particular, it is intuitive to expect that a system experiencing weak buoyant forcing under rapid rotation will exhibit fundamentally different dynamics compared to one dominated by strong buoyancy and only mildly affected by rotation. This raises several key questions: What precisely means “weak buoyant forcing” or “rapid rotation”? How can simulations be configured to meaningfully approximate the circulation within subsurface oceans? And what does it mean for a simulation to “meaningfully approximate” such environments? To address these questions in a more rigorous and quantitative manner, we introduce a set of non-dimensional control parameters that characterize the relative influence of buoyancy, Coriolis, and viscous forces. These parameters provide a systematic framework for classifying the dynamical regimes relevant to subsurface ocean circulation.

In the context of this section, the term dynamical regime refers to the flow geometry and the cooling patterns at the interface between the subsurface ocean and the overlying ice shell. This classification is specifically intended for application to icy moons and is distinct from frameworks based on heat transfer efficiency (e.g., Gastine et al., 2016; Cheng et al., 2018) or those focused on rotational constraints affecting mean flow scales (e.g., Christensen and Aubert, 2006). Nevertheless, as will be demonstrated in the following sections, this classification offers valuable insights into observed surface topography and provides predictive power for interpreting magnetic field measurements from upcoming space missions. It

should be noted that, due to limited knowledge of ocean composition and the exclusion of compositional buoyancy in this thesis, the following discussion does not consider the effects of saline buoyancy. For analyses of salt transport and double-diffusive processes in rotating convection, the reader is referred to e.g., Vance and Brown (2005); Liang et al. (2021); Zeng and Jansen (2021); Ashkenazy and Tziperman (2021) and references therein.

The simplest numerical models solve the Boussinesq approximation of the hydrodynamic equations describing thermal convection in a spherical shell that rotates around the z -axis at an angular speed Ω . The shell is bounded by an inner radius r_i and an outer radius r_o . The set of equations consists of the Navier-Stokes equation (1), which includes the Coriolis force, the incompressibility condition for the velocity field $\mathbf{v}$ (2), and a transport equation for temperature T (3).

The equations are typically expressed in a non-dimensional form, however there is no universally accepted scaling approach. As a result, various sets of non-dimensional parameters are used across different studies. Following Schmitz and Tilgner (2009, 2010), we adopt a scaling in which the shell thickness $D = r_o - r_i$ serves as the characteristic length scale, and time t is scaled by the thermal diffusion time D^2/κ , where κ is the thermal diffusivity (for approaches based on scaling by viscous time or rotational period, see, e.g., Christensen and Wicht, 2007; Gastine et al., 2016; Amit et al., 2020). Accordingly, the velocity is scaled by the ratio of the characteristic length to the time scale, κ/D . The temperature scale depends on the specific thermal boundary conditions imposed. Since in this thesis we consider only isothermal boundaries, temperature is scaled using the superadiabatic temperature difference ΔT between the inner and outer boundaries (for alternative scalings based on imposed heat flux, see, e.g., Mound and Davies, 2017; Terra-Nova et al., 2023; Lemasquerier et al., 2023). In this setting, the equations read

$$\frac{1}{Pr} \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) + \frac{2}{Ek} \mathbf{e}_z \times \mathbf{v} = -\nabla p + \nabla^2 \mathbf{v} + Ra T g \mathbf{e}_r, \quad (1)$$

$$\nabla \cdot \mathbf{v} = 0, \quad (2)$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \nabla^2 T, \quad (3)$$

where p is the pressure, $\mathbf{e}_z$ is the unit vector along the z -axis (axis of rotation) and g is the gravity. The non-dimensional control parameters are the Ekman number,

$$Ek = \frac{\nu}{\Omega D^2}, \quad (4)$$

the Rayleigh number,

$$Ra = \frac{\alpha g_o \Delta T D^3}{\nu \kappa}, \quad (5)$$

and the Prandtl number,

$$Pr = \frac{\nu}{\kappa}, \quad (6)$$

where ν is the kinematic viscosity, α is the thermal expansivity and g_o is the gravity at the outer boundary.

Within the framework outlined above, the Prandtl number characterizes the ratio of viscous to thermal diffusion effects, the Ekman number quantifies the relative importance of viscous forces compared to the Coriolis force, and the Rayleigh number measures the strength of buoyant forcing relative to viscous dissipation. Notably, both the Rayleigh and Ekman numbers depend on viscosity, with the Rayleigh number also incorporating thermal diffusivity. As a result, achieving realistic values for these parameters in numerical simulations is not computationally feasible. However, the primary concern is not the absolute deviation from realistic parameter values, but rather whether the simulations accurately capture the relative influence of buoyancy and Coriolis forces.

The Coriolis force has a stabilizing influence on the system, preventing the onset of convection at arbitrarily small Rayleigh numbers. More precisely, convection begins only when the Rayleigh number exceeds a critical threshold, denoted as Ra_c , below which ($Ra \leq Ra_c$) heat transport remains conductive. This critical value scales approximately as $Ra_c \sim Ek^{-4/3}$ (e.g., Chandrasekhar, 1961; Busse, 1970; Dormy et al., 2004). Assuming fixed Ek , upon increasing of Rayleigh number beyond Ra_c the system undergoes variety of changes in flow geometry and heat transfer characteristics ending with non-rotating convection at sufficiently high Ra (e.g., Liu and Ecke, 1997; Zhong et al., 2009; Schmitz and Tilgner, 2009; Gastine et al., 2016; Cheng et al., 2018; Plumley and Julien, 2019). In the following, we will discuss a selected subset of flow regimes applicable to the icy moons.

Soderlund (2019) reviewed estimates of the relevant physical parameters (assuming molecular values for viscosity and thermal diffusivity) used in the definitions of the Ekman, Rayleigh, and Prandtl numbers, and concluded that the subsurface oceans of Enceladus, Europa, Titan, and Ganymede all satisfy the supercritical condition $Ra \geq Ra_c$. The same conclusion was reached by Bire et al. (2022), who employed effective values for viscosity and diffusivity on the order of $O(10^{-3}) \text{ m}^2/\text{s}$ suggested by Kang et al. (2022a). Moreover, the estimates on the Rayleigh number suggest $Ra \geq 6Ra_c$, a threshold for non-linear, turbulent convection (Gastine et al., 2016) and $RaEk^2/Pr \leq 1$, a threshold for non-rotating convection (Gilman, 1977; Cheng et al., 2018). Therefore, our analysis is limited to turbulent rotating convection, specifically to the rapidly rotating and transitional regimes (see, e.g., Gastine et al., 2016). For discussions on weakly nonlinear turbulence and non-rotating convection, readers are referred to Gillet and Jones (2006) and Gastine et al. (2015), respectively.

In the rapidly rotating regime, the Coriolis force dominates and is balanced primarily by pressure gradient, resulting in columnar flow structures that are largely invariant along the planet’s rotation axis. Additionally, the combination of spherical geometry and non-linear turbulence drives an inverse energy cascade, leading to the formation of large-scale east–west (zonal) jets (e.g., Rhines, 1975; Busse, 1970; Christensen, 2002). In the rapidly rotating convection regime, zonal flows appear as a series of counter-rotating toroidal cylinders (e.g., Heimpel and Aurnou, 2007; Bire et al., 2022; Cabanes et al., 2024).

As the system enters the transitional regime and the Coriolis force loses its dominance, the organized pattern of toroidal cylinders breaks down. The flow reconfigures into two broad, counter-rotating jets near the equator: a prograde jet (aligned with the direction of rotation) emerges near the ocean’s upper boundary, while a retrograde jet forms near the bottom (e.g., Bire et al., 2022; Cabanes

et al., 2024). As buoyancy becomes increasingly influential (for example by a rising Rayleigh number at fixed Ekman number) three-dimensional turbulence develops, and the Coriolis force is no longer balanced by the pressure gradient. Enhanced turbulent mixing tends to homogenize angular momentum, resulting in the emergence of a three-jet structure: two prograde jets at higher latitudes and a retrograde jet near the equator (e.g., Aurnou et al., 2007; Yadav et al., 2016; Soderlund, 2019; Kvorka and Čadek, 2022). Under stronger buoyant forcing, large-scale meridional circulation cells develop across the mid-latitudes, accompanied by a radial outflow at the equator (e.g., Gilman, 1977; Soderlund, 2019; Šachl et al., 2024).

Recently, Ashkenazy and Tziperman (2021) identified a novel zonal flow configuration at a Prandtl number characteristic of freshwater, $Pr = 10$. While this pattern resembles the two-jet structure typically associated with the transition from a rotation-dominated to a rotation-influenced regime (see the discussion above), it exhibits a reversal in flow direction: the retrograde jet forms near the top of the ocean. Similar results were obtained in a systematic study presented in Chapter 4 of this thesis. This analysis suggests that the reversed two-jet configuration is a direct evolutionary outcome of the three-jet regime and emerges near the transition towards the non-rotating convection. Notably, the meridional and radial flows still resemble the pattern observed in the three jet configuration, however, with increased speed.

In summary, four distinct flow patterns characterized by different zonal circulation structures have been proposed as potential representations of subsurface ocean dynamics. In contrast, only two configurations have been identified for meridional and radial circulations: the rapidly rotating columnar regime and the transitional regime featuring circulation cells and equatorial outflows.

A careful reader may have noticed a key element absent from the preceding discussion: the boundary conditions. To fully define the system described by equations (1)–(3), appropriate boundary conditions must be specified to reflect the physical nature of the system’s interfaces. For example, a material boundary between the ocean and a silicate mantle or rocky core requires a different treatment than a phase boundary between solid ice and liquid water. In simulations of convection within icy satellites, the most commonly used mechanical boundary conditions are either stress-free and impenetrable (free-slip) or fixed (no-slip), typically paired with thermal boundary conditions specifying either a constant temperature (isothermal) or a fixed heat flux at the boundary.

In the simplest case, where boundary quantities are prescribed as homogeneous values, the qualitative behavior of the flow remains largely consistent across different boundary conditions. Asymptotic analysis by Calkins et al. (2015) demonstrated that in regimes dominated by the Coriolis force, solutions under free-slip conditions are similar at leading order for both fixed-temperature and fixed-flux thermal boundaries. Comparable findings were reported by Lemasquerier et al. (2023) for transitional regimes, and by Johnston and Doering (2009) for non-rotating turbulent convection with no-slip boundaries. Although both no-slip and free-slip boundaries yield qualitatively similar heat transfer behavior (e.g., Schmitz and Tilgner, 2010; Kvorka and Čadek, 2022), Yadav et al. (2016) observed more disrupted flow structures under no-slip conditions, primarily due to friction within boundary layers (e.g., Jones, 2007). Nevertheless, the fundamental flow

characteristics remain consistent with those observed under free-slip conditions.

An analysis of more complex cases, when one of the variables (typically heat flux or temperature) is allowed to vary in lateral direction over the boundary, is beyond the scope of this section, however, some results will be noted later in relation to surface topography formation. Before proceeding towards the implications of dynamical regime for observable quantities, we will discuss how a proper boundary condition is chosen for a given body internal structure. Starting from the thermal boundary conditions, the melting and freezing processes at the ice–ocean interface impose a temperature fixed at the melting point. This has led to the widespread adoption of an isothermal upper boundary condition for the ocean (e.g., Soderlund et al., 2014; Amit et al., 2020; Bire et al., 2022; Kvorka and Čadek, 2022; Terra-Nova et al., 2023; Bouffard et al., 2025). However, recent studies have shown that variations in the melting temperature, driven by topographic features at the ice–ocean interface, can have a non-negligible impact on ocean dynamics and heat transport (e.g., Labrosse et al., 2018; Kvorka and Čadek, 2024; Gastine and Favier, 2025). It is important to note, however, that in cases where such topography is absent or negligible (such as in the case of a convecting ice shell, Kihoulou et al., 2023) the assumption of a constant temperature remains valid.

Moreover, in sufficiently large icy bodies such as Titan and Ganymede, high-pressure (HP) ice layers are expected to form at the base of the ocean, and the lower boundary condition can be selected to reflect this phase transition. As HP ice is believed to undergo convection (e.g., Journaux et al., 2020), pressure driven temperature variations can be considered negligible. However, isothermal boundary temperature condition does not capture the heat and mass fluxes associated with melt extraction from the HP ice interior into the ocean (e.g., Choblet et al., 2017a). Despite this limitation, a constant temperature is commonly prescribed at the bottom of the ocean in models regarding Titan and Ganymede (e.g., Amit et al., 2020; Kvorka and Čadek, 2022, 2024; Cabanes et al., 2024; Gastine and Favier, 2025) with a notable exception of Terra-Nova et al. (2023). In contrast, when the ocean directly overlies a rocky core or silicate mantle, a flux-based boundary condition capturing the tidal heat generation pattern is more appropriate (e.g., Lemasquerier et al., 2023; Bouffard et al., 2025).

The no-slip boundary condition is generally considered appropriate for viscous fluids in contact with rigid surfaces (e.g., Christensen and Aubert, 2006; Gastine et al., 2016; Amit et al., 2020; Ashkenazy and Tziperman, 2021; Terra-Nova et al., 2023). However, as noted by Kuang and Bloxham (1997), this condition can lead to an overestimation of the influence of viscous boundary layers, particularly at high Ekman numbers. Analysis of velocity spectra by Cabanes et al. (2024) suggests that this overestimation becomes significant when $Ek > 10^{-5}$. As a result, several studies have adopted free-slip boundary conditions in their models to compensate for unrealistically high Ekman numbers used in simulations (e.g., Soderlund et al., 2014; Soderlund, 2019; Kvorka and Čadek, 2022). Notably, some authors apply free-slip conditions specifically at the ice–ocean interface (Bire et al., 2022, 2023), while others introduce velocity-dependent drag formulations to represent frictional effects (e.g., Ashkenazy et al., 2023; Kang, 2024).

3.1 Implications for motionally induced magnetic field

As previously noted, electromagnetic measurements played a crucial role in identifying subsurface oceans on Europa and Ganymede (Zimmer et al., 2000; Khurana et al., 2002; Kivelson et al., 2002), and have also been proposed as a method for detecting similar oceans on the icy moons of Uranus (Arridge and Eggington, 2021). These studies relate to magnetic fields induced by the motion of a conductive layer (a liquid ocean) within the external magnetic field generated by orbiting planet. More recently, attention has shifted to a different source of magnetic fields: those generated by fluid motion within the ocean itself (Vance et al., 2021; Šachl et al., 2024).

The primary mechanism behind motional induction of magnetic fields is the Lorentz force, expressed as $L = \mathbf{v}(\mathbf{r}, t) \times \mathbf{B}_0(t)$, where $\mathbf{v}(\mathbf{r}, t)$ denotes the flow velocity and $\mathbf{B}_0(t)$ represents the background magnetic field: Jupiter’s field in the case of Europa’s ocean, and Ganymede’s internal dipole field (Christensen, 2015) for its ocean. Since the Lorentz force depends linearly on both the magnitude and direction of the oceanic flow, future magnetic field measurements may offer valuable constraints on the ocean’s dynamical behavior. With upcoming missions equipped with magnetometers targeting Ganymede (Amtmann et al., 2024) and Europa (Kivelson et al., 2023), our discussion will focus on the proposed flow geometries within the subsurface oceans of these two moons.

In the case of Europa, proposed flow structures range from rotationally dominated columnar convection (Bire et al., 2022), to two- and three-jet configurations (Soderlund, 2019; Šachl et al., 2025), and even a reversed two-jet system (Ashkenazy and Tziperman, 2021). Notably, while Šachl et al. (2025) and Soderlund (2019) reported consistent results, Chapter 4 of this thesis expanded the dataset of Šachl et al. (2025) and showed that the reversed configuration found by Ashkenazy and Tziperman (2021) may also emerge under sufficiently strong internal heating.

The threshold for such heating, particularly the conditions under which a three-jet configuration transitions into a reversed two-jet system, lies largely beyond the scope of this thesis. However, Chapter 4 provides a partial answer by introducing an empirical parameter, $R_K = RaEk^{12/7}/Pr^{3/4}$, which effectively captures the overturning of the equatorial jet from prograde to retrograde at $R_K \approx 9$.

Šachl et al. (2025) demonstrated that a heat flux of 10 mW/m² is insufficient for the reversed configuration, whereas Ashkenazy and Tziperman (2021) observed reversed pattern within a range of 33-100 mW/m². The analysis in Chapter 4 supports these findings, showing that the $R_K \approx 9$ threshold correspond well to the parameter settings used in both studies ($R_K \leq 9$ for Šachl et al., 2025 and $R_K \geq 9$ for Ashkenazy and Tziperman, 2021). Given that Europa’s internal heat flux is poorly constrained, with estimates ranging from 10 to 200 mW/m² (e.g., Ashkenazy and Tziperman, 2021; Lemasquerier et al., 2023), all possible flow configurations should be considered when modeling Europa’s magnetic field. In the case of Ganymede, due to the large size (e.g., Kang and Jansen, 2022) and a large internal heating (Choblet et al., 2017a), only transitional (two-, three- and reversed-jet) configurations are expected (see also the chapter 4 of this thesis).

The flow pattern represents only one component required for estimating the magnetic field; the other crucial component is the magnitude of the flow speed. The model by Ashkenazy and Tziperman (2021) suggests flow speeds on the order

of a few centimeters per second for Europa. In contrast, Vance et al. (2021), using the flow models of Soderlund (2019) developed for $Ek = 3.10^{-4}$, estimated much higher velocities—up to 3 m/s for Europa and 6 m/s for Ganymede. This significant discrepancy was addressed by Cabanes et al. (2024) who conducted a systematic analysis of flow velocity spectra and found that, in the subsurface oceans of icy moons, the zonal (axisymmetric) flow component, shaped by rotational constraints, typically dominates over the less organized convective motions. Nevertheless, even this dominant zonal flow is not expected to exceed several tens of centimeters per second for Europa, and around 1.7 m/s for Ganymede.

Building on their findings, we developed a least-squares approach to estimate flow velocities and applied it to both Europa and Ganymede. Our results are consistent with the estimates by Cabanes et al. (2024), under the assumption that molecular values of viscosity and thermal diffusivity govern energy dissipation (cf. Wunsch and Ferrari, 2004; Zeng and Jansen, 2021; Kang et al., 2022c). A detailed discussion of these results, including the estimated lateral distribution and strength of the magnetic field, can be found in Chapters 3 and 4 of this thesis. This thesis does not account for additional factors that may influence oceanic flow and magnetic field generation, such as salinity effect (e.g., Ashkenazy and Tziperman, 2021), inhomogeneous heat flux at the ocean’s base resulting from tidal dissipation in the core (e.g., Lemasquerier et al., 2023; Bouffard et al., 2025), convection within high-pressure ice layers (e.g., Terra-Nova et al., 2023), temperature variations caused by ice-ocean interface deformation (Kvorka and Čadek, 2024; Gastine and Favier, 2025), or mechanically driven flows (e.g., Lemasquerier et al., 2017; Aygün and Čadek, 2023).

3.2 Implications for shape and topography

The potential presence of subsurface oceans beneath the ice shells of icy moons (as discussed in the first section), along with gravity measurements indicating non-hydrostatic shapes for Enceladus and Titan (see, e.g., Lefèvre et al., 2014; Čadek et al., 2016, 2017), has led to several hypotheses about the role of melting and freezing in shaping these ice shells. Nimmo and Bills (2010) proposed a model for Titan in which deformation results from melting and freezing at the ice-ocean interface, driven by tidal heating within the ice shell itself (see also Ojakangas and Stevenson, 1989). For Enceladus, Kang and Flierl (2020) suggested that variations in orbital eccentricity over the past billion years (e.g., Meyer and Wisdom, 2007) could have enabled an asymmetric deformation pattern, with one of the poles thinning due to enhanced tidal heating and localized melting. In contrast, for Europa, Ashkenazy et al. (2018) proposed a different mechanism, in which a strong equator-to-pole surface temperature gradient causes polar thickening of the ice shell due to enhanced freezing due to low surface temperatures in these regions (e.g., Ashkenazy, 2019).

In contrast to the passive role of the ice-ocean interface suggested in previous studies, Kang et al. (2022a) argued that oceanic heat transport could actively shape Enceladus’ topography. Assuming that the majority of the moon’s internal heat is generated within the ice shell, they proposed that water’s anomalous expansion, combined with salinity gradients, drives a large-scale circulation. This circulation extracts latent heat from freezing at one pole and transports it to

facilitate melting at the opposite pole. However, the follow-up studies by Kang et al. (2022c) and Kang and Jansen (2022) demonstrated that this mechanism is unlikely to operate on larger moons such as Titan and Ganymede, where high pressure suppresses the anomalous expansion of water.

An alternative hypothesis involving an active role of ocean flows and turbulent heat transfer driven by basal heating was proposed by Bouffard et al. (2025) who modeled rotating convection with an inhomogeneous heat flux imposed at the ocean’s lower boundary (for a model of core tidal deformation, see Choblet et al., 2017b). Their simulations showed that strong zonal currents in the equatorial region suppress vertical heat transport, while buoyant plumes of warm water are able to rise more freely at the poles. This results in enhanced polar heat transfer and leads to a thinner ice shell at the poles and a comparatively thicker shell at the equator.

In larger icy bodies such as Titan and Ganymede, the presence of high-pressure ice layers inhibits the formation of stable, large-scale heating patterns driven by convection (e.g., Journaux et al., 2020). As demonstrated by Kvorka et al. (2018), small-scale variations in heat flux originating from the ocean are unlikely to produce detectable topographic features, since the ice shell acts as a low-pass filter, effectively translating only long-wavelength flux anomalies into surface deformation. Nonetheless, oceanic convection may still generate measurable topography if it establishes a stable, monotonic heat flux profile from pole to equator.

Tilgner and Busse (1997) observed a long-wavelength, pole-to-equator heat flux pattern in simulations of rotating convection. In rapidly rotating systems, heat flux initially peaks at the equator due to equatorially trapped convection (Miquel et al., 2018), which enhances equatorial heat transport while polar regions remain largely conductive at moderately supercritical Rayleigh numbers ($Ra_c \leq Ra \leq 20Ra_c$, Gastine and Aurnou, 2023). As buoyant forcing increases, convection begins to develop in the polar regions. This transition shifts the dominant heat transport toward the poles, as the equatorial two-jet zonal flow configuration suppresses vertical heat transfer (e.g., Yadav et al., 2016). At even higher Rayleigh numbers, the development of a three-jet flow regime leads to the emergence of a mid-latitude cooling pattern, as shown in studies by Soderlund (2019); Amit et al. (2020); Kvorka and Čadek (2022).

Kvorka and Čadek (2022) unified the findings of Amit et al. (2020) and Soderlund (2019) and, using vast dataset covering the previously described flow regimes, proposed an empirical parameter $R_G = RaEk^{12/7}/Pr$ that best characterizes the transition from equatorial to polar cooling. Applying this parameter to Titan suggests that the moon lies within the polar cooling regime, with the predicted normalized heat flux closely matching the observed normalized topography (see also Chapter 1 of this thesis). However, direct application of this framework to other icy moons is limited, as Titan’s surface temperature variations are minimal (only a few Kelvins between pole and equator, Tokano, 2005) due to its dense atmosphere. In contrast, surface temperature gradients are more pronounced on other icy satellites and may play a dominant role in driving surface deformation, as shown for Europa (Ashkenazy et al., 2018) and discussed in Chapter 4 of this thesis for Ganymede.

The applicability of simple ocean models has been increasingly questioned

in recent years. Labrosse et al. (2018) proposed a heat transport mechanism driven by the pressure dependence of the melting temperature. Earlier models assumed the ice-ocean interface to be at a constant melting temperature; however, local variations in ice shell thickness alter the pressure, which in turn changes the melting temperature. Although this change is small (about 14 mK per 100 meters of topography, Kihoulou et al., 2023) it is comparable to the superadiabatic temperature contrast that drives convection, typically on the order of 0.1 K or less (e.g., Soderlund, 2019). Chapter 3 of this thesis presents a coupled model of ice shell deformation and ocean convection, showing that while heat transport along the boundary suppresses topography formation, it does not eliminate it entirely. A parameterization of this effect is introduced, along with an estimated value quantifying the strength of the additional heat transport that best fits Titan’s observed topography.

1 A numerical model of convective heat transfer in Titan’s subsurface ocean

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Abstract

It has been suggested that the long-wavelength topography of Titan is related to lateral variations in the heat flux from the ocean. Recent studies of the heat transfer in Titan’s ocean agree that the time-averaged heat flux can vary in latitude by tens of percent, but they predict different distributions of heat flux anomalies at the upper boundary of the ocean. In order to clarify this issue, we perform 115 numerical simulations of thermal convection in a rotating spherical shell, varying the mechanical boundary conditions and dimensionless input parameters (Rayleigh, Ekman and Prandtl numbers) by at least one order of magnitude. The results of the simulations are examined in terms of the modified transitional number, $R_G^* = RaEk^{12/7}Pr^{-1}$. Depending on the relative importance of rotation, the heat flux maximum is located either at the equator (equatorial cooling) or at the poles (polar cooling). We demonstrate that equatorial cooling occurs when $R_G^* < 1$ or $R_G^* > 10$ while polar cooling occurs when $R_G^* \in \langle 1, 10 \rangle$. Based on this result, we predict that Titan’s ocean is in the polar cooling mode and the heat flux distribution is controlled by zonal degree 2 and 4 harmonics. The predicted heat flux shows a high degree of similarity with the axisymmetric part of Titan’s long-wavelength topography, indicating a strong relationship between ocean dynamics and the processes in the ice shell.

1.1 Introduction

Several moons of the giant planets are thought to have liquid-water oceans buried beneath their icy crusts (Nimmo and Pappalardo, 2016). These oceans transport heat and chemicals from the deep interior to the surface, influence the tectonic evolution of the ice crust and are possible candidates for hosting life. Understanding the dynamics of subsurface oceans is therefore critical for comprehending the evolution of icy bodies and evaluating their habitability.

The subsurface oceans are inaccessible to direct observation, but they can be studied by numerical simulations. The simulations are carried out by solving the conservation equations for mass, momentum and energy applied to a rotating viscous fluid (see Appendix A for details). The main drawback of this approach is the order of magnitude difference between the actual values of the parameters that control the real ocean and those that can be achieved in numerical simulations. For example, in numerical simulations of thermal convection in planetary interiors, the Ekman number (Ek , the ratio of viscous to Coriolis forces) is usually five to ten orders of magnitude larger than its real value, while the Rayleigh number

(Ra , the ratio of the thermal diffusion time to the viscous buoyant rise time) is usually ten to fifteen orders of magnitude smaller than that expected in natural systems. In spite of this limitation, numerical simulations can provide valuable insight into the dynamics of rotating convective systems since the results obtained for unrealistic Ekman and Rayleigh numbers can be extrapolated using scaling laws. This approach has been used for several decades to investigate magnetic fields generated in molten planetary cores (see, e.g., Roberts and King, 2013, for a review) and can also be applied to subsurface oceans of icy moons (e.g. Soderlund et al., 2014; Soderlund, 2019; Amit et al., 2020).

Recent progress in computational algorithms (Schaeffer, 2013) together with increasing computational power have considerably widened the range of accessible Ekman and Rayleigh numbers and made it possible to test theoretical scaling laws and establish “empirical” scalings. Assuming a shell with an inner to outer radius ratio of 0.60, no-slip boundary conditions and gravity proportional to r^{-2} , Gastine et al. (2016) found five different scaling regimes: conductive, weakly nonlinear, rapidly rotating, transitional and non-rotating. Cheng et al. (2018) combined numerical simulations, laboratory experiments and asymptotic predictions and proposed a similar classification but using different terminology to describe the heat transfer modes (subcritical, columnar, plumes, geostrophic turbulence, unbalanced boundary layers, and non-rotating heat transfer). The classifications by Gastine et al. (2016) and Cheng et al. (2018) have been used as the basis for two studies investigating the spatial variations in the heat flux at the upper boundary of a subsurface water ocean (Soderlund, 2019; Amit et al., 2020). Latitudinal variations in heat flux from the ocean have been proposed to have contributed to the formation of chaos terrains on Europa (Soderlund et al., 2014), polar depressions on Titan (Kvorka et al., 2018) and ice thickness variations on Enceladus (Čadek et al., 2019b), and are likely to have played a significant role in the evolution of icy moons.

Soderlund (2019) and Amit et al. (2020) show that, depending on the relative importance of rotation, convection models predict three different cooling patterns: (i) the polar cooling, characterized by efficient heat transfer at high latitudes, (ii) the equatorial cooling, characterized by the maximum heat transfer near the equator, or (iii) the uniform cooling, corresponding to convection systems that are not affected by rotation. The studies agree that the lateral heat flux variations may be significant (up to $\sim 50\%$), but their results are contradictory. Whereas the model of Soderlund (2019) predicts the transition from equatorial to polar cooling with increasing influence of rotation, the model by Amit et al. (2020) indicates the opposite trend – the transition from polar to equatorial cooling. It is therefore difficult to draw conclusions about the cooling patterns relevant to icy moon oceans because the two sets of results do not allow a unique interpretation.

The study of Amit et al. (2020) focuses on Titan where the variations of oceanic heat flux have been estimated from analysis of topographic data (Kvorka et al., 2018). Amit et al. (2020) classify the heat transfer regime in Titan’s ocean as transitional to non-rotating and argue for enhanced polar cooling, which contrasts with the results of Soderlund (2019) suggesting weak equatorial cooling in this regime. This discrepancy may be related to differences in geometry and boundary conditions, or to the way in which the results are extrapolated to the real world. In both studies, convection is driven by a fixed temperature contrast between

the inner and outer boundary, but the results presented by Soderlund (2019) are obtained for a radius ratio of 0.90 and free-slip boundary conditions, while the numerical simulations discussed in Amit et al. (2020) are performed for a radius ratio of 0.80 and no-slip boundary conditions.

In macro-scale applications, no-slip is the proper boundary condition for a viscous fluid at a rigid boundary. Since the subsurface ocean is treated as a fluid shell with rigid and impermeable boundaries, the no-slip boundary condition seems to be the correct choice. However, as pointed out by Kuang and Bloxham (1997), the use of the no-slip condition may be problematic because the Ekman number used in numerical models is many orders of magnitude larger than in the real fluid. As a consequence, no-slip models tend to overestimate viscous effects, resulting in a thick Ekman layer (the boundary layer in which the frictional force is balanced by the Coriolis force). Since the net torque acting on the fluid shell is generally not equal to zero, the no-slip boundary conditions lead to a differential rotation of the ocean with respect to the ice crust and the silicate core. In contrast, the free-slip boundary condition effectively reduces the Ekman layer thickness to zero, and since there is no viscous coupling across the boundaries, the total angular momentum of the ocean is conserved (see also Kuang and Bloxham, 1999). The absence of friction forces in free-slip models is accompanied by the development of strong zonal flows, possibly influencing the heat transport in the ocean.

Current computational power does not allow numerical simulations of thermal convection with realistic Ekman and Rayleigh numbers. To avoid this problem, Soderlund (2019) and Amit et al. (2020) have performed a series of numerical simulations with parameters that are computationally tractable and interpreted the results using a scaling analysis. There are two problems that make it difficult to extrapolate the results of convection simulations to the real world. First, we do not know the superadiabatic temperature difference, ΔT , across the ocean. Given that $Ra \propto \Delta T$, this means that the Rayleigh number of the real ocean cannot be determined directly from the definition. Since the total heat output of the convective system is usually known, several methods have been proposed to estimate the Rayleigh number from the Nusselt number, Nu , characterizing the total heat transfer across the fluid domain. Soderlund (2019) and Amit et al. (2020) estimated the interval of possible values of Ra using the scaling laws for two end member cases: a rapidly rotating ocean and a non-rotating ocean (Gastine et al., 2015, 2016).

The second problem consists of finding the relationship between the numerical simulations performed for unrealistic values of control parameters and the real system, i.e. of determining the properties of the real system from the results of the numerical modeling. This can be done using a convective regime diagram (see, e.g., Fig. 1 in Soderlund, 2019 or Fig. 10 in Amit et al., 2020). The implicit assumption behind this approach is that each convective regime has its own specific cooling pattern. This assumption is likely to be satisfied for rapidly rotating convective systems but it may not be valid in the case of the transitional regime where the heat flux pattern does not show a clear trend (Amit et al., 2020). It is also worth mentioning that the convective regime diagrams themselves are uncertain to some extent since they can use different scaling laws to describe the same transition. For example, the transition to the non-rotating regime can be described by either the convective Rossby number ($\propto Ek$) or the transitional number R_G ($\propto Ek^{12/7}$, see

Tab. 1.2), which however differ by orders of magnitude at Ekman numbers relevant to ocean worlds (see red-shaded area in Fig. 1 of Soderlund (2019)). Another problem is that the empirical scaling relations are mostly based on numerical simulations with a radius ratio of 0.60 or smaller while the ratio of the inner to outer radius of a subsurface ocean is typically greater than 0.80.

Alternatively, one can characterize the system using a suitable ‘diagnostic’ parameter. This parameter must be sensitive to changes in the heat flux pattern, insensitive to order-of-magnitude changes in control parameters Ek and Ra , and must be chosen such that the heat flux characteristics can be unambiguously expressed in terms of this parameter. This approach was used by Amit et al. (2020) who analyzed the dependence of the heat flux characteristics on two diagnostic parameters: the local convective Rossby number, Ro_{loc} , and the transitional Rayleigh number, R_T (for the definitions, see Eqs. 15 and 16 in Amit et al., 2020).

The purpose of this paper is to clarify the discrepancy between the results of Soderlund (2019) and Amit et al. (2020) and to estimate the distribution of the heat flux coming from Titan’s subsurface ocean. To resolve this issue we perform a series of numerical simulations in 3d spherical geometry in which we vary Rayleigh, Ekman and Prandtl numbers (Prandtl number, Pr , is the ratio of momentum diffusivity to thermal diffusivity) and change the mechanical boundary conditions. The results of these simulations are interpreted in terms of different diagnostic parameters and used to estimate the Rayleigh number of Titan’s ocean. In particular, we attempt to answer the following two questions: (i) What is the most appropriate parameter for the classification of cooling patterns in convective systems weakly affected by rotation? (ii) How much is the extrapolation of the numerical models affected by the choice of the mechanical boundary condition? Answering these questions is essential for understanding the heat transfer in subsurface oceans of icy moons and the role it plays in the thermo-mechanical evolution of the ice crust.

1.2 Control and diagnostic parameters

The heat and mass transfer in the ocean is controlled by three dimensionless parameters: the Rayleigh, Ekman and Prandtl numbers (see Appendix 1.A for details). The values of the control parameters appropriate to Titan’s ocean are given in the third column of Tab. 1.1. While the Ekman and Prandtl numbers can easily be calculated using the formulas in Tab. 1.1, the accurate determination of the Rayleigh number is difficult since we do not know the superadiabatic temperature contrast ΔT in the subsurface ocean. This problem can be circumvented by using the total heat flow in combination with a scaling law that relates the control parameters to the Nusselt number (the ratio of total to conductive heat transfer across the ocean). The issue of determining the Rayleigh number from the total heat flow is discussed in Appendix 1.B.

The Prandtl number in Tab. 1.1 is defined as the ratio of kinematic viscosity ν to thermal diffusivity κ , where both ν and κ are considered to be material constants. However, in a number of studies, including Soderlund (2019) and Amit et al. (2020), Pr is identified with the turbulent Prandtl number and its value is set to 1. This choice is usually justified by the fact that momentum transfer and heat transfer are not independent because they both depend on the same turbulent

Parameter	Definition	Value on Titan	This study
Rayleigh number (Ra)	$\alpha g_o \Delta T D^3 / (\kappa \nu)$	$4 \cdot 10^{19} - 9 \cdot 10^{21}$	$2.1 \cdot 10^6 - 3.8 \cdot 10^7$
Ekman number (Ek)	$\nu / (\Omega D^2)$	$1 \cdot 10^{-12} - 4 \cdot 10^{-11}$	$10^{-5} - 10^{-3}$
Prandtl number (Pr)	ν / κ	10	1 – 10

Table 1.1 Parameters controlling heat and mass transfer in the ocean. Here, α denotes the thermal expansivity, g_o is the gravitational acceleration at the outer boundary, ΔT is the superadiabatic temperature contrast between the boundaries, D is the thickness of the ocean, κ is the thermal diffusivity, ν is the kinematic viscosity, and Ω is the rate of rotation. The values of Ra and Ek in the third column correspond to Titan where the thickness of the ocean is estimated to be between 100 and 500 km (see Tab. 1.3). The scaling used to determine the Rayleigh number is discussed in Appendix 1.B.

eddies (the principle known as the Reynolds analogy). In order to determine the sensitivity of the heat flux pattern to the Prandtl number, we consider three values: $Pr = 1$ (turbulent value), 3 and 10 (molecular value).

One of the goals of this paper is to estimate the latitudinal variations of the heat flux at the outer boundary of Titan’s ocean. The heat flux variations can be characterized by two parameters (Amit et al., 2020). The first one is the latitudinal position of the heat flux maximum. Based on the position of this maximum, convection models can be subdivided into two end-member types – the equatorial cooling models where the heat transfer is maximum near the equator, and the polar cooling models where the maximum heat transfer occurs at high latitudes. The cooling pattern can be quantified by the ratio

$$q^{h/l} = \frac{\langle q \rangle^h - \langle q \rangle^l}{\langle q \rangle^h + \langle q \rangle^l}, \quad (1.1)$$

where $\langle q \rangle^h$ is the average heat flux at high latitudes ($\theta > \theta_{tc}$), $\langle q \rangle^l$ is the average heat flux at low latitudes ($\theta < \theta_{tc}$), and $\theta_{tc} = \arccos(r_i/r_o)$ (for more details, see Eqs. (11)–(13) in Amit et al., 2020). The value of $q^{h/l}$ is positive for polar cooling and negative for equatorial cooling. The second parameter is the magnitude of the heat flux anomaly (Olson and Christensen, 2002), defined as the peak-to-peak amplitude of the zonally averaged heat flux, q^z , divided by twice the mean heat flux, $\bar{q}$,

$$q^* = \frac{q_{max}^z - q_{min}^z}{2\bar{q}}. \quad (1.2)$$

In order to answer the question of how the heat flux pattern depends on the thermal regime of the ocean, we need to know the criteria that separate the thermal regimes from each other. The transition to the non-rotating regime can be characterized by the convective Rossby number, Ro_c (Gilman, 1977), which is defined as the large scale ratio of the Coriolis force to the buoyancy force (Tab. 1.2). The convective Rossby number is a suitable measure for comparing weakly rotating systems but it fails as a diagnostic of the transition to the rapidly rotating regime which occurs at $Ro_c \ll 1$. According to King et al. (2010a), this transition is controlled by the competition between the viscous and thermal boundary layers which occurs at $RaEk^{7/4} \approx 1$ (hereafter denoted by R_K , see Tab. 1.2). A similar condition ($RaEk^{12/7} \sim 1 - 100$, R_G in Tab. 1.2) was obtained by Gastine et al.

Parameter	Definition	Value on Titan	This study
Convective Rossby number (Ro_c)	$(RaEk^2/Pr)^{1/2}$	0.04 – 0.1	0.05 – 2.2
R_G	$RaEk^{12/7}$	40 – 100	0.5 – 150
R_K	$RaEk^{7/4}$	13 – 42	0.4 – 120
Local convective Rossby number (Ro_{loc})	$RaEk^{8/5}/Pr^{3/5}$	185 – 380	0.4 – 150

Table 1.2 Diagnostic parameters used in this study. The values in the third column are obtained using the scaling described in Appendix 1.B. For a detailed discussion of different diagnostic parameters, see Soderlund (2019).

(2016) who analyzed more than 200 models covering a broad parameter range with Ek and Ra spanning seven orders of magnitude (see also Eq. 24 in Julien et al., 2012). The parameter R_G may also serve as an alternative to the convective Rossby number. As demonstrated by (Gastine et al., 2016, Fig. 16b), the Nusselt number (i.e., the total heat flow) is not affected by rotation when $R_G \gtrsim 100$. The last diagnostic parameter in Tab. 1.2 is the local convective Rossby number, which controls the breakdown of geostrophic balance in the thermal boundary layer (Julien et al., 2012; Gastine et al., 2016).

1.3 Results of numerical simulations

We investigate thermal convection in a subsurface ocean using the Navier-Stokes-Boussinesq model. The basic equations and the numerical method are described in Appendix A. The computational domain is a spherical shell of inner radius r_i and outer radius r_o . The radius ratio r_i/r_o is set to 0.8 in all simulations. Convection is driven by a fixed temperature contrast between the inner and outer boundary and the gravity acceleration is proportional to r^{-2} . All models presented in this section have free-slip boundaries. The results obtained for models with no-slip boundary conditions are discussed in section 1.4. The full list of the models explored in this study and the resulting values of Nu , Re (Reynolds number), q^* and $q^{h/l}$ are given in Tabs. 1.4 and 1.5 in Appendix 1.C.

Fig. 1.1 shows the heat flux characteristics q^* and $q^{h/l}$ introduced in section 1.2 as a function of diagnostic parameters Ro_c , Ro_{loc} , R_K and R_G . The results obtained for $Pr = 1$, 3 and 10 are plotted in blue, red and green, respectively. Inspection of the figure reveals that the time-averaged heat flux at the surface of the ocean strongly depends on the Prandtl number, which has been overlooked in previous studies (note that both Soderlund (2019) and Amit et al. (2020) use $Pr = 1$). The Prandtl number can influence not only the amplitude of the heat flux variations (panels a,c,e,g) but also the position of the global heat flux maximum (panels b,d,f,h). For example, for $R_K = 1$ (Fig. 1.1f), the model with $Pr = 10$ predicts enhanced polar cooling ($q^{h/l} \approx 0.4$) while the models with $Pr = 1$ and 3 predict equatorial cooling ($q^{h/l} \lesssim -0.4$).

Note that unlike the definitions of Ro_c and Ro_{loc} , where the Prandtl number appears in the denominator (Tab. 1.2), the definitions of R_K and R_G assume that the transition from the rapidly rotating to the non-rotating regime does not depend on the Prandtl number. Motivated by the fact that the buoyancy force in the momentum equation is scaled by Ra/Pr (see Eq. 1.4), we introduce the

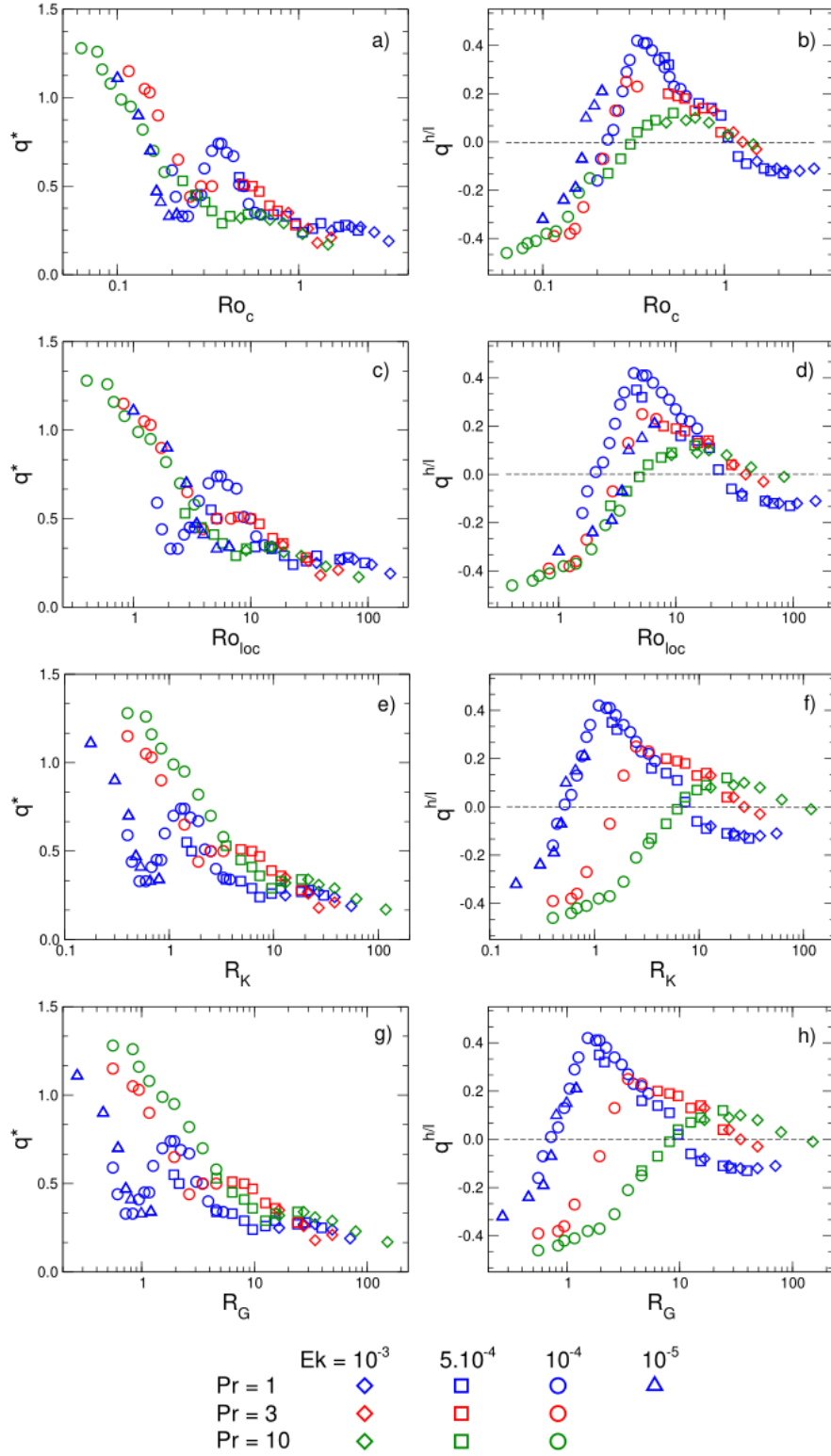


Figure 1.1 Heat flux characteristics q^* and $q^{h/l}$ plotted as a function of different diagnostic parameters. The dashed lines mark the transition from equatorial ($q^{h/l} < 0$) to polar ($q^{h/l} > 0$) cooling. For the definition of parameters Ro_c , Ro_{loc} , R_K and R_G , see Tab. 1.2.

modified dimensionless numbers R_K^* and R_G^* ,

$$R_K^* = R_K/Pr, \quad R_G^* = R_G/Pr, \quad (1.3)$$

with the aim to minimize the scatter of the results. As shown in Fig. 1.2, the classification of the heat flux pattern in terms of R_K^* and R_G^* is independent of the value of Pr , meaning that all models with the same R_K^* or R_G^* show either polar cooling or equatorial cooling, depending on the sign of $q^{h/l}$. The graphs obtained for R_K^* and R_G^* are almost the same, which is not surprising given the similar definitions of R_K and R_G (Tab. 1.1). In the following, we will use R_G^* which is based on simulations with $g \propto r^{-2}$ and $r_i/r_o = 0.60$ (Gastine et al., 2016), while R_K was numerically studied for an electrically conducting fluid, $g \propto r$ and $r_i/r_o = 0.35 - 0.4$ (King et al., 2010a), i.e. for the parameters corresponding to the Earth’s core rather than a subsurface ocean.

Inspection of Fig. 1.2b shows that the graph of $q^{h/l}$ intersects the horizontal axis at two points corresponding to a transition from equatorial to polar cooling (at $R_G^* \approx 1$) and from polar to equatorial cooling (at $R_G^* \approx 10$). The first transition represents a change from strong equatorial cooling at $R_G^* \lesssim 0.1$, where $q^{h/l} < -0.4$ and $q^* > 1$, to moderate to strong polar cooling at $R_G^* \gtrsim 1$, where $q^{h/l} = 0.1 - 0.4$ and $q^* = 0.3 - 0.6$. The transition is likely to be related to changes in the boundary layers, namely to the breakdown of the geostrophic turbulence followed by a regime of “unbalanced boundary layers” (Cheng et al., 2018). This interpretation is consistent with theoretical and experimental studies which show that this type of transition is described by the scaling law $Ra \propto Ek^{-1.80} - Ek^{-1.65}$ (Julien et al., 2012; Ecke and Niemela, 2014). The second transition (at $R_G^* \approx 10$) is less

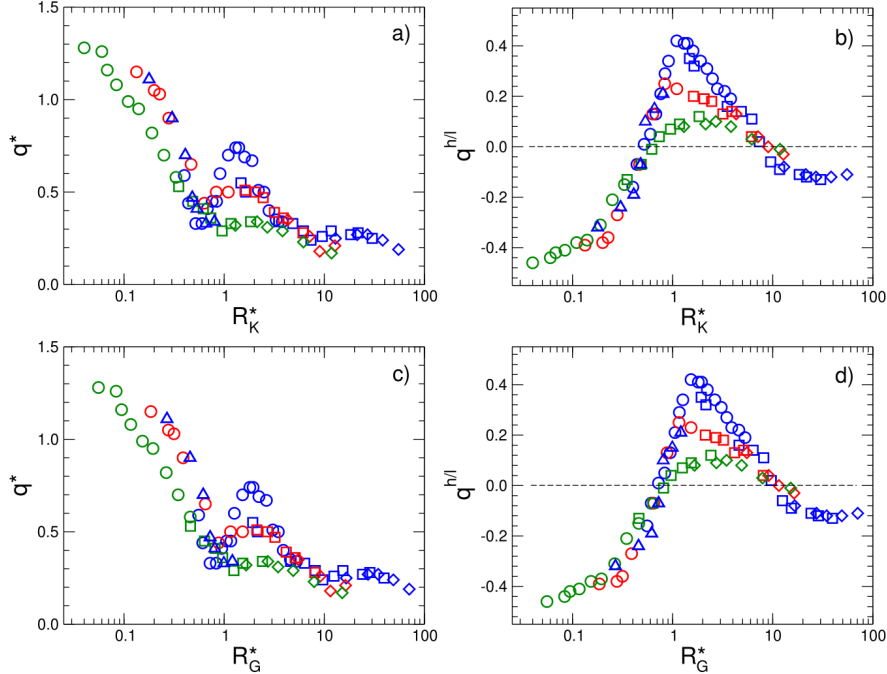


Figure 1.2 Heat flux characteristics q^* and $q^{h/l}$ plotted as a function of modified diagnostic parameters $R_K^* = R_K/Pr$ and $R_G^* = R_G/Pr$. The symbols are the same as in Fig. 1.1. The dashed lines in panels b and d mark the transition from equatorial ($q^{h/l} < 0$) to polar ($q^{h/l} > 0$) cooling.

pronounced than the first one and is not accompanied by a significant change in

q^* . As we will show later (Fig. 1.4), the heat flux pattern obtained for $R_G^* \approx 10$ has a strong degree 4 zonal component and the transition does not represent a qualitative change in heat flux structure but rather a competition between the polar and equatorial maxima. The changes in the heat flux pattern are likely to be associated with the decreasing influence of rotation on a global scale and are not necessarily related to the processes in the boundary layers.

The values of R_G^* for which $q^{h/l}=0$ are the same for different values of Ek and Pr , suggesting that the transitions between polar and equatorial cooling depend only on the diagnostic parameter R_G^* and do not depend on the specific choice of control parameters. Note that parameters Ro_c and Ro_{loc} predict the same transitions (Fig. 1.1b,d) but the position of the transition points depends on the Ekman number. For example, in the case of Ro_c (Fig. 1.1b), the transition from equatorial to polar cooling occurs at $Ro_c = 0.34$ for $Ek = 5 \cdot 10^{-4}$ (squares), at $Ro_c = 0.27$ for $Ek = 10^{-4}$ (circles) and at $Ro_c = 0.19$ for $Ek = 10^{-5}$ (triangles). We can see that the value of Ro_c at which the transition occurs decreases with decreasing Ek , suggesting that the transition value of Ro_c will be smaller than 0.1 for Ek relevant to Titan's ocean (Tabs. 1.1 and 1.2). An opposite trend is found for Ro_{loc} where the transition value increases as the Ekman number is decreased (compare the blue circles and triangles in Fig. 1.1d). This suggests that to characterize the transition from equatorial to polar cooling the diagnostic parameter must be proportional to $RaEk^\alpha$ where α is between 1.6 (corresponding to Ro_{loc}) and 2 (corresponding to Ro_c). These constraints are compatible with the theoretical ($\alpha = 1.67$) and experimental ($\alpha \in \langle 1.65, 1.80 \rangle$) results mentioned above and indicate that R_G^* ($\alpha = 1.71$) is a relevant parameter to assess the heat flux pattern in Titan's ocean.

While both Ro_c and Ro_{loc} fail to describe the first transition in a coherent way, the second transition can be equally well described by two conditions: $R_G^* = 10$ and $Ro_c = 1$ (see Fig. 1.1b). In Fig. 1.3, the transitions derived in this study are plotted in the parameter space with the x and y axis representing the influence of rotation (Ek) and the modified supercriticality of the Rayleigh number ($RaEk^{4/3}/Pr$), respectively (cf. Fig. 10 in Amit et al., 2020). We can see that the values of the parameters relevant to Titan (indicated in red) lie between the lines $R_G^* = 1$ and $R_G^* = 10$ suggesting that Titan's ocean is in the polar cooling mode. This conclusion is consistent with the analysis based on the convective Rossby number, although as discussed above, this parameter is not suitable for the classification of the heat flux pattern as the lower bound of the interval in which the ocean is in the polar cooling mode depends on the Ekman number. Three black dashed lines in the middle of Fig. 1.3 show the lower bounds of Ro_c obtained for Ek between 10^{-5} and $5 \cdot 10^{-4}$. The dependence of the transitional value of Ro_c on Ek can be estimated by considering that $Ro_c = R_G^{*1/2} Ek^{1/7}$ (see Tab. 1.2 and Eq. 1.3). Since the transition from equatorial to polar cooling occurs at $R_G^* \approx 1$ and this condition is independent of the choice of Ek , the transitional value of Ro_c is approximately equal to $Ek^{1/7}$, implying that for realistic values of the Ekman number ($Ek < 10^{-10}$), the transitional value of the Rossby number is smaller than 0.04.

Fig. 1.4 shows the zonally averaged heat flux at the upper boundary of the ocean plotted as a function of the latitude. The curves are computed for three different values of Pr and different combinations of parameters Ra and Ek ,

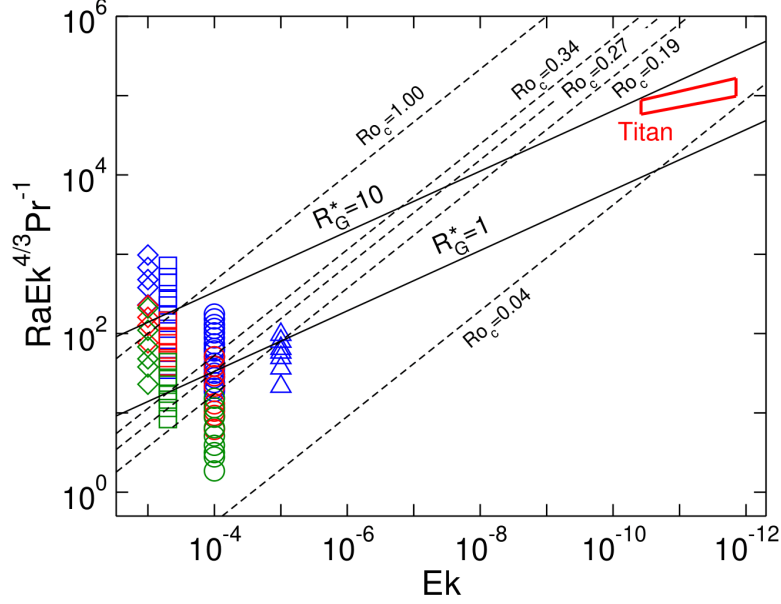


Figure 1.3 Boundaries between the areas of polar and equatorial cooling plotted as a function of parameters Ek (decreasing from left to right) and $RaEk^{4/3}/Pr$. The boundaries are shown for two diagnostic parameters: the modified transition number R_G^* (preferred solution) and the convective Rossby number Ro_c . The parameters relevant to Titan’s ocean (the red area in the graph on the right) lie between the lines $R_G^* = 1$ and $R_G^* = 10$ (full lines), suggesting that Titan’s ocean is in the polar cooling mode ($q^{h/l} > 0$). The dashed lines show the transitions obtained from the analysis of the convective Rossby number. While the upper bound of the interval in which $q^{h/l} > 0$ is uniquely defined by $Ro_c = 1$, the lower bound decreases as the Ekman number is decreased. The three dashed lines in the middle of the figure correspond to (from left to right) $Ek = 5 \cdot 10^{-4}$, 10^{-4} and 10^{-5} . For the ocean to be in the polar cooling mode, Ro_c must be smaller than 0.04 when $Ek \lesssim 10^{-10}$. The symbols on the left side of the graph represent the numerical simulation (see Fig. 1.1 for the legend).

corresponding to R_G^* between about 0.25 and 15. The heat transfer in the ocean is illustrated in Figs. 1.5 and 1.6 where we show the mean temperature and velocity fields for the models with $Pr = 1$ and 10, respectively. We will first discuss Figs. 1.4 and 1.5 and then compare Figs. 1.5 and 1.6.

The heat flux distribution obtained for $R_G^* < 0.5$ (Fig. 1.4a,b) is characterized by a strong degree 2 component and a pronounced equatorial cooling. The flow (Fig. 1.5a,b) is organized by the Coriolis force into narrow structures (Taylor columns) that are parallel to the rotation axis. The Reynolds stresses transfer the angular momentum of the cylindrical flow towards the equatorial region close to the outer boundary, giving rise to a prograde equatorial flow (Christensen, 2002) which tends to homogenize the temperature of the ocean near the equator (Guervilly and Cardin, 2017). The concentration of warm water outside the tangent cylinder (the cylinder of radius r_i whose axis is identical with the rotation axis) then leads to an efficient heat transfer at low latitudes. The cold region within the tangent cylinder is characterized by a stable thermal structure impeding the convective fluid motion (Olson et al., 1999).

As R_G^* increases, the magnitude of the heat flux in the equatorial region decreases, which is accompanied by an increase in heat transfer at high latitudes

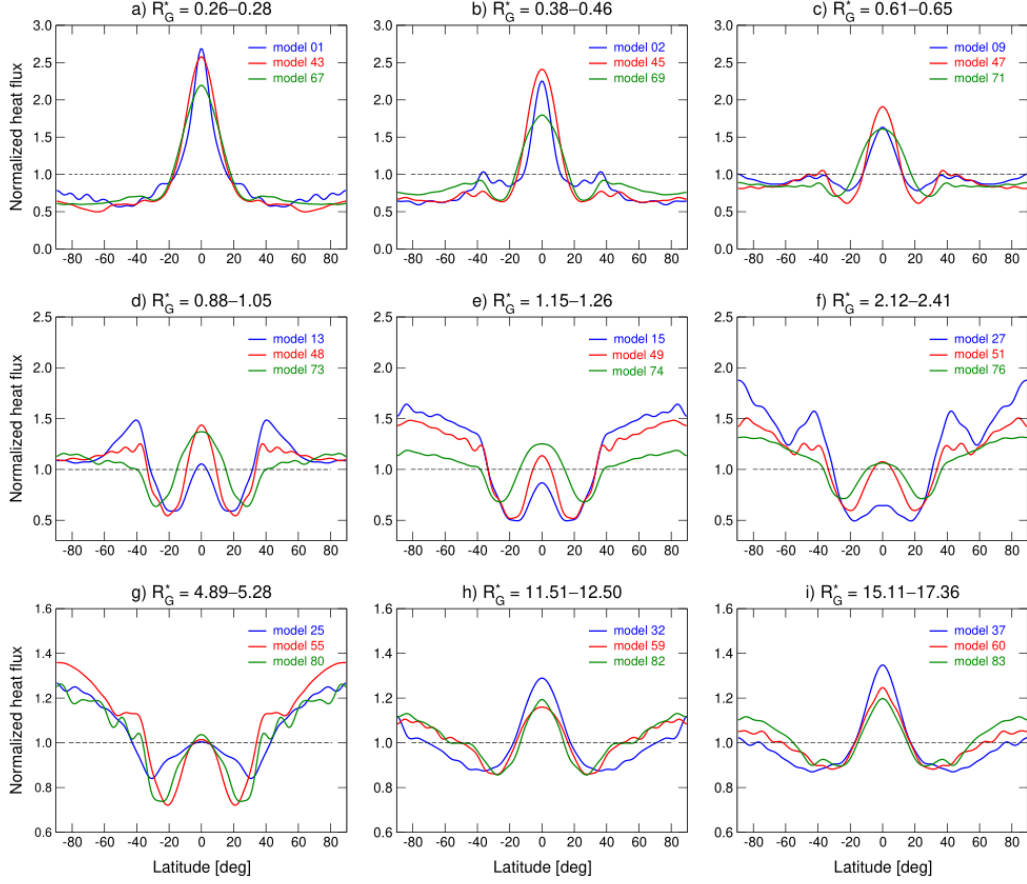


Figure 1.4 Time and zonally averaged heat flux at the upper boundary of the ocean plotted as a function of the latitude for different values of R_G^* . The blue, red and green curves correspond to $Pr = 1, 3$ and 10 , respectively. The heat flux is normalized such that the total heat flux is 1. The values of parameters Ra and Ek corresponding to the individual curves can be found in Tab. 1.5. Note that the plots have different vertical scales. Time averaging is performed over at least 0.02 viscous diffusion times.

(Fig. 1.4c–e). During the transition from the equatorial to polar cooling, the models show a complex heat flux pattern, with pronounced peaks at mid-latitudes (Fig. 1.4d), implying a strong degree 6 component. The change in the heat flux pattern is a consequence of the increasing influence of inertial and buoyancy forces (Fig. 1.5c–e). The cylindrical flow structures gradually break down and the convective heat transfer at high latitudes starts to play a dominant role. The heat transfer inside the tangent cylinder at mid-latitudes is enhanced by the convective heating of waters near the surface while the prograde zonal flow inhibits the convective transfer of heat in the equatorial region (Yadav et al., 2016).

The maximum polar cooling is obtained when $R_G^* = 2 - 6$ (Fig. 1.4f,g). The columnar flow induced by the Coriolis force has disappeared and the Reynolds stress is too weak to drive the prograde zonal flow (Fig. 1.5f,g). Homogenization of angular momentum (Aurnou et al., 2007) leads to the reversal of the equatorial zonal flow from prograde (Fig. 1.5f) to retrograde (Fig. 1.5g) and the increase in the magnitude of zonal flow velocity inside the tangent cylinder. The mean vertical velocity in the equatorial region is close to zero. However, this finding should be interpreted with caution since turbulent heat transfer due to non-axisymmetric, instantaneous radial velocities may still be significant (Soderlund et al., 2013), as

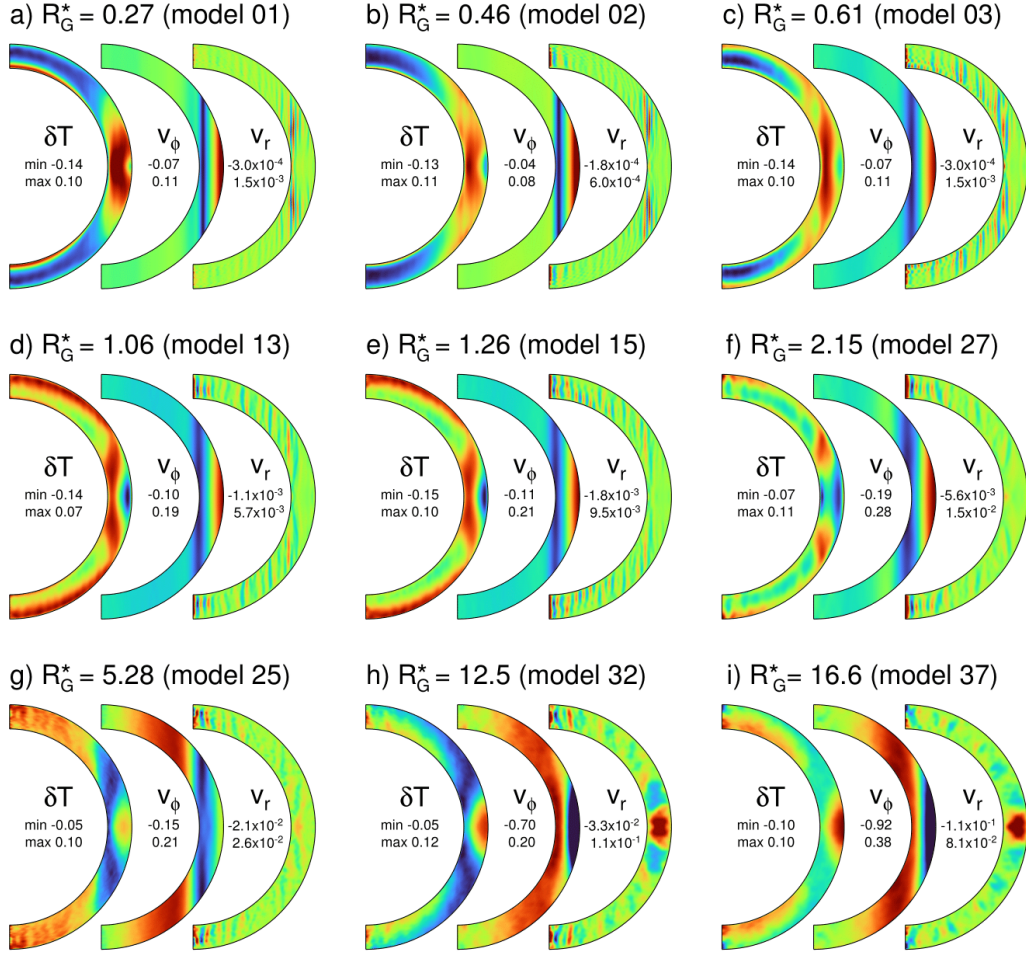


Figure 1.5 Temperature and velocity fields averaged over time and longitude. All models are computed for $Pr = 1$ and correspond to those shown in Fig. 1.4 (blue lines). Symbol δT denotes the dimensionless temperature variations taken relative to the average temperature profile while v_ϕ and v_r are the azimuthal and radial components of the flow velocity, respectively, given in Rossby number units, $U/(\Omega D)$, where U is the mean velocity in the ocean. The minimum and maximum values of δT , v_ϕ and v_r are given on the left-hand side of each cross section. Red (blue) denotes positive (negative) values.

indicated by the local increase in heat flux near the equator. The efficiency of convective heat transfer increases at high latitudes where the effect of rotation on the flow is smaller than in the equatorial region (Guervilly and Cardin, 2017), though not negligible (note that the mean radial velocities still exhibit a significant amount of axial alignment).

The transition at $R_G^* = 10$ is accompanied by an increase of the heat transfer efficiency near the equator and a return to the equatorial cooling (Fig. 1.4h,i). The heat flux is dominated by zonal degree 2 and 4 terms, with peaks at the poles and the equator and two minima at mid-latitudes. The amplitude of the heat flux variations decreases and the pattern becomes less sensitive to changes in R_G^* . The equatorial region is characterized by strong retrograde flow and Hadley-like meridional circulation (Soderlund, 2019), with warm water rising near the equator and cold water sinking at “subtropical” latitudes (Fig. 1.5h,i). At $R_G^* \approx 16$, the flows inside the tangent cylinder have no preferred spatial orientation, indicating

that the effect of rotation on the convection is negligible. When $R_G^* \gg 10$ the influence of rotation becomes negligible and the normalized heat flux is close to 1 (not shown here, but see Fig. 3a in Soderlund, 2019).

As discussed in connection with Figs. 1.1 and 1.2, the models with different Prandtl numbers give rather different values of q^* , especially for R_G^* between 1 and 10. Comparison of the curves in Fig. 1.4 shows that in most cases an increase in Pr causes a decrease in the amplitude of the heat flux peaks but only weakly affects the shape of the variations. The biggest difference is found for R_G^* between 1 and 3 where the models with $Pr = 10$ show a pronounced local maximum near the equator (Fig. 1.4d–f).

An increase in Pr generally leads to an enhancement of viscous forces and a reduction of turbulence (due to reduction of inertia forces). As a consequence, low Pr fluids produce more vigorous convection (higher Reynolds numbers), but lower heat transfer rates (lower Nusselt numbers) than moderate Pr fluids (for a review, see King and Aurnou, 2013). Inspection of Figs. 1.5 and 1.6 shows that the models with $Pr = 10$ follow the same trends as those with $Pr = 1$, but also reveals significant differences in the magnitude and spatial distribution of temperature and flow velocities. A full understanding of these differences would require a comprehensive analysis of the results which is beyond the scope of this study.

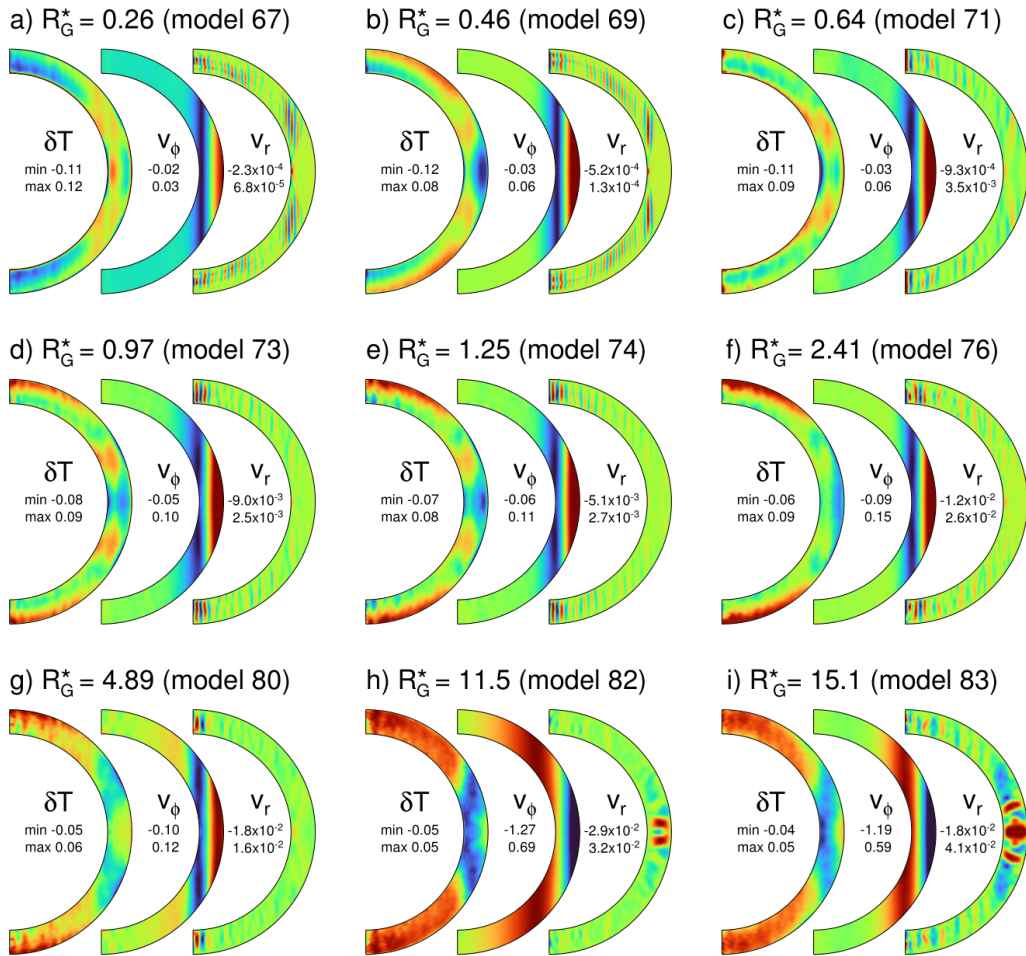


Figure 1.6 As in Fig. 1.5 but for $Pr = 10$.

1.4 Role of boundary conditions

As mentioned in the introduction, Soderlund (2019) and Amit et al. (2020) differ in their conclusions regarding Titan’s cooling pattern. While the former study suggests weak equatorial cooling, the results of the latter study are consistent with enhanced polar cooling. This discrepancy may be related to differences in boundary conditions: Unlike the study by Soderlund (2019), which investigates models with free-slip boundaries, the simulations by Amit et al. (2020) are performed with no-slip boundary conditions. It has been predicted that the thickness of the Ekman layer goes to zero for low Ekman numbers (Zhang and Jones, 1993), implying that the free-slip and no-slip models should give similar heat flux patterns for realistic values of Ek ($\sim 10^{-12} - 10^{-11}$). However, such values can hardly be reached by the present-day numerical simulations because of insufficient computing power. Although no-slip is the proper boundary condition for a viscous fluid at a rigid boundary, computationally feasible no-slip models may overestimate viscous effects (Kuang and Bloxham, 1997), resulting in a heat flux pattern that is different from that obtained for models with free-slip boundaries.

In Fig. 1.7, we compare the results obtained for models with no-slip (black) and free-slip (blue) boundary conditions. The models are computed for $Pr = 1$, i.e. the same as in the studies by Soderlund (2019) and Amit et al. (2020). The heat flux characteristics q^* and $q^{h/l}$ are plotted as a function of the parameter R_G^* but the same conclusions can be drawn for the convective Rossby number, Ro_c . Inspection of the figure shows that the two types of models give similar results for $R_G^* \lesssim 0.7$, but they predict significantly different heat flux characteristics for $R_G^* > 1$. While the no-slip models show the transition from equatorial to polar cooling with increasing R_G^* , the free-slip models show a more complex trend, suggesting the existence of a distinct cooling pattern in the transitional regime ($1 < R_G^* < 10$) manifested by an increase in heat flux at $R_G^* \approx 2$.

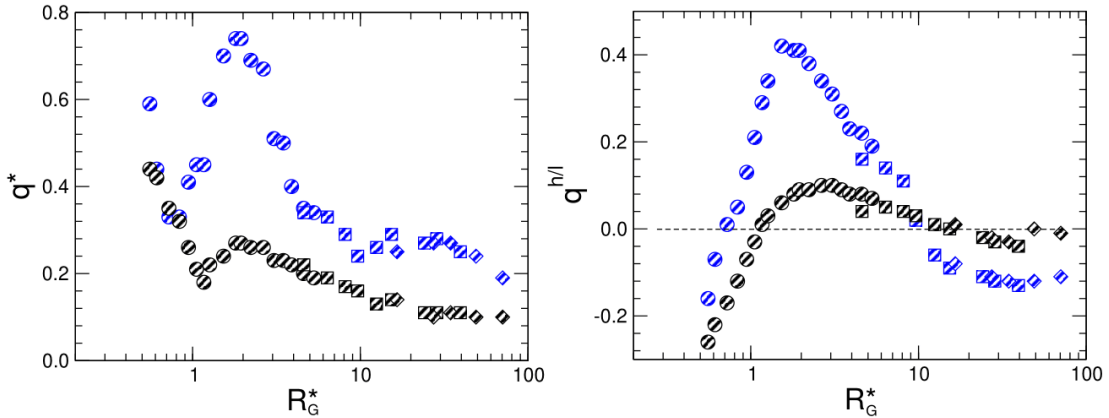


Figure 1.7 Heat flux characteristics q^* and $q^{h/l}$ as a function of the convective Rossby number computed for models with no-slip (black) and free-slip (blue) boundary conditions. The Prandtl number is set to 1 in all simulations. Circles, squares and diamonds correspond to $Ek = 10^{-4}$, $5 \cdot 10^{-4}$ and 10^{-3} , respectively.

Fig. 1.8 shows the zonally averaged heat flux at the upper boundary of the ocean plotted as a function of the latitude for six different values of R_G^* . The heat flux in models with small R_G^* (Fig. 1.8a) is characterized by a strong degree 2 component and a pronounced equatorial cooling. As R_G^* increases, the magnitude

of the heat flux in the equatorial region decreases, which is accompanied by an increase in convective heat transfer at high latitudes (Fig. 1.8b). During the transition from the equatorial to polar cooling, the no-slip models retain some resemblance to the fast-rotating models (note the broad heat flux maximum at the equator), in contrast to the free-slip models which show a more complex heat flux pattern, with two pronounced peaks at mid-latitudes. The polar cooling reaches a maximum at $R_G^* \approx 2$ for both the free-slip and no-slip models (Fig. 1.8c,d). However, while the heat flux curve obtained for the free-slip boundary condition shows a significant decrease near the equator and two pronounced maxima at the poles, and therefore corresponds to the spherical harmonics Y_{20} , the heat flux in the no-slip model is dominated by a degree 4 pattern, with a local maximum at the equator and two minima at mid-latitudes. As R_G^* further increases (Fig. 1.8e), the polar cooling becomes less efficient. In the case of the no-slip boundary condition, the shape of the heat flux curve remains the same as in the polar cooling mode shown in Fig. 1.8d, but the amplitude of the heat flux variations decreases. In contrast, the decrease of polar amplitudes in the free-slip models is accompanied by an increase of the heat transfer efficiency near the equator and a return to the equatorial cooling (Fig. 1.8f).

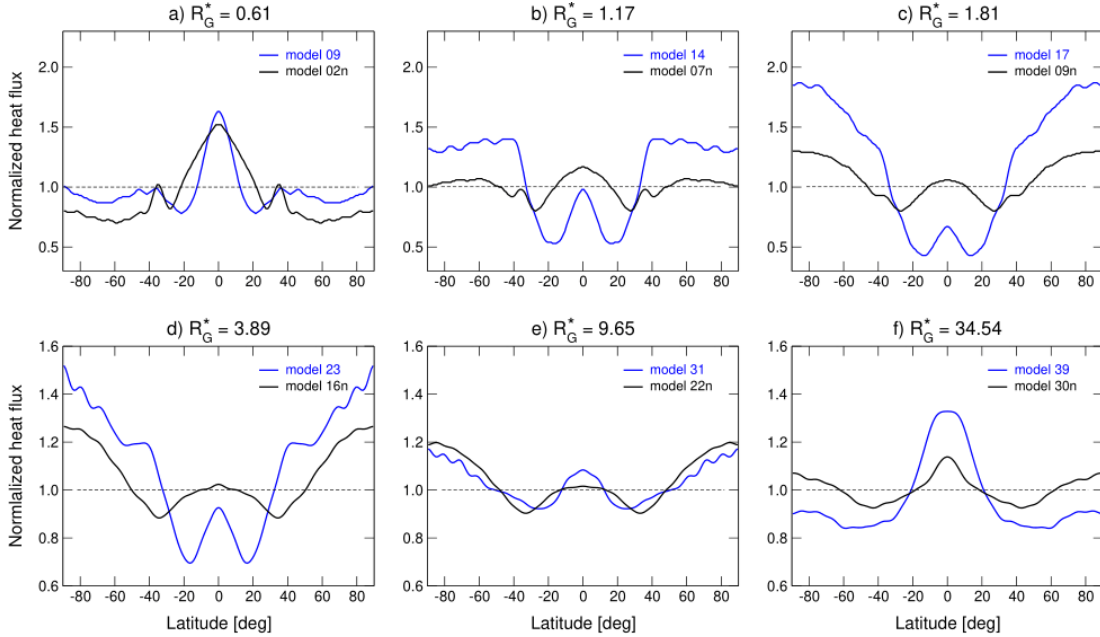


Figure 1.8 Time and zonally averaged heat flux at the upper boundary of the ocean plotted as a function of the latitude. The blue and black curves correspond to models with free-slip and no-slip boundary conditions, respectively. The models are computed for $Pr = 1$ and different values of the parameter R_G^* , Eq. (1.3). The values of parameters Ra and Ek corresponding to the individual curves can be found in Tabs. 1.4 and 1.5.

Our results suggest that the discrepancy between the conclusions of Soderlund (2019) and Amit et al. (2020) is related to the different boundary conditions. Based on the numerical simulations with free-slip boundary conditions corresponding to R_G^* between 2.5 and 490, Soderlund (2019) predicts the transition from equatorial to polar cooling with increasing influence of rotation (i.e. with decreasing R_G^* , see Fig. 3 therein). This is in good agreement with the results obtained in our study and shown in panels d–h of Fig. 1.8 (note that R_G^* in Fig. 3 in Soderlund,

2019 decreases from left to right). However, Soderlund (2019) fails to identify the transition from polar to equatorial cooling which occurs at $R_G^* \approx 1$, because this value is outside the range of tested parameters. The existence of this transition was demonstrated by Amit et al. (2020) who performed numerical simulations with no-slip boundary conditions for R_G^* ranging from 0.4 to 45 (see Fig. 7 therein). Our results confirm the general trend predicted for no-slip models by Amit et al. (2020), namely the transition from equatorial to polar cooling with increasing R_G^* , but give slightly different heat flux curves. This discrepancy may be related to the different lateral resolutions of the models or to the differences in gravitational acceleration ($g \propto r$ in Amit et al., 2020 while $g \propto r^{-2}$ in the present study).

1.5 Comparison with Titan’s topography

Fig. 1.9 shows two end-member models of how the heat flux from the ocean can influence the shape of the ice shell. The first model (Fig. 1.9a) assumes that the ice shell is rigid and the heat is transferred by conduction. A higher-than-average heat flux from the ocean causes the melting of ice and subsequent thinning of the ice shell. The decrease in pressure along the ice-water boundary is then compensated by negative surface topography, in agreement with the Airy compensation model (Kvorka et al., 2018; Čadek et al., 2019a).

The convection scenario (Fig. 1.9b) has not yet been investigated in detail, but its potential importance has been recognized recently in several studies of early magma oceans (e.g. Labrosse et al., 2018; Morison et al., 2019; Agrusta et al., 2019). The convecting ice shell is on average warmer and less viscous than in the conductive case, which causes that the molten ice in the region of the enhanced heat flux is continuously replenished by the low viscosity ice flowing from the interior of the ice shell. The shape of the bottom boundary remains unchanged and since the temperature of ice above the region of the enhanced heat flux is on average higher than elsewhere, the model is characterized by elevated topography, in agreement with the Pratt compensation model. The same heat flux anomaly can therefore produce either positive or negative topography, depending on which mode of heat transfer is dominant. The question of whether Titan’s ice shell is presently convecting or whether heat is transferred only by conduction is difficult to answer because the parameters controlling the onset of convection (grain size, thickness of the ice shell, temperature of the ocean, etc.) are not known with sufficient accuracy. In this section, we address this issue by comparing the results of our numerical simulations with Titan’s long-wavelength topography.

The map of Titan’s topography referenced to the geoid and expanded to spherical harmonic degree and order 5 is shown in Fig. 1.10a. The topography is characterized by depressions near the poles and several elevated areas at mid-latitudes. A significant portion (53%, measured by the L^2 -norm) of the topographic signal is produced by zonal degree 2 and degree 4 terms (Fig. 1.10b). The topography is highly compensated (Durante et al., 2019) and is not significantly affected by tidal heating in the ice shell (Čadek et al., 2021).

The results presented in section 1.3 suggest that the heat flux from Titan’s ocean is maximum at the poles and its pattern is dominated by the same harmonic components as the topography ($\ell = 2$ and 4 , $m = 0$, see Fig. 1.10c). Comparison of panels b and c of Fig. 1.10 reveals a high degree of similarity between some of

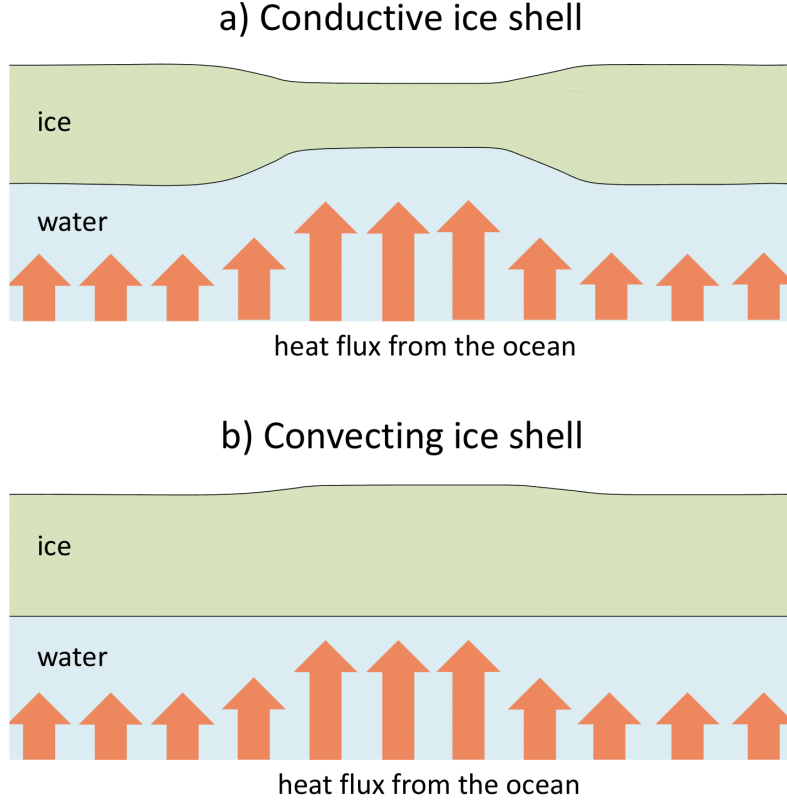


Figure 1.9 The response of the ice shell to variations in ocean heat flux is controlled by the viscosity of ice. a) If the ice shell is rigid and transfers the heat by conduction, lateral variations in heat flux cause thinning of the shell in regions of high heat flux and thickening of the shell in regions of low heat flux (Kvorka et al., 2018). b) If the viscosity of ice near the base of the ice shell is low and heat transfer occurs by convection, the shape of the bottom boundary remains essentially unchanged. The surface topography is smaller than in the conduction case and has the opposite sign (Kihoulou et al., 2021). We assume that in both cases the surface temperature is constant, in accordance with the conditions prevailing on Titan.

the heat flux models derived in this study and the equatorially symmetric part of Titan’s topography, thus supporting the hypothesis that the heat transfer in Titan’s ice shell occurs by conduction and the latitudinal variations in topography are caused by uneven heating at the ice-water boundary. The correlation between the topography and the heat flux is excellent for all models with $Pr = 10$ (plotted in green) and also for models 25 ($Pr = 1$) and 56 ($Pr = 3$). Note that models 25 and 79 are almost identical.

The magnitude of heat flux induced topography depends on the amplitude and wavelength of the heat flux variations and the viscosity of ice at the base of the ice shell, which in turn depends on the temperature of the ocean and the grain size of ice (see Fig. 5 in Kvorka et al., 2018). The total heat production of Titan is estimated to be between 250 and 545 GW (Kalousova and Sotin, 2020), corresponding to an average heat flux of $3.2 - 6.9 \text{ mW/m}^2$. The amplitude of the heat flux variations can be determined from Fig. 1.2. Taking into account that $R_G^* = 3 - 10$ on Titan (Tab. 1.3), we get $q^* = 0.25 - 0.5$, which multiplied by the average heat flux results in an amplitude of $0.8 - 3.4 \text{ mW/m}^2$. For simplicity, we will assume that the thickness of the ice shell is 100 km and the temperature

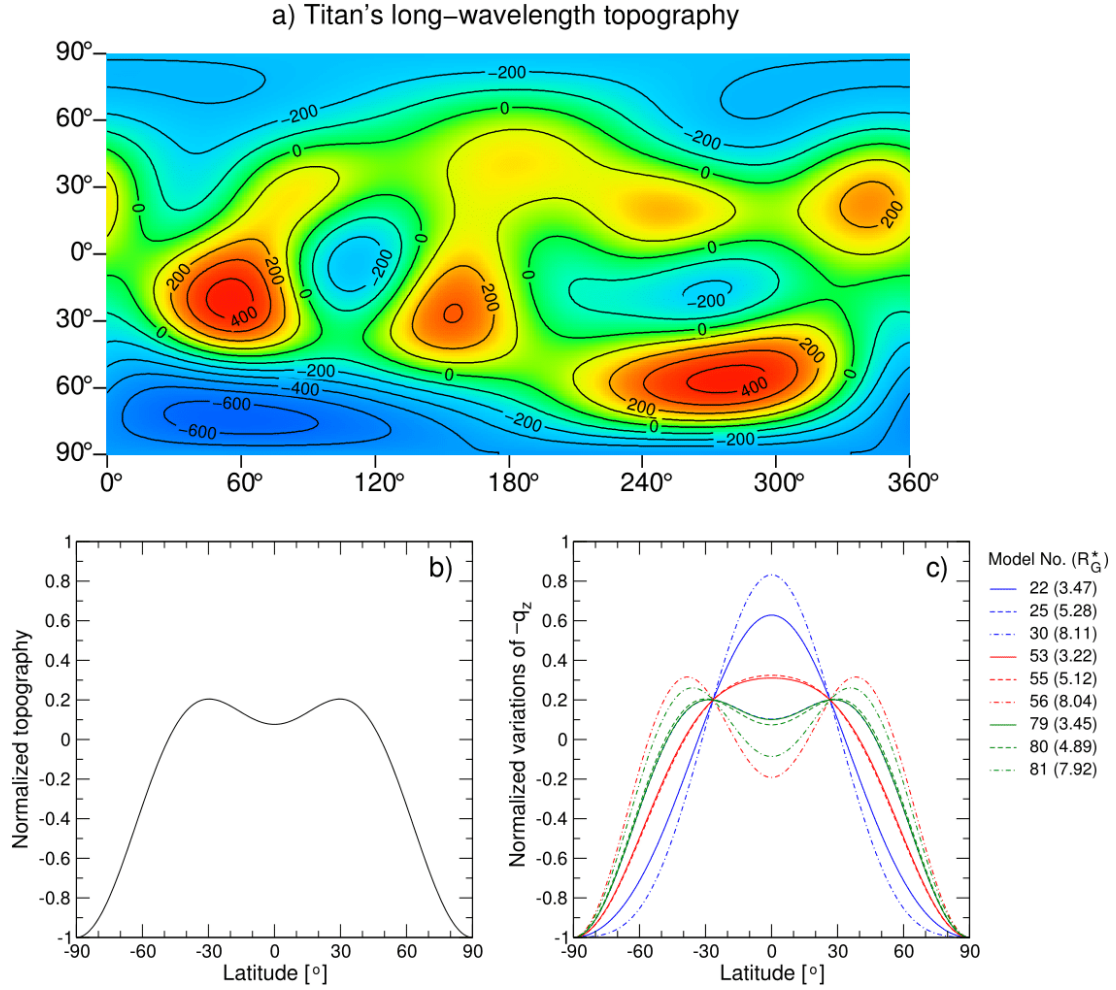


Figure 1.10 a) Topography of Titan (i.e., the height measured from the geoid) expanded to spherical harmonic degree and order 5. Topography is computed using the shape model by Corlies et al. (2017), gravity model by Durante et al. (2019) and the standard formula for the static tidal and centrifugal potentials. The contour interval is 100 m. b) Equatorially symmetric and zonally averaged topography of Titan as a function of latitude. c) Latitudinal variations in the heat flux computed for free-slip models with R_G^* ranging between 3 and 10 (cf. Tab. 1.3). The models with $Pr = 1, 3$ and 10 are plotted in blue, red and green, respectively. As in the case of panel b, we only plot the equatorially symmetric part of the field ($C_{20}Y_{20} + C_{40}Y_{40}$). To facilitate the comparison, topography and heat flux are normalized to 1 and q^z is plotted with the opposite sign.

of the ocean is 262 K (i.e., equal to the melting temperature of pure water ice). As shown by Kvorka et al. (2018), the topographic lows ($\lesssim -300$ m) observed in Titan's polar regions can be explained by a conductive ice shell model with 10 mm grain size and a heat flux amplitude of 1 mW/m², or 3 mm grain size and a heat flux amplitude of 3 mW/m². These values are well within the range of grain sizes expected on icy moons (Barr and McKinnon, 2007) and are likely compatible with the assumption of conduction dominated heat transfer (Běhouňková et al., 2013).

Although our model provides a plausible explanation of degree 2 and 4 zonal topography, it cannot account for the longitude dependent part of the topographic signal or the fact that the topography is more pronounced in the southern than

in the northern hemisphere. It is therefore likely that other effects, such as ethane precipitation (Choukroun and Sotin, 2012), climatically controlled erosion (e.g. Moore and Pappalardo, 2011) or convection in Titan’s high pressure ice layer (Choblet et al., 2017a), can also contribute to the development of the long-wavelength topography, or that some of the topographic features are of ancient origin.

Our results provide indirect evidence that the heat in Titan’s ice shell is transferred by conduction. However, it cannot be excluded that the ice shell is vigorously convecting and the heat flux from the ocean has little to no effect on the topography. If this is the case then it is likely that Titan’s surface is shaped by external processes, such as aeolian and fluvial erosion, methane and ethane precipitation, etc., and internal processes associated with lateral variations in ice properties, such as porosity or presence of methane clathrate (Čadek et al., 2021).

1.6 Conclusions

The evolution of Titan’s ice shell is primarily controlled by the heat transferred from the core to the surface. The heat produced in the deep interior is redistributed by ocean circulation, which is modulated by the Coriolis force, resulting in a latitudinal gradient in heat flux at the base of the ice shell. The recent studies by Soderlund (2019) and Amit et al. (2020) agree that the time-averaged heat flux can vary by tens of percent, but they differ in the predicted distribution of heat flux anomalies. In the present study, we show that this difference is related to the different boundary conditions used in the simulations. Although the models with free-slip and no-slip boundary conditions are expected to give similar results for $Ek \rightarrow 0$, they can predict different heat flux patterns for control parameters that are accessible with current computer power. Since the Ekman number used in numerical simulations is many orders of magnitude larger than in the real ocean, the no-slip models tend to overestimate viscous effects and, therefore, their results should be interpreted with caution.

One of the limitations of previous studies was that they did not investigate the dependence of the heat flux distribution on the Prandtl number. We show that even small changes in the Prandtl number can have a large impact on the magnitude of the heat flux anomaly. The effect of the Prandtl number is especially pronounced in the transitional regime where models with $Pr = 1$ show significantly stronger polar cooling than those with $Pr = 10$.

In order to determine the heat flux distribution at the upper boundary of Titan’s ocean, we constructed a set of about 80 ocean models, varying all relevant control parameters (Ra, Ek, Pr) by at least one order of magnitude. The heat flux characteristics of the models were interpreted in terms of the modified transitional number $R_G^* = RaEk^{12/7}Pr^{-1}$. Depending on the relative importance of rotation, the heat flux from the ocean is concentrated either near the equator (equatorial cooling) or at high latitudes (polar cooling). Our analysis suggests that equatorial cooling occurs when $R_G^* < 1$ or $R_G^* > 10$ while polar cooling occurs when $R_G^* \in (1, 10)$. Based on this result, we predict that the heat flux on Titan peaks near the poles and is dominated by zonal degree 2 and 4 terms. The predicted heat flux distribution is negatively correlated with the axisymmetric part of Titan’s long-wavelength topography, suggesting a coupling between ocean

dynamics and the evolution of the ice shell (Kvorka et al., 2018). It is likely that variations in the heat flux from the ocean also influence the evolution of other icy moons (e.g. Soderlund et al., 2014; Čadež et al., 2019b).

The approach developed in this work is applicable to icy moons where the ocean is underlain by a layer of high-pressure ice, which guarantees that the temperature at the bottom boundary of the ocean is constant. This is clearly not the case of Enceladus where the ocean circulation may be affected by uneven tidal heating in its porous core (Choblet et al., 2017b), or Europa where the temperature of the oceanic floor may vary due to magmatic processes in the silicate mantle (Běhouňková et al., 2021). Throughout the paper, we neglect the effect of salinity variations on the density of ocean water. As recently shown by Zeng and Jansen (2021), this effect may be important but it is difficult to implement due to the lack of knowledge about the composition of Titan’s ocean. In the case of polar cooling, the presence of freshwater lenses due to melting in the polar regions could lead to further increase in heat flux at high latitudes (Lobo et al., 2021).

Acknowledgements

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1.A Governing equation and numerical method

The models discussed in this paper have been obtained by simultaneously solving the Navier-Stokes equation including the Coriolis force, the transport equation for temperature and the mass conservation equation for an incompressible fluid. The variables in the equations are non-dimensionalized using the following scaling parameters: the thickness of the ocean, $D = r_o - r_i$, the superadiabatic temperature contrast, ΔT , and the viscous diffusion time, $\tau = D^2/\nu$, where ν is the kinematic viscosity. The governing equations then take the form (cf. Gastine et al., 2016):

$$-\nabla p' + \nabla^2 \mathbf{v}' - \frac{Ra}{Pr} T' f(r) \mathbf{e}_r - \frac{2}{Ek} \mathbf{e}_z \times \mathbf{v}' = \frac{\partial \mathbf{v}'}{\partial t'} + \mathbf{v}' \cdot \nabla \mathbf{v}' , \quad (1.4)$$

$$\frac{\partial T'}{\partial t'} = \frac{1}{Pr} \nabla^2 T' - \mathbf{v}' \cdot \nabla T' , \quad (1.5)$$

$$\nabla \cdot \mathbf{v}' = 0 . \quad (1.6)$$

Here, $t' = t/\tau$ is the dimensionless time, $\mathbf{v}' = \mathbf{v}(\tau/D)$ is the dimensionless velocity, $T' = (T - T_o)/\Delta T$ is the dimensionless temperature (with T_o the temperature at the outer boundary), $p' = p(\tau\rho/\nu)$ is the dimensionless pressure (with ρ the density), $f(r) = r_o^2/r^2$ is the function describing the radial variations of the gravitational acceleration, $\mathbf{e}_r$ is the radial unit vector, $\mathbf{e}_z$ is the unit vector parallel to the angular velocity vector, and Ra , Ek and Pr are the control parameters defined in Tab. 1.1. Note that Ek in Tab. 1.1 is a factor of two larger than that used in Soderlund (2019). The dimensionless temperature, T' , is fixed to 1 on

the inner boundary and to 0 on the outer boundary. Two types of mechanical boundary conditions are considered: no-slip,

$$\mathbf{v}' = \mathbf{0} , \quad (1.7)$$

and free-slip,

$$\mathbf{v}' \cdot \mathbf{e}_r = 0, \quad \boldsymbol{\sigma}' \cdot \mathbf{e}_r - (\mathbf{e}_r \cdot \boldsymbol{\sigma}' \cdot \mathbf{e}_r) \mathbf{e}_r = \mathbf{0} , \quad (1.8)$$

where $\boldsymbol{\sigma}'$ is the dimensionless stress.

Equations (1.4)–(1.6) are reformulated as a system of first order partial differential equations (i.e., in terms of variables $\mathbf{v}'$, $\boldsymbol{\sigma}'$, T' and $\nabla T'$) and solved using a pseudo-spectral method based on spherical harmonic expansions in angular coordinates and finite differences in radius. The expansions are truncated at degree $\ell_{max} = 120 - 215$ and the spherical harmonic coefficients are discretized in 65–97 unevenly spaced Chebyshev points in radius. At each time step, the nonlinear terms are evaluated on an auxiliary grid consisting of $3\ell_{max}/2 + 1$ Gauss-Legendre points in latitude and $3\ell_{max} + 1$ equally spaced points in longitude, and transformed back to the spectral domain using the Gauss-Legendre integration method and the fast Fourier transform (Martinec, 1989). The time derivatives are discretized using a hybrid approach in which the Crank-Nicolson method is applied to the diffusion terms and an explicit second order Adams-Bashforth method is applied to the nonlinear terms and the Coriolis force. The method described above has been implemented in the Fortran language and tested against the results of Gastine et al. (2016).

1.B Control parameters relevant to Titan

The evaluation of Pr and Ek is straightforward because the molecular values of ν and κ for water are known and the thickness of the ocean (D) can be estimated from geophysical observations (Tab. 1.3). The evaluation of the Rayleigh number,

$$Ra = \frac{\alpha g_o \Delta T D^3}{\kappa \nu} , \quad (1.9)$$

requires the knowledge of the adiabatic temperature contrast across the ocean (ΔT). An estimate of ΔT can be derived using the relationship between the Rayleigh number and the Nusselt number (Nu), whose value can be determined from the heat output of the icy satellite. Using the scaling laws for two end-members, corresponding to the rapidly rotating (RR) and non-rotating (NR) heat transfer regimes, Soderlund (2019) obtained

$$\Delta T_{NR} = 7.3 \left(\frac{\nu}{\alpha g_o \rho C_p} \right)^{1/4} \bar{q}^{3/4} , \quad (1.10)$$

$$\Delta T_{RR} = 2.1 \left(\frac{\Omega^4 \kappa}{\rho^2 C_p^2 \nu \alpha^3 g_o^3} \right)^{1/5} (\bar{q}^2 D)^{1/5} , \quad (1.11)$$

where $\bar{q}$ is the average heat flux. Eqs. (1.10) and (1.11) follow from the relationships $Nu = 0.07 Ra^{1/3}$ (Gastine et al., 2015) and $Nu = 0.15 Ra^{3/2} Ek^2$ (Gastine et al., 2016), respectively. The total heat production of Titan and the parameters of its ocean are not fully known (e.g. Vance et al., 2018). In this paper, we assume that $D \in \langle 100, 500 \rangle$ km, $\bar{q} \in \langle 3, 7 \rangle$ mW/m² (Kalousova and Sotin, 2020), $\rho = 1200$ kg m⁻³, $g_o = 1.4$ m s⁻², $\alpha = 3.2 \cdot 10^{-4}$ K⁻¹, $C_p = 2800$ J kg⁻¹ K⁻¹ and diffusivities ν and κ represent material parameters ($\nu = 1.8 \cdot 10^{-6}$ m²/s, $\kappa = 1.8 \cdot 10^{-7}$ m²/s). Substituting these values into Eqs. (1.9)–(1.10) then gives $Ra \in \langle 2.5 \cdot 10^{19}, 10^{23} \rangle$.

An alternative estimate of Ra can be obtained using the ‘diffusivity-free’ scaling proposed for free-slip models by Christensen (2002) and elaborated by Aubert (2005) and Christensen and Aubert (2006). Assuming that the diffusivities play a minor role in global heat transfer, Christensen and Aubert (2006) introduced a new type of the Nusselt number that characterizes the advected heat flow:

$$Nu^* = \frac{1}{4\pi r_o r_i} \frac{Q_{adv}}{\rho C_p \Omega D \Delta T} = (Nu - 1) Ek / Pr , \quad (1.12)$$

where r_o and r_i are the outer and inner radii of the ocean, Q_{adv} is the advected heat flow, i.e. the total heat flow Q corrected for the conductive heat flow Q_{cond} , and Nu is the Nusselt number defined as

$$Nu = \frac{Q}{Q_{cond}} = \frac{1}{4\pi r_o r_i} \frac{Q D}{\rho C_p \kappa \Delta T} . \quad (1.13)$$

Note that definition (1.13) is slightly different from the definition used by Soderlund (2019) and Amit et al. (2020), which does not include the correction for sphericity. Similarly, one can define a modified Rayleigh number that depends on Q_{adv} and does not depend explicitly on ΔT :

$$Ra_Q^* = \frac{1}{4\pi r_o r_i} \frac{\alpha g_o Q_{adv}}{\rho C_p \Omega^3 D^2} = Ra (Nu - 1) Ek^3 / Pr^2 . \quad (1.14)$$

Using an extensive set of simulations, Christensen and Aubert (2006) showed that the relationship between Nu^* and Ra_Q^* can be approximated by a simple power-law expression, valid for a broad range of parameters:

$$Nu^* = c Ra_Q^{*\gamma}, \quad (1.15)$$

where c and γ are parameters which can be determined using a regression analysis. Considering that $Ra_Q^*/Nu^* = Ek^2 Ra/Pr = Ro_c^2$, where Ro_c is the convective Rossby number (Tab. 1.2), Eq. (1.15) can be rewritten as

$$Ro_c^2 = b Ra_Q^{*\beta}, \quad (1.16)$$

where $b = 1/c$ and $\beta = 1 - \gamma$. Since $Ra = Ro_c^2 Pr / Ek^2 = b Ra_Q^{*\beta} Pr / Ek^2$, we can express the Rayleigh number as follows:

$$Ra = b \frac{\Omega^2 D^4}{\kappa \nu} \left(\frac{1}{4\pi r_o r_i} \frac{\alpha g_o Q}{\rho C_p \Omega^3 D^2} \right)^\beta, \quad (1.17)$$

where we assumed that $Q_{adv} \approx Q$.

The diffusivity-free scaling, Eq. (1.15), was derived for rapidly rotating dynamos with $r_i/r_o = 0.35$ (Christensen and Aubert, 2006). Therefore it is questionable whether this kind of scaling can be applied to Titan's ocean which is only weakly affected by rotation, $r_i/r_o \gtrsim 0.80$ and magnetic effects are negligible. In this paper, we determine the bounds of Ra from Eq. (1.17) but with parameters b and β derived from our own dataset. We first convert Ek , Ra , Pr and Nu in Tab. 1.5 to Ro_c and Ra_Q^* and plot the dependence of Ro_c^2 on Ra_Q^* (Fig. 1.11) to verify the validity of Eq. (1.16). Using the least squares method, we then obtain $b = 14.73 \pm 0.58$ and $\beta = 0.525 \pm 0.004$. These values correspond to $c = 0.068$ and $\gamma = 0.48$, which do not differ much from the values derived by Christensen and Aubert (2006).

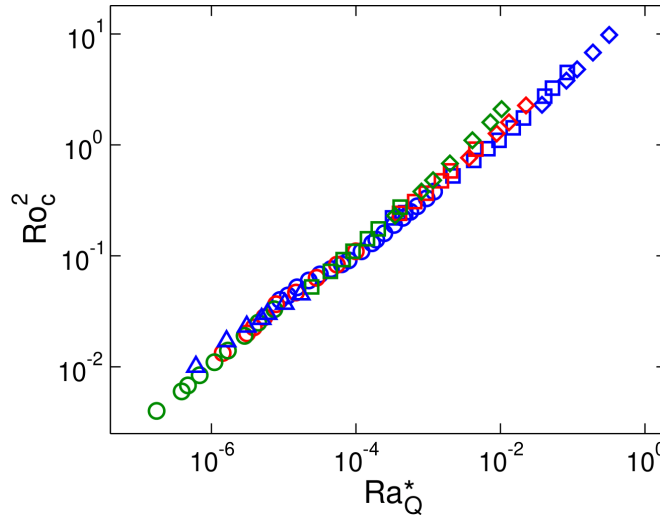


Figure 1.11 Square of the convective Rossby number as a function of the modified Rayleigh number. The data plotted in the figure can be found in Tab. 1.5.

The estimates of Ra obtained from Eq. (1.17) are presented in Tab. 1.3. The admissible values of Ra are found within the interval $\langle 4.5 \cdot 10^{19}, 10^{22} \rangle$, which lies

Parameter	Realistic range				
D [km]	100	200	300	400	500
Ra_Q^*	$(4.5 - 9.9) \cdot 10^{-7}$	$(1.2 - 2.6) \cdot 10^{-7}$	$(0.6 - 1.2) \cdot 10^{-7}$	$(3.2 - 7.1) \cdot 10^{-8}$	$(2.2 - 4.8) \cdot 10^{-8}$
Ek	$3.9 \cdot 10^{-11}$	$9.8 \cdot 10^{-12}$	$4.3 \cdot 10^{-12}$	$2.4 \cdot 10^{-12}$	$1.6 \cdot 10^{-12}$
Ra	$(4.5 - 6.8) \cdot 10^{19}$	$(3.5 - 5.4) \cdot 10^{20}$	$(1.2 - 1.8) \cdot 10^{21}$	$(2.9 - 4.4) \cdot 10^{21}$	$(5.7 - 8.6) \cdot 10^{21}$
Ro_{loc}	252 – 379	217 – 326	200 – 302	191 – 288	185 – 279
Ro_c	0.08 – 0.10	0.06 – 0.07	0.05 – 0.06	0.04 – 0.05	0.04 – 0.05
R_G^*	6.5 – 9.8	4.8 – 7.2	4.0 – 6.0	3.6 – 5.4	3.3 – 5.0
R_K^*	2.8 – 4.2	1.9 – 2.9	1.6 – 2.4	1.4 – 2.1	1.3 – 1.9

Table 1.3 Values of control and diagnostic parameters expected for different thicknesses of Titan’s ocean. The modified Rayleigh number, Ra_Q^* , and the Rayleigh number, Ra , are obtained from Eqs. (1.14) and (1.17), respectively, where $Q_{adv} = 250 - 545$ GW (Kalousová and Sotin, 2020), $r_o = 2500$ km and $r_i = r_o - D$. The remaining parameters are calculated using the definitions given in Tabs. 1.1 and 1.2 and Eq. (1.3).

well within the range of values derived from Eqs. (1.10) and (1.11). It has been argued that the diffusion-free scaling proposed by Christensen and Aubert (2006) is problematic because it is based on simulations where dissipation is far from negligible. According to Cheng and Aurnou (2016), the scaling is likely only valid for the systems where $Ra \sim Ra_{crit}$, i.e. for the heat transfer occurring very near to the onset of convection. If this was the case, the use of Eq. (1.17) in the context of the transitional regime would lead to a significant underestimation of the Rayleigh number. The fact that this approach gives similar values as those obtained independently from Eqs. (1.10) and (1.11) indicates that Eq. (1.17), where $b = 14.73$ and $\beta = 0.525$, can serve as a useful tool when evaluating the Rayleigh number in subsurface oceans of icy moons.

1.C List of simulations

We have performed 83 simulations with free-slip boundary conditions and 32 simulations with no-slip boundary conditions. The basic characteristics of the simulations are summarized in Tabs. 1.4 and 1.5. The symbol Re denotes the time-averaged convective Reynolds number (for the definition see Eq. 8 in Amit et al., 2020), ℓ_{max} is the cut-off degree and N_{rad} is the number of the Chebyshev points. All simulations presented in the paper are available on request.

Model No.	N_{rad}	ℓ_{max}	Pr	Ek	Ra	Nu	Re	q^*	$q^{h/l}$
1n	97	120	1	$1 \cdot 10^{-4}$	$4.0 \cdot 10^6$	5.97	208.72	0.44	-0.26
2n	97	120	1	$1 \cdot 10^{-4}$	$4.4 \cdot 10^6$	6.28	228.53	0.42	-0.22
3n	97	120	1	$1 \cdot 10^{-4}$	$5.2 \cdot 10^6$	7.14	272.14	0.35	-0.17
4n	97	120	1	$1 \cdot 10^{-4}$	$6.0 \cdot 10^6$	7.90	306.73	0.32	-0.12
5n	97	120	1	$1 \cdot 10^{-4}$	$6.8 \cdot 10^6$	8.75	334.81	0.26	-0.07
6n	97	120	1	$1 \cdot 10^{-4}$	$7.6 \cdot 10^6$	9.52	372.68	0.21	-0.03
7n	97	120	1	$1 \cdot 10^{-4}$	$8.4 \cdot 10^6$	10.26	407.91	0.18	0.01
8n	97	120	1	$1 \cdot 10^{-4}$	$9.1 \cdot 10^6$	10.87	432.71	0.22	0.03
9n	97	130	1	$1 \cdot 10^{-4}$	$1.1 \cdot 10^7$	12.17	493.91	0.24	0.06
10n	97	130	1	$1 \cdot 10^{-4}$	$1.3 \cdot 10^7$	13.31	557.66	0.27	0.08
11n	97	130	1	$1 \cdot 10^{-4}$	$1.4 \cdot 10^7$	13.83	588.17	0.27	0.09
12n	97	130	1	$1 \cdot 10^{-4}$	$1.6 \cdot 10^7$	14.65	650.86	0.26	0.09
13n	97	130	1	$1 \cdot 10^{-4}$	$1.9 \cdot 10^7$	15.74	714.57	0.26	0.10
14n	97	130	1	$1 \cdot 10^{-4}$	$2.2 \cdot 10^7$	16.63	795.72	0.23	0.10
15n	97	130	1	$1 \cdot 10^{-4}$	$2.5 \cdot 10^7$	17.42	866.33	0.23	0.09
16n	97	130	1	$1 \cdot 10^{-4}$	$2.8 \cdot 10^7$	18.15	919.09	0.22	0.08
17n	97	130	1	$1 \cdot 10^{-4}$	$3.3 \cdot 10^7$	19.18	1026.97	0.20	0.08
18n	97	140	1	$1 \cdot 10^{-4}$	$3.8 \cdot 10^7$	20.12	1075.46	0.19	0.07
19n	97	120	1	$5 \cdot 10^{-4}$	$2.1 \cdot 10^6$	9.30	263.15	0.22	0.04
20n	97	120	1	$5 \cdot 10^{-4}$	$2.9 \cdot 10^6$	10.65	317.78	0.19	0.05
21n	97	120	1	$5 \cdot 10^{-4}$	$3.7 \cdot 10^6$	11.70	373.34	0.17	0.04
22n	97	120	1	$5 \cdot 10^{-4}$	$4.4 \cdot 10^6$	12.52	421.91	0.16	0.03
23n	97	130	1	$5 \cdot 10^{-4}$	$5.7 \cdot 10^6$	13.78	486.19	0.13	0.01
24n	97	130	1	$5 \cdot 10^{-4}$	$7.0 \cdot 10^6$	14.92	547.64	0.14	0.00
25n	97	140	1	$5 \cdot 10^{-4}$	$1.1 \cdot 10^7$	17.64	703.41	0.11	-0.02
26n	97	140	1	$5 \cdot 10^{-4}$	$1.3 \cdot 10^7$	18.69	774.82	0.11	-0.03
27n	97	140	1	$5 \cdot 10^{-4}$	$1.8 \cdot 10^7$	20.59	908.05	0.11	-0.04
28n	97	120	1	$1 \cdot 10^{-3}$	$2.3 \cdot 10^6$	11.04	326.09	0.14	0.01
29n	97	120	1	$1 \cdot 10^{-3}$	$3.8 \cdot 10^6$	13.41	430.67	0.10	-0.02
30n	97	120	1	$1 \cdot 10^{-3}$	$4.8 \cdot 10^6$	14.43	485.84	0.11	-0.03
31n	97	120	1	$1 \cdot 10^{-3}$	$6.8 \cdot 10^6$	16.19	590.23	0.10	0.00
32n	97	120	1	$1 \cdot 10^{-3}$	$9.8 \cdot 10^6$	18.20	712.78	0.10	-0.01

Table 1.4 Results of no-slip simulations.

Model No.	N_{rad}	ℓ_{max}	Pr	Ek	Ra	Nu	Re	q^*	$q^{h/l}$
1	97	215	1	$1 \cdot 10^{-5}$	$1.0 \cdot 10^8$	7.10	1247.33	1.11	-0.32
2	97	215	1	$1 \cdot 10^{-5}$	$1.7 \cdot 10^8$	10.52	2153.53	0.90	-0.24
3	97	215	1	$1 \cdot 10^{-5}$	$2.3 \cdot 10^8$	14.39	2616.59	0.70	-0.19
4	97	215	1	$1 \cdot 10^{-5}$	$2.7 \cdot 10^8$	19.37	2871.43	0.47	-0.07
5	97	215	1	$1 \cdot 10^{-5}$	$3.0 \cdot 10^8$	21.01	3500.73	0.41	0.13
6	97	215	1	$1 \cdot 10^{-5}$	$3.7 \cdot 10^8$	28.97	4178.88	0.36	0.15
7	97	215	1	$1 \cdot 10^{-5}$	$4.5 \cdot 10^8$	39.24	4921.19	0.34	0.21
8	97	120	1	$1 \cdot 10^{-4}$	$4.0 \cdot 10^6$	3.22	360.27	0.59	-0.16
9	97	120	1	$1 \cdot 10^{-4}$	$4.4 \cdot 10^6$	3.60	396.74	0.44	-0.07
10	97	120	1	$1 \cdot 10^{-4}$	$5.2 \cdot 10^6$	3.95	485.07	0.33	0.01
11	97	120	1	$1 \cdot 10^{-4}$	$6.0 \cdot 10^6$	4.71	546.20	0.33	0.05
12	97	120	1	$1 \cdot 10^{-4}$	$6.8 \cdot 10^6$	5.67	606.49	0.41	0.13
13	97	120	1	$1 \cdot 10^{-4}$	$7.6 \cdot 10^6$	6.94	662.77	0.45	0.21
14	97	120	1	$1 \cdot 10^{-4}$	$8.4 \cdot 10^6$	8.58	713.20	0.45	0.29
15	97	120	1	$1 \cdot 10^{-4}$	$9.1 \cdot 10^6$	9.71	758.09	0.60	0.34
16	97	130	1	$1 \cdot 10^{-4}$	$1.1 \cdot 10^7$	11.85	886.62	0.70	0.42
17	97	130	1	$1 \cdot 10^{-4}$	$1.3 \cdot 10^7$	13.89	979.09	0.74	0.41
18	97	130	1	$1 \cdot 10^{-4}$	$1.4 \cdot 10^7$	14.69	1026.22	0.74	0.41
19	97	130	1	$1 \cdot 10^{-4}$	$1.6 \cdot 10^7$	16.46	1105.56	0.69	0.38
20	97	130	1	$1 \cdot 10^{-4}$	$1.9 \cdot 10^7$	18.78	1207.74	0.67	0.34
21	97	130	1	$1 \cdot 10^{-4}$	$2.2 \cdot 10^7$	21.25	1275.30	0.51	0.31
22	97	130	1	$1 \cdot 10^{-4}$	$2.5 \cdot 10^7$	23.60	1346.26	0.50	0.27
23	97	130	1	$1 \cdot 10^{-4}$	$2.8 \cdot 10^7$	26.22	1382.94	0.40	0.23
24	97	130	1	$1 \cdot 10^{-4}$	$3.3 \cdot 10^7$	29.82	1460.08	0.35	0.22
25	97	140	1	$1 \cdot 10^{-4}$	$3.8 \cdot 10^7$	32.77	1703.00	0.34	0.19
26	97	120	1	$5 \cdot 10^{-4}$	$8.8 \cdot 10^5$	3.36	244.43	0.70	0.39
27	97	120	1	$5 \cdot 10^{-4}$	$9.8 \cdot 10^5$	3.99	256.70	0.65	0.35
28	97	120	1	$5 \cdot 10^{-4}$	$2.1 \cdot 10^6$	9.38	398.56	0.34	0.16
29	97	120	1	$5 \cdot 10^{-4}$	$2.9 \cdot 10^6$	12.77	473.55	0.33	0.14
30	97	120	1	$5 \cdot 10^{-4}$	$3.7 \cdot 10^6$	15.50	534.36	0.29	0.11
31	97	120	1	$5 \cdot 10^{-4}$	$4.4 \cdot 10^6$	18.36	610.20	0.24	0.02
32	97	130	1	$5 \cdot 10^{-4}$	$5.7 \cdot 10^6$	22.10	685.10	0.26	-0.06
33	97	130	1	$5 \cdot 10^{-4}$	$7.0 \cdot 10^6$	24.79	838.93	0.29	-0.09
34	97	140	1	$5 \cdot 10^{-4}$	$1.1 \cdot 10^7$	30.77	1217.00	0.27	-0.11
35	97	140	1	$5 \cdot 10^{-4}$	$1.3 \cdot 10^7$	33.33	1477.21	0.28	-0.12
36	97	140	1	$5 \cdot 10^{-4}$	$1.8 \cdot 10^7$	38.42	1550.83	0.25	-0.13
37	97	120	1	$1 \cdot 10^{-3}$	$2.3 \cdot 10^6$	17.36	471.86	0.25	-0.08
38	97	120	1	$1 \cdot 10^{-3}$	$3.8 \cdot 10^6$	22.52	666.71	0.27	-0.11
39	97	120	1	$1 \cdot 10^{-3}$	$4.8 \cdot 10^6$	25.00	743.68	0.27	-0.12
40	97	120	1	$1 \cdot 10^{-3}$	$6.8 \cdot 10^6$	29.00	879.40	0.24	-0.12
41	97	120	1	$1 \cdot 10^{-3}$	$9.8 \cdot 10^6$	33.65	1063.69	0.19	-0.11
42	69	159	3	$1 \cdot 10^{-4}$	$4.0 \cdot 10^6$	4.22	117.26	1.15	-0.39
43	69	159	3	$1 \cdot 10^{-4}$	$6.0 \cdot 10^6$	5.68	170.70	1.05	-0.38
44	69	159	3	$1 \cdot 10^{-4}$	$6.8 \cdot 10^6$	6.10	202.00	1.03	-0.36
45	69	159	3	$1 \cdot 10^{-4}$	$8.4 \cdot 10^6$	6.79	263.71	0.90	-0.27

Table 1.5 Results of free-slip simulations.

Model No.	N_{rad}	ℓ_{max}	Pr	Ek	Ra	Nu	Re	q^*	$q^{h/l}$
46	69	159	3	$1 \cdot 10^{-4}$	$1.1 \cdot 10^7$	7.44	369.99	0.67	-0.08
47	76	159	3	$1 \cdot 10^{-4}$	$1.4 \cdot 10^7$	10.53	383.92	0.65	-0.07
48	76	159	3	$1 \cdot 10^{-4}$	$1.9 \cdot 10^7$	14.55	504.78	0.44	0.13
49	69	159	3	$1 \cdot 10^{-4}$	$2.5 \cdot 10^7$	20.64	609.23	0.50	0.25
50	69	159	3	$1 \cdot 10^{-4}$	$3.3 \cdot 10^7$	28.17	643.68	0.50	0.23
51	69	159	3	$5 \cdot 10^{-4}$	$2.9 \cdot 10^6$	10.96	215.54	0.51	0.20
52	69	159	3	$5 \cdot 10^{-4}$	$3.7 \cdot 10^6$	13.65	244.65	0.50	0.19
53	69	159	3	$5 \cdot 10^{-4}$	$4.4 \cdot 10^6$	16.37	264.39	0.47	0.18
54	69	159	3	$5 \cdot 10^{-4}$	$5.7 \cdot 10^6$	20.24	315.41	0.39	0.13
55	69	159	3	$5 \cdot 10^{-4}$	$7.0 \cdot 10^6$	22.00	330.67	0.36	0.14
56	69	159	3	$5 \cdot 10^{-4}$	$1.1 \cdot 10^7$	30.90	403.92	0.28	0.04
57	64	159	3	$1 \cdot 10^{-3}$	$2.3 \cdot 10^6$	15.41	194.48	0.35	0.13
58	64	147	3	$1 \cdot 10^{-3}$	$3.8 \cdot 10^6$	21.98	243.03	0.26	0.04
59	64	147	3	$1 \cdot 10^{-3}$	$4.8 \cdot 10^6$	25.63	286.39	0.18	0.00
60	64	147	3	$1 \cdot 10^{-3}$	$6.8 \cdot 10^6$	30.93	354.74	0.21	-0.03
61	69	159	10	$1 \cdot 10^{-4}$	$4.0 \cdot 10^6$	5.36	26.56	1.28	-0.46
62	69	159	10	$1 \cdot 10^{-4}$	$6.0 \cdot 10^6$	7.46	35.71	1.26	-0.44
63	69	159	10	$1 \cdot 10^{-4}$	$6.8 \cdot 10^6$	7.96	42.41	1.16	-0.42
64	69	159	10	$1 \cdot 10^{-4}$	$8.4 \cdot 10^6$	9.19	51.25	1.08	-0.41
65	69	159	10	$1 \cdot 10^{-4}$	$1.1 \cdot 10^7$	11.03	70.21	0.99	-0.38
66	69	159	10	$1 \cdot 10^{-4}$	$1.4 \cdot 10^7$	13.09	83.45	0.95	-0.37
67	69	159	10	$1 \cdot 10^{-4}$	$1.9 \cdot 10^7$	16.14	111.02	0.82	-0.31
68	81	159	10	$1 \cdot 10^{-4}$	$2.5 \cdot 10^7$	18.84	175.25	0.70	-0.21
69	69	159	10	$1 \cdot 10^{-4}$	$3.3 \cdot 10^7$	23.47	224.16	0.58	-0.15
70	69	159	10	$5 \cdot 10^{-4}$	$2.1 \cdot 10^6$	10.29	50.27	0.53	-0.13
71	69	159	10	$5 \cdot 10^{-4}$	$2.9 \cdot 10^6$	13.24	57.39	0.45	-0.07
72	69	159	10	$5 \cdot 10^{-4}$	$3.7 \cdot 10^6$	15.39	80.57	0.41	-0.01
73	69	159	10	$5 \cdot 10^{-4}$	$4.4 \cdot 10^6$	17.56	93.56	0.36	0.04
74	65	159	10	$5 \cdot 10^{-4}$	$5.7 \cdot 10^6$	21.24	103.41	0.29	0.07
75	69	159	10	$5 \cdot 10^{-4}$	$7.0 \cdot 10^6$	24.24	116.48	0.33	0.09
76	69	159	10	$5 \cdot 10^{-4}$	$1.1 \cdot 10^7$	30.77	153.27	0.34	0.12
77	65	147	10	$1 \cdot 10^{-3}$	$2.3 \cdot 10^6$	16.13	72.62	0.32	0.08
78	65	159	10	$1 \cdot 10^{-3}$	$3.8 \cdot 10^6$	22.20	90.79	0.34	0.09
79	65	159	10	$1 \cdot 10^{-3}$	$4.8 \cdot 10^6$	25.31	98.39	0.31	0.10
80	65	159	10	$1 \cdot 10^{-3}$	$6.8 \cdot 10^6$	30.39	119.13	0.29	0.08
81	73	186	10	$1 \cdot 10^{-3}$	$1.1 \cdot 10^7$	38.21	339.68	0.23	0.03
82	73	186	10	$1 \cdot 10^{-3}$	$1.6 \cdot 10^7$	46.43	350.49	0.17	0.01
83	73	186	10	$1 \cdot 10^{-3}$	$2.1 \cdot 10^7$	50.43	374.25	0.17	-0.01

Table 1.6 Results of free-slip simulations (continuation).

2 The role of subsurface ocean dynamics and phase transitions in forming the topography of icy moons

This chapter was published as Kvorka and Čadež (2024) in Icarus.

Abstract

The thermo-mechanical evolution of icy moons cannot be understood without taking into account the complex interaction between the solid ice crust and the liquid water ocean. This interaction is controlled by the heat flux from the ocean and the material properties of ice. In recent years, the mass and energy exchange between the crust and the ocean has been studied using two complementary approaches. While in the first approach, the heat flux from the ocean is governed by the internal dynamics of the ocean and is independent of the processes at the phase boundary, the other approach assumes that the heat flux from the ocean is controlled by variations in the melting temperature along the boundary and the role of the ocean in heat transfer is passive. Here we present a new method for modeling the heat transfer in the interior of icy moons. Our approach is based on solving the heat transfer equations simultaneously in the ice shell and the ocean, and is more general than the previous ones in that it consistently links the global ocean circulation with the melting temperature variations arising from the deformation of the ice-water phase boundary. The method is used to study the role of ocean circulation in the formation of surface and ice-water interface topographies. We show that the additional heat flux generated by variations in the melting temperature along the phase boundary counteracts the heat flux from the deep ocean but is not able to completely suppress it. The deep ocean heat flux is more reduced at high latitudes than near the equator, where the ocean circulation is dominated by strong zonal flows hampering the heat transfer in the meridional direction. Our simulations predict the shape and topography that are comparable in magnitude to those observed on Titan (~ 500 m) and explain the absence of a degree-2 sectoral component in the spherical harmonic expansion of Titan's topography.

2.1 Introduction

The outer shell of icy moons with subsurface oceans, such as Europa, Titan and Enceladus, differs from the lithosphere on Earth and other terrestrial planets, among other things, in that its lower boundary is a phase-change interface. The ice shell therefore represents an open thermodynamic system (e.g., Callen, 1985) that can exchange both energy and matter with the underlying ocean, in contrast to the silicate shell of a terrestrial planet which is usually modeled as a closed

system because the mass exchange with the iron core can be neglected in the first approximation. Since the boundary conditions are different for open and closed systems, concepts and methods developed for the study of the Earth (Airy isostasy, conductive and convective cooling, ocean circulation models, etc.) may not be directly applicable to icy moons.

While the role of melting and freezing processes in the evolution of icy moons has been recognized for many years (e.g., Pappalardo et al., 1999), numerical models including the thermal coupling between the ocean and the ice shell are still rare. Due to the difficulties associated with the modeling of two-phase systems, there is a tendency for the ice-ocean interaction to be analyzed under grossly simplified assumptions. For example, most studies of heat transfer in the ice shell do not include the effect of mass transport across the ice-water interface (i.e., the ice shell is treated as a closed system) and assume that the heat flowing into the ice shell from the ocean is homogeneously distributed (e.g., Ojakangas and Stevenson, 1989; Tobie et al., 2003; Showman and Han, 2004; Barr and Pappalardo, 2005; Mitri and Showman, 2005; Freeman et al., 2006; Nimmo and Bills, 2010; Běhounková et al., 2013; Peddinti and McNamara, 2019; Kalousová and Sotin, 2020; Green et al., 2021). Neither of these assumptions is justified. The crucial role of the open phase boundary in heat and mass transfer in planetary bodies has been demonstrated by, e.g., Labrosse et al. (2018), Kang et al. (2022b), Kihoulou et al. (2023), and will be discussed later. The existence of heat flux variations from the ocean, possibly affecting the processes in the ice shell, was predicted by regional models of hydrothermal circulation (Goodman et al., 2004; Goodman and Lenferink, 2012) and later demonstrated by global simulations of ocean dynamics (Soderlund et al., 2014; Soderlund, 2019; Amit et al., 2020; Kverka and Čadek, 2022; Terra-Nova et al., 2023). The latter models suggest that depending on the relative importance of rotation, the heat flux from the ocean may peak either at the equator (“equatorial cooling”) or at the poles (“polar cooling”). The existence of strong large-scale variations in the heat flux is also supported by the analysis of ice shell thickness variations on Enceladus (Čadek et al., 2019b) and provides a possible explanation for polar depressions on Titan (Kverka et al., 2018) and chaos features on Europa (Soderlund et al., 2014).

Recent studies have pointed to the important role of salinity variations in ocean dynamics and indicated a possible impact of melting ice on the global flow pattern (e.g., Ashkenazy and Tziperman, 2021; Lobo et al., 2021; Zeng and Jansen, 2021; Kang et al., 2022b; Bire et al., 2022). While the effect of salinity on ocean circulation may be of minor importance for large icy moons, such as Ganymede or Titan, it is likely to play a crucial role in the heat transfer on small icy moons, such as Enceladus, where the salinity of the ocean is expected to be low and heat transfer occurs at low temperature and pressure (Kang and Jansen, 2022). Several studies using the MIT Global Circulation Model (MITgcm) have addressed the problem of large ice thickness variations on Enceladus and demonstrated the importance of ice-water phase transition and salinity variations in shaping the topography of the phase boundary (Kang et al., 2022a; Kang and Jansen, 2022). The MITgcm approach takes into account the topography-induced variations of the melting temperature along the phase boundary and uses more realistic parameters of the ocean than alternative approaches to modeling ocean dynamics (e.g., Soderlund, 2019; Amit et al., 2020; Kverka and Čadek, 2022). On

the other hand, it neglects the flow of ice in the ice shell, which has been shown to have a significant impact on the evolution of ice thickness variations and which occurs even if the heat transfer in ice is dominated by conduction (Kvorka et al., 2018; Čadek et al., 2019a,b; Kihoulou et al., 2023).

Kvorka et al. (2018) attempted to answer the question of whether the global pattern of ocean circulation on large icy moons can be inferred from the observed topography. Using the boundary conditions appropriate for a phase boundary and assuming that the heat flux from the ocean is not affected by the processes in the ice shell, they demonstrated that a higher-than-average heat flux from the ocean triggers the melting of ice and induces a downward flow in the ice shell, leading to a reduction of ice thickness and the development of surface depression. On the other hand, the regions where the heat flux from the ocean is lower than average are characterized by the freezing of water, an upward ice flow and the development of topographic highs. Based on these results, Kvorka et al. (2018) concluded that the large-scale depressions in the polar regions of Titan are compatible with a polar cooling model of ocean circulation.

Independent of this research, a parallel line of investigation developed to focus on modeling the phase transitions in the context of the Earth’s core and early magma oceans (Alboussière et al., 2010; Deguen et al., 2013; Deguen, 2013; Labrosse et al., 2018; Morison et al., 2019; Agrusta et al., 2019). The novelty of these studies lies in the recognition that any change of the shape of the phase boundary inevitably leads to a redistribution of the heat flux in the liquid phase. Unlike Kvorka et al. (2018) who assume that the heat flux in the liquid phase (hereinafter the “ocean”) is independent of the deformation processes in the solid phase, Deguen et al. (2013) and Labrosse et al. (2018) argue that the heat flow from the ocean is controlled by variations in the melting temperature at the phase boundary that naturally occur when the solid layer is deformed and the pressure along the phase boundary is therefore not constant. Although the above mentioned studies differ in the mechanism generating the phase boundary topography (heat flux variations from the deep ocean in Kvorka et al. (2018) and convection in the solid layer in Labrosse et al. (2018)), the heat flux arising from melting temperature variations along the phase boundary is present in both approaches and can be significant. This concept has been overlooked by planetary scientists and its importance for understanding the dynamics of icy moons has only recently been recognized (Kihoulou et al., 2023; Lebec et al., 2023).

The concept was introduced in the studies by Alboussière et al. (2010), Deguen et al. (2013) and Deguen (2013) who postulated that the heat flux from the ocean, q , is proportional to the velocity, v , of convective motion in the ocean and the topography (the distance from an equipotential surface), t_b , of the phase boundary:

$$q \approx C_p \frac{dT_m}{dp} \rho_w^2 g v t_b \equiv \gamma t_b , \quad (2.1)$$

where C_p is the specific heat capacity, T_m is the melting temperature, p is the pressure, ρ_w is the density of the liquid phase and g is the gravity acceleration (for simplicity, we assume that dT_m/dp is much larger than the adiabatic temperature gradient). Unlike the MITgcm studies (e.g., Kang et al., 2022b,c), which include topography induced variations in the melting temperature but assume that the

ice shell is rigid, Alboussière et al. (2010), Deguen (2013) and Labrosse et al. (2018) take into account the flow of the solid phase and demonstrate that the mass transport through and the shape of the phase boundary depend on the viscosity of the solid layer. In their approach, the evolution of the phase boundary is controlled by two time scales: (i) the time scale of the heat transfer in the ocean induced by variations in the melting temperature along the phase boundary, $\tau_\phi = \rho_i L / \gamma$, where ρ_i is the density of the solid phase and L is the latent heat (cf. Eq. 2.1), and (ii) the time scale, τ_η , of the formation of topography t_b . If the topography of the phase boundary is generated by thermal convection in the solid layer, $\tau_\eta = \eta_i (\Delta \rho g D_i)^{-1}$, where η_i and D_i are the viscosity and the thickness of the solid layer, respectively, and $\Delta \rho$ is the density jump at the phase boundary.

The time scales τ_ϕ and τ_η represent competing mechanisms that determine the size of topography. While the inverse of τ_η characterizes the rate at which the phase-boundary topography is formed, the inverse of τ_ϕ represents the rate at which the topography is “eroded” by the heat flux driven by the temperature variations along the boundary (i.e., reduced by melting and freezing). The behavior of the phase boundary can then be characterized by the dimensionless number $\Phi = \tau_\phi / \tau_\eta$. When $\Phi \gg 1$, the rate at which topography is created is much higher than the rate at which topography is reduced by melting and freezing. As a consequence, the buoyancy of the topography blocks the mass flux across the phase boundary and the boundary acts as a material interface. In the case of $\Phi \ll 1$, the situation is opposite. The hindering effect of topography diminishes and the mass transport through the boundary becomes more efficient. As discussed by Labrosse et al. (2018), the decrease in Φ is accompanied by a reduction of the amplitude of the phase boundary topography. In the following, the drop in topography that occurs in response to melting temperature changes along a deformed phase boundary will be referred to as the DAL (Deguen-Alboussière-Labrosse) effect. Note that the specific form of τ_ϕ and τ_η varies depending on the factors responsible for the growth and reduction of topography t_b . If the heat transfer in the ice shell is dominated by conduction and the topography is generated by the heat flux from the deep ocean (as in the study by Kverka et al., 2018), $\tau_\phi = \min\{\rho_i L / \gamma, \eta_i (\Delta \rho g D_i)^{-1}\}$ and $\tau_\eta = \rho_i L t_b / \bar{q}$, where $\bar{q}$ is the average heat flux from the ocean. In this case, the flow in the ice shell tends to reduce the topography generated by the heat flux variations from the deep ocean and, therefore, has a similar effect as q in Eq. (2.1).

The role of the DAL effect in the formation of surface topography due to the heat flux variation from the deep ocean has been investigated by Kihoulou et al. (2023). Assuming that the heat flux entering the ice shell from below is a superposition of the heat flux originating in the deep ocean and the heat flux due to variations in the melting temperature along the phase boundary, Kihoulou et al. (2023) demonstrate that the DAL effect is strong enough to suppress topography formation even in the case where the velocity v in Eq. (2.1) is several orders of magnitude lower than that predicted by convection models (e.g., Soderlund, 2019). Based on this result, lateral variations in the heat flux from the ocean should produce only small changes in ice shell thickness, implying that satellite observation of the surface would tell us nothing about the state of the ocean. However, this finding contrasts with the data obtained by the Cassini spacecraft which suggest that the large-scale surface topography of icy moons ranges from

a few hundred meters to more than 1 km (e.g., Iess et al., 2010, 2014; Corlies et al., 2017; Durante et al., 2019; Zannoni et al., 2020). While the topography on Titan and Dione may be related to density anomalies in the ice shell (e.g., Choukroun and Sotin, 2012; Zannoni et al., 2020; Čadek et al., 2021), the models of Enceladus’ internal structure indicate that the thickness of the ice shell varies laterally by tens of km (e.g., Beuthe et al., 2016; Hemingway and Mittal, 2019; Čadek et al., 2016, 2019b).

The two approaches described above address different aspects of the heat transfer in planetary objects with a solid-liquid phase boundary. While in the model of Kvorka et al. (2018), the heat flux from the ocean is governed by the internal dynamics of the ocean and is independent of the processes at the phase boundary, Deguen et al. (2013) and Labrosse et al. (2018) assume that the heat flux from the ocean is controlled by variations in the melting temperature along the boundary and the role of the ocean in heat transfer is only passive. Kihoulou et al. (2023) attempted to bridge these two lines of research by assuming that the heat flux generated by the melting temperature variations can be superimposed over the heat flux from the deep ocean. However, as mentioned above, this approach leads to results which are in poor agreement with the data. This may mean that the effect of temperature variations on ocean circulation is relatively small and the flow in the real ocean is mainly driven by variations in salinity (e.g., Zeng and Jansen, 2021; Lobo et al., 2021; Kang et al., 2022c), or that the approach of Kihoulou et al. (2023) is still too simple to capture the complex interplay between the solid and liquid phases.

In this study, we present a new method for modeling the heat transfer in the interior of icy moons. Our approach is more general than the previous ones in that it consistently links the convective heat transfer in the ocean with the melting temperature variations arising from the deformation of the ice shell. The method is used to study the role of ocean circulation in the formation of surface and ice-water interface topographies. The goal of this study is to assess the relative importance of the DAL effect in the deformation of the phase boundary and to answer the question of whether the long-wavelength topography of icy moons may contain information about flow in the ocean.

Numerical simulations are performed for an icy moon whose size and internal structure are similar to those of Titan. This choice is motivated by the fact that Titan is one of the few icy satellites for which topography is known (Corlies et al., 2017; Durante et al., 2019) and the variations of the ice shell thickness are not likely to be large, making it easier to handle the boundary conditions at the ice-water interface (see section 2 for details). The origin of Titan’s long-wavelength topography remains unclear and is a subject of debate (e.g., Nimmo and Bills, 2010; Hemingway et al., 2013; Mitri et al., 2014; Lefèvre et al., 2014; Kvorka et al., 2018; Amit et al., 2020; Čadek et al., 2021; Kvorka and Čadek, 2022; Kihoulou et al., 2023). Unlike Enceladus, the role of salinity variations in ocean dynamics is likely to be small on Titan (Zeng and Jansen, 2021; Kang et al., 2022b), making it possible to reduce the number of free parameters.

Modeling the coupled evolution of the ice shell and the ocean is challenging for three reasons: First, the characteristic time scale of ice-flow creep is much longer than the time scale of mass transport in the ocean. To overcome this problem, we decompose the computational domain into solid and fluid sub-domains and

integrate the governing equations using the method based on the separation of time scales. Second, the ice-water phase boundary changes shape with time, which complicates the use of a standard spectral method. To overcome this issue, we expand the boundary condition at the real (non-spherical) boundary in a Taylor series and transform it into a form that is valid at the surface of a sphere. Third, numerical simulations of global ocean dynamics with realistic values of the Ekman (Ek) and Rayleigh (Ra) numbers are prohibitively expensive and out of reach for current computational algorithms. To circumvent this problem, we perform the simulations for the values of Ra and Ek that are numerically tractable and modify the boundary conditions at the ice-water interface so that the average heat flux from the ocean is the same as in the ice shell.

Several simplifying assumptions have been made to make the problem more amenable to solution. Perhaps the most important one is that we neglect the effect of salinity variations. This assumption is widely used in studies of subsurface ocean dynamics that employ the numerical tools originally designed to study planetary liquid cores (Soderlund et al., 2014; Soderlund, 2019; Amit et al., 2020; Kvorka and Čadek, 2022; Terra-Nova et al., 2023). The assumption is motivated by numerical convenience (reducing by one the number of parameters) and the lack of information about the composition of subsurface oceans, but it may be misleading when convection is driven by two different density gradients (e.g., Ashkenazy and Tziperman, 2021; Lobo et al., 2021; Zeng and Jansen, 2021; Kang et al., 2022c; Wong et al., 2022). We further assume that thermal and momentum diffusivities are constant throughout the ocean, the lower boundary of the ocean is spherical and the viscosity of ice is controlled by diffusion creep.

In this paper, we make a first step towards understanding the thermal coupling between the ice shell and the underlying ocean. We propose a theoretical framework that describes the dynamic interaction between ocean circulation and ice-shell deformation and present a set of numerical simulations to demonstrate the impact of ocean dynamics on the shape and topography of icy moons. Although the dataset is limited and, therefore, the results should be interpreted with caution, we believe that the effects discussed here can inspire future, more extensive research, which will lead to a deeper understanding of the processes occurring in the interior of icy moons.

2.2 Method

In the following, the bar above a function denotes the average over a spherical surface,

$$\bar{f}(t, r) = \frac{1}{4\pi} \int_0^\pi \int_0^{2\pi} f(t, r, \theta, \phi) d\phi \sin \theta d\theta , \quad (2.2)$$

where t is the time and r, θ, ϕ are the spherical coordinates. The subscripts i and w are used to distinguish between the variables in ice and water, respectively, while the subscript r denotes the radial part of a vector

$$v_r(t, r, \theta, \phi) = \mathbf{e}_r \cdot \mathbf{v}(t, r, \theta, \phi) , \quad (2.3)$$

where $\mathbf{e}_r$ is the radial unit vector.

2.2.1 Ice shell

Governing equations

We consider an incompressible ice shell of density ρ_i overlying an ocean of density ρ_w . We assume that the inertial forces associated with the flow of ice are negligible and for simplicity, we do not include the effect of tidal heating. The conservation laws of mass, momentum, and energy in the Boussinesq approximation then read

$$\nabla \cdot \mathbf{v}_i = 0 , \quad (2.4)$$

$$\nabla \cdot \boldsymbol{\sigma}_i + \rho_i g \alpha_i (T_i - T_{i0}) \mathbf{e}_r = \rho_i (g \mathbf{e}_r - \nabla V) , \quad (2.5)$$

$$\rho_i c_{pi} \frac{\partial T_i}{\partial t} = \nabla \cdot (k_i \nabla T_i) - \rho_i c_{pi} \mathbf{v}_i \cdot \nabla T_i , \quad (2.6)$$

where $\mathbf{v}_i$ is the flow velocity, $\boldsymbol{\sigma}_i$ is the Cauchy stress tensor, g is the mean value of gravitational acceleration, $g = g(r)$, V is the sum of the centrifugal potential, the static tidal potential and the gravitational potential due to the deformation, T_i is the temperature, α_i is the thermal expansivity, ρ_i is the density of ice at the temperature T_{i0} , c_{pi} is the heat capacity at constant pressure and k_i is the thermal conductivity. We assume that ice behaves as a viscous fluid on a long time scale,

$$\boldsymbol{\sigma}_i = -p_i \mathbf{I} + \eta_i (\nabla \mathbf{v}_i + (\nabla \mathbf{v}_i)^T) , \quad (2.7)$$

where p_i is the pressure, $\mathbf{I}$ is the identity tensor and η_i is the viscosity. The potential V in Eq. (2.5) can be expressed as

$$V(r, \theta, \phi, t) = r^2 \omega^2 \left(\frac{1}{3} - \frac{5}{6} P_{20}(\cos \theta) + \frac{1}{4} P_{22}(\cos \theta) \cos(2\phi) \right) + G \int_{\Omega} \frac{\rho' d\Omega'}{|\mathbf{r} - \mathbf{r}'|} , \quad (2.8)$$

where ω is the angular velocity, P_{20} and P_{22} are the associated Legendre polynomials, G is the gravitational constant, ρ' represents the density and the integration is carried out over the whole volume, Ω , of the deformed body. The first term on the right-hand side of Eq. (8) represents the centrifugal and tidal potentials (e.g., Kaula, 1964) while the second term describes the gravitational potential induced by the deformed body.

For simplicity, we omit the effect of tidal heating. Although this effect is likely to be of minor importance on Titan (e.g., Čadež et al., 2021), it can, in principle, produce a non-negligible topographic signal ($\lesssim 100$ m, see Figs. 2 and 3 in Kvorka et al., 2018). We further assume that the deformation of the ice-water boundary is accommodated by viscous flow in the ice shell, but the viscosity of ice is high enough to suppress convection. The question of whether Titan's ice shell is convecting or not has been a subject of debate (e.g., Sotin et al., 2021). The answer depends on the thickness of the ice shell and the viscosity of ice, which in turn depends on the grain size. Recently, Kihoulou et al. (2023) have shown that convection in Titan's 100-km thick ice shell occurs only if the viscosity at the base of the ice layer is lower than $2 \cdot 10^{15}$ Pa s, corresponding to a grain size of $\lesssim 2$ mm. Based on the models of grain size evolution (e.g., Barr and McKinnon, 2007) and the analysis of ice samples from the Antarctic ice sheet (Souchez et al., 2004; Durand et al., 2006), the grain size should be of the order of 1–100 mm. Here we use a value of 10 mm, corresponding to the geometric mean of the minimum and maximum values.

Linearized boundary conditions

The surface of the ice shell is described by function $r(t, \theta, \phi) = r_s + u_s(t, \theta, \phi)$, where r_s is the mean radius of the surface and $u_s(t, \theta, \phi)$ represents the deformation of the surface due to the centrifugal, tidal and buoyancy forces. In the following, u_s is referred to as “the shape of the surface”. Note that u_s is different from topography, which is defined as the distance from an equipotential surface (Fig. 2.1). The surface is isothermal and stress-free,

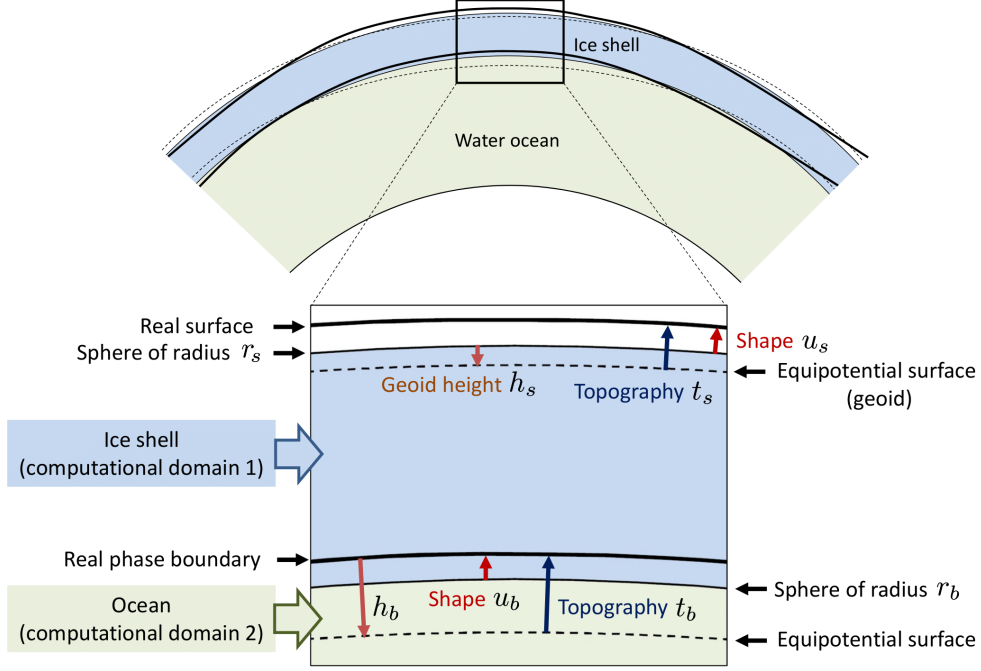


Figure 2.1 Sketch of the computational domain. The computational domain is divided in two sub-domains, the ice shell (blue) and the ocean (green). While the sub-domains have spherical boundaries (thin solid lines), the real boundaries of the ice shell (the surface and the ice-water phase boundary) are generally irregular (thick solid lines). The dashed lines indicate the positions of equipotential surfaces. The shapes (u_s, u_b), topographies (t_s, t_b) and the heights of the equipotential surfaces (h_s, h_b) are defined as positive if the arrows are pointing up.

$$T_i(r_s + u_s) = T_s , \quad (2.9)$$

$$\mathbf{n} \cdot \boldsymbol{\sigma}_i(r_s + u_s) = \mathbf{0} , \quad (2.10)$$

where $\mathbf{n}$ is the outer normal to the surface and T_s is the surface temperature. Under the assumption of small deformation ($u_s \ll r_s$, $\mathbf{n} \approx \mathbf{e}_r$), Eqs. (2.9) and (2.10) can be expanded in the first order Taylor series around r_s

$$T_i(r_s) + u_s \frac{\partial T_i}{\partial r}(r_s) = T_s , \quad (2.11)$$

$$\mathbf{e}_r \cdot \boldsymbol{\sigma}_i(r_s) + u_s \mathbf{e}_r \cdot \frac{\partial \boldsymbol{\sigma}_i}{\partial r}(r_s) = \mathbf{0} . \quad (2.12)$$

Neglecting the lateral variations in the temperature gradient and taking into account that the gradient of stress is approximately equal to the gradient of

hydrostatic prestress, we obtain

$$T_i(r_s) + u_s \frac{\partial \bar{T}_i}{\partial r}(r_s) = T_s , \quad (2.13)$$

$$\mathbf{e}_r \cdot \boldsymbol{\sigma}_i(r_s) + \rho_i \bar{g}(r_s) u_s \mathbf{e}_r = \mathbf{0} . \quad (2.14)$$

Unlike Eqs. (2.9) and (2.10), Eqs. (2.13) and (2.14) are formulated on a sphere, thus facilitating the application of a spectral method.

The bottom boundary of the ice shell is a phase boundary between ice and water. The temperature at the boundary is equal to the melting temperature of ice, which depends on the pressure in the ocean:

$$T_i(r_b + u_b) = T_m(p_w(r_b + u_b)) , \quad (2.15)$$

$$\mathbf{n} \cdot \boldsymbol{\sigma}_i(r_b + u_b) = -p_w(r_b + u_b) \mathbf{n} , \quad (2.16)$$

where r_b is the mean radius of the boundary, u_b is the shape of the boundary, p_w is the pressure in the ocean and T_m is the melting temperature of ice. Since the effect of salinity is neglected, T_m depends only on the pressure, $T_m = T_m(p_w)$. We assume that the stress due to the flow in the ocean can be neglected in the first order approximation and the pressure p_w satisfies the equation of hydrostatic equilibrium (cf. Eq. 2.5):

$$-\nabla (p_w - \rho_w V) = \rho_w g \mathbf{e}_r . \quad (2.17)$$

Integration of the equation yields

$$p_w(r_b + u_b) \approx \bar{p}_w(r_b) - \rho_w \bar{g}(r_b) u_b + \rho_w V(r_b) . \quad (2.18)$$

Invoking the Bruns formula for the geoid and applying it to Eq. (2.8), we get $V(r_b) \approx \bar{g}(r_b) h_b + r_b^2 \omega^2 / 3$, where $h_b = h_b(\theta, \phi)$ represents the undulations of the equipotential surface with the mean radius r_b . Eq. (2.18) then takes the form

$$p_w(r_b + u_b) = p_b - \rho_w \bar{g}(r_b) t_b , \quad (2.19)$$

where p_b is the average pressure at the ice/water phase boundary and $t_b = u_b - h_b$ is the topography. Expanding Eqs. (2.15) and (2.16) in the first order Taylor series around r_b and substituting Eq. (2.19) into Eq. (2.16) we obtain:

$$T_i(r_b) + u_b \frac{\partial \bar{T}_i}{\partial r}(r_b) = T_m(p_b) - \rho_w \bar{g}(r_b) t_b \frac{dT_m}{dp}(p_b) , \quad (2.20)$$

$$-\mathbf{e}_r \cdot \boldsymbol{\sigma}_i(r_b) - \rho_i \bar{g}(r_b) u_b \mathbf{e}_r = p_b \mathbf{e}_r - \rho_w \bar{g}(r_b) t_b \mathbf{e}_r , \quad (2.21)$$

where we assumed that the topographic slope is small and, therefore, $\mathbf{n} \approx \mathbf{e}_r$.

2.2.2 Evolution of the phase boundary

The upper boundary of the ice shell is a material surface, implying that the mass flux across the boundary is zero. For simplicity, we neglect the effect of precipitation (Choukroun and Sotin, 2012) and erosion (e.g., Hemingway et al., 2013; Kverka et al., 2018) and assume that the slope of the surface, ∇u_s , is small.

The equation governing the evolution of the surface then reads (e.g., Kvorka et al., 2018)

$$\frac{\partial u_s(t)}{\partial t} = \mathbf{e}_r \cdot \mathbf{v}_i(t, r_s) . \quad (2.22)$$

The bottom boundary of the ice shell is a phase boundary, which means that the ice shell can exchange both energy and matter with the ocean (for details, see, e.g., Müller, 1985 or Slattery et al., 2007). The evolution of the boundary can be described by the following equation (e.g., Kvorka et al., 2018):

$$\frac{\partial u_b(t)}{\partial t} = \mathbf{e}_r \cdot \mathbf{v}_i(t, r_b) - \frac{1}{L\rho_i} \mathbf{e}_r \cdot [\mathbf{q}_i(t, r_b) - \mathbf{q}_w(t, r_b)] , \quad (2.23)$$

where $\mathbf{q}_i$ and $\mathbf{q}_w$ is the heat flux in the ice shell and in the ocean, respectively, and L denotes the latent heat of ice. The first term on the right hand side of Eq. (2.23) corresponds to the deformation of ice due to viscous flow, while the second term describes the shape changes due to the phase transition (freezing and melting) occurring at the phase boundary. In steady state, $\partial u_s / \partial t = \partial u_b / \partial t = 0$, Eqs. (2.22) and (2.23) take the form

$$\mathbf{e}_r \cdot \mathbf{v}_i(t, r_s) = 0 , \quad L\rho_i \mathbf{e}_r \cdot \mathbf{v}_i(t, r_b) = \mathbf{e}_r \cdot [\mathbf{q}_i(t, r_b) - \mathbf{q}_w(t, r_b)] . \quad (2.24)$$

Note that unless $\mathbf{v}_i = \mathbf{0}$, the heat flux at the ice-ocean interface is discontinuous. Therefore, the steady state is actually a dynamic equilibrium between the viscous flows and the phase transition at the boundary, which has been overlooked in some previous studies (see, e.g., Ojakangas and Stevenson, 1989). As shown by Kihoulou et al. (2023), the effect of the ice flow is important, even if the viscosity of the ice at the melting point is high ($\sim 10^{20}$ Pa s) and the heat transfer in the ice shell is dominated by conduction.

We assume that the average thickness of the ice does not change in time. By averaging Eq. (2.23) over a sphere, we obtain:

$$\bar{q}_{r,i}(r_b) = \bar{q}_{r,w}(r_b) = \frac{Q}{4\pi r_b^2} , \quad (2.25)$$

where Q is the total heat flow across the ice shell obtained by solving Eqs. (2.4)-(2.8). We emphasize that Eq. (2.25) holds only for the average heat flux, while $\mathbf{q}_i(t, r_b) \neq \mathbf{q}_w(t, r_b)$. In analogy with previous studies of ocean dynamics (e.g., Amit et al., 2020; Kvorka and Čadež, 2022), we introduce the normalized heat flux

$$\mathbf{q}_w^*(t, r_b) = \frac{\mathbf{q}_w(t, r_b)}{\bar{q}_{r,w}(r_b)} . \quad (2.26)$$

Eq. (2.23) can then be rewritten as

$$\frac{\partial u_b(t)}{\partial t} = \mathbf{e}_r \cdot \mathbf{v}_i(t, r_b) - \frac{\bar{q}_{r,i}(r_b)}{L\rho_i} \mathbf{e}_r \cdot [\mathbf{q}_i^*(t, r_b) - \mathbf{q}_w^*(t, r_b)] , \quad (2.27)$$

where $\mathbf{q}_w^*(t, r_b)$ is obtained from a simulation of heat transfer in the ocean.

2.2.3 Water ocean

Governing equations

The problem of thermal convection in the ocean is solved in a dimensionless form and the heat flow from the ocean is scaled to match the total heat flow out of the moon. In the following, the tilde over a symbol denotes the dimensionless variables, while symbols without the tilde represent the dimensional quantities. The variables are non-dimensionalized by the following scaling parameters: the thickness of the ocean $D_w = r_b - r_c$, the mean superadiabatic temperature contrast $\Delta \bar{T}_w$, and the thermal diffusion time $\tau_w = D_w^2 / \kappa_w$, where r_c is the radius of the ocean bottom boundary and κ_w is the thermal diffusivity of water. The equations governing the heat transfer in the ocean then read (e.g., Schmitz and Tilgner, 2009, 2010):

$$\nabla \cdot \tilde{\mathbf{v}}_w = 0 , \quad (2.28)$$

$$\frac{1}{Pr} \left(\frac{\partial \tilde{\mathbf{v}}_w}{\partial \tilde{t}} + \tilde{\mathbf{v}}_w \cdot \nabla \tilde{\mathbf{v}}_w \right) + \frac{2}{Ek} \mathbf{e}_z \times \tilde{\mathbf{v}}_w = -\nabla \tilde{p}_w + \nabla^2 \tilde{\mathbf{v}}_w + Ra \frac{\tilde{r}_b^2}{\tilde{r}^2} \tilde{T}_w \mathbf{e}_r , \quad (2.29)$$

$$\frac{\partial \tilde{T}_w}{\partial \tilde{t}} + \tilde{\mathbf{v}}_w \cdot \nabla \tilde{T}_w = \nabla^2 \tilde{T}_w , \quad (2.30)$$

where T_w stands for the non-adiabatic temperature, $\mathbf{e}_z$ is the unit vector in the direction of the z -axis and Ek , Ra and Pr denote the Ekman, Rayleigh and Prandtl numbers, respectively,

$$Ek = \frac{\eta_w}{\rho_w \omega D_w^2} , \quad Ra = \frac{\alpha_w \rho_w \bar{g}(r_b) \Delta \bar{T}_w D_w^3}{\eta_w \kappa_w} , \quad Pr = \frac{\eta_w}{\rho_w \kappa_w} . \quad (2.31)$$

Boundary conditions

The upper boundary of the ocean is identical with the bottom boundary of the ice shell and its shape can thus be described by function $r(t, \theta, \phi) = r_b + u_b(t, \theta, \phi)$. The mass flux across the boundary must satisfy the following conservation condition (e.g. Slattery et al., 2007):

$$\rho_w \left(\mathbf{e}_r \cdot \mathbf{v}_w - \frac{\partial u_b}{\partial t} \right) = \rho_i \left(\mathbf{e}_r \cdot \mathbf{v}_i - \frac{\partial u_b}{\partial t} \right) . \quad (2.32)$$

As shown by Kihoulou et al. (2023), the velocity of ice, v_i , is of order of mm year^{-1} . Since the velocity v_w at the boundary is about the same as v_i and the average velocity in the ocean is of the order of mm s^{-1} (Soderlund, 2019; Kvorka and Čadek, 2022), the boundary can be treated as impermeable. The boundary conditions then take the form

$$\mathbf{e}_r \cdot \tilde{\mathbf{v}}_w = 0 , \quad (2.33)$$

$$\mathbf{e}_r \cdot \tilde{\boldsymbol{\sigma}}_w - (\mathbf{e}_r \cdot \tilde{\boldsymbol{\sigma}}_w \cdot \mathbf{e}_r) \mathbf{e}_r = \mathbf{0} , \quad (2.34)$$

where we neglect the friction at the boundary ($\tilde{\sigma}_{w,r\theta} = \tilde{\sigma}_{w,r\phi} = 0$) and $\tilde{\boldsymbol{\sigma}}_w = -\tilde{p}_w \mathbf{I} + \tilde{\nabla} \tilde{\mathbf{v}}_w + (\tilde{\nabla} \tilde{\mathbf{v}}_w)^T$ is the dimensionless Cauchy stress tensor. A recent study by Buffo et al. (2021) has suggested that ice-ocean interfaces on icy moons are not sharp but may take the form of a transition (“mushy”) layer. Although this layer

is likely to influence the mass transport between the ice shell and the ocean, the implementation of such a layer would be computationally demanding and, therefore, we omit it for simplicity. It should also be mentioned that there is currently no consensus on which boundary condition is appropriate for dimensionless models of ocean circulation. The free-slip boundary condition, Eq. (2.34), was used in studies by Soderlund (2019) and Kvorka and Čadek (2022), while Amit et al. (2020) and Terra-Nova et al. (2023) used the no-slip condition ($\mathbf{v}_w = \mathbf{0}$). No-slip is generally the proper boundary condition in macro-scale application. However, as pointed out by Kuang and Bloxham (1997), no-slip models tend to produce unrealistically thick Ekman boundary layers because Ek used in numerical models is several orders of magnitude larger than that corresponding to subsurface oceans (for comparison of the two types of boundary conditions, see, e.g., Kvorka and Čadek, 2022; Lemasquerier et al., 2023).

The thermal boundary condition for the convecting ocean is obtained by subtracting the adiabatic temperature from Eq. (2.15) (see, e.g., King et al., 2010b; Ricard et al., 2022).

$$T_w(r_b + u_b) = T_m(p_w(r_b + u_b)) - T_{ad}(p_w(r_b + u_b)) . \quad (2.35)$$

Assuming that $|u_b| \ll r_b$ and $\partial T_w / \partial r(r_b)$ can be approximated by $\partial \bar{T}_w / \partial r(r_b)$, Eq. (2.35) can be expressed as (Deguen, 2013)

$$T_w(r_b) + u_b \frac{\partial \bar{T}_w}{\partial r}(r_b) = T_b - \rho_w \bar{g}(r_b) t_b \left(\frac{dT_m}{dp}(p_b) - \frac{dT_{ad}}{dp}(p_b) \right) , \quad (2.36)$$

where $T_b = T_m(p_b) - T_{ad}(p_b)$. The latter assumption is clearly a simplification of reality. However, it is important from the numerical point of view because a nonlinear term in the boundary condition would be difficult to handle by a standard spectral method. Although the effect of the adiabatic gradient is significant in the Earth's metallic core (Deguen and Cardin, 2011), it is neglected here because dT_m/dp is about 10 times larger than dT_{ad}/dp in the ocean.

To non-dimensionalize Eq. (2.36), we replace T_m , u_b , t_b and r by their dimensionless counterparts,

$$\tilde{T}_w(\tilde{r}_b) = \frac{\tilde{r}_c}{\tilde{r}_b} Nu (\tilde{u}_b + K \tilde{t}_b) . \quad (2.37)$$

Here, Nu is the Nusselt number (the ratio of total to conductive heat transfer across the ocean),

$$Nu = -\frac{\tilde{r}_b}{\tilde{r}_c} \frac{\partial \tilde{T}_w}{\partial \tilde{r}}(\tilde{r}_b) , \quad (2.38)$$

and

$$K = \rho_w \bar{g}(r_b) \frac{dT_m}{dp}(p_b) / \frac{\partial \bar{T}_w}{\partial r}(r_b) = k_w \frac{\partial T_m}{\partial r}(r_b) / \bar{q}_{r,w}(r_b) , \quad (2.39)$$

where $\frac{\partial T_m}{\partial r}(r_b) = -\rho_w \bar{g}(r_b) \frac{dT_m}{dp}(p_b)$, $\bar{q}_{r,w} = -k_w \frac{\partial \bar{T}_w}{\partial r}(r_b)$ and k_w is the thermal conductivity of water. The first term on the right hand side of Eq. (2.37) corresponds to the temperature changes associated with the lateral variations in

ocean thickness. These changes are proportional to the deviation of the phase boundary from the spherical shape (u_b) and are scaled by the Nusselt number (Nu) because they depend on the superadiabatic temperature gradient. The second term on the right hand side of Eq. (2.37) represents the changes in the melting temperature due to the pressure variations induced by topography. These changes are proportional to topography (t_b) and are scaled by the inverse of the Clausius-Clapeyron slope, dT_m/dp , a factor not considered in the Boussinesq approximation, therefore necessitating the introduction of a new dimensionless parameter. The parameter K characterizes the relative importance of the two factors described above and correctly describes the ratio of horizontal to vertical forcing in the ocean (for details, see Appendix A). Note that a positive shape does not necessarily imply positive topography, and vice versa. This is because t_b also depends on the gravitational potential induced by the surface boundary undulation (i.e., by u_s), which is dynamically generated by the flow in the ice shell. Although u_b and t_b represent the same surface and are not independent of each other, they correspond to two different mechanisms responsible for the variations in temperature at the inner boundary of the computational domain.

For simplicity, we assume that the bottom boundary of the ocean does not deform and is spherical in shape. The boundary is isothermal ($T_w(r_c) = T_b + \Delta\bar{T}_w$), impermeable (see Eq. 2.33) and stress-free (see Eq. 2.34). The values of Ra , Ek , Pr and Nu corresponding to Titan are given in Tab. 2.1.

2.2.4 Computational strategy

The exchange of heat between the ocean and the ice shell is described by two parameters: the average heat flow across the boundary Q , Eq. (2.25), and the normalized variations of the heat flux from the ocean q_w^* , Eq. (2.26). Previous studies have demonstrated that heat flowing into the ice shell is not evenly distributed, but varies with the latitude and may peak either at the equator or at the poles depending on the relative importance of rotation (Soderlund et al., 2014; Soderlund, 2019; Amit et al., 2020; Bire et al., 2022; Terra-Nova et al., 2023). Kvorka and Čadek (2022) have shown that the heat flux distribution can be characterized by a dimensionless number

$$R_G^* = RaEk^{12/7}Pr^{-1}. \quad (2.40)$$

In other words, the latitudinal distribution of the heat flux does not depend on the values of the individual parameters but only on their combination given by Eq. (2.40).

Unlike the ice shell, simulations of ocean dynamics with realistic values of Ek and Ra are still prohibitively expensive and out of reach for current computational algorithms. Numerical simulations are therefore performed for values of Ek and Ra that are numerically tractable and the results are extrapolated to real conditions by using the scaling laws (e.g., Christensen and Aubert, 2006). Here we proceed as follows:

1. We choose R_G^* so that it corresponds to the conditions in Titan's ocean. Using the modified flux-based Rayleigh number (Christensen and Aubert, 2006; Kvorka and Čadek, 2022) and assuming $Q = 400$ GW and $D_w = 500$ km, we get $2.9 \leq R_G^* \leq 3.8$. Here we use $R_G^* = 3.3$.

Symbol	Parameter	Value	Reference	This study
r_s	Surface radius	2574.76 km	Corlies et al. (2017)	2575 km
ω	Angular speed	$4.6 \times 10^{-6} \text{ s}^{-1}$	Amit et al. (2020)	$4.6 \times 10^{-6} \text{ s}^{-1}$
D_i	Thickness of the ice shell	50–200 km	Hemingway et al. (2013)	75 km
D_w	Thickness of the ocean	90–650 km	Vance et al. (2018)	500 km
Ek	Ekman number	10^{-12} – 4×10^{-11}	Kvorka and Čadek (2022)	$(3\text{--}9) \times 10^{-4}$
Ra	Rayleigh number	4×10^{19} – 9×10^{21}	Kvorka and Čadek (2022)	$(5.49\text{--}36.1) \times 10^6$
Pr	Prandtl number	10	Soderlund (2019)	10
Q	Total heat flow	250–545 GW	Kalousová and Sotin (2020)	400 GW
Nu	Nusselt number	$(9.6\text{--}15.0) \times 10^7$	Kvorka and Čadek (2022)	24.1–45.6
R_G^*	Eq. (2.40)	1.98–8.68	this study	3.3
K	Eq. (2.39)	0.01–0.28	McDougall and Barker (2011)	0–10

Table 2.1 Parameters of Titan. Parameters Ek , Ra and Nu derived in Kvorka and Čadek (2022) correspond to an ocean thickness of 90–500 km and $r_o = 2500$ km.

2. We set $Pr = 10$ (roughly corresponding to water) and choose Ek so that the numerical simulation is feasible (Tab. 2.1). The value of Ra is then calculated from Eq. (2.40).
3. A numerical simulation is performed for Pr , Ek and Ra defined in point 2. Eqs. (2.28)–(2.30) are solved together with the equations governing the thermo-mechanical evolution of the ice shell (section 2.2.1, Eqs. 2.4–2.6). The two problems are coupled by conditions Eq. (2.23) and Eq. (2.37) and solved using the method based on the separation of time scales. The evolution of the system is represented by a sequence of equilibrium states (see the flowchart in Fig. 2.2), which makes it possible to use different time steps in the ice shell and the ocean.

The heat flux variation q_w^* in Eq. (2.23) is calculated from Eq. (2.26) while Nu in Eq. (2.37) is obtained from Eq. (2.38). Note that the Nusselt number in Eq. (2.38) does not correspond to the total heat flow Q in Eq. (2.25) because convection in the ocean is simulated for Ra that is orders of magnitude lower than in the real ocean. The value of K depends on the mean heat flux, the thermal conductivity of water and the Clausius-Clapeyron slope, and can easily be calculated from Eq. (2.39). If the ocean is made of fresh water and the thickness of the ice shell is 50–150 km, we obtain K ranging between 0.01 and 0.05. On the other hand, if the salinity of the ocean is high (20 wt %, see Vance et al., 2018), the parameter K can be as high as 0.3 (values obtained using Gibbs Seawater Toolbox, McDougall and Barker, 2011). However, since one of the goals of this study is to investigate the impact of the DAL effect on the dynamics of the coupled ice shell-ocean system, the parameter K is varied from 0 to 10.

The simulations of ocean circulation are performed by the semi-spectral code WhiteRabbit developed by J. Kvorka (publicly available at <https://github.com/kvorka/WhiteRabbit>). The spherical harmonic series are truncated at degree 213 and the spherical harmonic coefficients are discretized in 97 unevenly spaced Chebyshev points in radius. The details of the numerical method are described in Appendix A in Kvorka and Čadek (2022). To calculate the flow in the ice shell, the WhiteRabbit code was modified for the case of free boundaries and radially dependent material parameters.

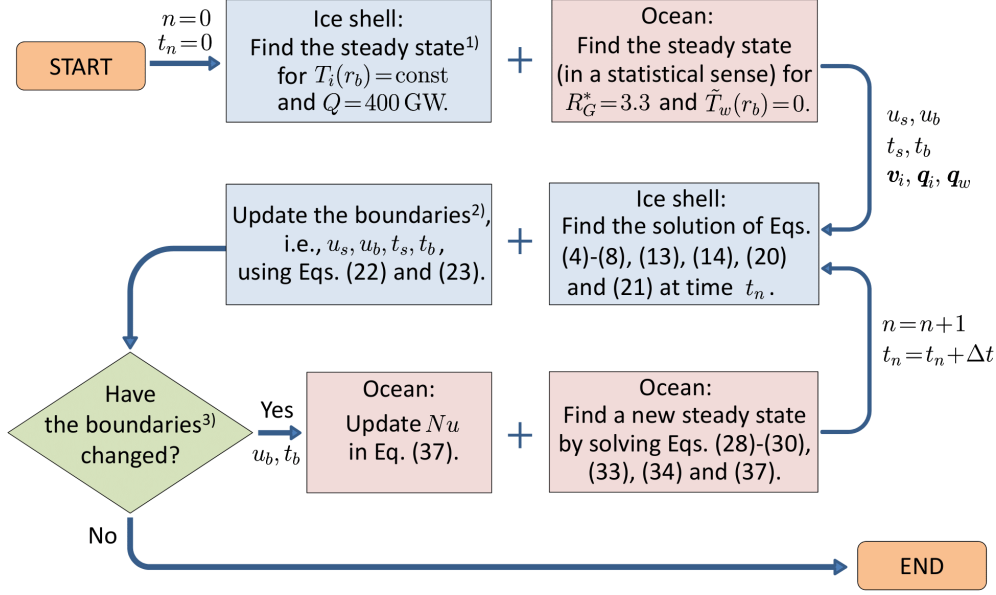


Figure 2.2 Flowchart of the proposed method. The evolution of the system is represented by a sequence of equilibrium states, which makes it possible to optimize the time steps in the ice shell and the ocean. Symbol t_n denotes the dimensional time in the time step n (not to be confused with t_s or t_b , which denote topographies of the boundaries) and Δt is the length of the time step used in computing the flow of ice. The time step is restricted by the Courant-Friedrichs-Lewy (CFL) condition and is about 1.3 kyr. The time step used in computing the heat transfer in the ocean is 7 orders of magnitude shorter than Δt . The typical time needed to reach the equilibrium state in the ice shell and the statistical steady state in the ocean is 100–500 Myrs and 0.001–0.01 thermal diffusion time, respectively. The symbol plus (+) between the boxes indicates that the activities described in the boxes are, or can be, performed simultaneously. ¹⁾ Since the viscosity of ice is rather high in our model, the heat transfer in the ice shell is dominated by conduction and, therefore, the initial shape of the ice shell is close to that in hydrostatic equilibrium. ²⁾ The shape of the ice shell (i.e., u_s and u_b) is calculated directly from Eqs. (2.22) and (2.23). The topographies are calculated from the equations $t_s = u_s - V(r_s)/g$ and $t_b = u_b - V(r_b)/g$, where V is given by Eq. (2.8). ³⁾ The simulation is stopped when $\|u_s(t_n) - u_s(t_{n-1})\|_2 / \|u_s(t_{n-1})\|_2 \leq \epsilon$ and $\|u_b(t_n) - u_b(t_{n-1})\|_2 / \|u_b(t_{n-1})\|_2 \leq \epsilon$, where $\epsilon = 10^{-6}$ and $\|\bullet\|_2$ denotes the L₂-norm.

2.3 Model parameters

All parameters used in this study are listed in Tabs. 2.1 and 2.2. The choice of the parameters controlling the heat transfer in the ocean (R_G^* , Pr , Ek , Ra and K) have already been discussed in section 2.2.4. Unlike the ocean where we vary the parameters Ek , Ra and K (Tab. 2.3) the material parameters of the ice shell are fixed and the equations governing the heat transfer are formulated in dimensional quantities. We assume that the size of ice grains is 10 mm (cf., e.g., Barr and McKinnon, 2007), implying that the viscosity at the base of the ice shell is about 10^{17} Pa s and the heat transfer in ice is dominated by conduction (Kihoulou et al., 2023). Although the convective heat transfer is negligible, the flow velocity near the phase interface is non-zero and the advection term in Eq. (2.6) cannot be neglected (see Fig. 8 and the related discussion in Kihoulou et al., 2023). The constitutive relations for α_i , k_i , c_{pi} and η_i are taken from Feistel and

Wagner (2006), Hobbs (1974) and Goldsby and Kohlstedt (2001). Since the lateral variations in temperature are small compared to the radial temperature contrast, the temperature $T_i(t, r, \theta, \phi)$ in the constitutive relations for α_i , k_i , c_{pi} and η_i can be replaced by the average temperature $\bar{T}_i = \bar{T}_i(t, r)$:

$$\alpha_i = \frac{1}{V_i} \frac{\partial V_i}{\partial \bar{T}_i}, \quad V_i(\bar{T}_i) = A_0 + A_3 \bar{T}_i^3 + A_4 \bar{T}_i^4 + A_5 \bar{T}_i^5 + A_6 \bar{T}_i^6 + A_7 \bar{T}_i^7, \quad (2.41)$$

$$k_i = \begin{cases} k_{i,1} + \frac{k_{i,2}}{\bar{T}_i}, & \text{for } r \leq r_s - 5 \text{ km} \\ 0.5 \text{ W m}^{-1} \text{ K}^{-1}, & \text{for } r > r_s - 5 \text{ km} \end{cases}, \quad (2.42)$$

$$c_{pi} = k_{cp,1} + k_{cp,2} \bar{T}_i, \quad (2.43)$$

$$\frac{1}{\eta_i} = \frac{2A_{d1}}{d^2 \bar{T}_i} \exp\left(-\frac{E_{d1}}{R \bar{T}_i}\right) + \frac{2A_{d2}}{d^3 \bar{T}_i} \exp\left(-\frac{E_{d2}}{R \bar{T}_i}\right), \quad (2.44)$$

where R is the universal gas constant, d is the grain size and A_0 – A_7 , $k_{i,1}$, $k_{i,2}$, $k_{cp,1}$, $k_{cp,2}$, A_{d1} , A_{d2} , E_{d1} and E_{d2} are constant parameters (see Tab. 2.2). A 5-km thick layer of low conductivity ($k_i = 0.5 \text{ W m}^{-1} \text{ K}^{-1}$) methane clathrate is considered near the surface (Kalousová and Sotin, 2020). For simplicity, we assume that the deformation is controlled by diffusion creep and the grain size does not evolve in time. As the material parameters depend only on radius, the equations governing the heat transfer in the ice shell (section 2.2.1) can be efficiently solved by standard spectral methods.

Symbol	Meaning	Value
$A_0 - A_7$	parameters in Eq. (2.41)	see Feistel and Wagner (2006)
$k_{i,1}$	parameter in Eq. (2.42)	$0.4685 \text{ W m}^{-1} \text{ K}^{-1}$
$k_{i,2}$	parameter in Eq. (2.42)	488.12 W m^{-1}
$k_{cp,1}$	parameter in Eq. (2.43)	$185 \text{ J kg}^{-1} \text{ K}^{-1}$
$k_{cp,2}$	parameter in Eq. (2.43)	$7.037 \text{ J kg}^{-1} \text{ K}^{-2}$
A_{d1}	parameter in Eq. (2.44)	$1.4 \times 10^{-7} \text{ Pa}^{-1} \text{ K m}^2 \text{ s}^{-1}$
A_{d2}	parameter in Eq. (2.44)	$3.0 \times 10^{-16} \text{ Pa}^{-1} \text{ K m}^3 \text{ s}^{-1}$
E_{d1}	parameter in Eq. (2.44)	59.4 kJ mol^{-1}
E_{d2}	parameter in Eq. (2.44)	49.0 kJ mol^{-1}
d	grain size	10 mm
T_m	melting temperature	262 K
dT_m/dp	inverse of Clausius-Clapeyron slope	-0.2 K MPa^{-1}
ρ_i	density of ice	920 kg m^{-3}
ρ_w	density of water	1200 kg m^{-3}
L	latent heat of ice	334 kJ kg^{-1}

Table 2.2 Material parameters used in this study.

2.4 Results

Fig. 2.3 illustrates the effect of boundary condition, Eq. (2.37), on the temperature and flow velocity in the ocean. When the right-hand side of Eq. (2.37) is set to zero, corresponding to the case where temperature at the upper boundary of the ocean is constant, the temperature field is characterized by an increase of temperature near the poles and a strong zonal flow at low latitudes (Fig. 2.3a). These characteristics are typical for models with polar cooling (cf. Soderlund, 2019; Amit et al., 2020; Kvorka and Čadež, 2022), which is not surprising as the parameters of the model ($Ek = 3 \times 10^{-4}$, $Ra = 3.61 \times 10^7$, $Pr = 10$) are chosen so that $R_G^* = RaEk^{12/7}Pr^{-1}$ is between 1 and 10 (Kvorka and Čadež, 2022). Panels b–f show the temperature and flow velocity fields in the ocean for models, where the temperature at the upper boundary of the computational domain varies depending on the shape and topography of the phase boundary. Comparison of these panels with panel a) suggests that the temperature variations associated with an irregular shape of the phase boundary and lateral variations in the melting temperature have a minor effect on the geometry of ocean circulation.

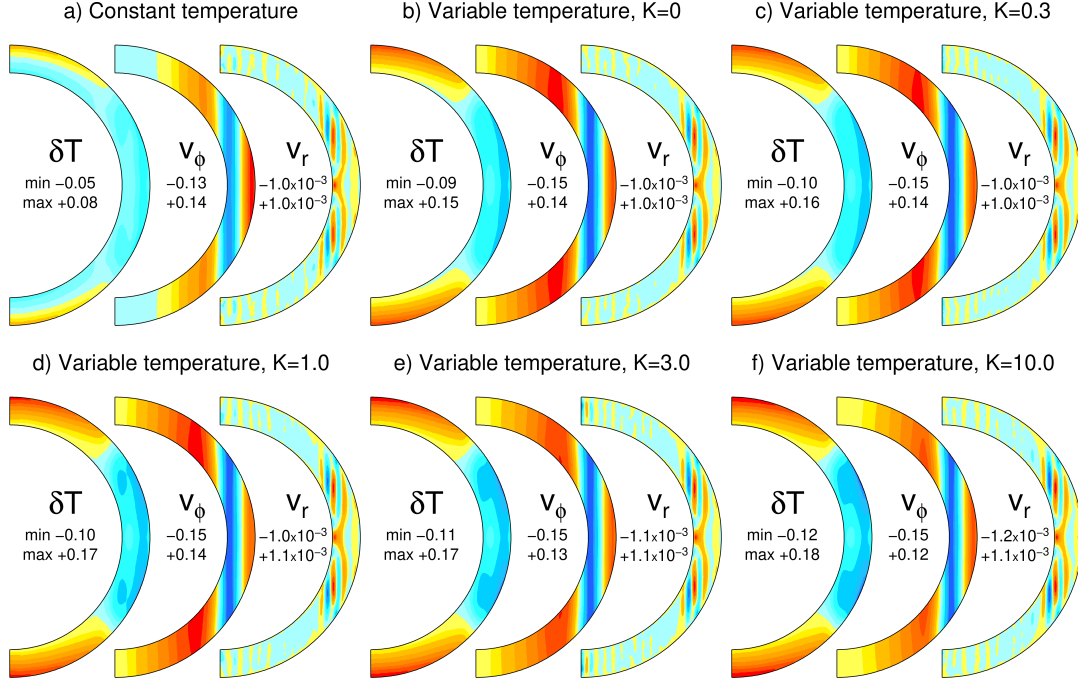


Figure 2.3 Temperature and velocity fields in the ocean averaged over time and longitude for models with $Ek = 3 \times 10^{-4}$ and different boundary conditions imposed at the ice-water interface (models 12–17 in Tab. 2.3). Time averaging is performed over at least 0.02 thermal diffusion times. Symbol δT denotes the temperature variations relative to the average temperature profile and v_ϕ and v_r are the azimuthal and radial components of the flow velocity, respectively, given in Rossby number units, $U/\omega D_w$, where U is the mean velocity in the ocean. The minimum and maximum values are given on the left-hand side of each cross section. Red (blue) denotes positive (negative) values.

Even though the long-term behavior of the ocean is little affected by the phase boundary, the effect of the phase change on the deformation of the ice shell can still be significant. Fig. 2.4 shows the topography of the ice-water interface

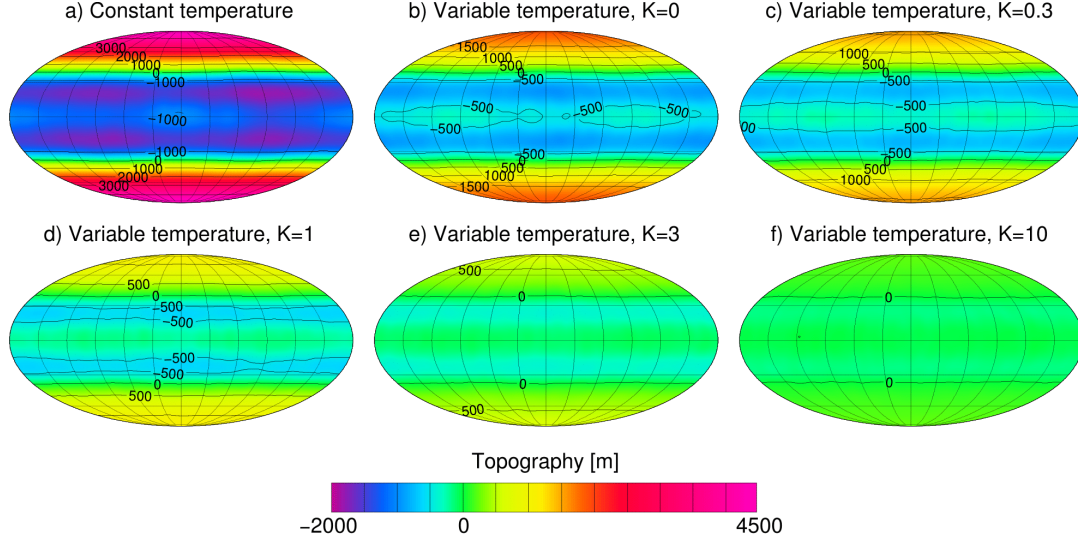


Figure 2.4 Phase boundary topography (i.e., the distance of the boundary from an equipotential surface) calculated for the same models as in Fig. 2.3. The contour interval is 500 m. Negative (positive) topography corresponds to an increase (decrease) in pressure and, consequently, to a decrease (increase) in the melting temperature.

calculated for the same models as in Fig. 2.3. When the temperature at the upper boundary of the oceanic computational domain is constant (Fig. 2.4a), the topography is characterized by significant elevations (4500 m) around the poles and depressions (-1900 m) at low latitudes. If we consider the effect of the shape (without including the changes in the melting temperature), the overall pattern remains about the same but the amplitude of the topography is reduced by a factor of two (Fig. 2.4b). Adding the effect of the pressure dependent melting temperature leads to a further reduction of the topography, as illustrated in Figs. 2.4c–f. The magnitude of topography drops from 1900 m ($K = 0$) to less than 250 m ($K = 10$), confirming the effectiveness of the topography-reducing mechanism described by Labrosse et al. (2018).

The DAL effect is illustrated in Fig. 2.5 where we compare the heat flux from the ocean with the topography of the ice-water boundary and the mass flux across the boundary. Inspection of panel a shows that the additional heat flux driven

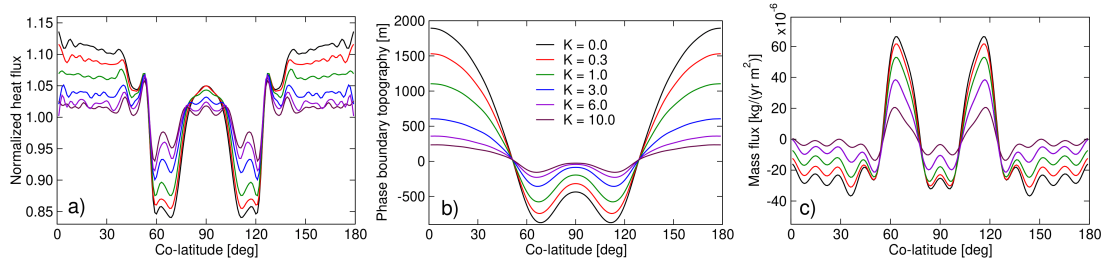


Figure 2.5 Heat flux from the ocean (a), phase boundary topography (b) and mass flux through the phase boundary (c) computed for $Ek = 3 \times 10^{-4}$ and different values of parameter K . The solutions are averaged over time and longitude and plotted as a function of co-latitude. The heat flux is normalized such that the total heat flux is 1.

by variations in the melting temperature along the phase boundary tends to counteract the heat flux from the deep ocean (corresponding to the curve obtained

for $K = 0$). This tendency is more pronounced for larger values of K . However, the heat flux variations from the deep ocean never completely disappear and remain significant ($\sim 5\%$) even for $K = 10$. Note that the heat flux anomalies are more reduced at high latitudes than near the equator, where the circulation is dominated by zonal jet flows (Fig. 2.3), which hamper the heat transfer in the latitudinal direction. Comparison of Fig. 2.5 with Fig. 2.3 shows that the parameter K has a negligible effect on the global circulation pattern but can significantly affect the variations in heat flux at the phase boundary.

As already discussed in connection with Fig. 2.4, the magnitude of topography decreases with increasing K . In addition to that, Fig. 2.5b shows that the phase transition acts as a low pass filter that transforms the heat flux from the ocean into a long-wavelength topographic signal. Small-scale variations in the heat flux have no impact on the phase boundary topography and, therefore, cannot be detected by analysis of the topographic signal. As in the case of the heat flux, the mass flux across the phase boundary (Fig. 2.5c) varies depending on the position with respect to the tangent cylinder (the cylinder with radius r_c whose axis is identical with the rotational axis). The phase boundary located inside the cylinder (co-latitudes $< 53^\circ$ and $> 127^\circ$) is characterized by a negative mass flux (melting of ice) that varies only weakly in the meridional direction and its amplitude is $\lesssim 20 \text{ kg m}^{-2}$ per million years. In contrast, the mass flux outside the tangent cylinder shows two sharp peaks centered at co-latitude of $90 \pm 27^\circ$ and separated by a shallow minimum that is similar in magnitude to those in polar regions. Comparison with Fig. 2.3 suggests that the peaks correspond to the transitions between a prograde zonal flow near the equator and retrograde zonal flows at higher latitudes. Note that the mass flux is negatively correlated with the heat flux (Fig. 2.5a) but its amplitude is not directly proportional to the amplitude of the phase boundary topography (Fig. 2.5b).

The surface topography (Fig. 2.6) is negatively correlated with the topography of the ice-water boundary. This result is in agreement with the recent study of Kihoulou et al. (2023), who have showed that the topography of a conductive ice shell roughly follows the Airy compensation scheme. The magnitude of the

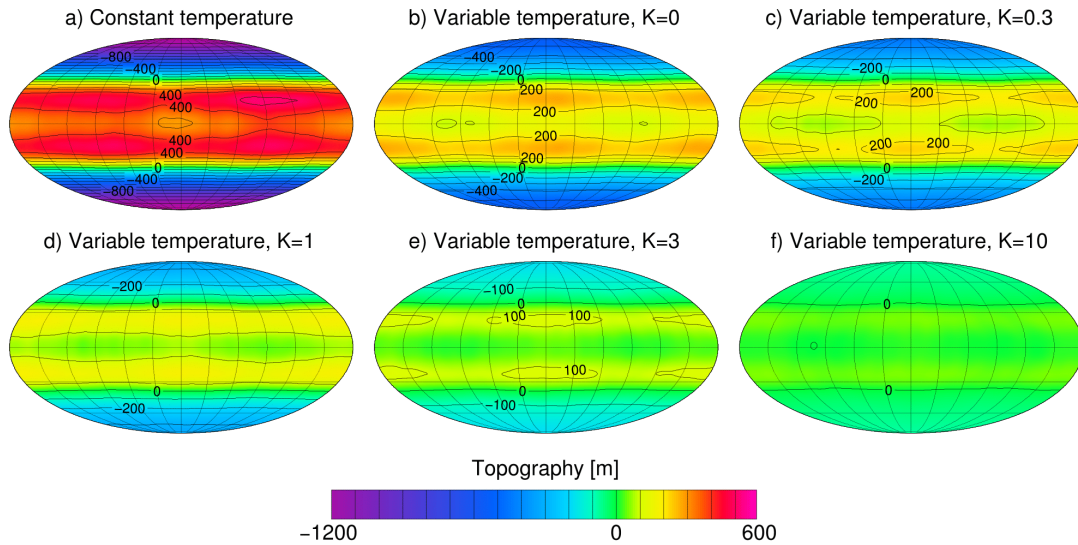


Figure 2.6 Surface topography (i.e., the distance of the surface from the geoid) calculated for the same models as in Fig. 2.3. The contour interval is 100 m.

topography decreases with increasing K , dropping below 130 m for $K = 10$ (see also Tab. 2.3). Note, however, that for $K \lesssim 3$, the amplitude of the topography is larger than about 500 m and therefore is comparable to that observed on Titan (see, e.g., Fig. 6d in Durante et al., 2019).

In contrast to the predicted topography, which is dominated by even zonal harmonics (coefficients C_{20} , C_{40} , C_{60} , ...), the shape calculated for the same models (Fig. 2.7) is more complex, showing a significant degree 2 order 2 component (C_{22}), typical of moons with synchronous rotation. While the amplitude of C_{20} decreases

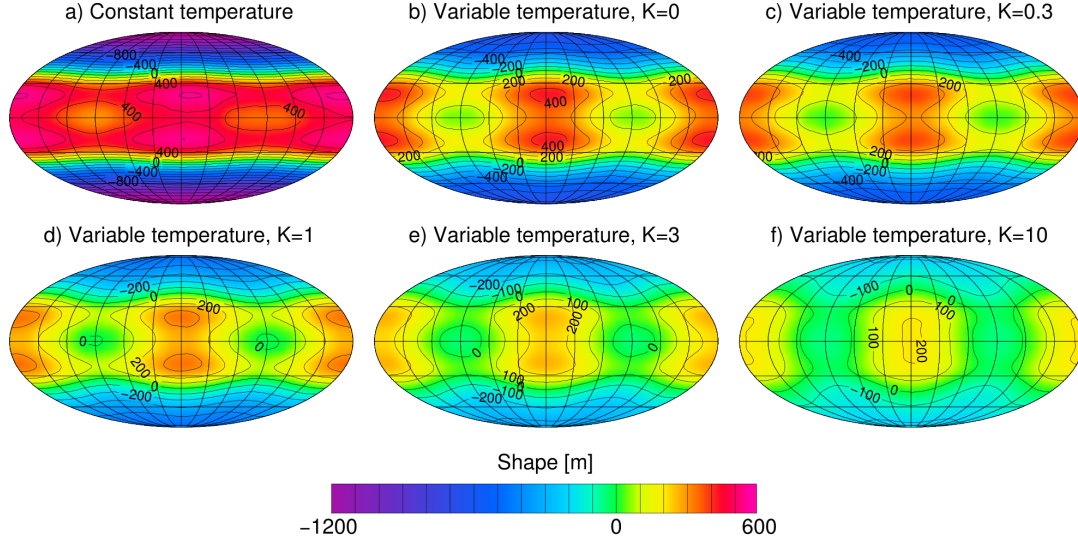


Figure 2.7 Shape of the surface (i.e., the height relative to the mean radius of the surface) calculated for the same models as in Fig. 2.3. The contour interval is 100 m.

with increasing K , the ratio $|C_{22}/C_{20}|$ shows the opposite trend, reaching the maximum value (0.22) for $K = 10$. The degree of similarity between the shape predicted by our method and the shape inferred from Cassini observations (Corlies et al., 2017) is discussed in detail in the next section.

Fig. 2.8 shows the solutions for $K = 0.3$ and three different values of Ek . The shapes and topographies obtained for these three models are almost identical (the difference in amplitude is less than 2%, see Tab. 2.3), indicating that the choice of the Ekman number may have only a moderate impact on the solution (provided that R_G^* and K do not change). The heat flux variations and the corresponding surface topography obtained for different values of Ek and K are shown in Fig. 2.9. While the topography (Fig. 2.9b) is very sensitive to the change of parameter K , the models with the same K and different Ek predict similar topography variations in the polar regions. However, significant discrepancies can be observed near the equator (compare, for example, the red curves in Fig. 2.9b) where t_b and u_b can be of opposite signs and, therefore, the shape and topography effects tend to counterbalance each other.

Finally, we examine the validity of the assumption made by Kihoulou et al. (2023) that the heat flux from the ocean, $q_{r,w}$, can be expressed as a superposition of the heat flux from the deep ocean, $q_{r,w}^0$, and the heat flux associated with the topography of the phase boundary:

$$q_{r,w} = q_{r,w}^0 - \gamma t_b , \quad (2.45)$$

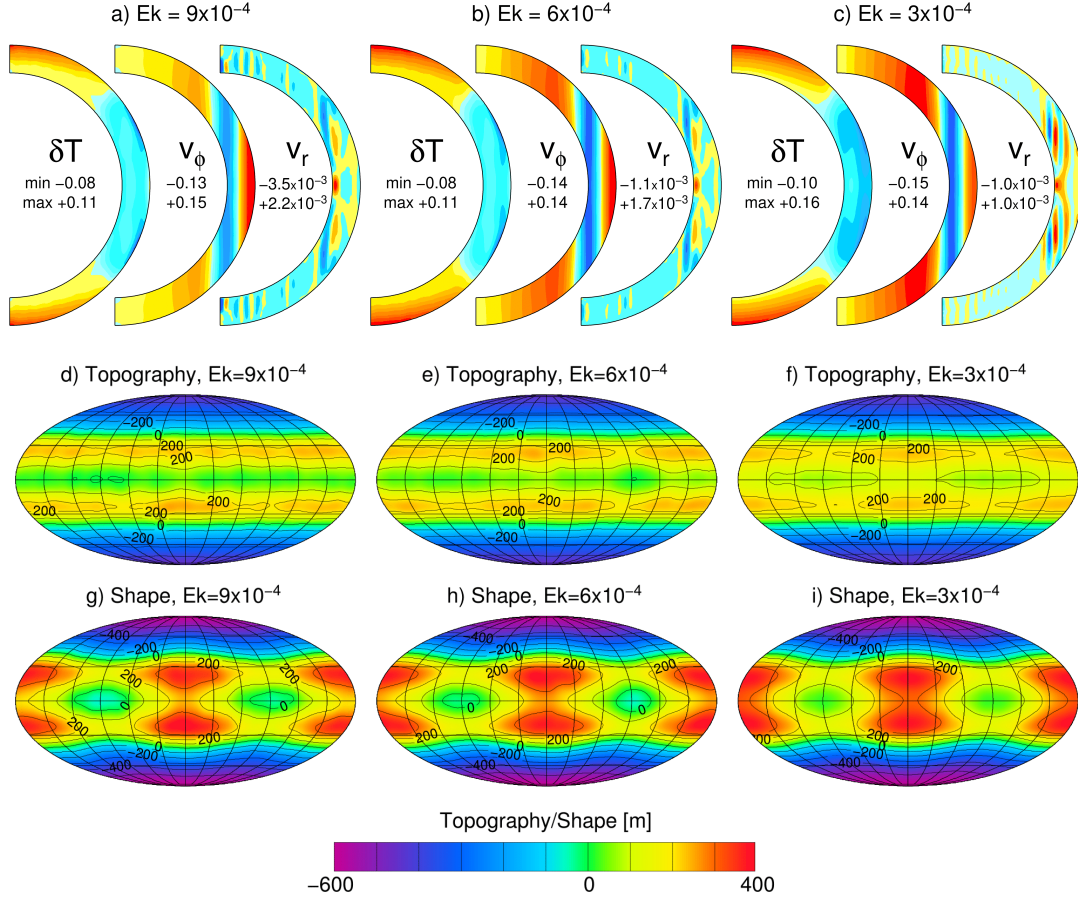


Figure 2.8 Comparison of the results obtained for $R_G^* = 3.3$, $K = 0.3$ and three different values of the Ekman number (models 3, 8 and 13 in Tab. 2.3). a)–c) Temperature and velocity fields averaged over time and longitude. d)–f) Surface topography. g)–i) Shape of the surface.

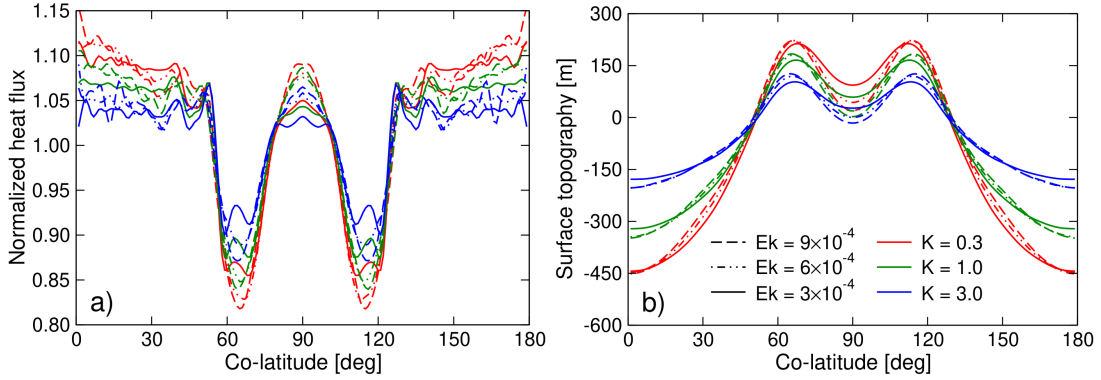


Figure 2.9 Heat flux from the ocean (a) and surface topography (b) computed for $K = 0.3, 0.1, 3.0$, and three different values of Ek (models 3, 4, 5; 8, 9, 10 and 13, 14, 15 in Tab. 2.3). The solutions are averaged over time and longitude and plotted as a function of co-latitude. The heat flux is normalized such that the total heat flux is 1.

where γ is a free parameter characterizing the efficiency of heat transfer in the ocean. The value of γ can be estimated from Eq. (2.1). Considering that the velocity in the ocean is of order $0.001\text{--}1\text{ m s}^{-1}$, we get γ ranging between 600 and $6 \times 10^5\text{ mW m}^{-3}$. As shown by Kihoulou et al. (2023), these values are large

enough to completely suppress the formation of topography. Unlike the approach of Kihoulou et al. (2023), in which γ is treated as an input parameter, our approach allows γ to be determined a posteriori from Eq. (2.45), because $q_{r,w}$ and t_b are known and $q_{r,w}^0$ is the heat flux obtained for $K = 0$:

$$\gamma = -\frac{q_{r,w} - q_{r,w}^0}{t_b} = -\frac{Q}{4\pi r_b^2} \frac{q_{r,w}^* - q_{r,w}^{*,0}}{t_b}. \quad (2.46)$$

Inspection of Fig. 2.10 shows that the values of γ inferred from our simulations are many orders of magnitude smaller than those derived from Eq. (2.1). The value of γ/K ranges from 10^{-4} to $5 \times 10^{-4} \text{ mW m}^{-3}$ and can be even negative near the equator where the circulation pattern is dominated by a zonal flow (see Fig. 2.3). This suggests that Eq. (2.45) is only a rough approximation of the real conditions and Eq. (2.1) should be used with caution, because the velocity near the boundary can be affected by global circulation.

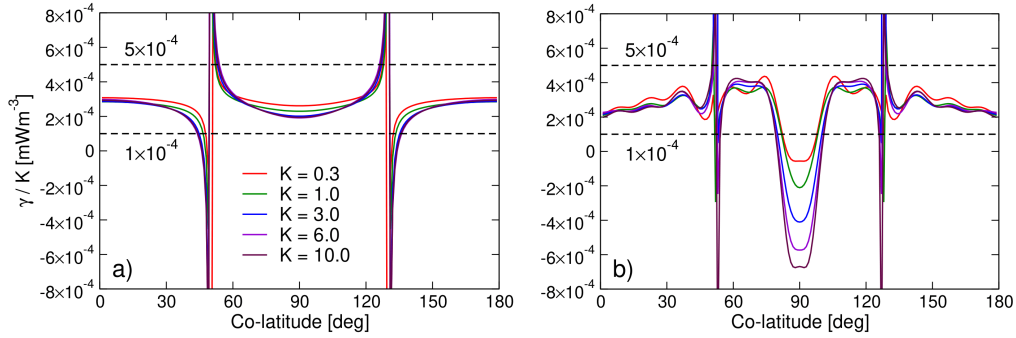


Figure 2.10 Parameter γ as a function of co-latitude θ for K ranging from 0.3 to 10. Parameter γ is calculated from Eq. (2.46) and scaled by K . Spherical harmonic expansions of $q_{r,w}$, $q_{r,w}^0$ and t_b in Eq. (2.46) are truncated at degree 4 (a) and 24 (b). The peaks at $\theta \approx 49^\circ$ and 131° correspond to $t_b \rightarrow 0$.

2.5 Comparison with observations

The long-wavelength shape of Titan derived from Cassini data (Corlies et al., 2017) is dominated by degree-2 components with elevations centered around the sub- and anti-Saturnian points and depressions in the polar region (Fig. 2.11a,b). A similar shape is also observed for other icy moons of Saturn, such as Enceladus or Dione (see, e.g., Nimmo et al., 2011; Tajeddine et al., 2017; Čadek et al., 2019b; Zannoni et al., 2020). This shape is typical of a tidally locked body because, in first approximation, the tidal potential can be expressed in terms of only two spherical harmonic functions, Y_{20} and Y_{22} . The values of the degree-2 harmonic coefficients of the shape are $C_{20} = -347 \pm 23 \text{ m}$ and $C_{22} = 63 \pm 10 \text{ m}$ (Corlies et al., 2017), corresponding to $C_{22}/C_{20} \approx 0.18 \pm 0.04$. Interestingly, the second largest coefficient in the spherical harmonic expansion is not C_{22} but $C_{40} = -164 \pm 38 \text{ m}$. Since Titan is a slowly rotating body ($P \approx 16$ days), the coefficient C_{40} is not directly affected by rotation and thus provides an important constraint on Titan's internal dynamics (e.g., Kvorka and Čadek, 2022). Titan's topography (Fig. 2.11d,e), defined here as the surface height relative to an equipotential, is calculated from

the shape model of Corlies et al. (2017), the gravity model of Durante et al. (2019) and the standard formula for the static tidal and centrifugal potentials (e.g., Čadež et al., 2021). The shape and topography models in Fig. 2.11 are expanded to

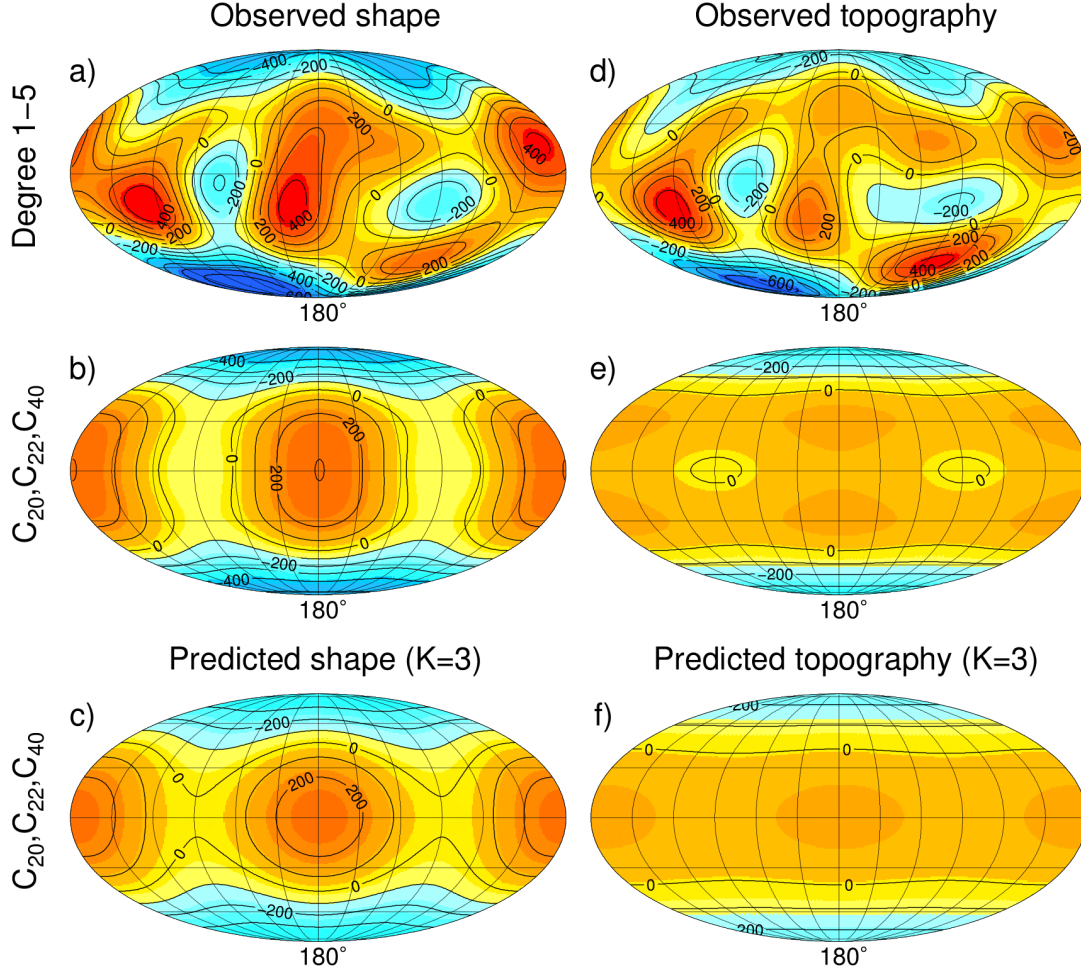


Figure 2.11 a) Shape of Titan expanded to spherical harmonic degree 5 (Corlies et al., 2017). b) As in panel a) but only for harmonic coefficients C_{20} , C_{22} and C_{40} . c) Shape of Titan (coefficients C_{20} , C_{22} and C_{40}) predicted for $K = 3$. d) Topography of Titan expanded to spherical harmonic degree 5 (based on the shape model by Corlies et al. (2017) and the gravity model by Durante et al. (2019)). e) As in panel d) but only for harmonic coefficients C_{20} , C_{22} and C_{40} . f) Topography of Titan (coefficients C_{20} , C_{22} and C_{40}) predicted for $K = 3$.

degree and order 5, corresponding to the cut-off of the gravity model developed by Durante et al. (2019). The topography is similar to the shape in that a significant portion of the topographic signal (53% measured by the L_2 -norm) is produced by zonal harmonic coefficients C_{20} (-176 ± 27 m) and C_{40} (166 ± 40 m), but it differs from it by an almost complete absence of the C_{22} term ($C_{22} = 11 \pm 10$ m), corresponding to $C_{22}/C_{20} = 0.057 \pm 0.065$). In the following, we refer to the models in Figs. 2.11b and 2.11e as the “observed” shape and topography to distinguish them from the shape and topography predicted by our method.

The shape and topography obtained for the model with $K = 3$ are shown in Figs. 2.11c and f, respectively. The model well captures the observed shape and topography at degree 2 but fails to predict the correct value of C_{40} (Fig. 2.12).

However, the overall agreement between the predicted and observed data is not

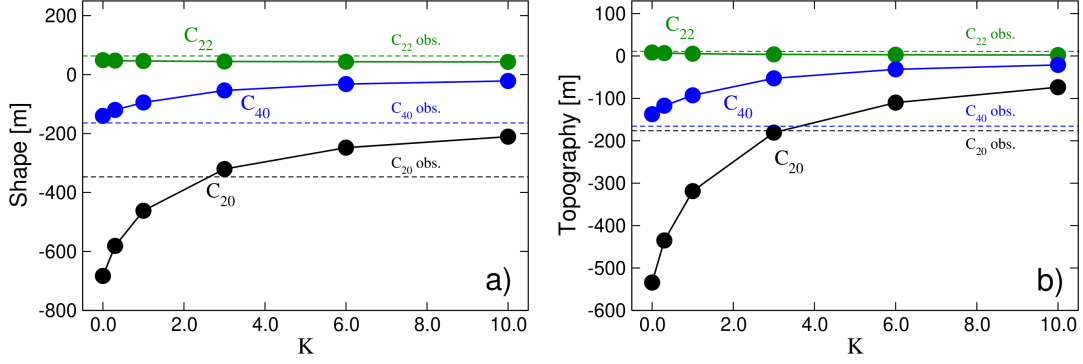


Figure 2.12 Spherical harmonic coefficients C_{20} , C_{22} and C_{40} of Titan’s shape (a) and topography (b) as a function of parameter K . The curves are computed for $R_G^* = 3.3$ ($Ek = 3 \times 10^{-4}$, $Ra = 3.61 \times 10^7$, $Pr = 10$) and grain size $d = 10$ mm. The horizontal dashed lines indicate the observed values. As in the study of Corlies et al. (2017), we use the non-normalized spherical harmonic coefficients, C_{nm} , which are related to the normalized coefficients, $\bar{C}_{nm}$, by $C_{nm} = \bar{C}_{nm}N_{nm}$, where $N_{20} = \sqrt{5}$, $N_{22} = \sqrt{5/12}$ and $N_{40} = 3$ (see also Park et al., 2016).

bad, considering that K is the only free parameter and all other parameters (R_G^* , ocean thickness D_w and grain size d) are fixed. Note that the best fitting value of K is about one to two orders of magnitude larger than the value corresponding to the real ocean (0.01–0.30, see Eq. 2.39). Inspection of Fig. 2.12 shows that models with $K = 0$ and $K = 0.3$ satisfactorily predict the coefficient C_{40} but give too large amplitudes of C_{20} . Previous studies have suggested that the ratio C_{40}/C_{20} depends on R_G^* (Kvorka and Čadek, 2022, Fig 10c) and the magnitude of the heat-flux generated topography decreases with decreasing viscosity of ice and, therefore, depends on grain size d (Kihoulou et al., 2023, Fig. 6). In the present study, both these parameters are kept constant ($R_G^* = 3.3$, $d = 10$ mm). A systematic exploration of the model space is beyond the scope of this study and will be presented in a next paper.

It cannot be ruled out that the observed coefficients C_{20} and C_{40} are affected by large scale processes in Titan’s atmosphere (e.g., Choukroun and Sotin, 2012; Čadek et al., 2021) and/or erosion (e.g., Hemingway et al., 2013; Kvorka et al., 2018). Another factor that can influence Titan’s shape and topography is tidal heating in the ice shell (Nimmo and Bills, 2010; Kvorka et al., 2018). The tidal heating primarily affects the coefficients C_{20} , C_{40} , C_{22} , C_{42} and C_{44} . The last two coefficients are of minor importance in the spherical harmonic expansion of Titan’s topography ($C_{42} = -3.2$ m, $C_{44} = 0.2$ m), which might indicate that the effect of tidal heating on the deformation of the ice shell is negligible (Čadek et al., 2021). However, since the DAL effect is stronger in the longitudinal than in the latitudinal direction, the argument based on the size of coefficients C_{22} , C_{42} and C_{44} may not be valid.

2.6 Conclusions

In this study, we have presented a novel method to model the thermal evolution of a coupled ice shell-ocean system. Our method differs from the previous studies in

that it consistently links the global ocean circulation with the melting temperature variations arising from the deformation of the ice shell. The method is used to investigate the interaction between the subsurface ocean dynamics and the ice flow in the outer solid shell. Specifically, we address the question of whether, and how much, the shape and topography of an icy moon is affected by the heat transfer in the ocean, or in other words, whether topographic data can contain information about physical processes in the ocean.

The ice shell-ocean interaction is determined by processes occurring at the ice-water phase boundary. Previous studies have argued that the topography of the phase boundary is likely to have small amplitudes because its formation is prevented by the heat flux associated with melting temperature variations, resulting from pressure changes along the deformed boundary (e.g., Labrosse et al., 2018; Kihoulou et al., 2023). If this view was correct, the ice thickness would have to be approximately the same over the entire surface of the moon. However, this is in contradiction with the shape and gravity data, which indicate that the amplitude of the degree-2 zonal topography on Titan and Enceladus is of the order of hundreds of meters (e.g., Iess et al., 2014; Tajeddine et al., 2017; Corlies et al., 2017; Durante et al., 2019). The fact that the surface topography of these moons is dominated by zonal harmonic terms suggests that either their ice shells follow the Pratt compensation scheme and the density of ice significantly varies in the latitudinal direction (e.g., Choukroun and Sotin, 2012; Čadek et al., 2021), or that the topography-reducing mechanism described by Labrosse et al. (2018) is less important than previously thought.

To quantify the topographic response to melting temperature changes along the phase boundary (here denoted as the Deguen-Alboussière-Labrosse or DAL effect), we have performed a series of numerical simulations that model the heat transfer in a slowly rotating ocean overlain by a viscous conductive ice shell. The parameters of the ocean are chosen so that the heat flux from the ocean increases towards the poles if the temperature at the upper boundary is constant (polar cooling, see, e.g., Kvorka and Čadek, 2022).

Our results show that the additional heat flux driven by variations in the melting temperature along the phase boundary tends to counteract the heat flux from the deep ocean but is not able to completely suppress it. The heat flux from the deep ocean is more reduced at high latitudes than near the equator, where the ocean circulation is dominated by zonal jet flows, which hamper the heat transfer in the latitudinal direction. Although the lateral variations in the ice shell thickness are reduced to some extent by the DAL effect, the large-scale surface topography induced by deep-ocean heat flux anomalies never completely disappears and is significant ($\sim 100\text{--}1000\text{ m}$) in all tested models. Our simulations predict the shape and topography that are comparable in magnitude to those observed on Titan and explain the absence of a degree-2 sectoral component in the spherical harmonic expansion of Titan’s topography.

The results suggest that the phase transition acts as a low pass filter that transforms the heat flux from the ocean into a large-scale topographic signal. Small-scale topographic variations in the heat flux have no impact on the surface topography and, therefore, cannot be found by observation. This also indicates that the small-scale features found on Titan and other icy moons, such as the 100-km-wide, 1-km-deep basins on Enceladus (Schenk and McKinnon, 2024; Park

et al., 2024), could not have been formed solely by thermal upwellings in the ocean.

The main difference between the model presented here and that of Labrosse et al. (2018) is that, in our model, the topography of the phase boundary is generated by variations in the heat flux from the deep ocean and the heat transport in the ice shell is dominated by conduction, while Labrosse et al. (2018) investigate the case where the phase boundary topography is generated by convection in the solid phase. In our model, the flow of ice tends to reduce the amplitude of topography and its effect on topography is thus opposite to that of the convective flow. This explains why our conclusions regarding the DAL effect are different from the findings of Labrosse et al. (2018). As shown by Kihoulou et al. (2023), the effect of the deep ocean heat flux variations on the topography of a convecting ice shell is negligible, which is in agreement with the results of Labrosse et al. (2018). This indicates that the strength of the DAL effect depends not only on the geometry of ocean circulation and the magnitude of the heat flux variations, but also on the mode of heat transfer in the ice shell.

The model presented in this study does not include the effect of salinity variations. Salinity variation can significantly influence the evolution of small icy moons, such as Enceladus, but are likely to play a limited role in ocean dynamics of large ice moons, such as Ganymede or Titan (Zeng and Jansen, 2021, Kang and Jansen, 2022). As recently discussed by, e.g., Kang et al. (2022a), the flow near the phase boundary on Enceladus is controlled by the interaction of three factors: (i) the variations in the melting temperature associated with the lateral changes in ice shell thickness (supporting the DAL effect), (ii) the negative thermal expansion coefficient of water near the melting point at low pressure (tending to suppress the convective heat transfer), and (iii) the variations in salinity due to melting of ice and freezing of water (acting against the DAL effect). Given the conditions in Enceladus' ocean, it is likely that the salinity variations generated in the ocean by melting are sufficient to suppress the DAL effect, thus maintaining Enceladus' large ice thickness variations (see also Kang, 2022; Kang et al., 2022c; Kang and Jansen, 2022). The impact of salinity variations on shaping Titan's topography is difficult to assess because the exact composition of the ocean is not known. As recently shown by Wong et al. (2022), double-diffusive convection, which occurs when both the temperature and concentration influence the density of the fluid, may result in very different heat and mass transfer modes depending on the Rayleigh number and the buoyancy ratio (the ratio of compositional to thermal forces). Since the buoyancy ratio depends on the composition of the ocean, the role of the DAL effect in the evolution of Titan's ice shell cannot be reliably assessed based on the current state of knowledge. Studies of another large icy moon, Europa, do not provide a definitive answer. While the results of Bire et al. (2023) indicate that a large moon's ocean supports buoyant convection regardless of salinity, Ashkenazy and Tziperman (2021) argue for an important role of ice melting and salinity variations in ocean circulations and suggest that Europa's ocean is weakly stratified.

The results of this study should be regarded as preliminary. Since the simulations of the coupled ice shell-ocean system are computationally demanding, the physical model is simplified in several respects and the simulations are performed only for a limited set of parameters. In spite of these limitations, we believe that

our study can inspire future, more extensive research, which will lead to a better understanding of ocean worlds in the outer Solar System.

Data availability

All data used for producing figures in this paper can be found online (<https://zenodo.org/records/8355226>, 10.5281/zenodo.8355226).

Acknowledgments

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2.A The ratio of horizontal to vertical forcing

The vertical forcing, F_v , in the real system is proportional to the density change across the ocean ($\rho_0 \alpha_w \Delta T_v$) and the volume of the convecting fluid ($\propto D_w^3$):

$$F_v \propto \rho_0 \alpha_w \Delta T_v D_w^3. \quad (\text{A.47})$$

On the other hand, the horizontal forcing, F_h , due to the lateral variations in the melting temperature is significant only in the vicinity of the boundary because the turbulent mixing in the bulk tends to suppress the horizontal temperature contrast, ΔT_m , in the deep ocean. In first approximation,

$$F_h \propto \rho_0 \alpha_w \Delta T_m D_w^2 \lambda_b, \quad (\text{A.48})$$

where λ_b is the thickness of the boundary layer. The temperature contrasts ΔT_v and ΔT_m can be estimated as follows:

$$\Delta T_v \approx \lambda_b \left. \frac{\partial \bar{T}_w}{\partial r} \right|_{r=r_b}, \quad (\text{A.49})$$

$$\Delta T_m \approx -\Delta t_b \left. \frac{\partial T_m}{\partial r} \right|_{r=r_b}, \quad (\text{A.50})$$

where Δt_b is the topography variation along the boundary and the superadiabatic temperature contrast is approximated from the change in temperature across the boundary layer. The latter approximation is rather crude but sufficient for scaling analysis (see also Gastine et al., 2015, 2016). The ratio of horizontal to vertical forcing in the real system is then

$$\frac{F_h}{F_v} \approx -\frac{\lambda_b \Delta T_m}{D_w \Delta T_v} \approx -\frac{\partial T_m / \partial r}{\partial \bar{T} / \partial r} \bigg|_{r=r_b} \frac{\Delta t_b}{D_w} \equiv K \frac{\Delta t_b}{D_w}. \quad (\text{A.51})$$

This means that the ratio of horizontal to vertical forcing in our model is about the same as in the real system, provided that K , Δt_b and D_w assume the realistic values.

2.B List of simulations

Model	R_G^*	Pr	Ek	Ra	K , Eq. (2.37)	Nu	Re	$q_w^{h/l}$	t_s^* [m]	u_s^* [m]
1	3.3	10	9×10^{-4}	5.49×10^6	$\tilde{T}_w(\tilde{r}_b) = 0$	24.1	1023.3	0.11	1164.6	1328.9
2	3.3	10	9×10^{-4}	5.49×10^6	0.0	24.2	1122.4	0.07	520.0	668.3
3	3.3	10	9×10^{-4}	5.49×10^6	0.3	24.1	1129.7	0.05	448.7	595.1
4	3.3	10	9×10^{-4}	5.49×10^6	1.0	24.1	1136.8	0.04	352.0	495.7
5	3.3	10	9×10^{-4}	5.49×10^6	3.0	24.2	1129.5	0.03	210.4	350.1
6	3.3	10	6×10^{-4}	1.10×10^7	$\tilde{T}_w(\tilde{r}_b) = 0$	30.6	1676.9	0.13	1239.5	1405.8
7	3.3	10	6×10^{-4}	1.10×10^7	0.0	30.4	1803.6	0.07	542.7	691.4
8	3.3	10	6×10^{-4}	1.10×10^7	0.3	30.4	1814.1	0.06	452.4	598.8
9	3.3	10	6×10^{-4}	1.10×10^7	1.0	30.4	1813.3	0.05	345.8	489.6
10	3.3	10	6×10^{-4}	1.10×10^7	3.0	30.2	1774.2	0.03	206.6	346.6
11	3.3	10	3×10^{-4}	3.61×10^7	$\tilde{T}_w(\tilde{r}_b) = 0$	46.1	3006.6	0.14	1302.0	1470.0
12	3.3	10	3×10^{-4}	3.61×10^7	0.0	45.3	3703.1	0.07	549.3	698.6
13	3.3	10	3×10^{-4}	3.61×10^7	0.3	45.3	3692.2	0.06	444.7	591.1
14	3.3	10	3×10^{-4}	3.61×10^7	1.0	45.3	3722.1	0.04	321.6	464.9
15	3.3	10	3×10^{-4}	3.61×10^7	3.0	45.3	3624.0	0.03	179.7	318.6
16	3.3	10	3×10^{-4}	3.61×10^7	6.0	45.6	3503.5	0.02	109.5	247.1
17	3.3	10	3×10^{-4}	3.61×10^7	10.0	45.4	3428.4	0.01	73.2	209.9

Table 2.3 Symbol Re denotes the Reynolds number, $Re = UD_w/\kappa_w$, where U is the mean flow velocity in the ocean, and t^* and u^* are the maximum values of $|t_s|$ and $|u_s|$, respectively.

3 Magnetic field induced by convective flow in Europa’s subsurface ocean

This chapter was published as Šachl et al. (2025) in Icarus.

Abstract

Movements of Europa in Jupiter’s magnetic field generate an induced magnetic field in the moon’s interior. Its measurements by the Galileo space probe led to the discovery of Europa’s subsurface ocean. However, interactions of the ocean flow with Jupiter’s background magnetic field also generate the motionally induced electrical currents in the ocean and the corresponding ocean-induced magnetic field (OIMF), which has not yet been studied in detail. A single study estimated the $\text{OIMF} \leq 20 \text{ nT}$ using a simplified scaling relation. In this paper, we revisit this estimate using a physically consistent modeling setup. Based on the numerical simulations of ocean convection, we show that two modes can exist in Europa’s ocean. Mode I is dominated by a prograde zonal flow at the equator with negligible radial and meridional flows. Mode II is characterized by Hadley-like meridional circulation cells in both hemispheres and a retrograde zonal flow at the equator. The scaling analysis based on our dataset strongly indicates that Mode II is appropriate for Europa’s ocean with velocities around 0.3 m/s . We then calculate Europa’s OIMF using a time-domain EM induction solver, which properly accounts for self-induction and diffusion of the magnetic field in the silicate and ice layers, and implicitly covers the full temporal spectrum. Our calculations suggest that even under the most favorable circumstances (150 km thick ocean with a conductivity of 18 S/m located under a 1 km thick ice layer) the magnitude of Europa’s OIMF forced by the flow in Mode II is approximately 1 nT , at the lower bound of the sensitivity of the Europa Clipper measurements and more than one order of magnitude smaller than previously predicted. The discrepancy is primarily caused by a more sluggish ocean flow and a correct treatment of EM induction. Moreover, Europa’s OIMF is affected by the electrical conductivity and thickness of ice and ocean, which we demonstrate in a parametric study.

3.1 Introduction

Electromagnetic (EM) methods played a crucial role in detecting the subsurface oceans on Europa and Ganymede (Zimmer et al., 2000; Khurana et al., 2002; Kivelson et al., 2002). Europa lacks an intrinsic magnetic field generated by dynamo action. However, as it passes through Jupiter’s magnetic field (JMF; see Section 3.2.2 for details), its time variations induce a secondary EM field, which was foreseen by Colburn and Reynolds (1985) and then measured by the Galileo probe at a single frequency corresponding to the synodic period. Zimmer

et al. (2000) studied the magnetic field perturbations measured by Galileo. They assumed a uniform JMF oscillating in time and a spherically symmetric conductive shell in Europa’s interior. A comparison of the predicted induced dipolar field with the measurements from two flybys indicated that Europa contains a conductive layer. By adding measurements from the flyby on January 2000, Kivelson et al. (2000) confirmed that the magnetic field perturbations vary in time, which ruled out the hypothesis of Kivelson et al. (1997b) that Europa has a permanent static dipole.

Despite these achievements, the potential of EM methods to probe the Jovian moons’ interior has not yet been fully explored. Two space missions to the Jovian system (NASA’s Europa Clipper and ESA’s Juice) are expected to bring new data in the early 2030s, providing unprecedentedly detailed information on the structure and temporal variations of the magnetic field in the vicinity of Europa and Ganymede. Furthermore, the EM response at two different frequencies could constrain ocean thickness and conductivity simultaneously (Khurana et al., 2009; Seufert et al., 2011).

In addition to the passive EM response of an interior ocean to the external field variations, the flow of the electrically conductive saline fluids also interacts with the ambient Jovian magnetic field, giving rise to an ocean-induced magnetic field (OIMF) by the process of motional induction. The magnetic Lorentz force, acting on charge carriers in the direction perpendicular both to the flow velocity and the ambient magnetic field, drives electric currents, which further interact with the conductive medium through inductive and galvanic coupling. Surprisingly, little attention has been given to Europa’s OIMF (EOIMF) so far. A notable exception is the study by Vance et al. (2021), who estimated the maximum amplitude of the radial EOIMF component to be 20 nT. However, their estimate was based on a simplified scaling relation in which they inserted typical values of the JMF, the electrical conductivity of the ocean, and the flow velocity according to the flow model of Soderlund (2019). The objective of this paper is to revisit their results using a physically consistent modeling approach.

The modeling of Earth’s OIMF is well established. The pioneering papers of Cox et al. (1970), Sanford (1971), Sanford (1982), and Larsen and Sanford (1985) were followed by the forward modeling studies of Stephenson and Bryan (1992), Flosadóttir et al. (1997), Vivier et al. (2004), and Manoj et al. (2006). Interest among the EM community in the OIMF was further stimulated by Tyler et al. (2003), who managed to extract the tidally driven OIMF (TIMF) from the CHAMP satellite data. Grayver and Olsen (2019) and Sabaka et al. (2020) later provided more precise TIMF models derived from Swarm data. The OIMF within Earth’s oceans was studied by Velínský et al. (2019) and Velínský et al. (2021). The TIMF was also used in the inverse modeling studies of Grayver et al. (2016), Grayver et al. (2017), and Šachl et al. (2022, 2024) to probe the 1-D and 3-D electrical conductivity of the upper mantle. In the context of this study, the most influential papers are Velínský et al. (2018) and Šachl et al. (2019), which studied the numerical aspects of OIMF modeling. Our modeling approach is designed accordingly.

Similarly to OIMF modeling, Earth’s ocean circulation modeling has a long history. Since the pioneering works of Bryan (1969), Semtner (1974), and Cox (1984), ocean general circulation models (OGCMs) have significantly evolved from

eddy-parametrized to eddy-permitting and eddy-resolving models with tuned parameterizations to include the subgrid-scale processes. MOM, NEMO, MITgcm (and ECCO), FESOM2, ICON, MPIOM, ROMS, and POP2 are some of the well-established state-of-the-art OGCMs. The OGCMs are available in various configurations, including regional and global models, coupled climate models, and ocean-only simulations. The models differ in their vertical coordinates (geopotential z , isopycnal ρ and terrain-following σ or s coordinate and their hybrids) and discretization techniques (mostly finite difference/element/volume methods). However, the transition from the Earth’s to planetary oceans is not so straightforward:

1. There is no free surface. The upper boundary of the subsurface ocean is in contact with a rigid ice shell.
2. The hydrostatic approximation, which is commonly applied, is invalid since the ocean is in a convective regime.
3. Similarly, the shallow-ocean approximation (Griffies, 2004) is invalid.
4. The ocean is global, and thus, the so-called Pole problem cannot be solved by relocating the coordinate-system singularity to the land.

Two different approaches are currently used to study the dynamics of subsurface oceans. Although both approaches solve the same governing equations, they differ in a number of details and are used to address different aspects of convective heat transfer. The first approach, hereafter referred to as DNS (Dimensionless Numerical Simulations), is based on the method developed to study the magnetic field generation in the Earth’s liquid core (e.g., Christensen and Wicht, 2007). This approach usually assumes that the material parameters of the ocean are constant and the buoyancy forces are proportional to temperature. The system can then be described by three dimensionless parameters, called the Rayleigh (Ra), Ekman (Ek), and Prandtl (Pr) numbers. The behavior of the system under study is usually estimated from the statistical processing of large datasets obtained for computationally feasible values of Ra and Ek and their extrapolation to realistic conditions (e.g., Amit et al., 2020; Kvorka and Čadek, 2022, 2024). The dynamics of Europa’s ocean has been addressed in several DNS studies (Soderlund et al., 2014; Soderlund, 2019; Lemasquerier et al., 2023; Cabanes et al., 2024). Other studies have investigated the heat transfer in the ocean of Saturn’s moon Titan and its link to ice thickness variations (Amit et al., 2020; Kvorka and Čadek, 2022; Terra-Nova et al., 2023). Although the simulations in the latter studies were performed for a relatively thick ocean, the datasets, and scaling laws presented provide useful insight into subsurface ocean dynamics and can also be applied to Europa.

The second approach, hereafter referred to as GCM (Global Circulation Models), is based on the methodology developed to study convection in Earth’s oceans (Marshall et al., 1997). In this approach, thermal and chemical density anomalies are consistently coupled through the equation of state, which is particularly important in studying small icy moons, such as Enceladus, where ocean salinity is likely to be relatively low (<30 g/kg, Zeng and Jansen, 2021). While the DNS approach solves the problem on a global scale and extrapolates the results obtained by

processing unrealistic datasets, the GCM approach uses a reduced geometry and incorporates the Reynolds averaging in the form of a prescribed viscosity model, thus avoiding the need to extrapolate the results (Visbeck et al., 1997). The GCM studies have mainly focused on Enceladus (e.g., Kang, 2022; Kang and Jansen, 2022) and Europa (Goodman et al., 2004; Goodman and Lenferink, 2012; Ashkenazy and Tziperman, 2021; Bire et al., 2022; Ashkenazy et al., 2023).

From the perspective of EOIMF, the flow patterns and speeds suggested for Europa’s ocean in the studies cited above appear contradictory. Soderlund (2019) proposed that flows in Europa’s ocean are dominated by Hadley-like circulation cells and three zonal jets with maximum speeds reaching approximately meters per second. In contrast, Ashkenazy and Tziperman (2021); Bire et al. (2022) and Cabanes et al. (2024) argued for a rapidly rotating ocean characterized by Taylor columns and two zonal jets in the equatorial region. Their models predict flow speeds ranging from less than 1 mm/s to 0.6 m/s. In both cases, the upper limit found for the flow speed is significantly higher than that obtained from the theoretical analysis of energetic constraints, which suggests that the mean flow speed in Europa’s ocean is between 1.5 and 6 cm/s (Jansen et al., 2023). The objective of this paper is to clarify the discrepancy among these results and to reassess the velocity estimates to appraise the prospects of EOIMF measurements for future missions.

The paper is organized as follows. In Section 3.2, we explain the theoretical aspects of our modeling approach. Section 3.2.1 is devoted to ocean modeling. We introduce the governing equations, discuss the boundary conditions, and examine the dependence of a flow pattern on the Rayleigh, Ekman, and Prandtl numbers. EM modeling is addressed in Section 3.2.2. We briefly discuss the EM induction equation, its mathematical form, and its numerical implementation. We then focus on the JMF and discuss its dipolar characteristics and evolution in time. The section concludes with a list of thickness models and a representative conductivity model supported by the actual knowledge of Europa’s interior. Section 3.3 is devoted to EOIMF calculated in the respective configurations. Finally, we summarize the results in Section 3.4.

3.2 Methodology

3.2.1 Ocean modeling

Governing equations and control parameters

In the present study, the flow in Europa’s ocean is investigated using the DNS approach. We consider the convection of a Boussinesq fluid contained in a spherical shell rotating at a constant angular speed, ω , around the z-axis. The outer radius, r_o , and the inner radius, r_i , of the shell are chosen so that $r_i/r_o = 0.9$. The boundaries of the shell are impermeable, free-slip, and isothermal (for the discussion of the boundary conditions, see Kvorka and Čadež, 2022). The equations

governing the heat and mass transport are expressed in the dimensionless form,

$$\nabla \cdot \mathbf{v} = 0 , \quad (3.1)$$

$$\frac{1}{Pr} \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) + \frac{2}{Ek} \mathbf{e}_z \times \mathbf{v} = -\nabla p + \nabla^2 \mathbf{v} + Ra T g \mathbf{e}_r , \quad (3.2)$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \nabla^2 T , \quad (3.3)$$

and solved using the pseudospectral method described in Kvorka and Čadek (2022). In the above equations, $\mathbf{v}(\mathbf{r}; t)$, $p(\mathbf{r}; t)$ and $T(\mathbf{r}; t)$ denote the dimensionless velocity, pressure and temperature, respectively, $g(r)$ is the gravitational acceleration ($g \propto r^{-2}$) and $\mathbf{e}_r$ is the radial unit vector. Convection is controlled by three dimensionless parameters: the Rayleigh, Ekman and Prandtl numbers,

$$Ek = \frac{\nu}{\omega D^2} , \quad Ra = \frac{\alpha g_o \Delta T D^3}{\nu \kappa} , \quad Pr = \frac{\nu}{\kappa} , \quad (3.4)$$

where ν is the kinematic viscosity, κ is the thermal diffusivity, g_o is the gravitational acceleration at the outer boundary of the ocean, D is the ocean thickness, ΔT is the superadiabatic temperature contrast across the ocean and α is the coefficient of thermal expansion.

The Ekman and Rayleigh numbers corresponding to the subsurface oceans of icy moons are of the order of $\lesssim 10^{-10}$ and $\gtrsim 10^{19}$, respectively (Soderlund, 2019). Simulations of ocean dynamics with such values of Ek and Ra are extremely expensive and out of reach for current computational methods. To avoid this difficulty, we perform a series of simulations for values of Ek and Ra that are numerically tractable and extrapolate the results to real conditions using the scaling laws (see Tab. 3.5 for the list of numerical simulations). Unlike the previous studies (e.g., Gastine et al., 2015, 2016; Kvorka and Čadek, 2022) that mostly examined convection in relatively thick shells ($r_i/r_o \leq 0.8$), the simulations are carried out for the radius ratio of 0.9 that is more relevant to Europa (cf. Soderlund, 2019). Our goal is to determine the geometry of Europa's ocean circulation and estimate the typical flow speed.

Depending on the relative importance of the Coriolis, buoyancy, and inertia forces (i.e., depending on the values of parameters Ek , Ra , and Pr), one can identify three convection regimes: rapidly rotating, transitional, and non-rotating. Each regime is characterized by specific flow patterns, as discussed in, e.g., Gastine et al. (2016), Cheng et al. (2018), Soderlund (2019), Amit et al. (2020), Kvorka and Čadek (2022) and Cabanes et al. (2024). The transition from one regime to another can be described by two dimensionless numbers, the local convective Rossby number (Julien et al., 2012),

$$Ro_{loc} = Ra Ek^{8/5} Pr^{-3/5} , \quad (3.5)$$

and the convective Rossby number (Gilman, 1977),

$$Ro_c = Ra^{1/2} Ek Pr^{-1/2} . \quad (3.6)$$

The transition from the rapidly rotating regime to the transitional regime occurs at $Ro_{loc} \approx 1$ while the transition from the transitional regime to the non-rotating

regime occurs at $Ro_c \approx 1$. In other words, the ocean is in the rapidly rotating regime if $Ro_{loc} < 1$, in the transitional regime if $Ro_{loc} > 1$ and $Ro_c < 1$, and in the non-rotating regime if $Ro_c > 1$. To calculate Ro_{loc} and Ro_c , we need to know the values of Pr , Ek and Ra . Using the same material parameters as Lemasquerier et al. (2023) and assuming that D is between 95 and 150 km (Casajus et al., 2021; Petricca et al., 2023), we obtain $Pr = 11.3$ and Ek ranging from $3.9 \cdot 10^{-12}$ to $9.5 \cdot 10^{-12}$. Unlike Pr and Ek , the Rayleigh number cannot be determined from Eq. (3.4) because the superadiabatic temperature contrast, ΔT , is unknown. As shown in detail in Appendix 3.A, the problem can be circumvented by using the total heat flow, Q , in conjunction with the statistical analysis of the dataset obtained by solving Eqs. (3.1)–(3.3) for different values of Ek and Ra (Tab. 3.5). Based on the model of tidal dissipation in Europa’s interior by Běhounková et al. (2021) and the study of Vance et al. (2018), we assume that Q is between 0.3 TW and 3.2 TW, which gives $Ra \in (1.3 \cdot 10^{21}, 2.7 \cdot 10^{22})$. Note that the range of Ra predicted here is compatible with but narrower than the range proposed by Soderlund (2019), who used the scaling laws developed by Gastine et al. (2015, 2016).

Flow geometry

Substituting the above values of Pr , Ek and Ra in Eqs. (3.5) and (3.6), we obtain $Ro_{loc} > 720$ and $Ro_c < 0.2$, implying that Europa’s ocean is in the transitional regime (see also Tab. 3.3). Previous studies (Soderlund, 2019; Amit et al., 2020; Kvorka and Čadek, 2022) have shown that the structure of flow in the transitional regime is not uniform but varies depending on the relative importance of rotation (see, e.g., Figs. 5 and 6 in Kvorka and Čadek, 2022). The flow patterns can be divided into two basic categories, denoted here as Mode I and Mode II. The flow in Mode I is still significantly affected by rotation. It is characterized by two zonal jets of opposite directions in the equatorial region and relatively small radial and meridional velocities (see Fig. 3.1a). As the influence of the Coriolis force decreases relative to the buoyancy force, the equatorial zonal jet at the outer boundary changes from prograde to retrograde. The overturn is accompanied by the formation of meridional Hadley-like circulation cells in both hemispheres and upwellings concentrated in the equatorial region (Fig. 3.1b). This new flow pattern is denoted as Mode II. Motivated by the discussion in Aurnou et al. (2007), we distinguish Mode II from Mode I based on the sign of the zonal flow velocity at the equator on the outer boundary.

Analysis of our dataset suggests that the transition from Mode I to Mode II can be described by the dimensionless parameter

$$R_M = RaEk^{12/7}Pr^{-3/4} . \quad (3.7)$$

Note that for $Pr = 1$, R_M is identical to the parameter proposed by Gastine et al. (2016) to describe the transition from rotating to non-rotating convection. Parameter R_M is also rather similar to parameter $R_G^* = RaEk^{12/7}Pr^{-1}$ used by Kvorka and Čadek (2022) to analyze latitudinal variations of the heat flux at the upper boundary of the ocean. As shown in Fig. 3.2, the transition from Mode I to Mode II occurs at $R_M = 8$. Substitution of Pr , Ek , and Ra obtained for Europa in the previous section gives $R_M > 27$, meaning that Mode II appropriately

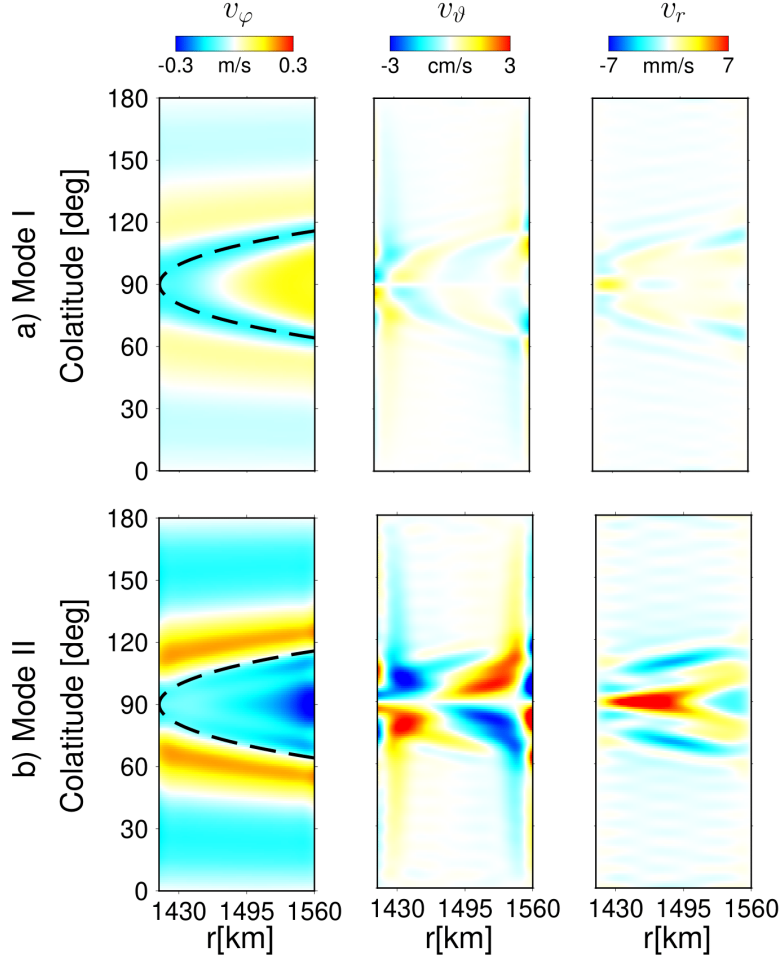


Figure 3.1 a) Flow velocity pattern typical of Mode I. The flow field [m/s] is obtained for model No. 31 and is scaled so that the mean speed, $U = 0.10$ m/s, corresponding to the ocean thickness of $D = 149$ km (model A in Tab. 3.8) and $Q = 0.3$ TW (the lowest value of Q considered in this study). b) As in panel a but for Mode II and model No. 32. The velocity is scaled so that $U = 0.28$ m/s, corresponding to the heat flow of 3.2 TW. The velocity fields are averaged over at least 0.02 viscous diffusion times and the longitude. The dashed lines mark the position of the tangent cylinder (the cylinder with radius r_i whose axis is identical to the rotation axis).

represents the flow in Europa’s ocean. In section 3.3, the induced magnetic field is evaluated for models No. 31 and 32 from Table 3.5 that can be considered as typical representatives of the flow geometry in Modes I and II, respectively.

Flow speed

In order to correctly predict the OIMF, we need to know not only the geometry of large-scale ocean circulation but also the flow speed. Vance et al. (2021) implemented the model of Soderlund (2019) under the assumption that the flow speeds obtained from numerical simulations are the same as those in the oceans of icy satellites. However, the speeds reported by Vance et al. (2021, $\lesssim 3$ m/s) are significantly larger than the values recently suggested for Europa’s ocean by Jansen et al. (2023, 1.5–6 cm/s), Cabanes et al. (2024, $\lesssim 0.6$ m/s) and Kang (2024, ~ 1 cm/s). Cabanes et al. (2024) argue that this discrepancy arises from the

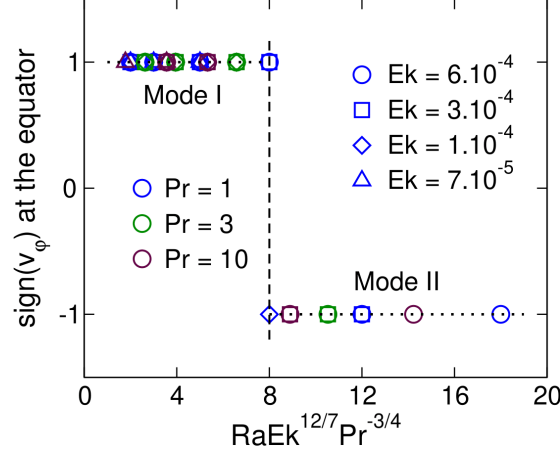


Figure 3.2 The sign of the zonal equatorial jet on the outer boundary as a function of parameter $R_M = RaEk^{12/7}Pr^{-3/4}$. Shapes and colors of the symbols indicate the values of parameters Ek and Pr used in the numerical simulation.

assumption made by Vance et al. (2021) and that any physical quantity, including the velocity, should be rescaled to a realistic setting. Following this argument, we have developed a scaling law for the Rossby number, Ro , which can be interpreted as a dimensionless representation of the mean flow speed ($Ro = UD^{-1}\omega^{-1}$ where U is the root-mean-square speed). The derivation is detailed in Appendix 3.A, and the scaling law is presented in Tab. 3.4 together with the results of other studies. Our scaling law predicts U in the range of 8–28 cm/s, in rough agreement with the estimate of Lemasquerier et al. (2023) who reported a range of 3.5–22 cm/s.

The velocity in models No. 31 and 32 is scaled using the Rossby number obtained for ocean thicknesses in Tab. 3.8 and the total heat flow of 0.3 and 3.2 TW, respectively. The radial, meridional, and zonal velocities are approximately 0.7–1.7 cm/s, 0.9–3.4 cm/s, and 14–32 cm/s, respectively. The lowest values are obtained for the thinnest and least heated ocean, with the thickness of 95 km and the total heat flow of 0.3 TW (heat flux of 10 mW/m² at the outer ocean’s boundary), while the highest values correspond to the ocean thickness of 149 km and the heat flow of 3.2 TW (125 mW/m²). For a detailed comparison with the results of previous studies, see Appendix 3.A.

3.2.2 EM modeling

Governing equations

The electromagnetic induction (EMI) equation governs the generation of EOIMF $\mathbf{B}(\mathbf{r}; t)$ due to the interactions of saline fluid flows with the JMF in Europa’s ocean,

$$\mu_0 \frac{\partial \mathbf{B}}{\partial t} + \nabla \times \left(\frac{1}{\sigma} \nabla \times \mathbf{B} \right) = \mu_0 \nabla \times \mathbf{L}, \quad (3.8)$$

where $\sigma(r)$ is the spherically symmetric electrical conductivity model of Europa’s interior, and μ_0 is the magnetic permeability of free space. The magnetic component of the Lorentz force acting on a unit charge,

$$\mathbf{L}(\mathbf{r}; t) = \mathbf{v}(\mathbf{r}; t) \times \mathbf{B}_0(t), \quad (3.9)$$

depends on the fluid velocity $\mathbf{v}(\mathbf{r}; t)$, and the JMF $\mathbf{B}_0(t)$ (see Section 3.2.2 for the details). In contrast to Eqs. (3.1)-(3.3), all quantities in Eqs. (3.8) and (3.9) are expressed in SI units and in the local Europa spherical coordinate system, $\mathbf{r} = (r, \vartheta, \varphi)$.

By neglecting the interactions of the small, motionally induced magnetic field with the electromagnetic environment of Europa, the EOIMF becomes a potential field in the moon’s exterior, with continuity condition applied across the surface,

$$\nabla^2 U = 0 \quad \text{for } r \geq R, \quad (3.10)$$

$$\mathbf{B} = -\nabla U \quad \text{at } r = R, \quad (3.11)$$

$$\lim_{r \rightarrow \infty} U = \mathcal{O}(r^{-2}), \quad (3.12)$$

where R is the radius of the surface. The magnetic field observable above the moon’s surface is then the sum of the JMF $\mathbf{B}_0$, the magnetospheric field, the field induced in Europa’s interior due to time-varying external field, and finally, the motionally induced contribution $\mathbf{B}$.

We solve the EMI equation (3.8) using the time-domain, spherical harmonic-finite element approach (Velínský and Martinec, 2005; Velínský et al., 2019; Velínský et al., 2021). The magnetic field is expanded in the series of vector spherical harmonic functions in the lateral coordinates (Varshalovich et al., 1988), and piecewise-linear 1-D finite elements in radius. For a spherically symmetric electrical conductivity profile $\sigma(r)$, the problem is decoupled in spherical harmonic degree and order. The forcing term L uses piecewise-constant radial parameterization and is non-zero only in the layers spanning the internal ocean of Europa. The Crank-Nicolson scheme is applied to the time-integration of the magnetic field from its initial condition. The method was originally developed for and validated on the modeling of OIMF on Earth (Šachl et al., 2019). We use homogeneous rather than stationary initial conditions to calculate the EOIMF due to shorter spin-up time t_{spin} . We tested that one year was sufficient for the spin-up process (not shown) and used this value in all presented calculations.

Jupiter’s magnetic field

Europa’s internal magnetic field is either weak or missing (Khurana et al., 1998; Kivelson et al., 2000). Therefore, we assume the ocean flows are interacting with the JMF. Additionally, we neglect the spatial variation of JMF across Europa’s interior, given that Europa’s size is negligible compared to its distance from Jupiter. We use the spherical-harmonic (SH) model of JMF of Connerney et al. (2022). Although small-scale JMF structures are present on the planet’s surface, the JMF is predominantly dipolar at Europa’s orbit since short wavelengths attenuate faster than long wavelengths (Figs. 3.3 and 3.4). Nevertheless, we also consider higher-order terms in our calculations. The JMF model of Connerney et al. (2022) is available up to SH degree 30, but we truncate the SH expansion at $j_{max} = 13$ based on the author’s recommendation.

The time variations of JMF experienced by Europa can be primarily attributed to the misalignment between Jupiter’s dipole and rotation axes and Europa’s movement along its orbit. The vector of the JMF horizontal component, given by the projection of JMF into Europa’s equatorial plane (EEP), rotates in EEP, and its magnitude varies between approximately one-fifth and one-third of the

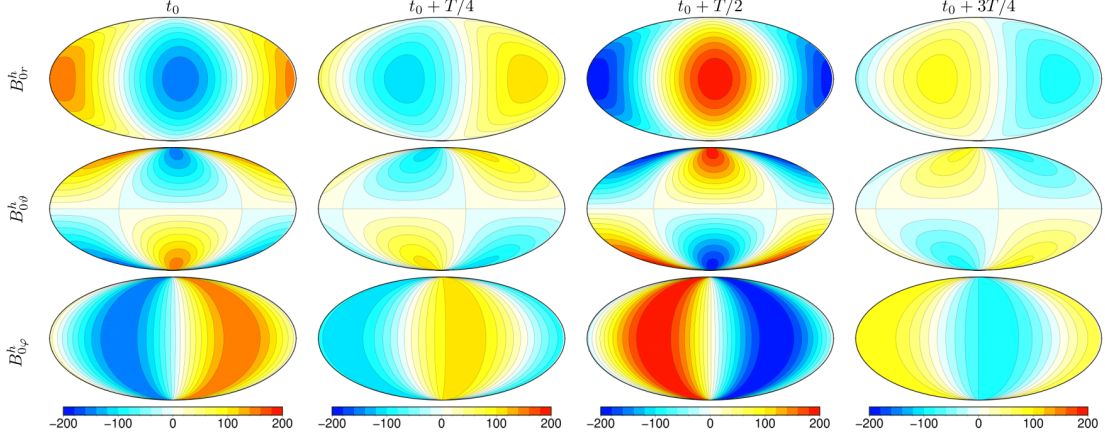


Figure 3.3 Horizontal JMF [nT] on Europa’s surface during the synodic period T starting from the initial time t_0 . The spherical harmonic expansion is truncated at $j_{max} = 13$. The horizontal JMF is obtained by the projection of JMF into Europa’s equatorial plane.

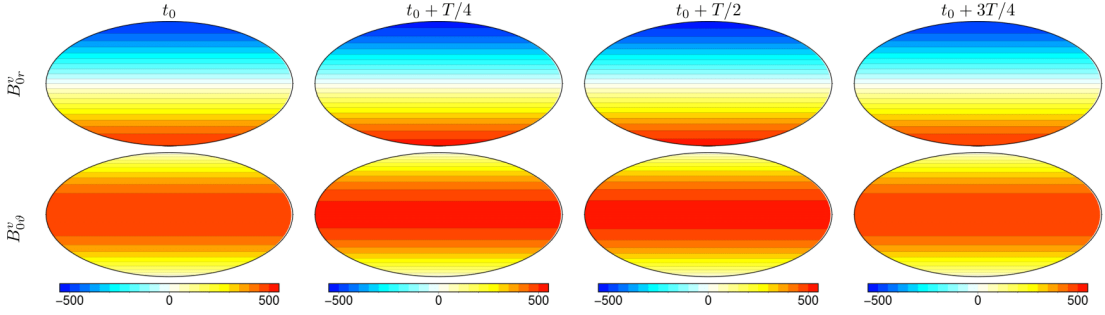


Figure 3.4 As in Fig. 3.3 but for the vertical JMF. The zonal component is zero. The vertical JMF is defined as the part of JMF parallel to Europa’s rotation axis.

vertical component that is perpendicular to EEP (Figs. 3.3 and 3.4). The secular variations of JMF are presumed to exist but are not provided by the model of Connerney et al. (2022).

The SH formalism is employed to calculate the JMF in Jupiter’s spherical coordinates. However, it is necessary to express it in Europa’s spherical coordinate system, where the EOIMF is calculated (see Section 3.2.2). Consequently, JMF components are transformed from spherical to Cartesian coordinates, rotated, and then transformed back to spherical coordinates.

Models of internal structure

We use a four-layer model of Europa’s interior, consisting of the iron-nickel core, silicate mantle, ocean, and ice. We assign a constant electrical conductivity to each layer. The iron-nickel core is supposed to be highly conductive. We set the core conductivity to 10^3 S/m and the core outer radius to 431 km (Sohl et al., 2002; Kimura, 2024) in all calculations. Different core parameters should not significantly affect the results since the EOIMF signals attenuate in the mantle. We test several plausible models of Europa’s interior regarding the conductivity and thickness of the mantle, ocean, and ice.

Let us begin with the thickness values. In recent years, considerable effort has

been made to constrain the ice thickness H and the ocean thickness D . Billings and Kattenhorn (2005) even introduced the term “great thickness debate” for studies devoted to ice thickness. Casajus et al. (2021) analyzed the gravity field using revisited Galileo data to constrain the hydrosphere (ice + ocean) thickness. They estimated hydrosphere thickness to be 125-150 km.

Petricca et al. (2023) presented the results of joint inversion of gravity and magnetic data. By combining two physical processes with different sensitivities, they could constrain both the ice and ocean thicknesses. In addition, they considered two induction amplitude values and three ocean chemical compositions. According to their calculations, the ice is 1-30 km thick, and the ocean is 80-155 km thick. Howell (2021) estimated the ice thickness to be 24.3 km, using a Bayesian analysis accounting for prior constraints based on geological analyses and geophysical parameters affecting possible solid-state convection. Lemasquerier et al. (2023) derived the ice thickness from ocean simulations. They predicted a 12 km thick ice shell for a thermal model dominated by tidal heating and a 24 km thick ice shell if radiogenic heating is dominant. Marusiak et al. (2021) estimated the ice thickness to be 5–30 km in their review paper. For completeness, note that the Juno team’s recent research (Levin et al., 2024) suggests that the minimum thickness of Europa’s ice is larger than the upper limit from the review by Marusiak et al. (2021).

This paper builds upon the results of Casajus et al. (2021) and Petricca et al. (2023). We combine the estimate of the hydrosphere thickness of Casajus et al. (2021) with the estimate of the ice thickness of Petricca et al. (2023) to construct four thickness models that are summarized in Tab. 3.1. Models A–D encompass

Model	H [km]	D [km]	$H + D$ [km]	U [m/s]
A	1	149	150	0.28
B	1	124	125	0.25
C	30	120	150	0.25
D	30	95	125	0.22

Table 3.1 Models of Europa’s hydrosphere considered in the study. The mean speed of the ocean’s flow, U , is obtained by employing the scaling law LS24 (see Appendix 3.A.2) and corresponds to the upper bound of the heat flux considered in this study ($q = 125 \text{ mW/m}^2$ at the outer ocean’s boundary).

the range of ice and ocean thicknesses estimated by the abovementioned studies.

Regarding the electrical conductivity, Europa’s mantle is predominantly composed of silicates, so Earth-like values of 10^{-2} S/m (Petricca et al., 2023) or 10^{-3} S/m (Marusiak et al., 2021) are expected. Additionally, we consider a nearly insulating mantle, using the value of 10^{-16} S/m consistent with the PlanetProfile outputs (Styczinski et al., 2023).

The conductivity of the ocean depends on its chemical composition. Europa’s ocean dominated by NaCl, MgSO_4 or carbonates has been discussed in the literature. The respective upper conductivity limits are 18 S/m, 6 S/m and 6 S/m at 0°C or 3.5 S/m at 5°C (Hand and Chyba, 2007; Kobe and Sheehy, 1948). In the previously mentioned study of Petricca et al. (2023), the reconstructed conductivity values were between ~ 1 S/m and these conductivity limits. That agrees with Naseem et al. (2023), who considered the ocean conductivity of 1–

10 S/m. Psarakis et al. (2024) conducted molecular dynamics simulations to derive electrical conductivity at high-pressure, low-temperature conditions. In Europa’s ocean, the pressure of 0–200 MPa and the temperature of 270–275 K (Vance et al., 2023) are expected. For the more likely SPC/E water model and 10 wt % NaCl ions, the ocean conductivity can be estimated to be approximately 7.5 S/m. In contrast, Castillo-Rogez et al. (2022) estimated the ocean conductivity to be 0.5–0.7 S/m. There is no reliable information on lateral variations of Europa’s electrical conductivity. For the saline ocean, it could be, in principle, related to the temperature and salinity obtained from the convection model using a suitable equation of state, but the uncertainties are presently too large to justify a computationally expensive 3-D EM modeling approach.

The ice is expected to be resistive. The conductivity values proposed by Petrenko and Schulson (1992) and Petricca et al. (2023) are 10^{-5} S/m and 10^{-6} S/m, respectively.

Consequently, we set the default values of mantle, ocean, and ice conductivity to $\sigma_m = 10^{-2}$ S/m, $\sigma_o = 10$ S/m and $\sigma_i = 10^{-5}$ S/m, respectively. Some of the alternative conductivity values cited above are considered in Section 3.3.4, where we study the EOIMF sensitivity to Europa’s conductivity structure. For example, the default ocean conductivity of 10 S/m is a reasonable upper limit corresponding to a salty highly conductive ocean. But we also test the true upper conductivity limit of 18 S/m according to Hand and Chyba (2007) and address the possibility of a much more resistive ocean by using 1 S/m motivated by Castillo-Rogez et al. (2022).

3.3 Results

3.3.1 EOIMF symmetries

Before proceeding to the discussion of the predicted EOIMF models, we will demonstrate the elementary physics of motional induction and relate the geometric symmetries of EOIMF to the underlying ocean flow patterns. The flow velocity models averaged over time are axially symmetric around Europa’s rotation axis (i.e., independent of the longitude) and mirror-symmetric about the equator (Fig. 3.1). This implies that the radial and zonal velocity components are symmetric, while the meridional component is antisymmetric about the equator: $v_r(r, \vartheta) = v_r(r, \pi - \vartheta)$, $v_\vartheta(r, \vartheta) = -v_\vartheta(r, \pi - \vartheta)$ and $v_\varphi(\vartheta) = v_\varphi(\pi - \vartheta)$, where ϑ denotes the colatitude. As for the horizontal JMF vector (Fig. 3.3), the meridional component is antisymmetric, and the radial and zonal components are symmetric, while in the case of the vertical JMF (Fig. 3.4), the meridional component is symmetric, and the radial component is antisymmetric.

Given the properties of the flow velocity and the JMF vector described above, one can expect that the resulting EOIMF vector will also have some level of symmetry. To investigate the EOIMF symmetries, we consider four combinations of flow velocity and JMF components (hereinafter referred to as Configurations 1–4, see Tab. 3.2), and for each of them, explore the symmetry properties of the corresponding EOIMF vector. In Tab. 3.2, $\mathbf{B}_0^h$ and $\mathbf{B}_0^v$ denote the horizontal and vertical components of JMF shown in Figs. 3.3 and 3.4, respectively, $\mathbf{v}^P = v_r \mathbf{e}_r + v_\vartheta \mathbf{e}_\vartheta$ is the poloidal velocity field associated with convective cells, and

Configuration	Jupiter's field	Flow	L_r	L_ϑ	L_φ	B_r	B_ϑ	B_φ
1	$\mathbf{B}_0^v$	$\mathbf{v}^T$	S	A	0	0	0	0
2	$\mathbf{B}_0^h$	$\mathbf{v}^P$	A	S	A	S	A	S
3	$\mathbf{B}_0^h$	$\mathbf{v}^T$	A	S	0	S	A	S
4	$\mathbf{B}_0^v$	$\mathbf{v}^P$	0	0	S	A	S	0
5	$\mathbf{B}_0$	$\mathbf{v}$	N	N	N	N	N	S

Table 3.2 Summary of the surface EOIMF symmetries. S = symmetric field, A = antisymmetric field, N = neither symmetric nor antisymmetric, 0 = field equals zero. The surface EOIMF in Configuration 1 vanishes since it is a toroidal field.

$\mathbf{v}^T = v_\varphi \mathbf{e}_\varphi$ is the toroidal velocity field associated with zonal jets. The expressions for the Lorentz force, $\mathbf{L}$, and $\nabla \times \mathbf{L}$, corresponding to each configuration, can be found in Appendix 3.B.1.

In interpreting the results presented in the last three columns of Tab. 3.2, it is essential to recognize that the vertical component of JMF is significantly larger than the horizontal component, and the circulation of Europa's ocean is dominated by the zonal flow; even in the case of Mode II, the poloidal flow which forms the convection cells in the meridional plane is an order of magnitude slower than the toroidal flow in the equatorial region. However, the interaction of the toroidal flow with the vertical JMF generates zero EOIMF, implying that the total EOIMF (denoted as Configuration 5 in Tab. 3.2) is mainly a superposition of Configurations 3 and 4 while Configuration 2 plays only a minor role. The zonal component of the total EOIMF is symmetric, but the radial and meridional components are neither symmetric nor antisymmetric since the symmetry properties of Configurations 2 and 3 differ from those of Configuration 4.

The EOIMF symmetries for Configurations 2–5 are illustrated in Fig. 3.5. Inspection of the figure shows that the EOIMF generated by convection cells and zonal jets exhibit comparable magnitudes even though the zonal jet speed is approximately one order of magnitude larger than the flow speed in convection cells. The EOIMF generated by convection cells manifests mainly in the radial and meridional components. The zonal component of EOIMF is mainly affected by the zonal flow, while the impact of convection cells is weak.

A remarkable feature demonstrated in Fig. 3.5 is the zonal stripes in the radial and meridional EOIMF components of Configuration 4. The origin of the zonal stripes is illustrated in Fig. 3.6. The Lorentz force generates zonal electrical currents that form a system of closed zonal loops, which in turn generate the EOIMF in the meridional plane. The total EOIMF is a superposition of partial EOIMFs from each loop, resulting in a system of narrow EOIMF stripes in the zonal direction. The EOIMF is constant in the zonal direction since both the vertical JMF and the velocity field are rotationally symmetric. However, the EOIMF exhibits relatively sharp sign changes in the meridional direction, reflecting the directional changes of the flow in the convection cells (upwellings and downwellings).

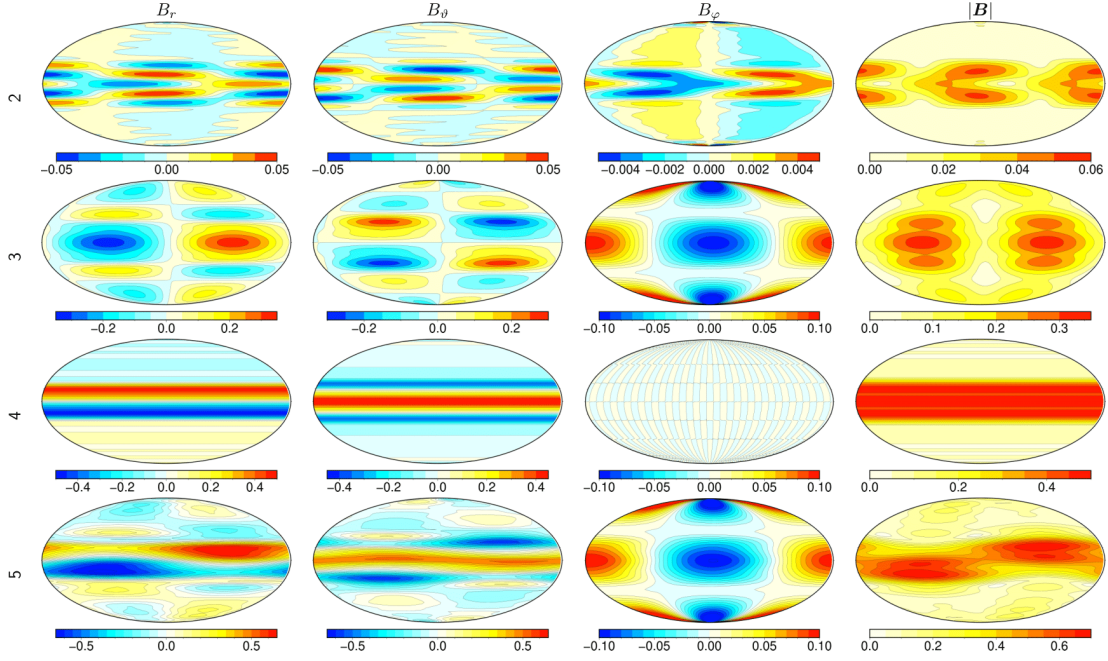


Figure 3.5 EOIMF [nT] for Configurations 2–5 (Tab. 3.2) calculated using thickness model A and flow Mode II in time $t = t_{spin} + 4$ h.

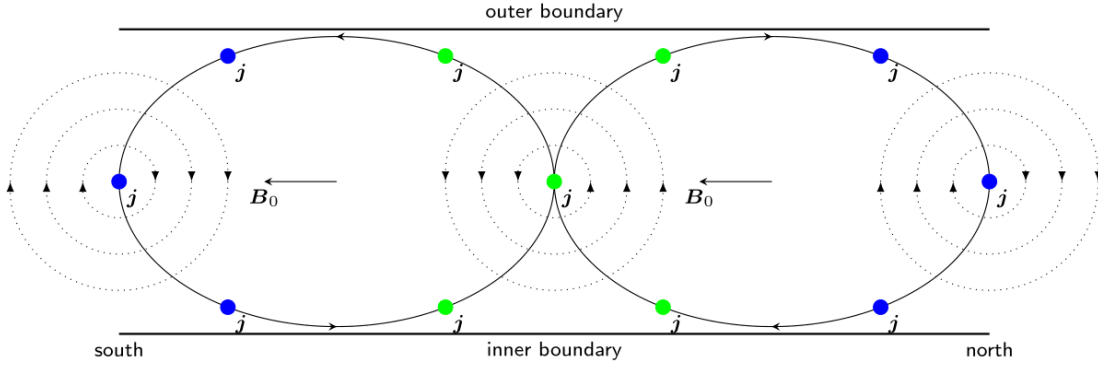


Figure 3.6 Sketch of the convection cells (solid ellipses), the electrical currents (color circles; green = out of the paper, blue = into the paper), and corresponding OIMF (dotted circles) in a meridional plane. The curvature of ocean boundaries is neglected for simplicity.

3.3.2 Comparison of EOIMF generated by Modes I and II

As discussed in section 3.2.1, two flow configurations, Mode I and Mode II, may exist in Europa's subsurface ocean. Fig. 3.7 shows EOIMFs generated by Modes I and II for the thickness model A. We can identify the zonal stripes in the EOIMF generated by Mode II but not by Mode I since the convection cells in Mode I are much weaker or missing. The zonal components generated by the respective modes have different signs, indicating that the zonal jets flow in opposite directions in Modes I and II. Additionally, the EOIMF generated by Mode II is stronger; its magnitude is approximately three times larger than that of Mode I.

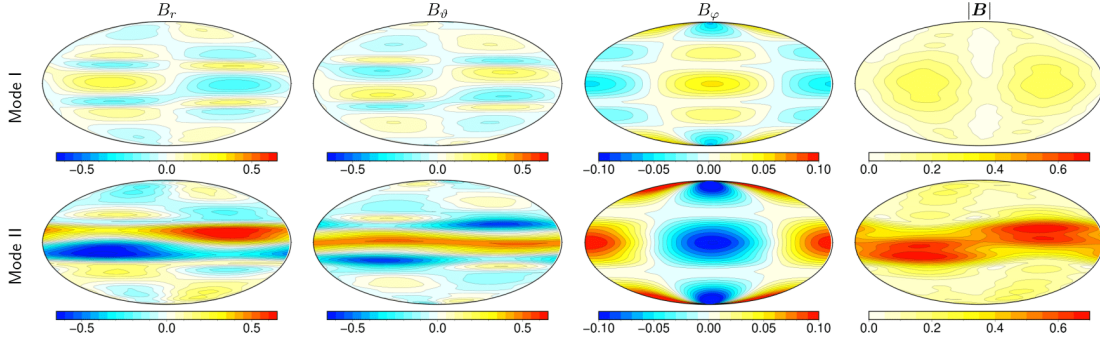


Figure 3.7 EOIMFs [nT] calculated using thickness model A and one of two possible flow configurations shown in Fig. 3.1 in time $t = t_{spin} + 4$ h.

3.3.3 EOIMF sensitivity to ice and ocean thicknesses

Here, we focus on the sensitivity of EOIMF to the ocean and ice thicknesses. Fig. 3.8 shows the EOIMF surface amplitude calculated for the thickness models A–D (Tab. 3.1) using the default mantle, ice, and ocean conductivities from the previous section. The convection is assumed to be in Mode II. As we can see, the

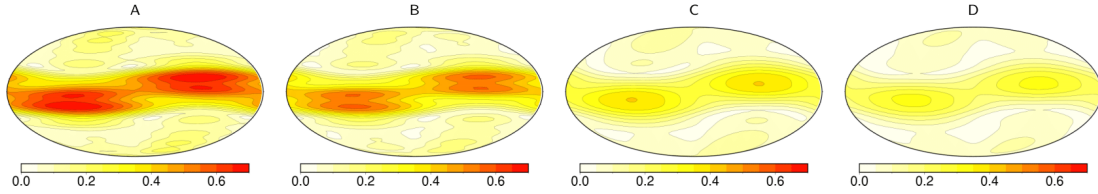


Figure 3.8 EOIMF strength [nT] for thicknesses A–D from Tab. 3.1 and flow Mode II. Snapshots in time $t = t_{spin} + 4$ h.

EOIMF is stronger if the ocean is thicker and the ice is thinner. The reason is that the ocean flow is more vigorous in the thicker ocean to balance the decrease in the conductive heat flux. In addition, the thicker ice damps the field by shifting the source away from the surface (see Section 3.3.4).

3.3.4 EOIMF sensitivity to mantle, ocean and ice conductivity

Here, we explore the dependence of EOIMF on the electrical conductivity of ice, ocean, and mantle. Fig. 3.9 summarizes the EOIMF obtained by a series of calculations in which one conductivity value was perturbed from the default model. The thicknesses are fixed to model A, and the convective flow is in Mode II.

Mantle conductivity has a minimal effect on the EOIMF (Fig. 3.9, mantle). If we decrease mantle conductivity by two orders of magnitude to $\sigma_m = 10^{-4}$ S/m, the EOIMF remains almost unchanged. A further decrease of mantle conductivity to $\sigma_m = 10^{-6}$ S/m results in a mild weakening of the EOIMF, around 10% in maximum values. In the case of a resistive mantle, $\sigma_m = 10^{-16}$ S/m, the EOIMF strength decreases by approximately 50%. However, this conductivity value is most likely too low.

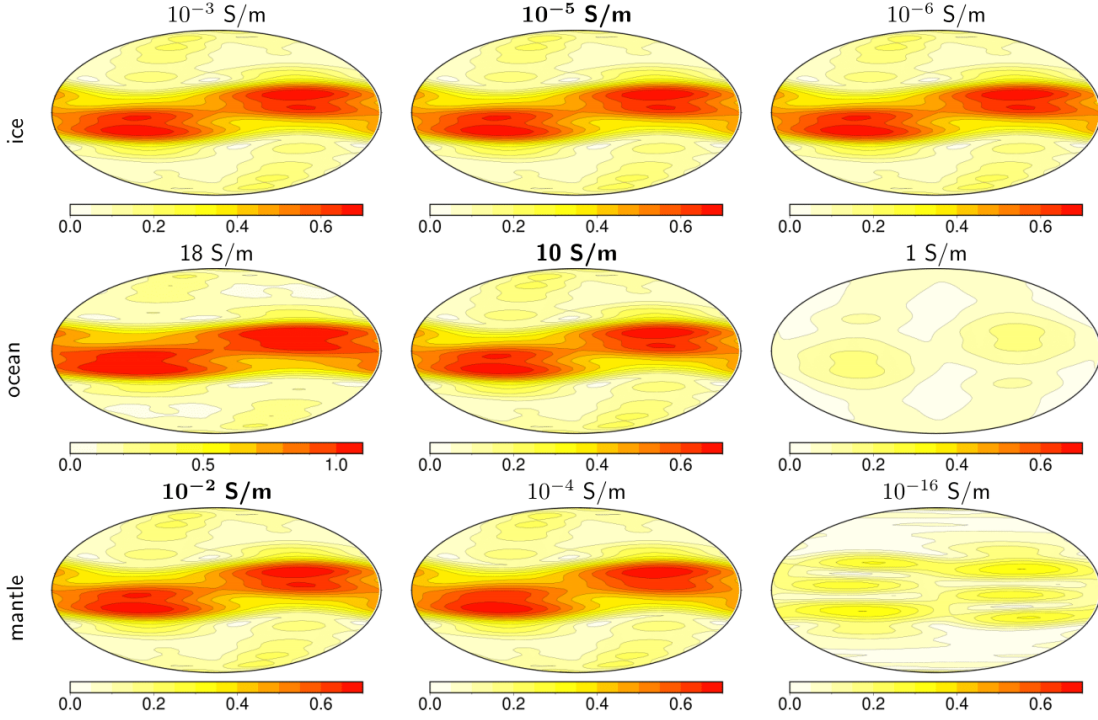


Figure 3.9 EOIMF strength [nT] as a function of mantle, ocean, and ice conductivity for thickness model A. The default conductivity values are in bold. Snapshots in time $t = t_{spin} + 4$ h.

Ocean conductivity has the expected effect on the EOIMF; a more conductive ocean implies a stronger EOIMF (Fig. 3.9, ocean). Indeed, within the ocean layer, the second term on the left-hand side of the EMI equation (3.8) can be written as $-\sigma^{-1}\Delta\mathbf{B}$. In other words, it is a diffusion term inversely proportional to conductivity. The EOIMF strength reaches 1 nT if $\sigma_o = 18$ S/m; the color scale is extended in the corresponding panel. However, the value of 18 S/m corresponds to an extremely conductive ocean, which is probably unrealistic.

Finally, ice conductivity has a negligible effect on the EOIMF in model A (Fig. 3.9, ice). However, this is not unexpected, given that the ice layer is only 1 km thick, and thus, its impact is minimal. Therefore, we repeat the numerical experiment for model C, where the ice thickness is 30 km (Fig. 3.10). Ice conductivity matters in model C. The EOIMF is stronger if ice is more conductive. Nevertheless, even if we consider a conductivity of $\sigma_i = 10^{-3}$ S/m, which is probably an exaggerated value, the EOIMF in model C remains weaker than in model A. It is worth noting that, due to change in the ocean's thickness (D), the mean speed in model C is 0.25 m/s, compared to 0.28 m/s in model A (see Tab. 3.1), which partially explains the lower value of the magnetic field.

3.4 Conclusions

We have presented the first numerical calculations of Europa's ocean-induced magnetic field (EOIMF) generated by the convective flow in Europa's subsurface ocean in the presence of Jupiter's magnetic field. Our calculations indicate that even for the most favorable ocean and ice conductivities and thicknesses, the

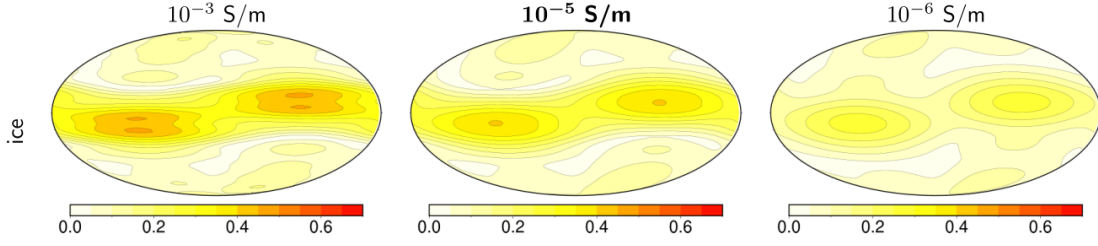


Figure 3.10 EOIMF strength [nT] as a function of ice conductivity for thickness model C. The default conductivity value is in bold. Snapshots in time $t = t_{spin} + 4$ h.

magnitude of EOIMF is below 1 nT on the Europa’s surface, which is more than one order of magnitude less than estimated by Vance et al. (2021). The discrepancy is primarily attributable to the complete solution of the electromagnetic induction equation in our study (versus a simple scaling relation in Vance et al. (2021)) and the more sluggish ocean flow used to compute the EOIMF, which is a consequence of a newly derived scaling relation for the velocity field. Furthermore, we have demonstrated that the convection cells in the meridional planes may generate equally strong EOIMF as the zonal jets despite the order of magnitude smaller velocities in the convection cells.

While the ocean flow averaged over time is symmetric about the equator, only the zonal component of the calculated EOIMF is symmetric, but the radial and meridional components show no symmetry. Nevertheless, partial EOIMFs generated by the vertical/horizontal JMF and the zonal/meridional-plane flow are symmetric or antisymmetric. In the presence of vertical JMF, the flow in convection cells generates rotationally symmetric EOIMF, which reflects (to a certain degree) the geometry of the flow: meridional velocities in the radial component and vice versa. Consequently, the structures in the radial and meridional components of the total EOIMF may resemble zonal stripes.

These results have significant consequences for the Juice and Europa Clipper missions. If we successfully retrieved the EOIMF signal from the satellite measurements of the magnetic field, we could use it to address additional Level-1 scientific questions (Roberts et al., 2023; Van Hoolst et al., 2024), such as identifying the ocean flow mode from the EOIMF structure. However, our results suggest that the EOIMF signal is on the edge of detectability of the upcoming satellite missions. For example, the Europa Clipper mission has a required sensitivity of magnetic field measurements of 1.5 nT (Kivelson et al., 2023). The EOIMF signal will be further attenuated with the increasing distance from Europa’s surface. For a typical Juice flyby at $h = 400$ km, the respective attenuation factors $(a/(a+h))^{j+2}$ for the SH degrees $j = 2, \dots, 6$ are 0.40, 0.32, 0.25, 0.20 and 0.16. The Europa Clipper mission is expected to get as close as 25 km, implying the attenuation factors 0.94, 0.92, 0.91, 0.89 and 0.88 for the same SH degrees. Last but not least, isolating the EOIMF from the aggregate magnetic imprint of the complex Europa’s interactions with Jupiter’s magnetosphere will represent a challenging problem of its own.

Acknowledgments

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Data availability

Data can be downloaded from <https://zenodo.org/records/13838989>.

Use of AI

During the preparation of this work the author(s) used Grammarly and DeepL in order to improve grammar and readability.

3.A Ocean model

3.A.1 Determining the Rayleigh number

In the following, we assume that any quantity X characterising the ocean model can be expressed in the form (e.g. Christensen and Aubert, 2006)

$$X \sim Pr^a Ek^b Ra_Q^c, \quad (\text{A.13})$$

where a, b, c are scaling exponents and Ra_Q is the flux-based Rayleigh number,

$$Ra_Q = \frac{1}{4\pi r_o(r_o - D)} \frac{\alpha g_o Q_{adv}}{\rho_w C_p \omega^3 D^2} = Ra(Nu - 1) Ek^3 Pr^{-2}. \quad (\text{A.14})$$

Here, r_o is the ocean outer boundary radius, Q_{adv} is the advected heat flow, ρ_w is the water density, C_p is the thermal capacity of water at constant pressure and Nu is the Nusselt number,

$$Nu = \frac{1}{4\pi r_o(r_o - D)} \frac{QD}{\rho_w C_p \kappa \Delta T}, \quad (\text{A.15})$$

where Q is the total heat flow through the ocean, containing the advected and the convective part.

Since the Rayleigh number cannot be determined directly from Eq. (3.4) in the main text because of the lack of information on ΔT , we derive a scaling law for Ra in the form of Eq. (A.13) using the least squares method applied to the dataset in Tab. 3.5 (see also Fig. 3.11). In Tab. 3.3, we compare the scaling law derived in this study with those used in the literature and present the corresponding

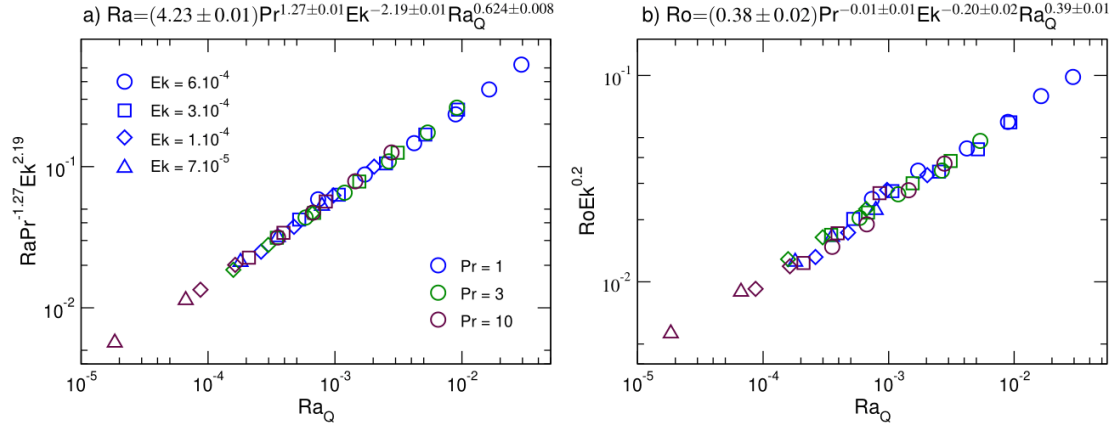


Figure 3.11 Graphs illustrating the scaling laws derived for a) Ra and b) Ro from the dataset in Tab. 3.5 using the least squares method. Since Ro depends only weakly on the Prandtl number, the dependence on Pr is omitted. Shapes and colors of the symbols indicate the values of parameters Ek and Pr used in the numerical simulation.

ranges of Ra , Ro_{loc} and Ro_c evaluated based on the parameter values detailed in the main text. The estimated Rayleigh numbers span approximately two orders of magnitude, $9.6 \cdot 10^{19} - 3.9 \cdot 10^{22}$, which is comparable to the range of $10^{20} - 10^{22}$ derived by Soderlund (2019), where the lower bound corresponds to the scaling law for the rapidly rotating convection (TG16) while the upper bound is obtained from the scaling law for a non-rotating ocean (TG15). The values in Tab. 3.3 indicate that Europa's ocean is in the transitional regime, as $Ro_{loc} \gg 1$ and $Ro_c \ll 1$.

Notation, reference	Radius ratio	Scaling law	Ra	Ro_{loc}	Ro_c
CH02, Christensen (2002)	0.35	$Ra = 12.99 Ek^{-2.00} Pr^{1.00} Ra_Q^{0.45}$	$3.7 \cdot 10^{20} - 4.7 \cdot 10^{21}$	203–607	0.05–0.08
TG15, Gastine et al. (2015)	0.60	$Ra = 7.35 Ek^{-2.25} Pr^{1.5} Ra_Q^{0.75}$	$1.5 \cdot 10^{21} - 3.9 \cdot 10^{22}$	827–5068	0.11–0.23
TG16, Gastine et al. (2016)	0.60	$Ra = 2.14 Ek^{-2.00} Pr^{0.8} Ra_Q^{0.40}$	$9.6 \cdot 10^{19} - 1.1 \cdot 10^{21}$	52–144	0.03–0.04
KC22, Kverka and Čadek (2022)	0.80	$Ra = 14.73 Ek^{-2.00} Pr^{1.0} Ra_Q^{0.53}$	$9.6 \cdot 10^{19} - 1.4 \cdot 10^{21}$	52–177	0.03–0.04
LS24, this study	0.90	$Ra = 4.61 Ek^{-2.19} Pr^{1.27} Ra_Q^{0.62}$	$1.3 \cdot 10^{21} - 2.7 \cdot 10^{22}$	728–3416	0.10–0.18

Table 3.3 Selected scaling laws proposed for estimating the Rayleigh number (column 3) and the values of Ra , Ro_{loc} , and Ro_c corresponding to Europa's ocean (columns 4-6). For the definitions of Ro_{loc} and Ro_c , see Eqs. (3.5) and (3.6) in the main text. The original scaling laws presented in the cited literature have been rewritten to the form of Eq. (A.13), where we assumed $Nu \gg 1$ in Eq. (A.14). The scaling laws CH02 and KC22 were derived for models with free-slip boundaries without assuming a priori a specific convection regime. In contrast, TG15 and TG16 were derived for non-rotating and rapidly rotating no-slip models, respectively. LS24 is derived from simulations with free-slip boundaries placed in the transitional regime. The presented scaling law also differs in the value of Pr for which they are applicable: CH02, KC22, and LS24 were derived considering a range of Pr , whereas TG15 and TG16 assumed $Pr = 1$. The dependence on Pr in TG15 and TG16 results from using a molecular value, $Pr = 10$, in Eq. (A.14), a practice frequently used in the literature (e.g., Soderlund et al., 2014; Soderlund, 2019; Amit et al., 2020; Cabanes et al., 2024).

Therefore, the scaling laws TG15 and TG16 are not applicable because they are relevant to non-rotating and rapidly rotating regimes, respectively. The scaling laws CH02 and KC22 may only hold within a limited range near the onset of convection (Cheng and Aurnou, 2016) and, thus, may be inappropriate for Europa.

The scaling law LS24 used in this study bridges the gap between the non-diffusive scaling laws suitable for rapidly rotating oceans on the one side and non-rotating oceans on the other, reducing the range of admissible Ra by an order of magnitude. LS24 is suitable for subsurface oceans in the transitional regime, such as Europa, unlike TG15 and TG16, which have been derived for different convection regimes.

3.A.2 Velocity of ocean flow

The velocity of the ocean flow can be estimated using Eq. (A.13) with $X = Ro$, where Ro is the Rossby number, defined as the root mean square velocity of the ocean flow, U , scaled by ωD

$$Ro = \frac{U}{\omega D} . \quad (\text{A.16})$$

The scaling law for Ro is again obtained by the least squares analysis of the dataset in Tab. 3.5 ($Ro = 0.38Ek^{-2}Ra_Q^{0.39}$, see Fig. 3.11b). In order to determine U , we first calculate Ro from the above scaling law into which we substitute the values of Ra_Q , Eq. (A.14), and Ek , Eq. (3.4), corresponding to Europa's ocean, and then calculate U from Eq. (A.16). Values for U obtained in this way range from 8 to 28 cm/s. This range is consistent with the findings of Lemasquerier et al. (2023), who reported velocities in a range of 3.5–22 cm/s for heterogeneous heat flux conditions at the bottom boundary. The analysis of energetic constraints by Jansen et al. (2023) predicts velocities of 1.5–6 cm/s, slightly lower than those obtained by LS24. Considering the eddy viscosities of $O(10^{-3}) \text{ m}^2/\text{s}$ Bire et al. (2022) found velocities in the range of mm/s. In contrast to other GCM studies that have implemented the eddy viscosity concept (Kang et al., 2022b; Kang and Jansen, 2022; Ashkenazy and Tziperman, 2021; Ashkenazy et al., 2023) and assumed varying horizontal and vertical viscosities, Bire et al. (2022) assume a constant viscosity throughout the ocean volume. This allows the results of Bire et al. (2022) to be directly compared with those obtained in the present study using the DNS approach. Considering the eddy viscosity and diffusivity, as in Bire et al. (2022), we get $Pr = 1$ and $Ek \approx 10^{-9}$ – 10^{-8} . For these values, the LS24 scaling law predicts velocities ranging from 1 to 8 cm/s. These velocities are consistent with Jansen et al. (2023) but approximately ten times higher than those derived by Bire et al. (2022) for a freshwater ocean. For completeness, it should also be mentioned that Ashkenazy and Tziperman (2021) used the MITgcm code in a Europa setting, incorporating salinity and phase boundary effects, and reported velocities of the order of cm/s. For detailed information on the results discussed above, see Tab. 3.4.

Notation, reference	Radius ratio	Method, scaling law	U
CH02, Christensen (2002)	0.35	DNS, $Ro = 0.65 Ra_Q^{0.20}$	3–7 cm/s
TG15, Gastine et al. (2015)	0.60	DNS, $Ro = 0.88 Ek^{-0.1} Pr^{0.7} Ra_Q^{0.36}$	15–46 cm/s
AS21, Ashkenazy and Tziperman (2021)	0.92	GCM	$\sim$ cm/s
BI22, Bire et al. (2022)	0.76–0.96	GCM	$\sim$ mm/s
JA23, Jansen et al. (2023)	0.92	energetic constraints	1.5–6 cm/s
LE23, Lemasquerier et al. (2023)	0.90	DNS, not reported	3.5–22 cm/s
CA24, Cabanes et al. (2024)	0.80	DNS, not reported	21–61 cm/s
LS24, this study	0.90	DNS, $Ro = 0.38 Ek^{-0.2} Ra_Q^{0.39}$	8–28 cm/s

Table 3.4 Estimates of the mean flow speed in Europa’s ocean (column 4) derived from published scaling laws, energetic constraints, and GCM simulations.

3.B EOIMF

3.B.1 The Lorentz force for Configurations 1–4

In this section, we provide the analytical expressions for the Lorentz force, $\mathbf{L}$, and the right-hand side ($\nabla \times \mathbf{L}$) of Eq. (3.8) corresponding to Configurations 1–4 defined in Tab. 3.2. In Configuration 1, the Lorentz force is

$$\mathbf{L}_1 = v_\varphi B_{0\vartheta}^v \mathbf{e}_r + v_\varphi B_{0r}^v \mathbf{e}_\vartheta, \quad (\text{B.17})$$

where L_{1r} is symmetric and $L_{1\vartheta}$ is antisymmetric since the product of two symmetric (or two antisymmetric) fields is symmetric and the product of one symmetric and one antisymmetric field is antisymmetric. The right-hand side $\mathbf{R}_1$ of the EMI equation is then

$$\mathbf{R}_1(r, \vartheta) = \nabla \times \mathbf{L}_1 = \frac{1}{r} \left[L_{1\vartheta} - \frac{\partial L_{1r}}{\partial \vartheta} \right] \mathbf{e}_\varphi, \quad (\text{B.18})$$

which is antisymmetric with respect to the equator since a meridional derivative of a symmetric function is antisymmetric (and vice versa).

In Configuration 2, the Lorentz force is

$$\mathbf{L}_2 = v_\vartheta B_{0\varphi}^h \mathbf{e}_r - v_r B_{0\varphi}^h \mathbf{e}_\vartheta + (v_r B_{0\vartheta}^h - v_\vartheta B_{0r}^h) \mathbf{e}_\varphi, \quad (\text{B.19})$$

where $L_{2\vartheta}$ is symmetric and L_{2r} and $L_{2\varphi}$ are antisymmetric. The curl of the Lorentz force is

$$\begin{aligned} \mathbf{R}_2 = & \frac{1}{r \sin \vartheta} \left[\frac{\partial}{\partial \vartheta} (L_{2\varphi} \sin \vartheta) - \frac{\partial L_{2\vartheta}}{\partial \varphi} \right] \mathbf{e}_r + \frac{1}{r} \left[\frac{1}{\sin \vartheta} \frac{\partial L_{2r}}{\partial \varphi} - \frac{\partial}{\partial r} (r L_{2\varphi}) \right] \mathbf{e}_\vartheta + \\ & + \frac{1}{r} \left[\frac{\partial}{\partial r} (r L_{2\vartheta}) - \frac{\partial L_{2r}}{\partial \vartheta} \right] \mathbf{e}_\varphi, \end{aligned} \quad (\text{B.20})$$

where R_{2r} and $R_{2\varphi}$ are symmetric and $R_{2\vartheta}$ is antisymmetric.

In Configuration 3, the Lorentz force is

$$\mathbf{L}_3 = v_\varphi B_{0\vartheta}^h \mathbf{e}_r + v_\varphi B_{0r}^h \mathbf{e}_\varphi, \quad (\text{B.21})$$

where $L_{3\vartheta}$ is symmetric and L_{3r} is antisymmetric. The curl of the Lorentz force is

$$\mathbf{R}_3 = -\frac{1}{r\sin\vartheta}\frac{\partial L_{3\vartheta}}{\partial\varphi}\mathbf{e}_r + \frac{1}{r\sin\vartheta}\frac{\partial L_{3r}}{\partial\varphi}\mathbf{e}_\vartheta + \frac{1}{r}\left(L_{3\vartheta} - \frac{\partial L_{3r}}{\partial\vartheta}\right)\mathbf{e}_\varphi, \quad (\text{B.22})$$

where R_{3r} and $R_{3\varphi}$ are symmetric and $R_{3\vartheta}$ is antisymmetric.

In Configuration 4, the Lorentz force is symmetric,

$$\mathbf{L}_4 = (v_r B_{0\vartheta}^v - v_\vartheta B_{0r}^v)\mathbf{e}_\varphi, \quad (\text{B.23})$$

and its curl is

$$\mathbf{R}_4 = \frac{1}{r\sin\vartheta}\frac{\partial}{\partial\vartheta}(L_{4\varphi}\sin\vartheta)\mathbf{e}_r - \frac{1}{r}\frac{\partial}{\partial r}(rL_{4\varphi})\mathbf{e}_\vartheta, \quad (\text{B.24})$$

where $R_{4\vartheta}$ is symmetric and R_{4r} is antisymmetric.

3.B.2 Comparison with the model of Vance et al. (2021)

In this section, we compare our EOIMF results with the estimate provided by Vance et al. (2021). For this purpose, we consider the same flow model as in Vance et al. (2021), hereinafter referred to as Sod19, and use it to calculate the corresponding EOIMF by solving the EMI equation (see section 3.2.2). The flow model was first presented by Soderlund (2019) and we obtained it by solving Eqs. (3.1)–(3.3) for $Ek = 3 \cdot 10^{-4}$, $Ra = 3.4 \cdot 10^7$, $Pr = 1$ and $r_i/r_o = 0.9$ using our numerical code. Comparison of Sod19 (Fig. 3.12) with the model used in the present study (Fig. 3.1b) shows that both models are characterized by a strong retrograde zonal flow outside the tangential cylinder, a pair of prograde jets at higher latitude, and a non-negligible circulation in the meridional plane. However, the models differ in the flow speed, which is about ten times higher for Sod19 than for the model used in this study.

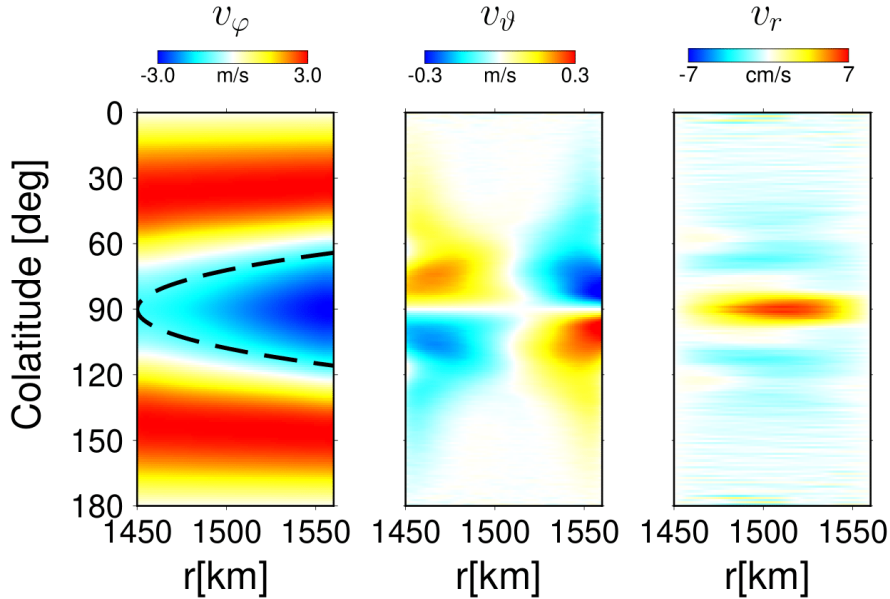


Figure 3.12 Flow model Sod19.

The EOIMF obtained for Sod19 by solving the EMI equation for the thickness model A is presented in Fig. 3.13. The figure is organized in the same way as

Fig. 3.5 in the main text, which allows a detailed comparison of both fields. While the differences in EOIMF patterns are minor, the magnitude of EOIMF obtained for Sod19 ($\lesssim 7$ nT) is almost an order of magnitude larger than that predicted for our flow model and a factor of three smaller than that reported by Vance et al. (2021) (20 nT, see Table 5 therein). The former discrepancy is related to the fact that Vance et al. (2021) used Sod19 without scaling the flow speed, whereas the speed in our flow model is corrected using the appropriate scaling law (see section 3.A.2). The latter discrepancy can be attributed to the differences in the methods used to calculate the motionally induced field strength. The estimate of Vance et al. (2021) is calculated using the formula $B \sim \mu_0 \sigma D U B_0$, unlike our EOIMF estimate, which is based on the solution of the general EMI equation.

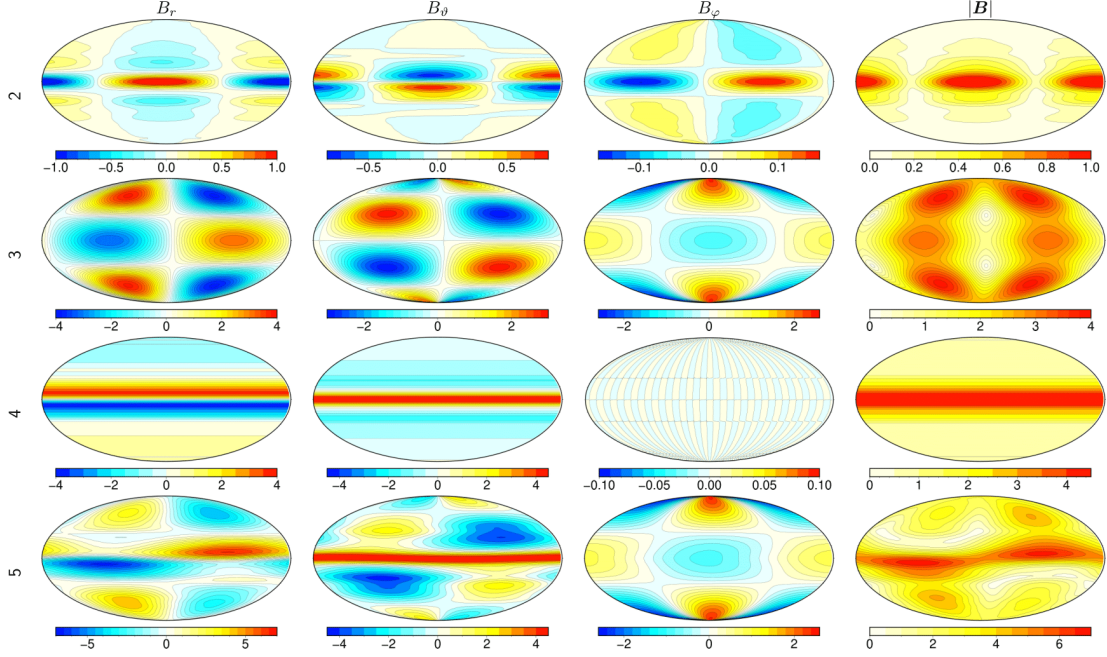


Figure 3.13 EOIMF [nT] four hours after the spin up calculated for the flow model Sod19. The figure is organized in the same way as Fig. 3.5 in the main text.

3.C Table of ocean simulations

Model No.	N_r	j_{max}	Pr	Ek	Ra	Ro_{loc}	Ro_c	R_M	Nu	Ro
1	61	157	1	$6 \cdot 10^{-4}$	$6.67 \cdot 10^5$	4.67	0.49	2.00	5.11	0.11
2	61	157	1	$6 \cdot 10^{-4}$	$1.00 \cdot 10^6$	7.00	0.60	3.00	7.95	0.15
3	73	177	1	$6 \cdot 10^{-4}$	$1.67 \cdot 10^6$	11.67	0.77	5.00	11.67	0.19
4	73	177	1	$6 \cdot 10^{-4}$	$2.67 \cdot 10^6$	18.68	0.98	8.00	15.49	0.26
5	73	207	1	$6 \cdot 10^{-4}$	$4.00 \cdot 10^6$	28.02	1.20	12.00	18.95	0.35
6	85	257	1	$6 \cdot 10^{-4}$	$6.00 \cdot 10^6$	42.02	1.47	18.00	22.68	0.43
7	73	177	1	$3 \cdot 10^{-4}$	$2.19 \cdot 10^6$	5.05	0.44	2.00	8.84	0.10
8	73	177	1	$3 \cdot 10^{-4}$	$3.28 \cdot 10^6$	7.58	0.54	3.00	12.09	0.14
9	73	207	1	$3 \cdot 10^{-4}$	$5.47 \cdot 10^6$	12.64	0.70	5.00	17.05	0.17
10	81	237	1	$3 \cdot 10^{-4}$	$8.76 \cdot 10^6$	20.22	0.89	8.00	21.78	0.22
11	81	285	1	$3 \cdot 10^{-4}$	$1.31 \cdot 10^7$	30.32	1.09	12.00	26.29	0.30
12	73	217	1	$1 \cdot 10^{-4}$	$1.44 \cdot 10^7$	5.73	0.38	2.00	18.17	0.08
13	73	247	1	$1 \cdot 10^{-4}$	$2.16 \cdot 10^7$	8.60	0.46	3.00	22.01	0.11
14	73	377	1	$1 \cdot 10^{-4}$	$3.60 \cdot 10^7$	14.33	0.60	5.00	27.00	0.18
15	97	413	1	$1 \cdot 10^{-4}$	$5.76 \cdot 10^7$	22.92	0.76	8.00	35.32	0.21
16	81	257	1	$7 \cdot 10^{-5}$	$2.65 \cdot 10^7$	5.97	0.36	2.00	19.80	0.08
17	81	257	1	$7 \cdot 10^{-5}$	$3.98 \cdot 10^7$	8.95	0.44	3.00	25.98	0.11
18	73	347	1	$7 \cdot 10^{-5}$	$6.63 \cdot 10^7$	14.92	0.57	5.00	34.69	0.15
19	65	177	3	$6 \cdot 10^{-4}$	$2.00 \cdot 10^6$	7.25	0.49	2.63	12.23	0.09
20	65	177	3	$6 \cdot 10^{-4}$	$3.00 \cdot 10^6$	10.87	0.60	3.95	16.54	0.12
21	73	197	3	$6 \cdot 10^{-4}$	$5.00 \cdot 10^6$	18.11	0.77	6.58	22.13	0.15
22	73	197	3	$6 \cdot 10^{-4}$	$8.01 \cdot 10^6$	28.98	0.98	10.53	27.96	0.21
23	73	227	3	$3 \cdot 10^{-4}$	$6.57 \cdot 10^6$	7.84	0.44	2.63	17.69	0.09
24	73	227	3	$3 \cdot 10^{-4}$	$9.85 \cdot 10^6$	11.76	0.54	3.95	23.17	0.11
25	73	227	3	$3 \cdot 10^{-4}$	$1.64 \cdot 10^7$	19.61	0.70	6.58	31.55	0.15
26	85	267	3	$3 \cdot 10^{-4}$	$2.63 \cdot 10^7$	31.37	0.89	10.53	39.57	0.20
27	85	267	3	$1 \cdot 10^{-4}$	$4.32 \cdot 10^7$	8.89	0.38	2.63	32.96	0.08
28	85	267	3	$1 \cdot 10^{-4}$	$6.48 \cdot 10^7$	13.34	0.46	3.95	41.45	0.10
29	85	297	3	$1 \cdot 10^{-4}$	$1.08 \cdot 10^8$	22.23	0.60	6.58	55.25	0.14
30	73	177	10	$6 \cdot 10^{-4}$	$6.67 \cdot 10^6$	11.73	0.49	3.56	24.68	0.07
31	85	237	10	$6 \cdot 10^{-4}$	$1.00 \cdot 10^7$	17.59	0.60	5.33	30.99	0.08
32	85	237	10	$6 \cdot 10^{-4}$	$1.67 \cdot 10^7$	29.32	0.77	8.89	40.08	0.12
33	97	267	10	$6 \cdot 10^{-4}$	$2.67 \cdot 10^7$	46.91	0.98	14.23	48.36	0.16
34	85	237	10	$3 \cdot 10^{-4}$	$2.19 \cdot 10^7$	12.70	0.44	3.56	35.50	0.06
35	85	237	10	$3 \cdot 10^{-4}$	$3.28 \cdot 10^7$	19.04	0.54	5.33	44.27	0.09
36	97	285	10	$3 \cdot 10^{-4}$	$5.47 \cdot 10^7$	31.74	0.70	8.89	57.30	0.14
37	97	267	10	$1 \cdot 10^{-4}$	$1.44 \cdot 10^8$	14.39	0.38	3.56	60.53	0.06
38	97	267	10	$1 \cdot 10^{-4}$	$2.16 \cdot 10^8$	21.59	0.46	5.33	75.77	0.08
39	97	285	10	$7 \cdot 10^{-5}$	$1.33 \cdot 10^8$	7.50	0.25	1.78	40.45	0.04
40	113	321	10	$7 \cdot 10^{-5}$	$2.65 \cdot 10^8$	14.99	0.36	3.56	73.29	0.06

Table 3.5 List of numerical simulations used to derive the scaling laws for Europa’s ocean. Symbols N_r and j_{max} denote the number of the Chebyshev points and the cut-off degree used to discretize Eqs. (3.1)–(3.3) in the radial and lateral directions, respectively. The results of the simulations are represented by the Nusselt (Nu) and Rossby (Ro) numbers.

4 Convective flow in Ganymede’s subsurface ocean: Implications for the induced magnetic field and topography

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Abstract

Subsurface oceans of icy moons are inaccessible to direct observation and numerical modeling is currently the only way to study the dynamics of these systems. Here, we present the first comprehensive study of Ganymede’s ocean based on the numerical simulations of thermal convection in a rotating spherical shell, and discuss the implications of the flow circulation for Ganymede’s long-wavelength topography and induced magnetic field. In order to determine the structure of the flow in Ganymede’s ocean, we have performed 128 numerical simulations, varying all relevant control parameters (Ra , Ek , Pr) by at least one order of magnitude. Based on this data set, we predict that the ocean circulation is characterized by a retrograde equatorial jet, meridional circulation cells at low latitudes and narrow upwellings and downwellings in the polar regions. The mean speed of the ocean flow ranges from a few cm/s for a thin (< 100 km) ocean to 0.8 m/s for a 500 km thick ocean. The time-averaged heat flux from the ocean varies with the latitude reaching the maximum at the poles and the minimum at mid-latitudes. Assuming that the heat transfer in the ice shell is dominated by conduction, we determine Ganymede’s long-wavelength topography generated by the uneven heat flux from the ocean. We predict that Ganymede’s polar regions are flat or even elevated and the global topographic pattern is dominated by an equatorial depression and elevations at mid-latitudes. Preliminary calculations of the magnetic field, induced by the flow of the salty ocean in the presence of Ganymede’s internal magnetic field, predict sufficiently large amplitudes to allow their detection by the Juice and Europa Clipper spacecrafts. We demonstrate that the pattern of the flow-induced magnetic field strongly depends on the geometry of ocean circulation, suggesting that accurate magnetic measurements can provide constraints on the dynamics of Ganymede’s ocean.

4.1 Introduction

Jupiter’s moon Ganymede is the largest natural satellite in the Solar System. It is a fully differentiated body with a liquid metallic core, a silicate mantle and a ~ 800 km thick hydrosphere (Schubert et al., 2004; Vance et al., 2018). Analysis of the magnetic field and thermodynamic considerations suggest that Ganymede has a thick ocean trapped between two layers of ice (Khurana et al., 1998; Kivelson et al., 2002; Vance et al., 2021). The exact composition of the

ocean is not known, but the spectroscopic observations of Ganymede’s surface made by the Galileo spacecraft (McCord et al., 2001) indicate that the ocean is likely to contain a significant amount of magnesium sulfate (MgSO_4) and possibly also sodium sulfate (Na_2SO_4). The thickness of the ocean is not well constrained. It has been estimated to be 200–300 km based on the Galileo electromagnetic data (Kivelson et al., 2002) and 275–460 km based on the material properties and thermodynamic modeling of water ice phases Ih, II, III, V and VI and seawater of different composition (Vance et al., 2018, 2021). Two space missions will explore Ganymede in the early 2030s: NASA’s Europa Clipper will conduct several close flybys of Ganymede beginning 2030 and a little later ESA’s Jupiter Icy Moon Explorer (Juice) will orbit the moon for a period of about one year. The missions will study the surface’s geochemistry and geomorphology through various imaging techniques and are expected to provide detailed information on Ganymede’s shape and its gravitational and magnetic fields.

Previous studies have suggested that the shape and the tectonic evolution of outer ice shells of icy moons are closely linked to the dynamics of the subsurface ocean (e.g., Goodman et al., 2004; Soderlund et al., 2014; Kvorka et al., 2018). It has been shown that the heat flux from the ocean is not homogeneous but varies with the latitude due to the Coriolis effect (e.g., Soderlund, 2019; Amit et al., 2020; Kvorka and Čadek, 2022), which can lead to lateral changes in ice thickness, affecting the topography and geoid at low harmonic degrees (Čadek et al., 2019; Kvorka and Čadek, 2024). Recent studies by Vance et al. (2021) and Šachl et al. (2025) have indicated that the flow in an electrically conductive subsurface ocean can generate a non-negligible magnetic field that can be, at least in principle, detected by spacecraft magnetometers. These findings underscore the need for developing a realistic model of Ganymede’s ocean which could be taken into account in interpreting the data from space missions.

Compared to the oceans on Enceladus and Europa, which have been investigated in a number of studies (e.g., Soderlund et al., 2014; Ashkenazy and Tziperman, 2021; Bire et al., 2022; Ashkenazy et al., 2023; Kang and Jansen, 2022; Lemasquerier et al., 2023; Zhang et al., 2024; Kang, 2024; Šachl et al., 2025), Ganymede’s ocean has so far received little attention. Although the ocean is similar in some respects to the ocean on Titan (thick ocean between two ice layers), the available studies are either based on a limited number of simulations (Soderlund, 2019; Cabanes et al., 2024) or the simulations have been performed for only large ocean thicknesses (Amit et al., 2020; Kvorka and Čadek, 2022; Terra-Nova et al., 2023). In this paper, we present the first comprehensive study of Ganymede’s ocean. Based on the set of 128 numerical simulations of thermal convection in a rotating spherical shell, we propose a new model of Ganymede’s ocean circulation and discuss its implications for the long-wavelength topography and the induced magnetic field.

4.2 Modeling the dynamics of subsurface oceans

Subsurface oceans of icy moons are inaccessible to direct observation and numerical modeling is currently the only way to study the dynamics of these systems. If the material properties of the ocean are homogeneous and the buoyancy force is proportional to temperature, the dynamics of the ocean can be fully

described by four dimensionless parameters: the radius ratio r_i/r_o and the Rayleigh, Ekman and Prandtl numbers (see also section 4.3.1):

$$Ek = \frac{\nu}{\omega D^2}, \quad Ra = \frac{\alpha g_o \Delta T D^3}{\nu \kappa}, \quad Pr = \frac{\nu}{\kappa}, \quad (4.1)$$

where r_i and r_o are the inner and outer radii of the ocean, respectively, D is the ocean thickness, ν is the kinematic viscosity, ω is the angular velocity, α is the coefficient of thermal expansion, g_o is the gravitational acceleration at the outer boundary of the ocean, ΔT is the superadiabatic temperature contrast across the ocean, and κ is the thermal diffusivity. The Rayleigh and Ekman numbers typical of subsurface oceans are of the order of 10^{16} – 10^{24} and 10^{-13} – 10^{-10} , respectively (Soderlund, 2019). Since the numerical simulations of global ocean dynamics with realistic values of Ra and Ek are out of reach of current computational algorithms, the subsurface oceans have been studied using two approaches, each based on different assumptions and simplifications.

The first approach, hereinafter referred to as DNS (Dimensionless Numerical Simulations), relies on the numerical simulations performed for computationally feasible values of Ra and Ek (typically $Ra \leq 10^9$ and $Ek \geq 10^{-6}$). It assumes that the changes in the character of ocean circulations associated with the increase of Ra and the decrease of Ek in the numerical models can be extrapolated to the conditions of the subsurface ocean (i.e., $Ra \geq 10^{16}$ and $Ek \leq 10^{-10}$). The extrapolation is usually based on the statistical processing of a large set of numerical models and the transition between different convection regimes is characterized by suitable “diagnostic” parameters, such as the convective Rossby number, Ro_c , or the local convective Rossby, Ro_{loc} (see below for details). Since Ra , Ek and Pr are assumed to be spatially constant, the flow can be computed using a spectral method (e.g., Christensen and Wicht, 2007). The approach has been used in a number of studies addressing the heat transfer in the oceans of Europa and Titan (e.g., Soderlund et al., 2014; Amit et al., 2020; Terra-Nova et al., 2023; Lemasquerier et al., 2023; Kvorka and Čadek, 2024; Šachl et al., 2025), and investigating how the style of the convective heat transfer depends on the control parameters (e.g., Gastine et al., 2016; Soderlund, 2019; Kvorka and Čadek, 2022; Cabanes et al., 2024). Recent studies have also explored the impact of uneven heat production in the silicate interiors of Europa and Enceladus on ocean circulation and its role in the evolution of the ice shell (Lemasquerier et al., 2023; Bouffard et al., 2025). The DNS studies neglect the effect of salinity on the density of water, partly because the exact chemical composition of the oceans is not known, and partly because adding a new parameter would complicate the scaling analysis.

The second approach, hereinafter referred to as the GCM (Global Circulation Models), is based on the methodology developed to study the dynamics of Earth’s oceans. The flow in the subsurface ocean is calculated with the effective viscosity, which characterizes the transport and dissipation of energy on the scales that cannot be resolved by the numerical model (e.g., Visbeck et al., 1997). The default effective viscosity and diffusivity in the GCM models are set to represent the high-frequency turbulence driven by tides and libration (see Wunsch and Ferrari, 2004, for a review). The values are estimated from the tidal dissipation rate (e.g., Kang et al., 2022a) and range from 10^{-4} m/s² to 10^{-2} m/s² (e.g., Ashkenazy and

Tziperman, 2021; Kang, 2022; Bire et al., 2022). Since the flow is solved using the finite volume method, the material properties of water can vary in space and time. Unlike DNS, the GCM approach does not require large sets of simulations, which allows the models to have a higher resolution than the DNS models. Recent GCM studies have employed the non-hydrostatic version of the MITgcm code (Marshall et al., 1997) to explore the impact of salinity variations and satellite size on the ocean dynamics (e.g, Ashkenazy and Tziperman, 2021; Lobo et al., 2021; Zeng and Jansen, 2021; Kang, 2022). Their results suggest that the effect of salinity might play a crucial role in the heat transfer in small icy moons, possibly explaining the hemispheric asymmetry observed on Enceladus (Kang et al., 2022a,c). In contrast, such asymmetry is not expected to occur on larger bodies (Kang and Jansen, 2022). For Europa, Ashkenazy and Tziperman (2021) simulated salinity-driven convection and proposed that efficient meridional heat transport could lead to a relatively uniform ice shell thickness, with ocean flow speeds one to two orders of magnitude lower than previously estimated by Soderlund (2019). Bire et al. (2022) employed a model based on the deep Cartesian beta-plane approximation (Dellar, 2011), implemented using the Oceananigans.jl framework (Ramadhan et al., 2020). They explored freshwater ocean dynamics across a range of aspect ratios (0.76–0.96) and low Ekman numbers (10^{-8} to 10^{-5}), previously unexplored in this context. The results of Bire et al. (2022) support the presence of Jovian-like convection in the subsurface oceans (Ashkenazy and Tziperman, 2021), characterized by a turbulent geostrophic flow that gives rise to Taylor columns and counter-rotating zonal jets (cf. Soderlund, 2019; Šachl et al., 2025).

In this study, we do not consider the possibility that Ganymede’s ocean is stratified. As Ganymede is a large body, the pressure in the ocean is high enough to suppress the formation of a stable layer below the outer ice shell associated with the anomalous behavior of water at low pressures and temperatures (Melosh et al., 2004). We also neglect the effect of double-diffusive convection (Vance and Brown, 2005) and salinity-driven processes (Ashkenazy and Tziperman, 2021). Based on the estimate of the super-criticality of Ganymede’s ocean by Soderlund (2019), we assume that convection adopts a three-dimensional structure and fills the entire volume of the ocean.

Depending on the relative importance of the Coriolis force, convection in a rotating, subsurface ocean can occur in three regimes: rapidly rotating, transitional, and non-rotating (Gastine et al., 2016; Cheng et al., 2018). The transitions between the regimes are governed by two dimensionless numbers, the local convective Rossby number, Ro_{loc} (Julien et al., 2012), and the convective Rossby number, Ro_c (Gilman, 1977):

$$Ro_{loc} = RaEk^{8/5}Pr^{-3/5}, \quad Ro_c = Ra^{1/2}EkPr^{-1/2}. \quad (4.2)$$

The *rapidly rotating regime* is bounded by $Ro_{loc} \leq 1$. The flow in this regime is characterized by a balance between the pressure gradient and the Coriolis force (“geostrophic flow”), resulting in the formation of Taylor columns aligned with the rotation axis and series of zonal jets outside the tangent cylinder (the cylinder of radius r_i aligned with the rotation axis). The distribution and the size of the jets have been recently investigated by Bire et al. (2022) and later by Cabanes et al. (2024) who derived a theoretical upper bound for the speed of jets based on the analysis of the velocity spectra. Recent DNS studies have suggested

that the rate of heat transfer in the rapidly rotating regime significantly varies with the latitude reaching a maximum near the equator (Soderlund, 2019; Amit et al., 2020; Kvorka and Čadek, 2022; Gastine and Aurnou, 2023). The equatorial cooling is a consequence of the fact that in a rapidly rotating shell, the onset of convection inside the tangent cylinder (i.e., in the polar region) requires much larger convective forcing than near the equator (Jones et al., 2000; Dormy et al., 2004; Gastine and Aurnou, 2023). This means that under certain conditions, heat can be transferred by conduction inside and by convection outside the tangent cylinder, resulting in strong latitudinal variations in the heat flux.

Most of the icy moons’ oceans are in *the transitional regime* (Soderlund, 2019). The regime is bounded by $Ro_{loc} \geq 1$ and $Ro_c \leq 1$ (Gilman, 1977; Cheng et al., 2018) and is characterized by the decreasing importance of the Coriolis force and the growth of turbulence with increasing Ro_c . As the effect of rotation decreases, the vigor of convection inside the tangent cylinder increases and the heat flux maxima, located at the equator in the rapidly rotating regime, move towards the poles: the global cooling pattern changes from equatorial to polar (Yadav et al., 2016; Soderlund, 2019; Amit et al., 2020; Kvorka and Čadek, 2022). The ocean circulation is characterized by large, alternating zonal jets that can fill the entire shell. At low Ro_c , the flow is dominated by a prograde flow at the equator near the outer boundary (e.g., Soderlund, 2019; Kvorka and Čadek, 2022). As Ro_c increases, the equatorial flow changes from prograde to retrograde (e.g., Aurnou et al., 2007; Yadav et al., 2016). This overturn is accompanied by the formation of Hadley-like circulation cells at low latitudes and the emergence of prograde jets at higher latitudes (Soderlund, 2019; Šachl et al., 2025). As Ro_c increases, the ocean moves to *the non-rotating regime*, and for $Ro_c \gg 1$, the effect of rotation on the convective heat transfer becomes negligible (Cheng et al., 2018).

The study of subsurface ocean circulation is complicated by two factors: the salinity of the ocean water and the fact that the upper boundary of the ocean (and, on Titan and Ganymede, perhaps the bottom boundary as well) is a phase-change interface. The exact composition of the subsurface oceans is not known. It depends on the size and composition of the silicate mantle or core and the efficiency of the mass transport across the bottom boundary of the ocean (Soderlund et al., 2020). It has been suggested that salinity plays a crucial role in the formation of ice thickness variations on Enceladus (e.g., Lobo et al., 2021; Zeng and Jansen, 2021; Kang et al., 2022a,c; Kang and Jansen, 2022). The role of salinity variations in ocean dynamics of large icy moons is a subject of debate (see, e.g., Zeng and Jansen, 2021; Ashkenazy and Tziperman, 2021; Wong et al., 2022; Kang et al., 2022a; Bire et al., 2023; Bouffard et al., 2025).

The presence of a phase transition on the boundary of the ocean means that the ocean is an open thermodynamic system which can exchange both energy and matter with its surroundings. The phase transition affects the ocean circulation in two ways. First, the melting of ice and the freezing of water change the salinity distribution near the boundary and thus generate additional buoyancy forces (e.g., Lobo et al., 2021; Kang et al., 2022a,c). Second, variations in the melting temperature along the boundary associated with lateral changes in ice thickness can generate a flow that tends to equilibrate the temperature at the boundary (e.g., Labrosse et al., 2018; Kang and Jansen, 2022; Kihoulou et al., 2023). The inclusion of this flow in a model of ocean dynamics is complicated by the fact that

the ice shell deforms and therefore the flow in the ocean must be coupled with the creep of ice (Kvorka and Čadek, 2024).

4.3 Model of Ganymede's ocean

4.3.1 Governing equations

We consider the convection of a Boussinesq fluid contained in a spherical shell rotating at a constant angular speed, ω , around the z-axis. The outer radius, r_o , and the inner radius, r_i , of the shell are chosen to correspond to the models of Ganymede's structure proposed by Vance et al. (2018) (see Table 4.1). The boundaries of the shell are impermeable, free-slip, and isothermal. The equations governing the heat and mass transport are expressed in the dimensionless form,

$$\nabla \cdot \mathbf{v} = 0 , \quad (4.3)$$

$$\frac{1}{Pr} \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) + \frac{2}{Ek} \mathbf{e}_z \times \mathbf{v} = -\nabla p + \nabla^2 \mathbf{v} + Ra T g \mathbf{e}_r , \quad (4.4)$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \nabla^2 T , \quad (4.5)$$

and solved using the pseudospectral method developed by Kvorka and Čadek (2022). In the above equations, Pr , Ek and Ra denote the Prandtl, Ekman and Rayleigh numbers, respectively (Eq. 4.1), $\mathbf{v}$ is the velocity, p is the pressure, T is the temperature, g represents the radial changes in the gravitational acceleration ($g \propto r^{-2}$), $\mathbf{e}_r$ is the radial unit vector, and $\mathbf{e}_z$ is the unit vector parallel to the rotation axis.

4.3.2 Diagnostic parameters

The analysis presented below is based on 128 numerical simulations performed for $r_i/r_o = 0.80, 0.85$ and 0.90 , $Pr = 1, 3$ and 10 , $Ek \in \langle 6 \cdot 10^{-5}, 6 \cdot 10^{-4} \rangle$ and $Ra \in \langle 6.67 \cdot 10^5, 2.65 \cdot 10^8 \rangle$, see Tables 4.1–4.3. The results of the simulations are characterized by three dimensionless numbers, the Nusselt number, Nu , the Rossby number, Ro , and the transitional number, R_M , introduced by Šachl et al. (2025):

$$Nu = \frac{Q}{Q_{cond}} = \frac{1}{4\pi r_o r_i} \frac{QD}{\rho C_p \kappa \Delta T} , \quad Ro = \frac{U}{\omega D} , \quad R_M = Ra Ek^{12/7} Pr^{-3/4} , \quad (4.6)$$

where Q is the total heat flow, Q_{cond} is the conductive heat flow, ρ is the density of water, C_p is the heat capacity at constant pressure and U is the root-mean-square speed of the flow.

The Nusselt number represents the enhancement in heat transfer due to convection as compared to conduction. Since the value of the Rayleigh number, Ra , corresponding to the real ocean cannot be determined from Eq. (4.1) because the superadiabatic temperature contrast, ΔT , is not known, Ra is often estimated using the power law relations $Nu \propto Ra^\beta Ek^\gamma Pr^\delta$ and their theoretical limits derived for the rapidly rotating and non-rotating oceans (Soderlund, 2019; Cabanes et al., 2024). The disadvantage is that the range of admissible values of Ra obtained

in this way is rather wide ($Ra = 10^{20}$ – 10^{24} for Ganymede’s ocean, see Soderlund, 2019). In order to enhance the accuracy of the estimate, we transform Nu to the “flux-based” Rayleigh number, $Ra_Q = Ra(Nu - 1)Ek^3Pr^{-2}$ (Christensen and Aubert, 2006; Yadav et al., 2013; Kvorka and Čadek, 2022), and calculate Ra from the equation

$$Ra = \alpha Ra_Q^\beta Ek^\gamma Pr^\delta, \quad (4.7)$$

where parameters α , β , γ and δ are determined by the least squares method (see Appendix 4.A). The advantage of this approach is that all parameters on the right hand side of Eq. (4.7) are known or can be estimated. The data used in the regression analysis correspond to the transitional regime, i.e., parameters Ra , Ek and Pr in the numerical simulations are chosen so that $Ro_c < 1$ and $Ro_{loc} > 1$. Unlike the approach mentioned above, which provides the upper and lower bounds for Ra , the specific form of Eq. (4.7) derived here allows the Rayleigh number to be determined unambiguously provided that the total heat flow, Q , and the material parameters of the ocean are known and the subsurface ocean is in the transitional regime. The values of Ra obtained for different thicknesses of Ganymede’s ocean are listed in Table 4.1. Using the values of Pr , Ek and Ra given in Table 4.1, we obtain $Ro_c \leq 0.4$ and $Ro_{loc} \gtrsim 10^3$, implying that Ganymede’s ocean is likely in the transitional regime (see Fig. 4.1B for a graphical representation).

	r_o [km]	D [km]	H [km]	r_i/r_o	Pr	Ek	Ra_Q	Ra	U [m/s]	R_M
A	2629	518	5	0.80	10	$6.7 \cdot 10^{-13}$	$(1.4\text{--}5.8) \cdot 10^{-9}$	$(1.2\text{--}2.6) \cdot 10^{23}$	0.46–0.78	28–63
B	2608	493	26	0.81	10	$7.4 \cdot 10^{-13}$	$(1.6\text{--}6.7) \cdot 10^{-9}$	$(1.0\text{--}2.2) \cdot 10^{23}$	0.44–0.75	29–64
C	2564	361	70	0.86	10	$1.4 \cdot 10^{-12}$	$(3.0\text{--}11.9) \cdot 10^{-9}$	$(4.4\text{--}9.8) \cdot 10^{22}$	0.20–0.33	37–83
D	2500	119	134	0.95	10	$1.3 \cdot 10^{-11}$	$(2.6\text{--}10.6) \cdot 10^{-8}$	$(1.1\text{--}2.4) \cdot 10^{21}$	0.07–0.13	42–95
E	2478	24	157	0.99	10	$3.1 \cdot 10^{-10}$	$(6.3\text{--}25.2) \cdot 10^{-7}$	$(7.6\text{--}16.9) \cdot 10^{18}$	0.02–0.03	67–150

Table 4.1 Radius of the ocean outer boundary, r_o , ocean thickness, D , ice shell thickness, H , and radius ratio, r_i/r_o , of Ganymede’s ocean as proposed by Vance et al. (2018). The radius of Ganymede is fixed at 2634 km. The value of the Prandtl number, $Pr = 10$, corresponds to the molecular values of viscosity, $\nu = 1.8 \cdot 10^{-6} \text{ m}^2 \text{ s}^{-1}$, and thermal diffusivity, $\kappa = 1.8 \cdot 10^{-7} \text{ m}^2 \text{ s}^{-1}$ (Soderlund, 2019; Amit et al., 2020; Kvorka and Čadek, 2022). The Ekman number, Ek , is calculated for a rotation speed of 10^{-5} s^{-1} . The flux-based Rayleigh number, Ra_Q , is determined from Eq. (4.12) where $Q \in \langle 400, 1600 \rangle \text{ GW}$ (Choblet et al., 2017a). The Rayleigh number, Ra , and the root mean square speed, U , are calculated from Eqs. (4.7) and (4.8), respectively (see also Appendix 4.A). Calculations were performed for $\alpha = 2 \cdot 10^{-4} \text{ K}^{-1}$, $\rho = 1150 \text{ kg m}^{-3}$, $C_p = 3500 \text{ J kg}^{-1} \text{ K}^{-1}$, and $g_o = 1.4 \text{ m s}^{-2}$.

The Rossby number is a dimensionless measure of the mean flow speed. The physical value of the speed, U , which can be inferred from the Rossby number is important for two reasons: First, the flow in an electrically conductive ocean can produce an induced magnetic field, which can be measured by space missions and used to constrain the geometry of ocean circulation (Vance et al., 2021; Šachl et al., 2025). The magnitude of the induced magnetic field is roughly proportional to the flow speed (see Eq. 8 in Šachl et al., 2025) and, in the case of Ganymede, is expected to be large enough to imprint an observable signature on the magnetic field generated in Ganymede’s core (Vance et al., 2021). The second reason is that the flow velocity (both the speed and the direction) affects the shape of the boundary between the outer ice shell and the ocean (e.g., Kang et al., 2022a;

Kihoulou et al., 2023; Kvorka and Čadek, 2024). The shape changes at the phase boundary cause the creep of ice, leading to the formation of topographic features at the surface (e.g., Kvorka et al., 2018). As the size and the distribution of these features depends on the heat flux from the ocean (Čadek et al., 2019), analysis of topography can, in principle, provide information about processes in the ocean.

In this study, the Rossby number (and thus U) is estimated from the equation

$$Ro = \alpha_1 Ra_Q^{\beta_1} Ek^{\gamma_1} Pr^{\delta_1}, \quad (4.8)$$

where parameters α_1 , β_1 , γ_1 and δ_1 are determined from the data sets in Tables 4.1–4.3 using the least squares method (see Appendix 4.A for details). This approach was first used by Christensen and Aubert (2006) to demonstrate that Ro is independent of the Prandtl number and the magnetic Prandtl number in dynamo models. Recently, Šachl et al. (2025) have used the same method to place constraints on the flow speed in the subsurface ocean of Europa and Abbate and Aurnou (2023) applied this technique to analyze the velocities observed in their experimental study of convection within a rotating cylinder. Table 4.2 presents a comparison of the root mean square (rms) speed in Ganymede’s ocean obtained using different methods. Since it is likely that Ganymede’s ocean is enclosed between two layers of ice (Vance et al., 2018), only studies that assume a constant temperature at both boundaries are included. Depending on the extrapolation method, the speed estimates fall into three categories:

(i) VA21 assumes that the speeds derived from the numerical simulations are realistic despite the fact that the prescribed control parameters have unrealistic values. For Ganymede’s ocean, this method yields the speed ranging from approximately 20 cm for a 24 km thick ocean to nearly 4.5 m/s for a 518 km thick ocean (the values have been obtained using the same method and parameters as in Vance et al. (2021)). The main drawback of this approach is that it relies on a single simulation by Soderlund (2019) that is assumed to represent Ganymede’s ocean. While the flow structures predicted by Soderlund (2019) are consistent with the models obtained for Titan and Europa by Kvorka and Čadek (2022) and Šachl et al. (2025), the estimated flow speeds are likely to be overestimated by an order of magnitude (Ashkenazy and Tziperman, 2021; Cabanes et al., 2024; Šachl et al., 2025).

(ii) The CA24 estimate is derived under the assumption that the heat transfer in the subsurface ocean is controlled by zonostrophic convection. In this regime, the kinetic energy spectra show universal scaling with the wave number k : the zonal component scales as k^{-5} , while the residual (non-zonal) component follows a $k^{-5/3}$ scaling (see, e.g., Sukoriansky et al., 2002). Cabanes et al. (2024) numerically calibrated these power laws using simulations with low Ekman numbers ($Ek \leq 10^{-5}$) in a geometry representative of subsurface oceans ($r_i/r_o = 0.80$) and assumed that the power injected by buoyancy is dissipated by the Ekman friction in the boundary layers (cf., Jansen et al., 2023; Kang, 2024). The zonal flow speed obtained in this way for Ganymede does not exceed 1.66 m/s, while the residual speed is less than 6 cm/s (see Table 2 in Cabanes et al., 2024). These bounds correspond to an ocean thickness of 518 km and a heat flux of approximately 9.3 TW. The peak zonal and residual speeds obtained for the reference ocean configuration used in the present study, i.e., model C in Table 4.1 and $Q = 1$ TW, are 61 cm/s and 2.7 cm/s, respectively, and thus consistent with the results

presented in Figure 4.2 below. A limitation of this approach is the questionable suitability of the investigated range of parameters. Since the simulations with low Ekman numbers are computationally demanding, Cabanes et al. (2024) did not achieve the equatorial zonal flow overturn expected at high supercriticality (e.g., Heimpel and Aurnou, 2007), meaning that some models relevant to icy moons have remained unexplored.

(iii) The third category is represented by two methods, CH02 and KV25, which are based on the assumption that the Rossby number can be expressed as a power function of control parameters Ra , Ek and Pr (Eq. 4.8). This concept was derived from force balance arguments and numerically validated for both the rapidly-rotating and non-rotating regimes (see Gastine et al., 2015, 2016, for a review) but it is far from obvious that it is also valid in the transitional regime. The CH02 scaling assumes that the viscosity and thermal diffusivity have no influence on the system. As a result, the exponents γ_1 and δ_1 in Eq. (4.8) are set to zero (e.g., Christensen, 2002; Christensen and Aubert, 2006; Yadav et al., 2013). The non-diffusive scaling law has recently been used by Kvorka and Čadež (2022) and Lemasquerier et al. (2023), who proposed scaling laws for the subsurface oceans of Titan and Europa, respectively. Although their results show an apparent collapse of the data onto a single line, suggesting a good agreement between the data and the scaling law, the use of the non-diffusive scaling is problematic because the data are affected by a “shingling” effect (Cheng and Aurnou, 2016). Therefore, in the present study (KV25), we impose no a priori constraints on exponents γ_1 and δ_1 in Eq. (4.8). Even though our analysis is carried out for a limited range of control parameters, the results are consistent with those obtained using the more sophisticated approach of Cabanes et al. (2024) discussed above.

Notation, reference	Radius ratio	Method, scaling law	U
CH02, Christensen (2002)	0.35	DNS, $Ro = 0.65Ra_Q^{0.20}$	1–7 cm/s
VA21, Vance et al. (2021)	0.90	DNS, no scaling	20–445 cm/s
CA24, Cabanes et al. (2024)	0.80	DNS, decay of energy spectra	30–166 cm/s
KV25, this study	0.80–0.90	DNS, see Appendix 4.A	2–78 cm/s

Table 4.2 The mean flow speed in Ganymede’s ocean derived from different scaling laws.

The values of the exponents $\alpha_1 - \delta_1$ derived in this study are presented in Appendix 4.A. Inspection of Table 4.3 shows that the exponents change with the relative thickness of the ocean. The largest changes are found for δ_1 , which controls the dependence of the Rossby number (and thus the speed U) on the Prandtl number. While for $r_i/r_o = 0.80$, the Rossby number is weakly dependent on the Prandtl number ($Ro \propto Pr^{-0.1}$), the value of δ_1 obtained for $r_i/r_o = 0.90$ is comparable to the statistical error ($\delta_1 = 0.01 \pm 0.01$). A more pronounced dependence, $Ro \propto Pr^{-1/4}$, was found by Christensen (2002) for $r_i/r_o = 0.35$ and interpreted as a consequence of the dependence of non-zonal flow on the Prandtl number. Aubert (2005) proposed a scaling law for zonal jets that was independent of the Prandtl number while Fan et al. (2024) estimated that the dependence of the zonal Rossby number on the Prandtl number has the form

$Ro_Z \propto Pr^{-0.13}$. This indicates that the speed of zonal jets is poorly correlated with the Prandtl number and, unlike the non-zonal component, may not depend on it. The diminishing influence of the Prandtl number on the Rossby number with increasing radius ratio can then be attributed to increasing fraction of zonal flows in the kinetic energy budget (Yadav et al., 2016).

4.3.3 Flow characteristics

As already mentioned in the introduction, ocean circulation in the transitional regime features strong zonal jets driven by Reynolds stresses (see Busse, 1983, 1994 for a detailed discussion). Zonal jets are among the most studied effects in rotating convection (e.g. Gilman, 1977; Christensen, 2001, 2002; Aurnou et al., 2007; Gastine et al., 2013; Yadav et al., 2016; Bire et al., 2022; Cabanes et al., 2024, and references therein). Aurnou et al. (2007) and Yadav et al. (2016) suggested that the direction of the equatorial zonal jet flips from prograde to retrograde at $Ro_c \approx 1$. Gastine et al. (2013) identified a parameter controlling the equatorial zonal jet overturn based on the simulations of the dynamics of gas giants in the anelastic approximation. Using a large set of simulations, Šachl et al. (2025) have demonstrated that in the case of the incompressible Boussinesq approximation, the change of the flow direction can be described by the dimensionless number $R_M = RaEk^{12/7}/Pr^{3/4}$.

Šachl et al. (2025) investigated the magnetic field induced by convective flow in the subsurface ocean of Europa ($r_i/r_o = 0.90$). Based on the sign of the velocity vector at the equator, Šachl et al. (2025) categorized the flow patterns into two groups, denoted as Mode I and Mode II. The circulation in Mode I is characterized by a prograde zonal jet near the upper boundary in the equatorial region and relatively small meridional and radial velocities. In contrast, Mode II is characterized by a retrograde equatorial jet, meridional circulation cells in both hemispheres and upwellings at the equator. Šachl et al. (2025) demonstrated that the transition from Mode I to Mode II occurs at $R_M \approx 8$ for $r_i/r_o = 0.90$.

In this study, we have extended the dataset of Šachl et al. (2025) to include the ocean thickness range expected on Ganymede (see Table 4.1 for current models of ocean thickness and Tables 4.1–4.3 for the list of numerical simulations). Our analysis shows that for r_i/r_o between 0.80 and 0.90, the reversal in zonal jet direction, i.e., the transition from Mode I to Mode II, occurs at $R_M \approx 9$ (see Fig. 4.1). Note that $R_M \approx 9$ and $Ro_c \approx 1$ yield nearly identical results within the parameters range used in most DNS studies ($5 \cdot 10^{-5} \leq Ek \leq 10^{-3}$ and $Pr = 1$, see the double-dot-dashed and dotted lines in Fig. 4.1A). For comparison, we also include the results of two GCM studies (Bire et al., 2022; Ashkenazy and Tziperman, 2021). Despite the differences between the DNS and GCM approaches, such as the heat flux prescribed at the bottom of the ocean in both GCM models, different mechanical boundary conditions (no-slip in Ashkenazy and Tziperman, 2021, combination of free-slip and no-slip in Bire et al., 2022) and the inclusion of salinity in Ashkenazy and Tziperman (2021), $R_M = 9$ correctly predicts the equatorial flow direction.

Fig. 4.2 shows the time-averaged velocity distributions typical of Mode I (top panels) and Mode II (middle and bottom panels). Similarly to the flow patterns reported by Soderlund (2019) for $Pr = 1$, the time-averaged flow is dominated by

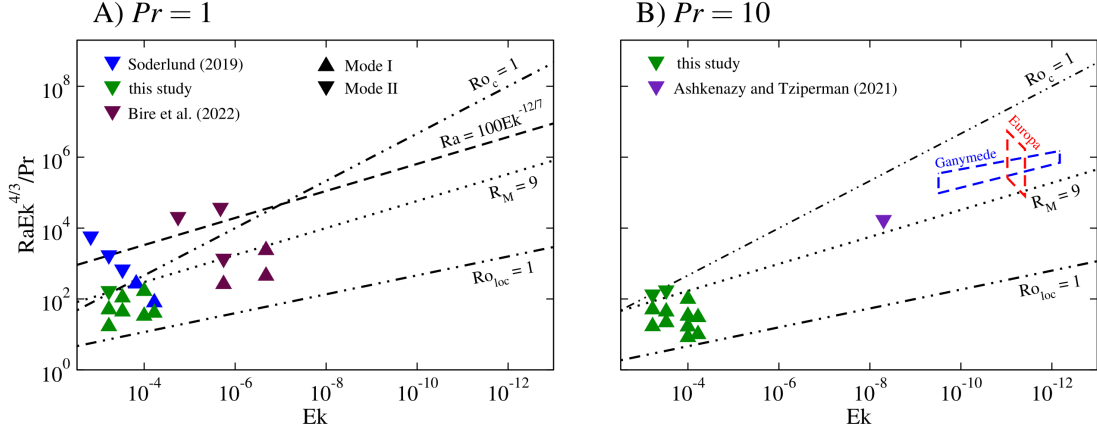


Figure 4.1 Direction of the equatorial flow obtained from DNS (green and blue) and GCM (violet and magenta) numerical simulations plotted as a function of the Ekman number and the supercriticality $RaEk^{4/3}/Pr$ for A) $Pr=1$ and B) $Pr=10$. Numerical models predicting the prograde (Mode I) and retrograde (Mode II) directions are represented by the up- and down-pointing triangles, respectively. The boundary between Mode I and Mode II derived in this study ($R_M=9$) is plotted by the dotted line. The double-dot-dashed lines ($Ro_{loc}=1$ and $Ro_c=1$) show the boundaries of the transitional regime. The dashed line in panel A, representing $Ra=100Ek^{-12/7}$, marks the upper boundary of the rotationally influenced heat transfer derived for $Pr=1$ by Gastine et al. (2016). The blue and red lines in panel B correspond to the parameters relevant to Ganymede’s (see Table 4.1) and Europa’s (Šachl et al., 2025) ocean, respectively.

strong zonal jets and weaker meridional circulation at low latitudes. In Mode I, convective flow outside the tangent cylinder is organized into two zonal jets of opposite direction, with speeds reaching up to 0.3 m/s (panel A). This flow structure is similar to that corresponding to the upper limit of the rapidly rotating convection (see Fig. 5 in Cabanes et al., 2024), indicating that convection outside the tangent cylinder is still dominated by the Coriolis force. Convection inside the tangent cylinder is characterized by broad retrograde zonal flows with speeds of 0.1 m/s at high latitudes and weak retrograde zonal flows at mid-latitudes. Note that the structure of the flow inside the tangent cylinder in Mode I is significantly different from that observed in models of rapidly rotating convection, which is characterized by a cascade of zonal jets with alternating direction (e.g., Bire et al., 2022; Cabanes et al., 2024). The meridional component of the flow velocity (Fig. 4.2B) is less than 4 mm/s inside the tangent cylinder and about 1 cm/s near the tangent cylinder surface. The speed of the radial flow (Fig. 4.2C) is of a similar magnitude. The speeds obtained here for convection in Mode I are consistent with the theoretical bounds derived by Cabanes et al. (2024) who predict $v_\phi \in \langle 0.3, 1.7 \rangle$ m/s, and v_r and $v_\theta \in \langle 1, 6 \rangle$ cm/s.

Detailed analysis of our results suggests that depending on the value of R_M and Pr , convective flow in Mode II can take two forms, denoted here as IIa and IIb. Modes IIa and IIb differ in the structure of zonal flows (cf. panels D and G in Fig. 4.2) and the vigour and lateral extent of the meridional circulation (panels E,F and H,I). The zonal flow in Mode IIa is characterized by two prograde jets localized at the inner boundary of the tangent cylinder and a nearly homogeneous retrograde flow outside the tangent cylinder. The meridional and radial flow in

Mode II is dominated by a system of Hadley-like circulation cells at low latitudes (Fig. 4.2, panels E, F, H and I). Comparison with other studies suggests that the structure of the zonal flow for R_M ranging from 8 to 10 does not depend on the Prandtl number (see, e.g., Fig. 2Ad in Soderlund (2019) and Fig. 6b in Bire et al. (2022) for $Pr = 1$, and Fig. 6h in Kvorka and Čadek (2022) and Fig. 4.2D in this study for $Pr = 10$). However, as the effect of rotation decreases (i.e., as R_M increases), the results obtained for different values of Pr diverge. The models with $Pr = 10$ (Mode IIb, this study) predict two zonal jets of opposite direction outside the tangent cylinder and a weak homogeneous retrograde flow inside the tangent cylinder (Fig. 4.2G). A similar distribution of zonal flows was obtained by Ashkenazy and Tziperman (2021) for $Pr = 10$, $Ek = 5 \cdot 10^{-9}$ and $R_M \approx 20$ in a 2-dimensional GCM simulation using the effective viscosity. In contrast, the models with $Pr = 1$ predict a broad prograde zonal flow inside the tangent cylinder and a retrograde zonal flow near the equator (Yadav et al., 2016; Soderlund, 2019; Kvorka and Čadek, 2022). It should be noted, however, that although the results presented in these studies were obtained for $R_M \gtrsim 10$, the simulations were performed for $Ro_c \gtrsim 1$ (i.e., for convection at the transition between the transitional and non-rotating regimes). Therefore, the results of these simulations may not be relevant to convection in Ganymede’s ocean, which is characterized by $Ro_c < 0.3$ and is likely in the transitional regime. This together with the large values of R_M (see Table 4.1) indicates that the convection flow in Ganymede’s ocean is in Mode IIb. As we show in section 4.4.2, the induced magnetic fields produced by convection in Modes I, IIa and IIb significantly differ from each other in size and structure meaning that the geometry of ocean circulation can, in principle, be constrained by magnetic measurements.

The flow patterns shown in Fig. 4.2 are computed for the radius ratio of 0.85. The results obtained for $r_i/r_o = 0.80$ and 0.90 are presented in Appendix 4.B. Comparison of Fig. 4.2 with Fig. 4.7 shows that the choice of radius ratio has only a minor impact on the geometry of ocean circulation but can significantly affect the flow speed.

4.3.4 Heat transfer characteristics

Studies investigating the heat transfer in subsurface oceans agree that the time-averaged heat flux from the ocean can vary in latitude by tens of percent (e.g., Soderlund et al., 2014; Soderlund, 2019; Amit et al., 2020; Kvorka and Čadek, 2022). The uneven heat flux from the ocean affects the evolution of the outer ice shell and can be manifested by meridional variations of topography (e.g., Kvorka et al., 2018; Kang et al., 2022a; Kang and Jansen, 2022; Kihoulou et al., 2023; Kvorka and Čadek, 2024). Depending on the relative importance of the Coriolis effect, the heat flux maximum is located either at the poles (“polar cooling”) or near the equator (“equatorial cooling”). However, there is currently no consensus on which parameter should be used to describe the transition from equatorial to polar cooling. The analysis of Kvorka and Čadek (2022) based of the statistical processing of about 80 numerical simulations with free-slip boundary conditions suggests that this transition is controlled by parameter $R_G^* = RaEk^{12/7}/Pr$ and occurs when $R_G^* \approx 1$. Using a series of no-slip simulations, Hartmann et al. (2024) has demonstrated that the maximum polar cooling is reached when the thickness

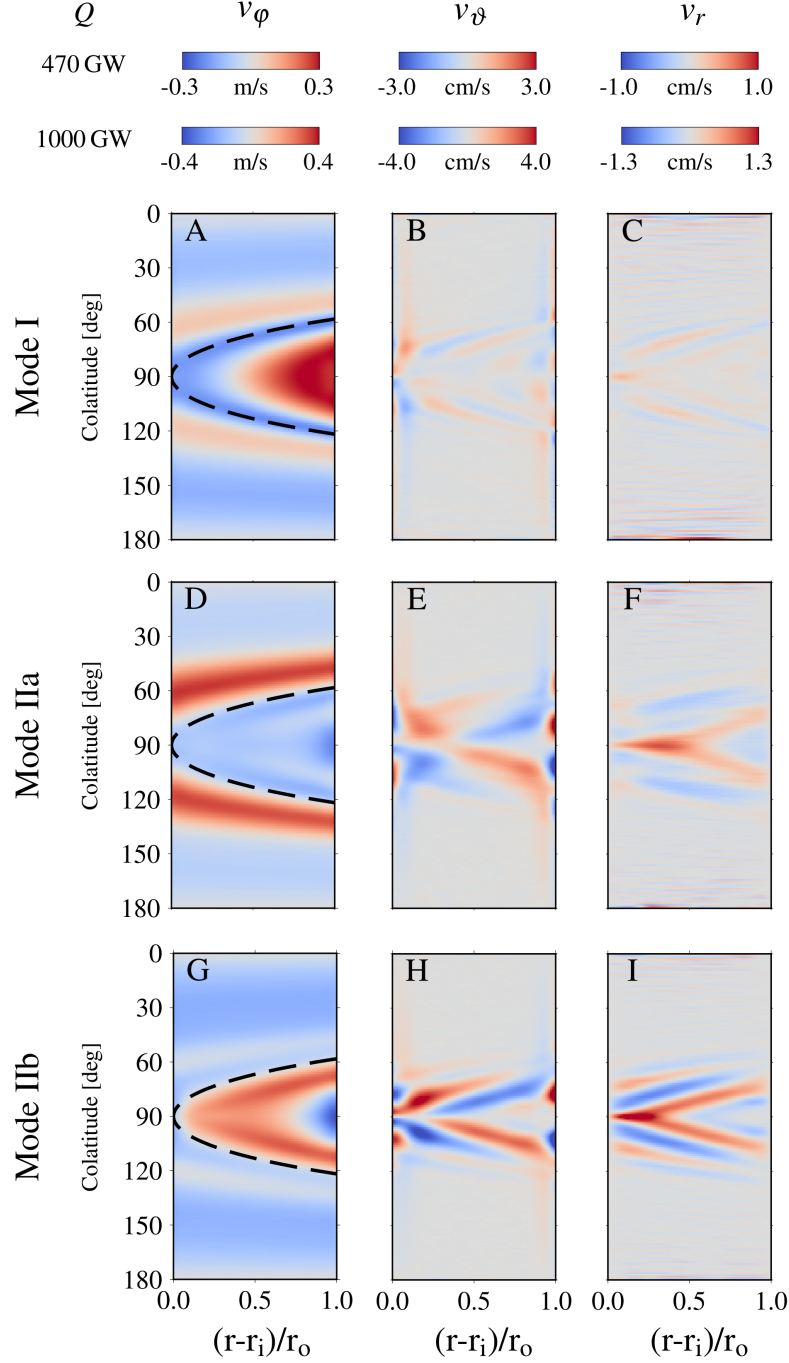


Figure 4.2 Zonal (v_ϕ), meridional (v_θ) and radial (v_r) components of the flow velocity plotted as a function of the dimensionless radius and the colatitude for numerical simulations representing convection in Modes I (top), IIa (middle) and IIb (bottom). The velocity fields are calculated for $r_i/r_o = 0.85$ (model C in Table 4.1) and correspond to models No. 9, 11 and 12 in Tab. 4.2. Black dashed lines in panels A, D and I show the position of the tangent cylinder. The velocity is averaged over at least 0.02 viscous diffusion times and longitude and scaled so that the mean speed, U , is equal to 0.21 cm/s and 0.28 cm/s, corresponding to $Q = 470$ GW (Hay and Matsuyama, 2019) and 1000 GW (Choblet et al., 2017a), respectively.

of the thermal boundary layer is the same as the thickness of the viscous (Ekman) boundary layer, and estimated that the transition from equatorial to polar cooling

occurs at $RaEk^{3/2} \approx 1$ (see also King et al., 2012; King and Aurnou, 2013). Similar results were reported by Amit et al. (2020) using a pseudospectral method and Bire et al. (2022), who employed a non-traditional beta-plane approximation (Dellar, 2011). It is worth noting that the models describing “the boundary layer crossing” mostly predict transition parameter $RaEk^\gamma$ with $\gamma > 3/2$. For example, King et al. (2010a) obtained $\gamma = 7/4$ using no-slip models, Julien et al. (2012) found $\gamma = 5/3$ for both free-slip and no-slip models, and Ecke and Niemela (2014) observed the range $1.65 \leq \gamma \leq 1.80$ in experiments with a rotating helium gas. Using the values of Ek , Ra and Pr in Table 4.1, one can show that $R_G^* > 1$ and $RaEk^\gamma > 1$ for all γ from the interval $\langle 1.5, 1.8 \rangle$.

The upper boundary of the polar cooling regime is unclear. Kvorka and Čadek (2022) have suggested that the transition from the polar to the second equatorial cooling occurs at $R_G^* \approx 10$. However, for simulations that use $Ek \geq 10^{-4}$, this condition is indistinguishable from the condition $Ro_c \approx 1$ (see Figs. 1b and 2b in Kvorka and Čadek, 2022). The possibility that Ro_c is the parameter that controls the transition to the second equatorial cooling was supported by the results of Hartmann et al. (2024, Figs. 3b,e). As the value of Ro_c obtained for Ganymede’s ocean is less than 0.3 it is likely that the heat transfer in the ocean is dominated by polar cooling.

Fig. 4.3 shows the distribution of the heat flux at the upper boundary of the ocean computed for the same models as in Fig. 4.2. The heat transfer in Mode I is characterized by the enhanced heat flux in the polar regions and relatively low cooling rates at low latitudes. The heat flux distribution is dominated by a zonal degree 2 term while the degree 4 term responsible for the heat flux minima at mid-latitudes is relatively weak. As the effect of rotation decreases and the convection system moves to Mode IIa, the latitudinal variations in the heat flux decrease and a new local maximum develops near the equator due to the meridional circulation at low latitudes. The transition from Mode IIa to Mode IIb is accompanied by the re-amplification of the lateral heat flux variations and the increase in the heat flux at the equator. The ocean remains in the polar cooling mode but the heat flux distribution has a strong zonal degree 4 component.

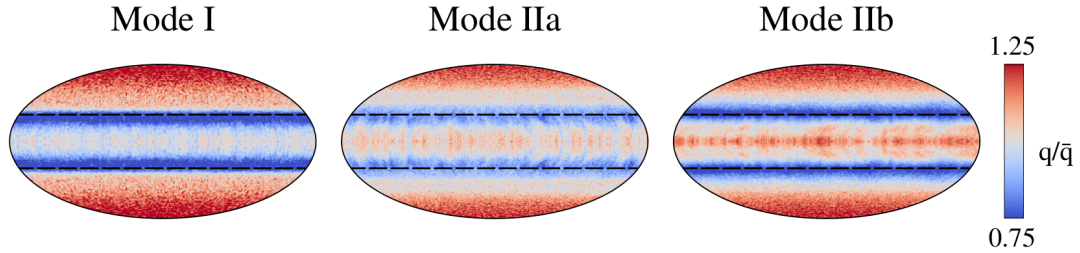


Figure 4.3 Radial component of the heat flux at the upper boundary of the ocean for the same models as in Fig. 4.2. The heat flux is averaged over at least 0.02 viscous diffusion times and normalized to the mean value. The dashed lines mark the intersection of the ocean surface with the tangent cylinder.

4.4 Implications for topography and magnetic field

4.4.1 Topography

It has been suggested that the long-wavelength topography of some icy moons is related to lateral variations in the heat flux from the ocean (e.g., Kvorka et al., 2018; Čadek et al., 2019). According to this concept, a higher-than-average heat flux triggers the melting of ice, leading to the downward flow in the ice shell and the development of surface depression. On the other hand, a lower-than-average heat flux causes the thickening of the ice shell and is manifested by the development of topographic highs. If this concept is correct then the global pattern of ocean circulation can be inferred from the observed topography. If we neglect the effect of salinity, the interaction between the ocean and the ice shell is controlled by three competing processes: (i) the heat flux associated with the global ocean circulation, (ii) the creep in the ice shell, and (iii) the additional heat flux associated with temperature variations at the base of the ice shell (e.g., Labrosse et al., 2018; Kang et al., 2022b; Kang and Jansen, 2022; Kvorka and Čadek, 2024; Gastine and Favier, 2025). Since the interface between the ice shell and the ocean is a phase boundary, the temperature at the base of the ice shell is equal to the melting temperature of ice. This temperature is constant only if the shape of the boundary coincides with the equipotential surface, i.e., if the pressure along the boundary is constant. If the topography of the ice-water interface, i.e., the distance of the interface from the equipotential surface, is nonzero, the temperature along the phase interface varies due to the pressure dependence of the melting temperature. The variations in the melting temperature along the boundary give rise to an additional heat flux, which is proportional to the topographic amplitude and the flow speed of water (for a detailed discussion, see, e.g., Labrosse et al., 2018; Kihoulou et al., 2023; Kvorka and Čadek, 2024).

Kihoulou et al. (2023) analyzed the formation of the surface topography assuming that the total heat flux from the ocean, q , can be expressed in the form

$$q = \tilde{q}_o - \tilde{\gamma} t_{i/o} , \quad (4.9)$$

where $\tilde{q}_o$ is the laterally varying heat flux from the deep ocean, $t_{i/o}$ is the topography of the phase boundary and γ is a parameter depending on the flow speed. Kihoulou et al. (2023) found that the magnitude of the surface topography is negligible if the heat transfer in the ice shell is dominated by convection because the effect of heat flux variations from the ocean is compensated by the other two processes. If the heat transfer in ice is dominated by conduction, the magnitude of the topography primarily depends on the heat flux driven by variations in the melting temperature at the phase boundary, which are controlled by parameter γ . However, even if $\gamma = 0$, the effect of an uneven (aspherical) phase boundary cannot be neglected because the lateral variations in ocean thickness, induced by the deformation of the ice shell, affect the heat flux $\tilde{q}_o$ from the deep ocean (Kvorka and Čadek, 2024; Gastine and Favier, 2025). To correct the heat flux models presented above for the effect of the ice shell deformation, we use Eq. (37) derived by Kvorka and

Čadek (2024) and modify Eq. (4.9) as follows:

$$q \approx q_o - \frac{\gamma}{K} u_{i/o} - \gamma t_{i/o} , \quad (4.10)$$

where q is the total heat flux entering the ice shell, q_o is the heat flux from the ocean obtained from the model with a spherical boundary (Fig. 4.3), $u_{i/o}$ is the shape of the phase boundary (defined as the distance from a sphere), $t_{i/o}$ is the topography of the phase boundary (defined as the distance from an equipotential surface) and K describes the ratio of horizontal to vertical forcing in the ocean (for details, see section 2.3.2 in Kvorka and Čadek (2024)). To calculate the surface topography corresponding to the flow models presented in Fig. 4.2, we must first correct the resulting heat flux (q_o , Fig. 4.3) for the effect of lateral changes in ocean thickness ($-\gamma u_{i/o}/K$) and the heat flux generated by temperature changes along the phase interface ($-\gamma t_{i/o}$). Note that even if the ice shell is in hydrostatic equilibrium (and therefore $t_{i/o} = 0$), $u_{i/o}$ is nonzero because Ganymede's equilibrium shape is affected by rotation and tides. Using the model that couples the ice shell dynamics with the ocean circulation on Titan, Kvorka and Čadek (2024) obtained $\gamma \approx 3.10^{-4} \text{ mW/m}^3$ and $K \approx 3$.

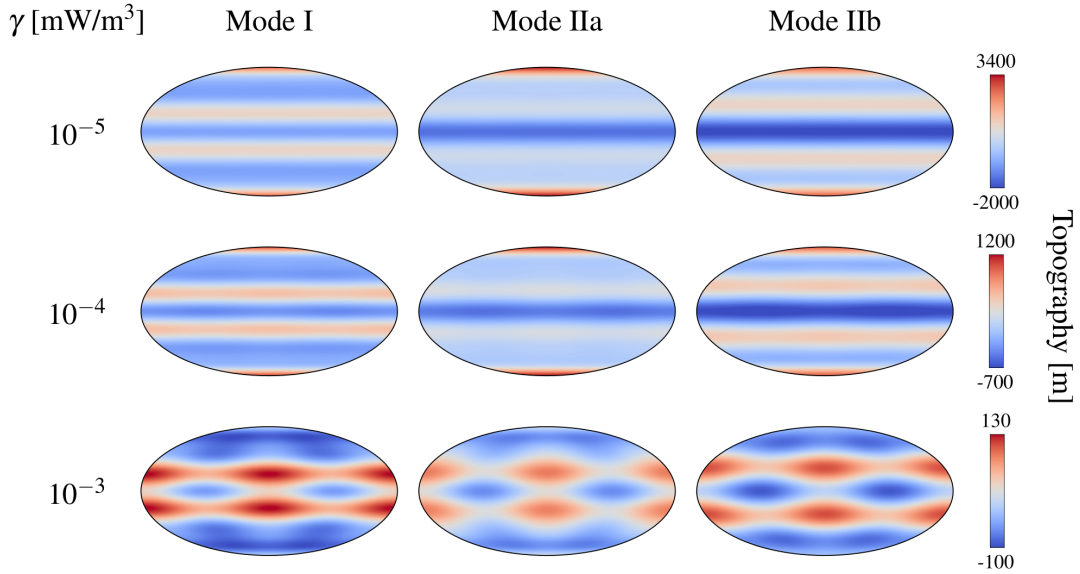


Figure 4.4 Ganymede's long-wavelength topography ($l = 2 - 10$) predicted for model C in Table 4.1. The constitutive equations used to describe the material properties of ice can be found in section 2.1 in Kihoulou et al. (2023). The viscosity of ice is evaluated for a grain size of 10 mm. The average temperature at the ice-water interface is set to 265.6 K (Marcolli, 2017) and the surface temperature is calculated for a surface albedo of 0.43 (Bockelée-Morvan et al., 2024) using the method of Nadeau and McGehee (2017).

Fig. 4.4 shows Ganymede's topography computed for $K = 3$ and three values of γ using the method described by Kvorka et al. (2018) and Kvorka and Čadek (2024). The model takes into account the meridional variations of Ganymede's surface temperature and the complex material properties of ice (see Kihoulou et al., 2023, for further details). For simplicity, we neglect the lateral variations in albedo (Bockelée-Morvan et al., 2024) and assume that the viscosity and thermal expansivity of ice depend only on the radius.

The convective heat transfer in Ganymede’s ocean generates an outward heat flux pattern with minima at mid-latitudes and peaks at the poles (Fig. 4.3). The heat flux distribution does not significantly differ from that predicted for Titan (Kvorka and Čadek, 2022, 2024) and, therefore, it could be expected that the enhanced heat flux at high latitudes will be manifested by polar depressions. However, as shown in Fig. 4.4, this is not the case. Since the surface temperature on Ganymede significantly decreases towards the poles (unlike Titan where the surface temperature is more or less constant), the heat emitted from the polar regions to the space can be larger than the heat entering the ice shell from the ocean. This results in the thickening of the ice shell at high latitudes and the formation of polar elevations. As a consequence, the pressure at the phase boundary increases, lowering the melting temperature near the poles. The decrease of the temperature induces a flow towards the poles that tends to reduce the temperature gradient along the phase boundary and restore the hydrostatic equilibrium in the ice shell. The strength of this effect (denoted as the DAL effect by Kvorka and Čadek, 2024) increases with increasing γ .

Our results indicate that for $\gamma \leq 10^{-4}$ (Fig. 4.4 top and middle), the surface of the ice shell can be divided into two regions: a polar region, where topographic elevation is maintained by surface temperature variations, and an equatorial region, characterized by two topographic peaks separated by a depression caused primarily by the oceanic heat flux. When γ is increased to 10^{-3} (Fig. 4.4 bottom), the polar elevations are nearly eliminated by the DAL effect and the topography is dominated by two ridges located approximately above the intersection of the phase boundary with the tangent cylinder. The irregular shape of the ridges is related to the fact that Ganymede is tidally locked and its hydrostatic shape can be approximated by a triaxial ellipsoid (see also Kvorka and Čadek, 2024). The amplitude of the topography decreases with increasing γ and ranges from more than 3 km for $\gamma = 10^{-5}$ to less than 130 m for $\gamma = 10^{-3}$ (cf. Kihoulou et al., 2023). It should be mentioned that the values of $\gamma \approx 10^{-4}$ found by Kvorka and Čadek (2024) corresponds to an ocean with $R_M \approx 5.9$, and hence is relevant to Mode I. The value of $K = 3$ used to compute the topography was determined by fitting the long-wavelength topography of Titan and may not be relevant to Ganymede. In addition, the magnitude of the topography depends on the choice of the grain size (Kihoulou et al., 2023), which may vary spatially and range from a few to tens of mm (Barr and McKinnon, 2007). The results shown in Fig. 4.4 should be regarded as an illustration of the potential impact that the subsurface ocean may have on the evolution of the surface. A more comprehensive model accounting for the interaction between the ice shell and the ocean (Kvorka and Čadek, 2024; Gastine and Favier, 2025) lies beyond the scope of this study and will be addressed in the future research.

Topographic data that will be acquired by the Juice mission can provide information about the strength of the DAL effect (by constraining the topography near the poles) and about the thickness of the ocean (by determining the location of the longitudinal ridges). However, it is far from clear whether these data will be sufficient to distinguish between Modes I, IIa and IIb because the three flow models predict similar topographic patterns and the analysis of the observed topography can be complicated by the topographic signal associated with the tectonic evolution of the ice shell.

4.4.2 Motionally induced magnetic field

The Galileo spacecraft discovered that Ganymede has a dipole magnetic field generated in Ganymede’s metallic core (Kivelson et al., 1996; Gurnett et al., 1996). The field has a strength of about 720 nT at Ganymede’s equator and 1440 nT at the poles (Kivelson et al., 1997a, 1998, 2002), which is significantly more than the strength of the Jovian magnetic field at the distance of Ganymede (about 120 nT, Kivelson et al., 1997a). The interaction of the flow in Ganymede’s ocean with the ambient magnetic field generates electrical currents, which in turn generate an ocean-induced magnetic field (OIMF). This motionally induced magnetic field contains information about the ocean circulation and its measurement by future space missions can provide a better understanding of the dynamic processes in Ganymede’s ocean. Using a simplified scaling relation, Vance et al. (2021) estimated that the radial component of the OIMF is in the range of 8–330 nT.

In this section, we present the OIMF maps corresponding to Modes I, IIa and IIb. The time-averaged ocean induced magnetic field, $\mathbf{B}$, is obtained by solving the equation of the electromagnetic induction in the steady-state approximation (for details, see Šachl et al., 2025):

$$\nabla \times \left(\frac{1}{\sigma} \nabla \times \mathbf{B} \right) = \mu_0 \nabla \times (\mathbf{v} \times \mathbf{B}_0) , \quad (4.11)$$

where $\mathbf{B}_0$ is Ganymede’s internal dipole magnetic field, tilted with respect to the rotational axis of Ganymede by 176° (Weber et al., 2022), $\mathbf{v}$ is the zonally and time averaged flow velocity, σ is the electrical conductivity of Ganymede’s interior, and μ_0 is the magnetic permeability of free space. We assume that $\mathbf{B} \ll \mathbf{B}_0$ and neglect the effect of Jupiter’s magnetic field. Eq. (4.11) is solved using the spherical harmonic-finite element method (Velínský and Martinec, 2005) for model C in Table 4.1 and $Q = 1000$ GW. The electrical conductivity of Ganymede’s interior is not well constrained. The magnitude of $\mathbf{B}$ is strongly affected by the electrical conductivity of the ocean, which depends on the composition of the ocean and may vary from approximately 0.1 S/m to 18 S/m, corresponding to the NaCl saturation limit (Hand and Chyba, 2007). The latest estimates from improved magnetic field models constrained by Galileo and Juno data (Jia et al., 2025), and from high-pressure, low-temperature measurements of aqueous magnesium sulfate (Pan et al., 2020), tend to agree that the electrical conductivity of Ganymede’s ocean is less than 1 S/m.

Fig. 4.5 shows the OIMF computed for $\sigma_{\text{ice}} = 10^{-5}$ S/m (Petrenko and Schulson, 1992), $\sigma_{\text{ocean}} = 10$ S/m (Hand and Chyba, 2007; Petricca et al., 2023), $\sigma_{\text{mantle}} = 0.01$ S/m (Petricca et al., 2023), and $\sigma_{\text{core}} = 1000$ S/m (corresponding to a highly conductive iron core). Since the toroidal magnetic field is locked inside the ocean, and inaccessible to direct observations, only the poloidal part is shown, upward-continued to an altitude of 200 km. The spatial structure is dominated by spherical harmonic order 1 in the longitudinal direction, caused by interaction of zonally symmetric flows with the horizontal, non-axisymmetric component of Ganymede’s dipole field. It is important to note that since the viscosity and thermal diffusivity used in this study are much larger than in the real ocean, our model does not allow for an accurate representation of the small-scale, time-varying features of the flow field. The magnetic field shown in Fig. 4.5 is therefore calculated from the zonally and time averaged flow velocity and thus corresponds to the induced

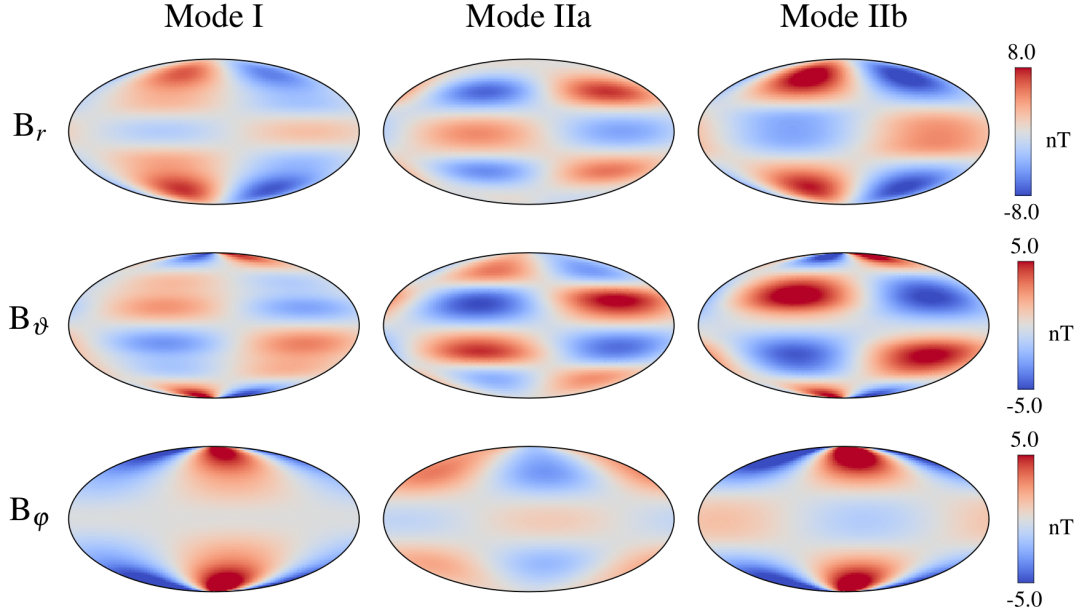


Figure 4.5 Radial (B_r), meridional (B_θ) and zonal (B_ϕ) components of the OIMF calculated for convection in Modes I, IIa and IIb (Fig. 4.2), structure model C (Table 4.1) and $\mathbf{B}_0$ from Weber et al. (2022). The field is shown at an altitude of 200 km above the surface of Ganymede.

magnetic field averaged over time. The instantaneous pattern of the magnetic field can contain a strong short-wavelength component and be significantly more complex than that shown in the present study.

The predicted signals have a strength of about 8 nT, which is significantly less than the upper estimate of Vance et al. (2021) but within the sensitivity of the magnetic field measurements (Kivelson et al., 2023; Ellmeier et al., 2024). However, in the case of significantly lower electrical conductivity of the ocean, the amplitudes can be further reduced by another order of magnitude, to the very edge of detectability by future measurements. Although the strength of the predicted OIMF is almost independent of the mode of ocean circulation, the geometry of flow strongly affects the OIMF pattern, suggesting that accurate measurements of the magnetic field can provide constraints on the dynamics of Ganymede’s ocean. Note, for example, that the magnetic field obtained for Mode IIa is anti-correlated with those obtained for Modes I and IIb, and the component B_θ predicted for Mode I has a more complex structure compared to Modes IIa,b. A detailed analysis of Ganymede’s OIMF will be presented in a separate study.

4.5 Conclusions

In order to determine the structure of the flow in Ganymede’s ocean, we performed 128 numerical simulations, varying all relevant control parameters (Ra , Ek , Pr) by at least one order of magnitude. Based on this data set, we developed a scaling law that relates the Rayleigh number, Ra , to Ganymede’s total heat production, Q , and estimated the mean flow speed in the ocean. Assuming that Q can range between 400 and 1600 GW and varying the radius ratio, r_i/r_o , from 0.80 to 0.99, we found that the value of Ra is in the interval between 10^{19} (corresponding

to $r_i/r_o = 0.99$) and 10^{23} (corresponding to $r_i/r_o = 0.80$). Using these values of the Rayleigh number and the corresponding values of the Ekman number, we demonstrated that Ganymede's ocean is likely in the transitional regime and the ocean circulation is characterized by a retrograde equatorial jet, meridional circulation cells at low latitudes and narrow upwellings and downwellings in the polar regions (Fig. 4.2, Modes IIa,b). The mean speed of the ocean flow ranges from a few cm/s for a thin (< 100 km) ocean to 0.8 m/s for a 500 km thick ocean (Table 4.1). The time-averaged heat flux from the ocean varies with the latitude reaching the maximum at the poles and the minimum at mid-latitudes (Fig. 4.3).

Assuming that the heat transfer in the ice shell is dominated by conduction and using the parameters that describe the ice-ocean interaction on Titan, we determined Ganymede's long-wavelength topography generated by the uneven heat flux from the ocean (Fig. 4.4). Unlike Titan, where the surface temperature is nearly constant and the topography is characterized by polar depressions, the temperature on Ganymede's surface significantly decreases towards the poles, leading to an increase of the cooling rate at high latitudes that may compensate the topographic effect of the heat flux from the ocean. We predict that Ganymede's polar regions are flat or even elevated and the global topographic pattern is dominated by an equatorial depression flanked by the ridges correlating with the position of the tangent cylinder.

Preliminary calculations of the magnetic field, induced by the flow of the salty ocean in the presence of Ganymede's internal magnetic field, predict sufficiently large amplitudes to allow their detection by the Juice and Europa Clipper spacecrafts. Differences in the spatial patterns of OIMF calculated for individual ocean flow modes could in principle allow their discrimination. A detailed parametric study exploring the parameter space of ice and ocean conductivities and thicknesses, as well as the combined effect of Ganymede's internal and Jupiter's external magnetic field on the OIMF is underway.

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Data availability

Data can be downloaded from <https://zenodo.org/records/14725726>.

4.A Extrapolation technique

Each numerical simulation is characterized by three input parameters (Pr , Ek and Ra) and two output parameters – the Nusselt number (Nu) and the Rossby number (Ro). We assume that these parameters are related by Eq. (4.7) where the flux-based Rayleigh number, Ra_Q , can be expressed in term of the advected heat flow, Q_{adv} , and parameters Ra , Ek , Pr and Nu as follows (Christensen and Aubert, 2006):

$$Ra_Q = \frac{1}{4\pi r_o r_i} \frac{\alpha g_o Q_{adv}}{\rho C_p \omega^3 D^2} = Ra(Nu - 1)Ek^3 Pr^{-2} . \quad (4.12)$$

The unknown parameters α , β , γ and δ in Eq. (4.7) are determined from the data given in Tables 4.1–4.3 by the least squares method (Fig. 4.6). The values of the parameters together with the corresponding standard deviations are given in Table 4.3. It is worth noting that parameters β , γ and δ are nearly independent of the radius ratio. The value of Ra_Q corresponding to the actual conditions in Ganymede’s ocean can be calculated from Eq. (4.12) by assuming that $Q_{adv} \approx Q$ where Q is the total heat flow. The Rayleigh number of Ganymede’s ocean is then calculated using Eq. (4.7) where $Pr = 10$ and Ek is given by Eq. (4.1). The scaling law for the Rossby number, Eq. (4.8), is derived in a similar way.

a) $Ra = \alpha Ra_Q^\beta Ek^\gamma Pr^\delta$				
r_i/r_o	α	β	γ	δ
0.80	6.00 ± 0.50	0.58 ± 0.01	-2.16 ± 0.03	1.13 ± 0.02
0.85	5.05 ± 0.50	0.58 ± 0.02	-2.17 ± 0.03	1.16 ± 0.03
0.90	4.23 ± 0.01	0.62 ± 0.01	-2.19 ± 0.01	1.27 ± 0.01

b) $Ro = \alpha_1 Ra_Q^{\beta_1} Ek^{\gamma_1} Pr^{\delta_1}$				
r_i/r_o	α_1	β_1	γ_1	δ_1
0.80	0.33 ± 0.05	0.37 ± 0.01	-0.23 ± 0.03	-0.10 ± 0.02
0.85	0.37 ± 0.07	0.38 ± 0.01	-0.21 ± 0.03	-0.07 ± 0.03
0.90	0.38 ± 0.02	0.39 ± 0.01	-0.20 ± 0.02	-0.01 ± 0.01

Table 4.3 Exponents in the scaling laws for a) the Rayleigh number (Eq. 4.7) and b) the Rossby number (Eq. 4.8) determined from the data in Tables 4.1–4.3 by the least squares method (see also Fig. 4.6).

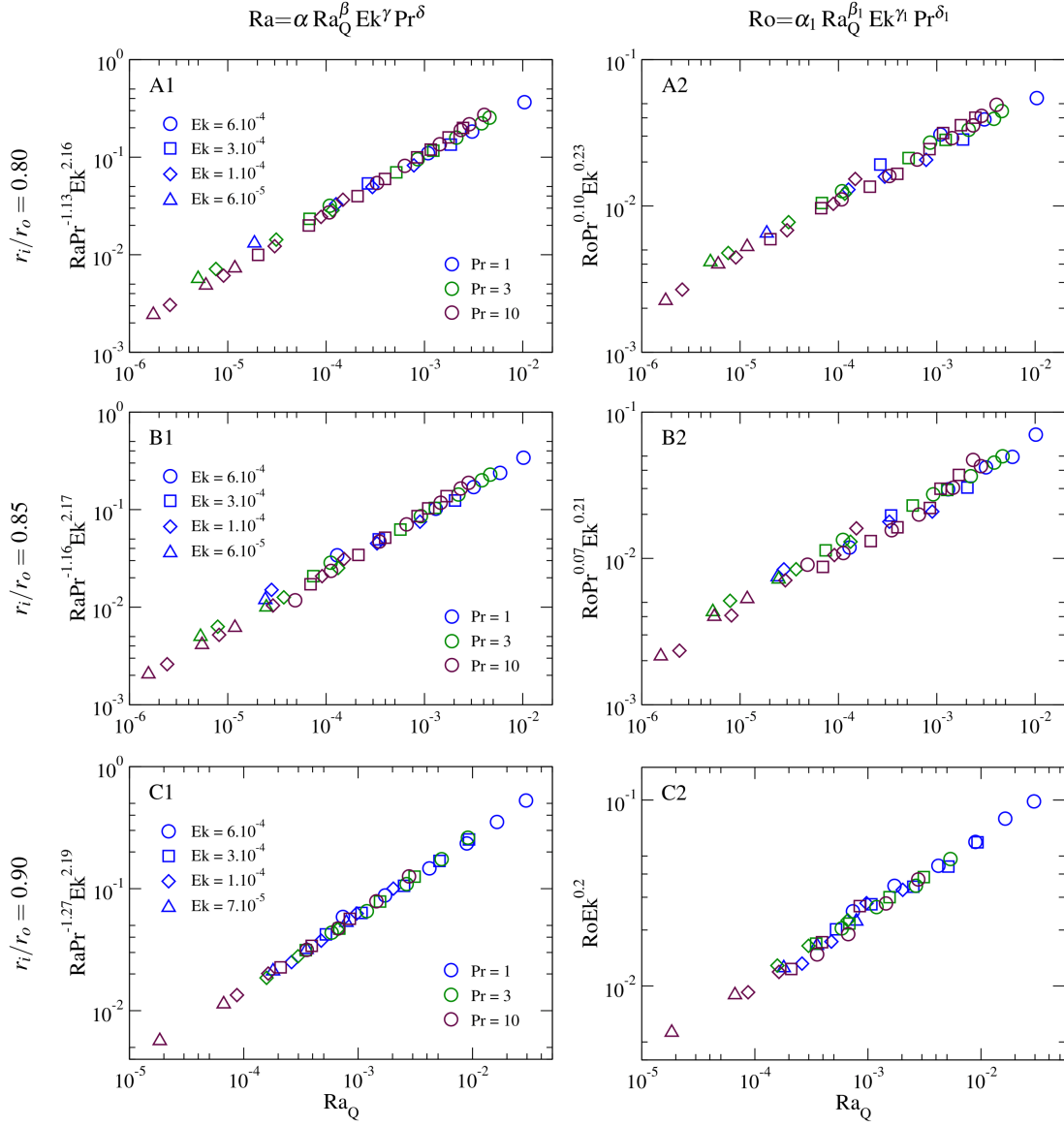


Figure 4.6 Graphs illustrating the scaling laws for Ra (left) and Ro (right) derived using the least squares method for $r_i/r_o = 0.8$ (top), 0.85 (middle) and 0.90 (bottom). The data used for the least squares regression are given in Tabs. 4.1–4.3 and the resulting values of α – δ and α_1 – δ_1 are listed in Tab. 4.3. Shapes and colors of the symbols indicate the values of parameters Ek and Pr used in the numerical simulation.

4.B Results for $r_i/r_o = 0.80$ and 0.90

The models discussed in the main text correspond to the radius ratio of 0.85. The results obtained for two different choices of r_i/r_o are presented in Figs. 4.7 and 4.8. Comparison of Fig. 4.7 with Fig. 4.2 in the main text shows that the flow speed strongly decreases with increasing r_i/r_o (see also U in Tab. 4.1). However, the structure of the flow do not change, except that the lateral extent of the meridional circulation near the equator decreases as the relative width of TC increases.

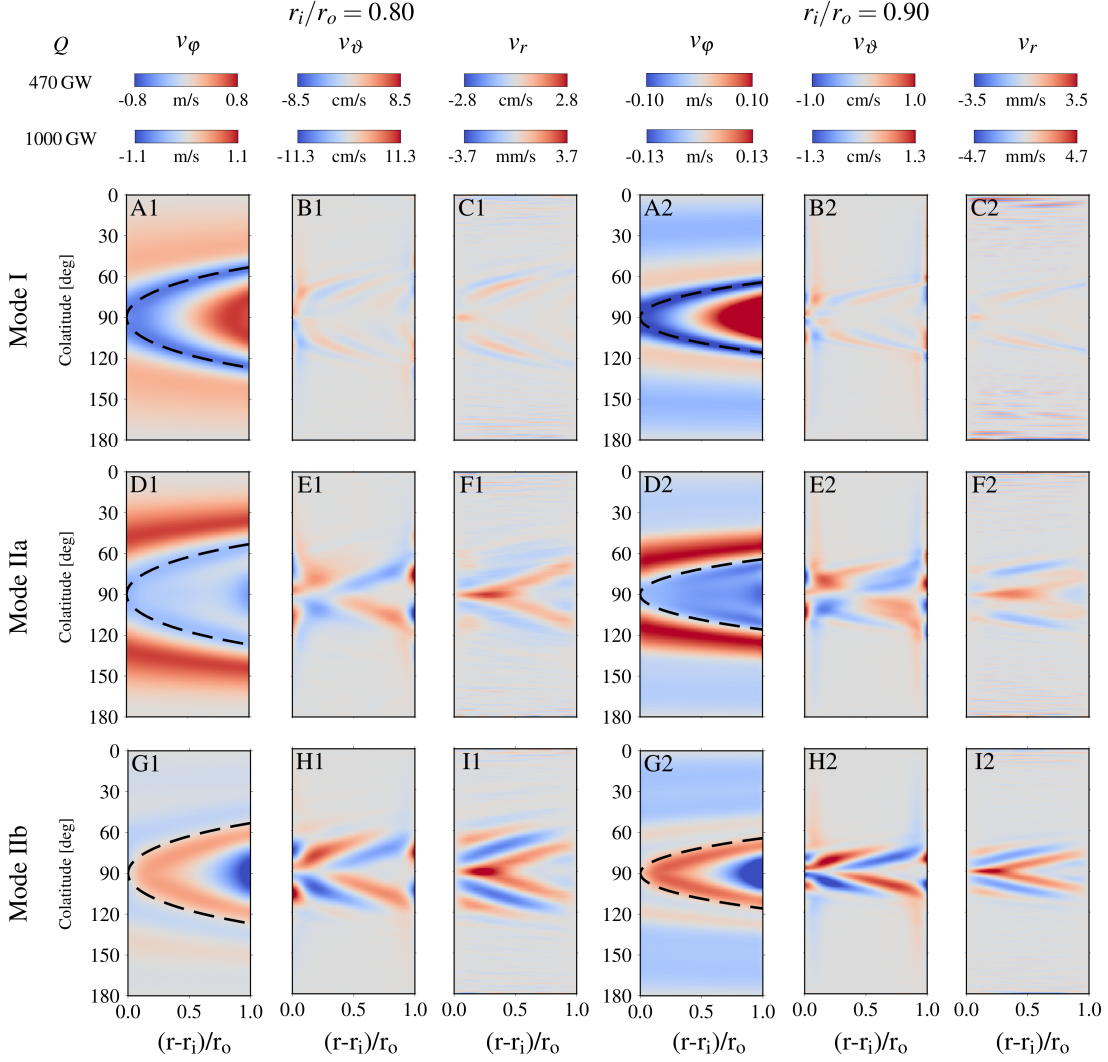


Figure 4.7 As in Fig. 4.2 but for the radius ratio of 0.80 (left) and 0.90 (right). The velocity fields correspond to simulations No. 10, 12 and 13 in Table 4.1 for $r_i/r_o = 0.80$ and No. 34, 36 and 37 in Tab. 4.3 for $r_i/r_o = 0.90$.

A similar trend is observed for the heat flux (Fig. 4.8) where the decrease in the extent of the meridional circulation leads to a relative increase in the heat flux near the equator. For completeness, we also show how the radius ratio affects the transition from Mode I to Mode II.

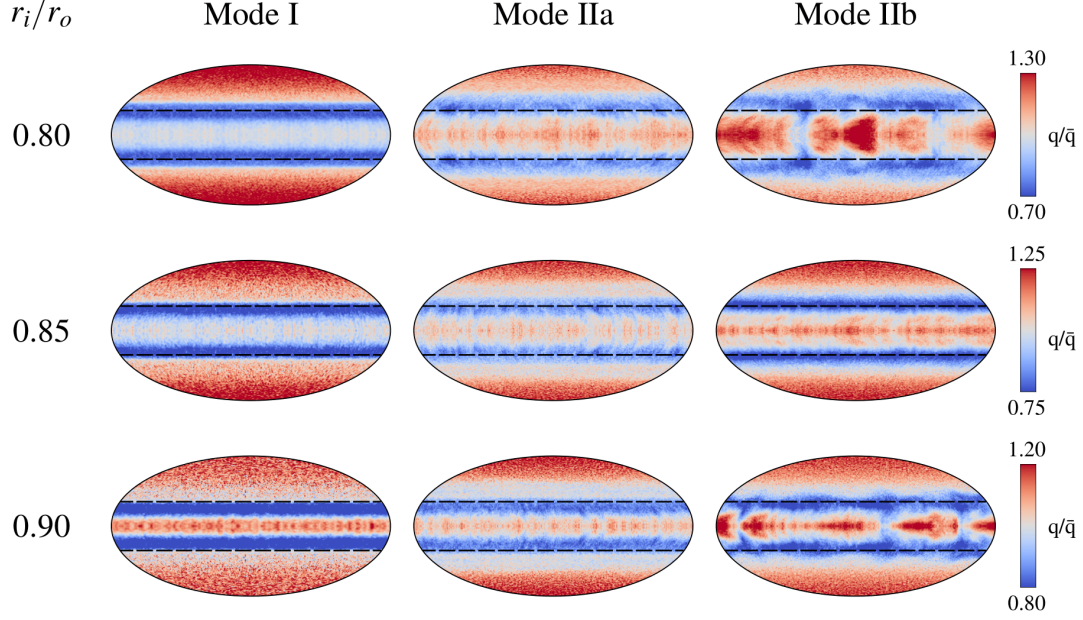


Figure 4.8 Heat flux distribution at the upper boundary of the ocean obtained for $r_i/r_o = 0.80$ (top), 0.85 (middle) and 0.90 (bottom) and convection modes I (left), IIa (middle) and IIb (right). The heat flux distributions correspond to simulations (from left to right) No. 10, 12 and 13 in Tab. 4.1 (top), 9, 11 and 12 in Tab. 4.2 (middle), and 34, 36 and 37 in Tab. 4.1 (bottom).

Close inspection of Fig. 4.9 reveals that the value of R_M at which the transition occurs decreases from about 9.5 for $r_i/r_o = 0.80$ to about 8 for $r_i/r_o = 0.90$.

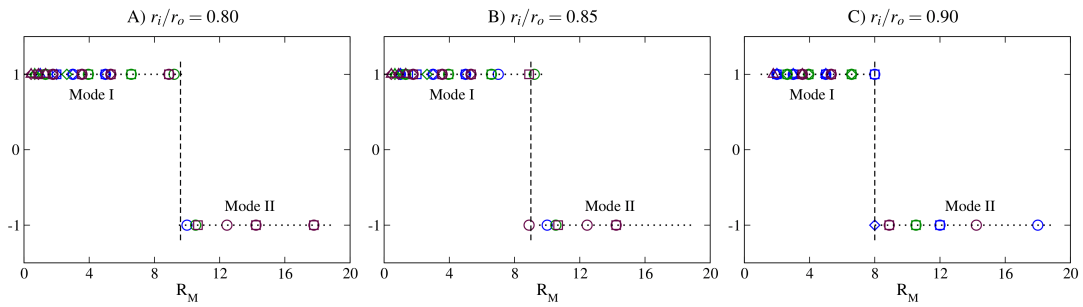


Figure 4.9 The sign of the equatorial zonal jet as a function of parameter $R_M = RaEk^{12/7}Pr^{-3/4}$. Shapes and colors of the symbols indicate the values of parameters Ek and Pr used in the numerical simulation.

4.C List of simulations

Model No.	N_r	j_{max}	Pr	Ek	Ra	Ro_{loc}	Ro_c	R_M	Nu	Ro	Mode
0	61	159	10	$6 \cdot 10^{-4}$	$3.34 \cdot 10^6$	5.86	0.35	1.78	15.94	0.05	I
1	73	177	10	$6 \cdot 10^{-4}$	$6.67 \cdot 10^6$	11.73	0.49	3.56	23.87	0.07	I
2	73	177	10	$6 \cdot 10^{-4}$	$1.00 \cdot 10^7$	17.59	0.60	5.33	30.22	0.09	I
3	73	177	10	$6 \cdot 10^{-4}$	$1.67 \cdot 10^7$	29.32	0.77	8.89	40.83	0.13	I
4	85	213	10	$6 \cdot 10^{-4}$	$2.33 \cdot 10^7$	41.05	0.92	12.45	47.33	0.16	IIa
5	81	213	10	$6 \cdot 10^{-4}$	$2.67 \cdot 10^7$	46.91	0.98	14.23	50.61	0.18	IIb
6	79	213	10	$6 \cdot 10^{-4}$	$3.34 \cdot 10^7$	58.64	1.10	17.78	57.35	0.22	IIb
7	61	159	10	$3 \cdot 10^{-4}$	$5.47 \cdot 10^6$	3.17	0.22	0.89	14.76	0.03	I
8	67	177	10	$3 \cdot 10^{-4}$	$1.09 \cdot 10^7$	6.35	0.31	1.78	23.59	0.05	I
9	73	177	10	$3 \cdot 10^{-4}$	$2.19 \cdot 10^7$	12.70	0.44	3.56	36.34	0.07	I
10	73	177	10	$3 \cdot 10^{-4}$	$3.28 \cdot 10^7$	19.04	0.54	5.33	45.76	0.09	I
11	81	213	10	$3 \cdot 10^{-4}$	$5.47 \cdot 10^7$	31.74	0.70	8.89	58.09	0.13	I
12	85	237	10	$3 \cdot 10^{-4}$	$6.57 \cdot 10^7$	38.09	0.77	10.67	66.30	0.16	IIa
13	85	237	10	$3 \cdot 10^{-4}$	$8.76 \cdot 10^7$	50.78	0.89	14.23	75.42	0.18	IIb
14	89	297	10	$3 \cdot 10^{-4}$	$1.09 \cdot 10^8$	63.48	0.99	17.78	84.50	0.21	IIb
15	73	213	10	$1 \cdot 10^{-4}$	$1.80 \cdot 10^7$	1.80	0.13	0.44	15.36	0.02	I
16	73	213	10	$1 \cdot 10^{-4}$	$3.60 \cdot 10^7$	3.60	0.19	0.89	26.17	0.03	I
17	81	237	10	$1 \cdot 10^{-4}$	$7.20 \cdot 10^7$	7.20	0.27	1.78	42.64	0.05	I
18	89	285	10	$1 \cdot 10^{-4}$	$1.44 \cdot 10^8$	14.39	0.38	3.56	62.61	0.07	I
19	117	447	10	$1 \cdot 10^{-4}$	$2.16 \cdot 10^8$	21.59	0.46	5.33	69.72	0.10	I
20	81	237	10	$6 \cdot 10^{-5}$	$4.32 \cdot 10^7$	1.91	0.12	0.44	19.80	0.02	I
21	89	267	10	$6 \cdot 10^{-5}$	$8.64 \cdot 10^7$	3.81	0.18	0.89	33.12	0.03	I
22	97	317	10	$6 \cdot 10^{-5}$	$1.30 \cdot 10^8$	5.72	0.22	1.33	43.18	0.04	I
23	73	177	1	$6 \cdot 10^{-4}$	$1.00 \cdot 10^6$	7.00	0.60	3.00	6.03	0.17	I
24	85	213	1	$6 \cdot 10^{-4}$	$1.67 \cdot 10^6$	11.67	0.77	5.00	9.47	0.22	I
25	97	267	1	$6 \cdot 10^{-4}$	$3.34 \cdot 10^6$	23.35	1.10	10.00	15.43	0.30	IIa
26	85	237	1	$3 \cdot 10^{-4}$	$2.19 \cdot 10^6$	5.05	0.44	2.00	5.51	0.12	I
27	97	253	1	$3 \cdot 10^{-4}$	$5.47 \cdot 10^6$	12.64	0.70	5.00	13.50	0.18	I
28	85	237	1	$1 \cdot 10^{-4}$	$1.44 \cdot 10^7$	5.73	0.38	2.00	9.78	0.11	I
29	97	237	1	$1 \cdot 10^{-4}$	$2.16 \cdot 10^7$	8.60	0.46	3.00	14.69	0.13	I
30	97	267	1	$1 \cdot 10^{-4}$	$3.60 \cdot 10^7$	14.33	0.60	5.00	22.67	0.17	I
31	97	253	1	$6 \cdot 10^{-5}$	$1.73 \cdot 10^7$	3.04	0.25	1.00	6.00	0.06	I
32	73	177	3	$6 \cdot 10^{-4}$	$1.00 \cdot 10^6$	3.62	0.35	1.32	5.53	0.06	I
33	73	197	3	$6 \cdot 10^{-4}$	$3.00 \cdot 10^6$	10.87	0.60	3.95	12.86	0.13	I
34	85	213	3	$6 \cdot 10^{-4}$	$5.00 \cdot 10^6$	18.11	0.77	6.58	18.55	0.16	I
35	85	237	3	$6 \cdot 10^{-4}$	$7.00 \cdot 10^6$	25.36	0.92	9.21	23.70	0.19	I
36	97	267	3	$6 \cdot 10^{-4}$	$8.01 \cdot 10^6$	28.98	0.98	10.53	24.86	0.22	IIa
37	73	177	3	$3 \cdot 10^{-4}$	$3.28 \cdot 10^6$	3.92	0.31	1.32	7.89	0.06	I
38	85	213	3	$3 \cdot 10^{-4}$	$9.85 \cdot 10^6$	11.76	0.54	3.95	18.49	0.12	I
39	97	253	3	$3 \cdot 10^{-4}$	$1.64 \cdot 10^7$	19.61	0.70	6.58	25.95	0.16	I
40	73	213	3	$1 \cdot 10^{-4}$	$1.08 \cdot 10^7$	2.22	0.19	0.66	7.31	0.04	I
41	81	237	3	$1 \cdot 10^{-4}$	$2.16 \cdot 10^7$	4.45	0.27	1.32	13.97	0.06	I
42	97	253	3	$1 \cdot 10^{-4}$	$4.32 \cdot 10^7$	8.89	0.38	2.63	25.20	0.09	I
43	97	253	3	$6 \cdot 10^{-5}$	$2.59 \cdot 10^7$	2.36	0.18	0.66	9.03	0.03	I

Table 4.1 List of numerical simulations with the radius ratio of 0.80. Symbols N_r and j_{max} denote the number of the Chebyshev points and the cut-off degree used to discretize the governing equations in the radial and lateral directions, respectively. The results of the simulations are represented by the transition parameter R_M , the Nusselt number (Nu) and the Rossby number (Ro).

Model No.	N_r	j_{max}	Pr	Ek	Ra	Ro_{loc}	Ro_c	R_M	Nu	Ro	Mode
0	67	159	10	$6 \cdot 10^{-4}$	$3.34 \cdot 10^6$	5.86	0.35	1.78	16.56	0.04	I
1	73	177	10	$6 \cdot 10^{-4}$	$6.67 \cdot 10^6$	11.73	0.49	3.56	25.14	0.06	I
2	73	177	10	$6 \cdot 10^{-4}$	$1.00 \cdot 10^7$	17.59	0.60	5.33	31.45	0.08	I
3	73	177	10	$6 \cdot 10^{-4}$	$1.67 \cdot 10^7$	29.32	0.77	8.89	41.57	0.12	IIa
4	81	213	10	$6 \cdot 10^{-4}$	$2.33 \cdot 10^7$	41.05	0.92	12.45	47.42	0.19	IIa
5	81	237	10	$6 \cdot 10^{-4}$	$2.67 \cdot 10^7$	46.91	0.98	14.23	49.79	0.21	IIb
7	73	177	10	$3 \cdot 10^{-4}$	$1.09 \cdot 10^7$	6.35	0.31	1.78	24.48	0.04	I
8	73	177	10	$3 \cdot 10^{-4}$	$2.19 \cdot 10^7$	12.70	0.44	3.56	37.03	0.06	I
9	73	177	10	$3 \cdot 10^{-4}$	$3.28 \cdot 10^7$	19.04	0.54	5.33	45.98	0.08	I
10	85	237	10	$3 \cdot 10^{-4}$	$5.47 \cdot 10^7$	31.74	0.70	8.89	58.64	0.10	I
11	89	237	10	$3 \cdot 10^{-4}$	$6.00 \cdot 10^7$	34.80	0.73	9.75	62.25	0.12	IIa
12	89	237	10	$3 \cdot 10^{-4}$	$6.57 \cdot 10^7$	38.09	0.77	10.67	62.59	0.14	IIb
13	89	297	10	$3 \cdot 10^{-4}$	$8.76 \cdot 10^7$	50.78	0.89	14.23	72.00	0.17	IIb
14	73	213	10	$1 \cdot 10^{-4}$	$1.80 \cdot 10^7$	1.80	0.13	0.44	14.43	0.01	I
15	73	213	10	$1 \cdot 10^{-4}$	$3.60 \cdot 10^7$	3.60	0.19	0.89	23.67	0.02	I
16	81	237	10	$1 \cdot 10^{-4}$	$7.20 \cdot 10^7$	7.20	0.27	1.78	41.02	0.04	I
17	89	285	10	$1 \cdot 10^{-4}$	$1.44 \cdot 10^8$	14.39	0.38	3.56	64.13	0.06	I
18	117	447	10	$1 \cdot 10^{-4}$	$2.16 \cdot 10^8$	21.59	0.46	5.33	71.48	0.09	I
19	81	237	10	$6 \cdot 10^{-5}$	$4.32 \cdot 10^7$	1.91	0.12	0.44	17.72	0.01	I
20	89	267	10	$6 \cdot 10^{-5}$	$8.64 \cdot 10^7$	3.81	0.18	0.89	30.29	0.03	I
21	89	317	10	$6 \cdot 10^{-5}$	$1.30 \cdot 10^8$	5.72	0.22	1.33	43.26	0.03	I
22	73	177	1	$6 \cdot 10^{-4}$	$3.34 \cdot 10^5$	2.33	0.35	1.00	2.80	0.06	I
23	73	177	1	$6 \cdot 10^{-4}$	$1.00 \cdot 10^6$	7.00	0.60	3.00	7.03	0.14	I
24	85	213	1	$6 \cdot 10^{-4}$	$1.67 \cdot 10^6$	11.67	0.77	5.00	9.78	0.20	I
25	85	213	1	$6 \cdot 10^{-4}$	$2.33 \cdot 10^6$	16.34	0.92	7.00	12.66	0.23	I
26	97	267	1	$6 \cdot 10^{-4}$	$3.34 \cdot 10^6$	23.35	1.10	10.00	15.11	0.38	IIa
27	85	213	1	$3 \cdot 10^{-4}$	$2.19 \cdot 10^6$	5.05	0.44	2.00	6.81	0.11	I
28	85	237	1	$3 \cdot 10^{-4}$	$5.47 \cdot 10^6$	12.64	0.70	5.00	14.85	0.17	I
29	85	213	1	$1 \cdot 10^{-4}$	$7.20 \cdot 10^6$	2.87	0.27	1.00	4.87	0.06	I
30	97	253	1	$1 \cdot 10^{-4}$	$2.16 \cdot 10^7$	8.60	0.46	3.00	16.26	0.12	I
31	97	253	1	$1 \cdot 10^{-4}$	$3.60 \cdot 10^7$	14.33	0.60	5.00	26.02	0.14	I
32	97	237	1	$6 \cdot 10^{-5}$	$1.73 \cdot 10^7$	3.04	0.25	1.00	7.45	0.06	I
33	73	177	3	$6 \cdot 10^{-4}$	$1.00 \cdot 10^6$	3.62	0.35	1.32	5.63	0.06	I
34	73	197	3	$6 \cdot 10^{-4}$	$3.00 \cdot 10^6$	10.87	0.60	3.95	13.76	0.12	I
35	85	213	3	$6 \cdot 10^{-4}$	$5.00 \cdot 10^6$	18.11	0.77	6.58	19.46	0.16	I
36	85	237	3	$6 \cdot 10^{-4}$	$7.00 \cdot 10^6$	25.36	0.92	9.21	23.85	0.20	I
37	97	267	3	$6 \cdot 10^{-4}$	$8.01 \cdot 10^6$	28.98	0.98	10.53	25.28	0.22	IIa
38	73	177	3	$3 \cdot 10^{-4}$	$3.28 \cdot 10^6$	3.92	0.31	1.32	8.53	0.06	I
39	85	213	3	$3 \cdot 10^{-4}$	$9.85 \cdot 10^6$	11.76	0.54	3.95	20.26	0.12	I
40	100	253	3	$3 \cdot 10^{-4}$	$1.64 \cdot 10^7$	19.61	0.70	6.58	26.97	0.15	I
41	73	213	3	$1 \cdot 10^{-4}$	$1.08 \cdot 10^7$	2.22	0.19	0.66	7.59	0.03	I
42	85	213	3	$1 \cdot 10^{-4}$	$2.16 \cdot 10^7$	4.45	0.27	1.32	16.47	0.05	I
43	97	253	3	$1 \cdot 10^{-4}$	$4.32 \cdot 10^7$	8.89	0.38	2.63	28.65	0.08	I
44	89	237	3	$6 \cdot 10^{-5}$	$2.59 \cdot 10^7$	2.36	0.18	0.66	9.49	0.03	I
45	97	285	3	$6 \cdot 10^{-5}$	$5.18 \cdot 10^7$	4.71	0.25	1.32	20.82	0.05	I

Table 4.2 As in Tab. 4.1 but for $r_i/r_o = 0.85$.

Model No.	N_r	j_{max}	Pr	Ek	Ra	Ro_{loc}	Ro_c	R_M	Nu	Ro	Mode
1	61	157	1	$6 \cdot 10^{-4}$	$6.67 \cdot 10^5$	4.67	0.49	2.00	5.11	0.11	I
2	61	157	1	$6 \cdot 10^{-4}$	$1.00 \cdot 10^6$	7.00	0.60	3.00	7.95	0.15	I
3	73	177	1	$6 \cdot 10^{-4}$	$1.67 \cdot 10^6$	11.67	0.77	5.00	11.67	0.19	I
4	73	177	1	$6 \cdot 10^{-4}$	$2.67 \cdot 10^6$	18.68	0.98	8.00	15.49	0.26	I
5	73	207	1	$6 \cdot 10^{-4}$	$4.00 \cdot 10^6$	28.02	1.20	12.00	18.95	0.35	IIa
6	85	257	1	$6 \cdot 10^{-4}$	$6.00 \cdot 10^6$	42.02	1.47	18.00	22.68	0.43	IIa
7	73	177	1	$3 \cdot 10^{-4}$	$2.19 \cdot 10^6$	5.05	0.44	2.00	8.84	0.10	I
8	73	177	1	$3 \cdot 10^{-4}$	$3.28 \cdot 10^6$	7.58	0.54	3.00	12.09	0.14	I
9	73	207	1	$3 \cdot 10^{-4}$	$5.47 \cdot 10^6$	12.64	0.70	5.00	17.05	0.17	I
10	81	237	1	$3 \cdot 10^{-4}$	$8.76 \cdot 10^6$	20.22	0.89	8.00	21.78	0.22	IIa
11	81	285	1	$3 \cdot 10^{-4}$	$1.31 \cdot 10^7$	30.32	1.09	12.00	26.29	0.30	IIa
12	73	217	1	$1 \cdot 10^{-4}$	$1.44 \cdot 10^7$	5.73	0.38	2.00	18.17	0.08	I
13	73	247	1	$1 \cdot 10^{-4}$	$2.16 \cdot 10^7$	8.60	0.46	3.00	22.01	0.11	I
14	73	377	1	$1 \cdot 10^{-4}$	$3.60 \cdot 10^7$	14.33	0.60	5.00	27.00	0.18	I
15	97	413	1	$1 \cdot 10^{-4}$	$5.76 \cdot 10^7$	22.92	0.76	8.00	35.32	0.21	IIa
16	81	257	1	$7 \cdot 10^{-5}$	$2.65 \cdot 10^7$	5.97	0.36	2.00	19.80	0.08	I
17	81	257	1	$7 \cdot 10^{-5}$	$3.98 \cdot 10^7$	8.95	0.44	3.00	25.98	0.11	I
18	73	347	1	$7 \cdot 10^{-5}$	$6.63 \cdot 10^7$	14.92	0.57	5.00	34.69	0.15	I
19	65	177	3	$6 \cdot 10^{-4}$	$2.00 \cdot 10^6$	7.25	0.49	2.63	12.23	0.09	I
20	65	177	3	$6 \cdot 10^{-4}$	$3.00 \cdot 10^6$	10.87	0.60	3.95	16.54	0.12	I
21	73	197	3	$6 \cdot 10^{-4}$	$5.00 \cdot 10^6$	18.11	0.77	6.58	22.13	0.15	I
22	73	197	3	$6 \cdot 10^{-4}$	$8.01 \cdot 10^6$	28.98	0.98	10.53	27.96	0.21	IIa
23	73	227	3	$3 \cdot 10^{-4}$	$6.57 \cdot 10^6$	7.84	0.44	2.63	17.69	0.09	I
24	73	227	3	$3 \cdot 10^{-4}$	$9.85 \cdot 10^6$	11.76	0.54	3.95	23.17	0.11	I
25	73	227	3	$3 \cdot 10^{-4}$	$1.64 \cdot 10^7$	19.61	0.70	6.58	31.55	0.15	I
26	85	267	3	$3 \cdot 10^{-4}$	$2.63 \cdot 10^7$	31.37	0.89	10.53	39.57	0.20	IIa
27	85	267	3	$1 \cdot 10^{-4}$	$4.32 \cdot 10^7$	8.89	0.38	2.63	32.96	0.08	I
28	85	267	3	$1 \cdot 10^{-4}$	$6.48 \cdot 10^7$	13.34	0.46	3.95	41.45	0.10	I
29	85	297	3	$1 \cdot 10^{-4}$	$1.08 \cdot 10^8$	22.23	0.60	6.58	55.25	0.14	I
30	73	177	10	$6 \cdot 10^{-4}$	$6.67 \cdot 10^6$	11.73	0.49	3.56	24.68	0.07	I
31	85	237	10	$6 \cdot 10^{-4}$	$1.00 \cdot 10^7$	17.59	0.60	5.33	30.99	0.08	I
32	85	237	10	$6 \cdot 10^{-4}$	$1.67 \cdot 10^7$	29.32	0.77	8.89	40.08	0.12	IIa
33	97	267	10	$6 \cdot 10^{-4}$	$2.67 \cdot 10^7$	46.91	0.98	14.23	48.36	0.16	IIb
34	85	237	10	$3 \cdot 10^{-4}$	$2.19 \cdot 10^7$	12.70	0.44	3.56	35.50	0.06	I
35	85	253	10	$3 \cdot 10^{-4}$	$3.28 \cdot 10^7$	19.04	0.54	5.33	44.27	0.09	I
36	97	285	10	$3 \cdot 10^{-4}$	$5.47 \cdot 10^7$	31.74	0.70	8.89	57.30	0.14	IIa
37	97	321	10	$3 \cdot 10^{-4}$	$8.50 \cdot 10^7$	49.30	0.87	13.81	68.21	0.16	IIb
38	97	267	10	$1 \cdot 10^{-4}$	$1.44 \cdot 10^8$	14.39	0.38	3.56	60.53	0.06	I
39	97	267	10	$1 \cdot 10^{-4}$	$2.16 \cdot 10^8$	21.59	0.46	5.33	75.77	0.08	I
40	97	285	10	$7 \cdot 10^{-5}$	$1.33 \cdot 10^8$	7.50	0.25	1.78	40.45	0.04	I
41	113	321	10	$7 \cdot 10^{-5}$	$2.65 \cdot 10^8$	14.99	0.36	3.56	73.29	0.06	I

Table 4.3 As in Tab. 4.1 but for $r_i/r_o = 0.90$.

Conclusion

In this thesis, we investigated the thermal convection in the subsurface oceans of icy moons, examining its impact on the ice shell deformation and the potential for indirect detection of ocean flows by future missions. We provided new insights into the variability of the heat flux coming from a convective ocean to the overlying ice shell, identifying a possible mechanism for surface topography formation through melting and freezing at the phase boundary. Additionally, we studied the structure and speed of convective flows, putting a strong focus on their detectability through motionally induced magnetic field. By examining extensive datasets spanning multiple orders of magnitude in control parameters, we derived a novel set of scaling laws for heat transfer and flow speed within the transitional regime of rotating convection, thus relevant to icy moon oceans. We identified multiple regimes of heat transfer and flow geometry and new non-dimensional parameters that accurately characterize these transitions were introduced.

Our results indicate that two distinct regimes of ocean cooling exist: polar cooling, characterized by peak heat transfer at the poles, and equatorial cooling, where heat flux is maximized at the equator. Extrapolating our datasets to parameters values relevant to Titan suggests that the oceanic heat flux peaks at the poles, potentially driving the formation of large topographic depressions in the polar regions. This aligns with available observations and supports the idea that variations in oceanic heat output play a key role in shaping the surface of icy moons.

The above hypothesis was tested beyond the classical Boussinesq approximation in the case of Titan. To achieve this, we developed a comprehensive model integrating ice shell deformation with underlying ocean convection, enabling us to quantify the impact of pressure-driven changes in melting temperature on the system (DAL effect). Due to the high computational cost, simulations of the DAL effect were limited to the transitional convection regime. Our results show that while heat flux from the deep ocean drives the formation of polar depressions, the higher melting temperature at the poles (resulting from lower pressure) promotes heat advection toward the equator, suppressing the formation of topography. A parametric study for Titan identified the parameter values that best match observed shape and topography. Building on these results, we developed a simplified parametrization of the DAL effect to quantify shape- and topography-induced variations in the oceanic heat flux at long wavelengths. This parametrization was then applied to Ganymede, with its predictions awaiting to be tested by the upcoming Juice mission.

Apart from investigating the role of oceanic convection in surface deformation, we examined the potential signature of ocean flows in the magnetic field. Focusing on the transitional convection regime, we identified three distinct flow modes. Mode I is characterized by two strong equatorial zonal jets moving in opposite directions, with the prograde jet near the phase boundary and the retrograde jet deeper in the ocean. Mode IIa features three zonal jets: one retrograde jet filling the equatorial region and two prograde jets at higher latitudes. Finally, Mode IIb, which is the opposite of Mode I, has two jets in the equatorial region, but with the retrograde jet near the interface. Given that the JUICE and Europa Clipper

missions carry magnetometers, our research focused on Europa and Ganymede. For Europa, our predictions suggest that the induced magnetic field from ocean flows may be too weak to be detected by the spacecraft's magnetometers, as it probably does not exceed 2 nT. However, for Ganymede, the results are more promising, with the motionally induced magnetic field potentially reaching up to 15 nT.

Observational data from upcoming missions will be essential for validating and refining the models developed in this thesis, as several questions remain unresolved. Future research should focus on enhancing numerical models with higher resolution and more accurately constrained material properties, particularly with respect to ice rheology and the effects of salinity in the ocean. Additionally, tidal deformation of the rocky cores may cause significant variations in heat flux, potentially influencing convection patterns in a manner akin to the DAL effect. The impact of tidally driven flows on short timescales, along with the effective viscosity of the ocean, also remains uncertain and warrants further exploration. In conclusion, this thesis demonstrates that convection within the subsurface oceans of icy moons plays a crucial role in the thermal evolution of icy bodies in the Outer Solar System, affecting surface deformation, induced magnetic fields, and the transport of chemicals from the interior to the icy shells. Our findings contribute to the broader field of planetary geophysics and lay the groundwork for future research into the dynamic interiors and habitability of ocean worlds.

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List of Figures

1.1	Heat flux characteristics q^* and $q^{h/l}$ plotted as a function of different diagnostic parameters.	30
1.2	Heat flux characteristics q^* and $q^{h/l}$ plotted as a function of modified diagnostic parameters $R_K^* = R_K/Pr$ and $R_G^* = R_G/Pr$	31
1.3	Boundaries between the areas of polar and equatorial cooling plotted as a function of parameters Ek and $RaEk^{4/3}/Pr$	33
1.4	Time and zonally averaged heat flux at the upper boundary of the ocean plotted as a function of the latitude for different values of R_G^*	34
1.5	Temperature and velocity fields in rotating sphericall shell averaged over time and longitude, $Pr = 1$	35
1.6	Temperature and velocity fields in rotating sphericall shell averaged over time and longitude, $Pr = 10$	36
1.7	Heat flux characteristics q^* and $q^{h/l}$ as a function of the convective Rossby number computed for models with no-slip and free-slip boundary conditions.	37
1.8	Time and zonally averaged heat flux at the upper boundary of the ocean plotted as a function of the latitude.	38
1.9	The response of the ice shell to variations in ocean heat flux.	40
1.10	Topography of Titan.	41
1.11	Square of the convective Rossby number as a function of the modified Rayleigh number.	46
2.1	Sketch of the computational domain for ice-ocean coupling.	58
2.2	Flowchart of the method proposed for coupled ice-ocean computation.	65
2.3	Temperature and velocity fields in Titan's ocean (ice-ocean coupled computations).	67
2.4	Titan's phase boundary topography (ice-ocean coupled computations).	68
2.5	Heat flux from Titan's ocean (ice-ocean coupled computations).	68
2.6	Titan's surface topography (ice-ocean coupled computations).	69
2.7	Shape of Titan's surface (ice-ocean coupled computations).	70
2.8	Comparison of the results obtained for different Ek (ice-ocean coupled computations).	71
2.9	Comparison of the heat flux and topography obtained for different Ek (ice-ocean coupled computations).	71
2.10	Parameter γ as a function of co-latitude θ (ice-ocean coupled computations).	72
2.11	Shape and topography of Titan truncated to the degree 5 (ice-ocean coupled computations).	73
2.12	Shape and topography of Titan truncated to the degree 2 (ice-ocean coupled computations).	74
3.1	Flow velocity patterns typical of Mode I and Mode II.	85
3.2	The sign of the zonal equatorial jet on the outer boundary as a function of parameter R_M	86
3.3	Horizontal Jupiter's magnetic field.	88

3.4	Vertical Jupiter's magnetic field.	88
3.5	Europa's motionally induced magnetic field in various configurations.	92
3.6	Sketch of the electrical currents.	92
3.7	Europa's motionally induced magnetic field for various ice thicknesses.	93
3.8	Europa's motionally induced magnetic field strength for Mode II flow pattern.	93
3.9	Europa's motionally induced magnetic field strength as a function of electrical conductivity of mantle, ocean, ice.	94
3.10	Europa's motionally induced magnetic field strength as a function of electrical conductivity of ice.	95
3.11	Graphs illustrating the scaling laws derived for Rayleigh and Rossby number.	97
3.12	Flow model of Soderlund (2019).	100
3.13	Europa's motionally induced magnetic field derived from the flow model of Soderlund (2019).	101
4.1	Diagram of equatorial flow obtained from various DNS methods.	113
4.2	Zonal (v_ϕ), meridional (v_ϑ) and radial (v_r) components of the flow velocity predicted for radius ratio 0.85.	115
4.3	Radial component of the heat flux at the upper boundary of the ocean predicted for Ganymede.	116
4.4	Ganymede's long-wavelength topography ($l = 2 - 10$) predicted for model C in Tab. 4.1.	118
4.5	Radial, meridional and zonal components of the magnetic field generated by ocean convection in Modes I, IIa and IIb.	121
4.6	Graphs illustrating the scaling laws for Ganymede.	124
4.7	Zonal (v_ϕ), meridional (v_ϑ) and radial (v_r) components of the flow velocity predicted for radius ratio 0.80, 0.90.	125
4.8	Heat flux distribution at the upper boundary of the ocean obtained for various radius ratios.	126
4.9	The sign of the equatorial zonal jet as a function of parameter $R_M = RaEk^{12/7}Pr^{-3/4}$	126

List of Tables

1.1	Parameters controlling heat and mass transfer in Titan's ocean. . .	28
1.2	Diagnostic parameters appropriate for Titan's ocean.	29
1.3	Values of control and diagnostic parameters expected for different thicknesses of Titan's ocean.	47
1.4	Results of no-slip simulations with radius ratio 0.80.	48
1.5	Results of free-slip simulations with radius ratio 0.80.	49
1.6	Results of free-slip simulations with radius ratio 0.80 (continuation). .	50
2.1	Control and diagnostic parameters of Titan.	64
2.2	Material parameters of ice.	66
2.3	Table of ice-ocean coupling models for Titan.	78
3.1	Considered models of Europa's hydrosphere.	89
3.2	Induced magnetic field symmetries.	91
3.3	Selected scaling laws proposed for estimating the Rayleigh number. .	97
3.4	Selected scaling laws proposed for estimating the Rossby number. . .	99
3.5	List of numerical simulations used to derive the scaling laws for Europa's ocean.	102
4.1	Considered models of Ganymede hydrosphere.	109
4.2	Flow speeds: Ganymede	111
4.3	Scaling laws proposed for Ganymede.	123
4.1	List of numerical simulations with the radius ratio of 0.80.	127
4.2	List of numerical simulations with the radius ratio of 0.85.	128
4.3	List of numerical simulations with the radius ratio of 0.90.	129

List of Publications

1. J. Kvorka, O. Čadek. A numerical model of convective heat transfer in Titan's subsurface ocean. *Icarus*, 376:114853, 2022. doi: 10.1016/j.icarus.2021.114853
2. J. Kvorka, O. Čadek. The role of subsurface ocean dynamics and phase transitions in forming the topography of icy moons. *Icarus*, 412:115985, 2024. doi: 10.1016/j.icarus.2024.115985
3. L. Šachl, J. Kvorka, O. Čadek, J. Velímský. Magnetic field induced by convective flow in Europa's subsurface ocean. *Icarus*, 429:116375, 2025. doi: 10.1016/j.icarus.2024.116375
4. J. Kvorka, O. Čadek, L. Šachl, J. Velímský. Convective flow in Ganymede's subsurface ocean: Implications for the induced magnetic field and topography. Under review in *Icarus* (submitted on 27.1.2025).