

**FACULTY
OF MATHEMATICS
AND PHYSICS**
Charles University

BACHELOR THESIS

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Magnetic field in oceans: Parametric study and visualisation

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Study programme: Physics

Prague 2025

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This bachelor thesis would not have been possible without the financial support provided by the European Space Agency (ESA), Contract No. 4000143627/24/I-EB Swarm for Ocean Dynamics, part of the ESA Solid Earth Magnetic Science Cluster, EO for Society programme.

A significant contribution was the spectral analysis programming done by Rasmus Møller Blangsbøll, which was instrumental in generating most of the plots in this thesis.

My deepest thanks go to my supervisor for having infinite patience, to all the people at the Department of Geophysics for always treating me so nicely, to all my friends for giving me so much support over the years, and to my family for always being there for me. Thank you!

Title: Magnetic field in oceans: Parametric study and visualisation

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Abstract: The Ocean-Induced Magnetic Field (OIMF) holds great potential for advancing large-scale ocean monitoring, as it is sensitive to ocean flow and electrical conductivity throughout the full ocean depth. This thesis is motivated by the ongoing ESA project Swarm for Ocean Dynamics. Our task in the project is to investigate, how various modelling parameters affect the simulated OIMF. We find that the choice of the ocean model is the primary factor determining the OIMF spatio-temporal characteristics. The influence of other input parameters, such as the electrical conductivity of the Earth's mantle, or the choice of the model of the main geomagnetic field, have only a secondary effect. We have also investigated the impact of the spatial resolution and the initial condition on the OIMF numerical simulations, which offer limited benefits relative to their computational cost. Our results help identify optimal model setups for efficient OIMF simulation and contribute to future research in this field.

Keywords: electromagnetic induction, ocean circulation

Název práce: Magnetické pole v oceánech: Parametrická studie a vizualizace

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Abstrakt: Magnetické pole indukované oceánem (anglicky OIMF) má velký potenciál pro rozvoj monitorování oceánů ve velkém měřítku, protože je citlivé na proudění oceánu a elektrickou vodivost v celé hloubce oceánu. Tato práce je motivována probíhajícím projektem ESA s názvem Swarm for Ocean Dynamics. Naším úkolem v projektu je zkoumat, jak různé modelovací parametry ovlivňují simulované OIMF. Zjistili jsme, že klíčovým faktorem určujícím prostorově-časové charakteristiky OIMF je konkrétní volba oceánského modelu. Vliv ostatních vstupních parametrů, jako je elektrická vodivost zemského pláště nebo volba modelu hlavního geomagnetického pole, má pouze druhotný dopad. Dále jsme zkoumali vliv prostorového rozlišení a počátečních podmínek na numerické simulace OIMF, které poskytují jen omezený přínos vzhledem ke své výpočetní náročnosti. Naše výsledky pomáhají identifikovat optimální nastavení modelu pro efektivní simulaci OIMF a přispívají k budoucímu výzkumu v této oblasti.

Klíčová slova: elektromagnetická indukce, oceánské proudění

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Introduction

Magnetic fields in the oceans, commonly referred to as Ocean-Induced Magnetic Fields (OIMF), represent a fascinating area of study. This phenomenon is governed by fundamental principles of electrodynamics, particularly Faraday’s law of induction, and Biot-Savart law. The OIMF data can be utilized to estimate oceanic properties in remote or deep-sea regions where direct measurements are difficult or even impossible to obtain. The most significant long-term goal is to employ OIMF in characterizing global ocean circulation and supporting large-scale ocean monitoring efforts, since the OIMF is sensitive to the water flow in the whole depth of the ocean, not just on the surface, in contrary to satellite measurements (e.g., salinity, temperature, altimetry) or *in situ* observations (e.g., ships, buoys). While this idea is highly promising, it also poses a big scientific challenge.

For example, thanks to buoys, many oceanic parameters, like temperature, salinity, and flow velocity, are already well-mapped on the surface and in small depths of the ocean. Even though buoy data already partially accommodates our needs, the amount of it is low, and is only point-wise, therefore not sufficient for large-scale monitoring of the ocean. More data from deeper oceanic regions is crucial for further advancement.

This topic becomes even more anticipated with the fact that ESA’s *Swarm for Ocean Dynamics* mission, consisting of three satellites equipped with magnetometers, has been continuously measuring the Earth’s magnetic field since 2013. ESA’s Swarm flotilla is made up of the satellites Alpha and Charlie, which are on nearly circular orbits at an altitude of around 462 km above the Earth’s surface, and Bravo, which orbits at a higher altitude of around 510 km (*Swarm Frequently Asked Questions [online]* 2025).

The first interest in the OIMF emerged in the 1970s through fundamental research by Cox (1970), Sanford (1971), Sanford (1982), and Larsen and Sanford (1985). The earliest numerical simulations of OIMF were carried out by Stephenson and Bryan (1992) and Flosadóttir et al. (1997). In parallel, Tyler et al. (1997) utilized the Ocean and IsoPYCnal Coordinates (OPYC) model to predict its OIMF signature. Subsequent efforts to estimate the strength of the OIMF at satellite altitudes, specifically around 400 km, where the CHALLENGING Minisatellite Payload (CHAMP), Swarm, and other low-orbit magnetometric missions operate, were made by Vivier et al. (2004), using simulations from the Océan PARallélisé (OPA), Hamburg Ocean Primitive Equation (HOPE), and Estimating the Circulation and Climate of the Ocean (ECCO) models. Further investigations by Manoj et al. (2006) examined the magnetic signatures generated by oceanic electric currents derived from both the ECCO and OCCAM (Ocean Circulation and Climate Advanced Modelling project) models, assessing signals at sea level and at approximately 430 km, the orbital height of the lower pair of the Swarm satellites.

Across these studies, a small signal, in the order of around 1 nT was consistently predicted at satellite altitude, with the Antarctic Circumpolar Current (ACC) producing the most pronounced effect due to its massive volume transport and its alignment with the geomagnetic pole.

Detecting this signal is highly challenging, as the OIMF is only one component of the Earth’s complex geomagnetic system. Other sources—including the Earth’s

main magnetic field, crustal/lithospheric magnetization, solar-quiet ionospheric currents, and auroral electrojet ionospheric currents (Finlay et al., 2024)—contribute significantly to the total measured magnetic field. These sources typically generate magnetic field strengths at least an order of magnitude greater than that of the OIMF, complicating its isolation in satellite data. To put this into perspective, the Earth’s core magnetic field has a strength of approximately 40 000 nT at altitudes of around 400 kilometres.

Nowadays, we are already able to describe tidal flows, a specific type of oceanic flow, very accurately, and subsequently also its Tidal-Induced Magnetic Field (TIMF). This accomplishment is largely due to the fact that tides represent an easier type of movement, primarily two-dimensional, meaning the entire mass of water moves barotropically due to the Moon’s gravitational force. At a given location, the velocities are independent of depth. In general, barotropic equations governing ocean flow involve three unknowns: the two horizontal velocity components and the sea surface elevation. The latter is directly measured by satellite altimetry, and through the continuity equation, it indirectly constrains the velocity field as well. Satellite altimetry is in these days well-established and is key to accurate description of the tides. Magnetic fields induced by tidal flows are routinely extracted from satellite, coastal and island observatory data (Sabaka et al., 2016; Grayver and Olsen, 2019; Grayver et al., 2024; Tyler and Trossman, 2024). This success is largely due to the fact that tidal flows occur at well-known frequencies, which provides strong a priori information that greatly aids in isolating their signals from the data. Satellite observations of the TIMF are inverted in terms of the conductivity of the upper mantle (Grayver et al., 2017; Šachl et al., 2022; Šachl et al., 2024).

In recent years, advancements have been made towards detection of the global ocean circulation signal using data from seafloor voltage cables by Velínský et al. (2021). Here, it is important to mention that in contrast to tides, the global ocean circulation is driven by interactions with the atmosphere, has a much more complex structure, is vertically stratified, and also possesses a radial (vertical) component. Its temporal variations are long-term and include additional phenomena such as El Niño–Southern Oscillation (ENSO). In principle, this study has shown that long-term variations in oceanic flow correlate with the voltage across a cable integrated along its length, but unfortunately, this provides only integral information that reflects the water transport along the entire length of the cable. There are also only a few such cables, therefore it cannot be used for systematic ocean description.

The extraction of the actual OIMF signal from Swarm measurements is currently being undertaken by researchers at the Technical University of Denmark (DTU) (Finlay et al., 2024), in the framework of the Swarm for Ocean Dynamics project, solved by DTU Space Copenhagen and Charles University Prague, and funded by the European Space Agency. Our role is to predict the expected magnetic field strengths via numerical simulations and quantify its sensitivity to input parameters, thereby helping the Danish team to identify the relevant signals more efficiently in the Swarm magnetic data. In a nutshell, our aim is to identify systematic spatio-temporal characteristics of the OIMF that could aid in its detection.

Closely related to this matter, since the underlying physics is the same, is the

exciting research conducted by scientists studying the OIMF of the icy moons of Jupiter and Saturn (Kureš, 2023; Šachl et al., 2025). The JUICE (ESA) and Europa Clipper (NASA) interplanetary missions are currently on their ways to the Jovian system, which they will reach in the early 2030s. In the best-case scenarios, the OIMF signals could provide additional constraints on the internal structure and ocean dynamics of the icy moons, in particular Europa and Ganymede. However, isolation of the OIMF signal from observed data will be even more challenging than for the Earth oceans.

As illustrated, this field of study is very lively and developing fast, even though there is still a lot to be done. The goal of this bachelor thesis is therefore to take a step on this long journey.

In Chapter 1, we introduce the underlying principles of the OIMF and provide a theoretical derivation of the key equations that form the basis of our approach, and present our workflow. Chapter 2 focuses on the input variables to the electromagnetic induction (EMI) equation, namely the ocean flow velocities and electrical conductivities of seawater and solid Earth. In Chapter 3, we present the core of our computational method, the solver used to predict the OIMF. Chapter 4 outlines the different simulation scenarios conducted to meet the objectives of this study. Chapter 5 presents and analyzes the outcomes of our computations. Finally, the Conclusion summarizes our findings and concludes the thesis.

1 Theoretical background

1.1 Understanding the phenomenon

The generation of the Ocean-Induced Magnetic Fields (OIMF) can be explained with the help of basic concepts in electrodynamics. Faraday's law of induction states that a moving conductor (saltwater) in the presence of a magnetic field (core field) induces a current in the conductor.

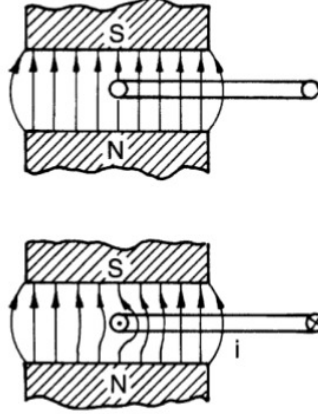


Figure 1.1 Faraday's law of induction. Taken from Davidson (2017).

Mathematically, the induction effect is described by the Maxwell-Faraday equation,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (1.1)$$

relating a spatially varying non-conservative electric field $\mathbf{E}(\mathbf{r}, t)$ to a time-varying magnetic field $\mathbf{B}(\mathbf{r}, t)$, where $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$ denote a general electric and magnetic field, respectively.

From Biot-Savart law one knows that once an electric current flows through a conductor (oceans) it creates a magnetic field (the OIMF) as a result.

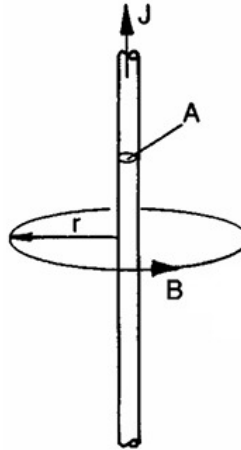


Figure 1.2 Biot-Savart law. Taken from Davidson (2017).

This law can be quantified by the following equation, holding for a general magnetic field $\mathbf{B}(\mathbf{r}, t)$:

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}, \quad (1.2)$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m (*Vacuum permeability [online]* 2025) is the vacuum magnetic permeability, π is the well-known mathematical constant, I is the electrical current flowing through the conductor, $d\mathbf{l}$ is an infinitesimal length of the conductor, $\hat{\mathbf{r}}$ is a unit vector pointing in the direction of the examined point, and r^2 is the square of the radius vector.

In summary, the Earth possesses a magnetic field generated by its core. As electrically conductive ocean water, due to its salinity and temperature, moves through this magnetic field, it induces electric currents. These currents, in turn, produce a secondary magnetic field, the OIMF, which can be detected and measured.

1.2 Underlying equations

Having gained a qualitative understanding of the phenomenon, we now proceed to derive the underlying equation that enables the computation of the magnetic field. The following equations are necessary for the derivation:

Gauss's law for magnetism:

$$\nabla \cdot \mathbf{B}_{oi} = 0, \quad (1.3)$$

where $\mathbf{B}_{oi}(\mathbf{r}, t)$ is the Ocean-Induced Magnetic Field. The absence of magnetic monopoles, a fundamental principle of magnetism, applies universally to all magnetic fields, including $\mathbf{B}_{oi}$.

Faraday's law of induction:

$$\nabla \times \mathbf{E}_{oi} = -\frac{\partial \mathbf{B}_{oi}}{\partial t}, \quad (1.4)$$

where $\mathbf{E}_{oi}(\mathbf{r}, t)$ is the electric field generated by the oceans.

Ampère's circuital law:

$$\nabla \times \mathbf{B}_{oi} = \mu_0 \mathbf{j}, \quad (1.5)$$

where in our case, for ocean water, sediments, and silicate lithosphere and mantle, the magnetic permeability is the same as of vacuum. $\mathbf{j}(\mathbf{r}, t)$ is the electric current density, for which

Ohm's law for a moving conductor holds:

$$\mathbf{j} = \sigma(\mathbf{E}_{oi} + \mathbf{u} \times \mathbf{B}_M), \quad (1.6)$$

where $\sigma(\mathbf{r}, t)$ is the electrical conductivity, $\mathbf{u}(\mathbf{r}, t)$ is the velocity of the ocean flow, and $\mathbf{B}_M(\mathbf{r}, t)$ is the *ambient magnetic field*, which in our case is represented by the main magnetic field of the Earth generated in its core. If we would want to be exact, on the right-hand side of the cross product, we would have to write

$\mathbf{u} \times (\mathbf{B}_M + \mathbf{B}_{oi} + \dots)$, including all components of the geomagnetic field. But since these components are negligible compared to $\mathbf{B}_M$, they are omitted, leaving only $\mathbf{B}_M$. The cross product $\mathbf{u} \times \mathbf{B}_M$ describes the Lorentz force exerted on charge carriers as they move through a continuum.

In the Maxwell's equations we assume quasi-stationarity, meaning the Maxwell displacement current density $\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} = j_M$ is negligible, where ε_0 is the permittivity of vacuum. The quasi-static assumption indicates that the changes are happening slowly enough, which our oceanic currents satisfy. This can be shown by an order-of-magnitude comparison of the conductivity and displacement current terms, $\sigma \gg \varepsilon_0/T$, where σ is the typical conductivity of water ($\sim 10^0$ S/m) and ε_0 is the permittivity of vacuum ($\sim 10^{-11}$ F/m). T is the typical timescale we are working with, say 1 day or longer, i.e., $\sim 10^5$ s. The inequality clearly holds.

With these core relationships established, we can derive the final equation, which will be solved using the `elmgtD` solver presented in Chapter 3. We will start our derivation by substituting equation (1.6) into equation (1.5). All subsequent steps are shown on the right, and are denoted by a '/' symbol.

$$\begin{aligned}
& \nabla \times \mathbf{B}_{oi} = \mu_0 \sigma (\mathbf{E}_{oi} + \mathbf{u} \times \mathbf{B}_M) \quad /:\sigma \quad \wedge \quad /-\mu_0 \mathbf{E}_{oi} \\
& \frac{1}{\sigma} \nabla \times \mathbf{B}_{oi} - \mu_0 \mathbf{E}_{oi} = \mu_0 (\mathbf{u} \times \mathbf{B}_M) \quad / \nabla \times \quad / \mu_0 \text{ is const} \\
& \nabla \times \left(\frac{1}{\sigma} \nabla \times \mathbf{B}_{oi} \right) - \mu_0 \underbrace{\nabla \times \mathbf{E}_{oi}}_{-\frac{\partial \mathbf{B}_{oi}}{\partial t}} = \mu_0 \nabla \times (\mathbf{u} \times \mathbf{B}_M) \quad / \sigma \text{ not!} \\
& \quad \quad \quad - \frac{\partial \mathbf{B}_{oi}}{\partial t} \quad \quad \quad / \text{using (1.4)}
\end{aligned}$$

After these steps, we arrive at our final equation (1.7), where a driving force appears on the right-hand side of the equation, originating from Lorentz force in Ohm's law.

$$\boxed{\nabla \times \left(\frac{1}{\sigma} \nabla \times \mathbf{B}_{oi} \right) + \mu_0 \frac{\partial \mathbf{B}_{oi}}{\partial t} = \mu_0 \nabla \times (\mathbf{u} \times \mathbf{B}_M)} \quad (1.7)$$

As it is commonly known, a partial differential equation is incomplete without an initial and a boundary condition. As the initial condition, we take the homogeneous Neumann initial condition, meaning $\frac{\partial \mathbf{B}_{oi}}{\partial t} = 0$ at $t = 0$. Alternatively, a non-homogeneous Dirichlet initial condition can be also used, $\mathbf{B}_{oi} = \mathbf{B}_{oi}^0$ at $t = 0$. This is useful for extending a previously calculated solution for an additional time interval. As for the boundary condition, we assume that on the surface of the Earth, $\mathbf{B}_{oi}$ is continuous; outside of the Earth's surface, i.e., in the isolating atmosphere, $\mathbf{B} = -\nabla U$, where

$$U(\mathbf{r}, t) = a \sum_{j=1}^{\infty} \sum_{m=0}^j \left(\frac{a}{r} \right)^{j+1} P_{jm}(\cos \theta) [g_{jm}(t) \cos(m\phi) + h_{jm}(t) \sin(m\phi)] \quad (1.8)$$

is the analytical solution of the Laplace equation for the scalar magnetic potential U , where g_{jm} and h_{jm} are the spherical harmonic coefficients (Finlay et al., 2024). This is expressed in spherical coordinates (r, θ, ϕ) , where r is the radial distance,

θ is the polar angle (or in other words, colatitude corresponding to the meridional direction, positive southwards), and ϕ is the azimuthal angle (or in other words, longitude, corresponding to the zonal direction, positive eastwards).

The series does not contain terms corresponding to the external field $(r/a)^j$, and thus the condition $\lim_{r \rightarrow \infty} U = 0$ corresponds to the boundary condition in the absence of external field sources. The coefficients g_{jm} and h_{jm} are also already Schmidt quasi-normalized (Winch et al., 2005).

1.3 Methodology

Equation (1.7) is a first-order differential equation in time, and a second-order in space that cannot be solved analytically for non-constant conductivity and must therefore be approached numerically. To do so, a non-trivial and non-linear computational pipeline is required, as illustrated in Figure 1.3.

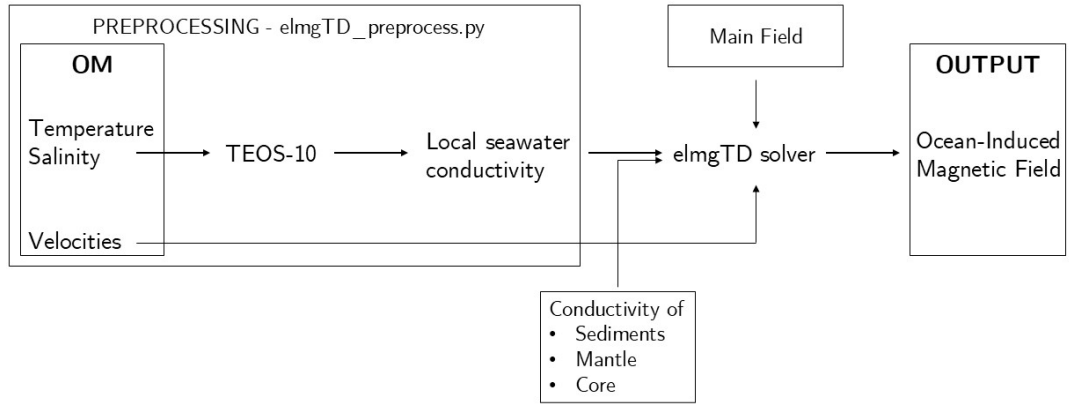


Figure 1.3 Workflow of the forward modelling chain. See details in text.

The general direction of our workflow is from left to right with some supplementing inputs showed vertically. On the left side of Figure 1.3, one can see the **Preprocessing** stage. This step consists of multiple parts, from which the first is the Ocean Model (**OM**), which is discussed in more depth in Section 2.1. From the Ocean Model, we need the 4-D spatio-temporal distribution of three key variables: temperature, salinity, and velocity. Using the **TEOS-10** equation of state (*TEOS-10 [online]* 2025), **Local seawater conductivity** is calculated from the temperature and salinity values of the OM for every spatio-temporal coordinate. This conductivity is combined with electrical conductivity of solid Earth domains, namely the **Conductivity of Sediments, Mantle, and Core**, which are discussed in Section 2.2.1. Once all relevant conductivities are obtained, they serve as input to the **elmgTD solver**. However, before the governing equation (1.7) can be solved, two additional inputs are required. The first is the ocean velocity field, obtained from the OM, which is described in detail in Section 3.1. The second input is the core magnetic field, which is showed in the **Main Field** box in Figure 1.3, and is described in Section 3.2. Last but not least, as the final result we get the OIMF (**Ocean-Induced Magnetic Field**), which is our aim.

The two main components of our approach, the Preprocessing and the **elmgTD** solver are described in Chapters 2 and 3, respectively.

2 Preprocessing

The goal of the preprocessing is the calculation of the electrical conductivity of the ocean from the input Ocean Model. This task is done by the code written in Python, `elmgTD_preprocess.py`. All the code snippets in this chapter come from this file and are openly available on GitHub (Velínský, 2025).

2.1 Ocean Models

In order to investigate the sensitivity of OIMF to the input Ocean Model, we carry out a series of OIMF simulations, using different products provided by the Copernicus Marine Environment Monitoring Service (CMEMS) and Estimating the Circulation and Climate of the Ocean (ECCO) services, which are described below.

Listing 2.1 Code snippet: Ocean Model selection and accessing

```
# Select ocean model
# OM= 1 # CMEMS from internet
# OM= 3 # CMEMS from files
OM= 2 # ECCO2

.
.
.

# read ocean model
if (OM == 1):
    print ('Reading CMEMS online',flush=True)
    OceanModel= get_CMEMS(lat=lat,lon=lon,
        start_datetime=locstart,end_datetime=locend)
elif (OM == 3):
    print ('Reading CMEMS from files',flush=True)
    OceanModel= get_CMEMS_fromfiles(lat=lat,lon=lon,
        CMEMS_Path=CMEMS_Path, start_datetime=locstart,
        end_datetime=locend)
elif (OM == 2):
    print ('Reading ECCO2',flush=True)
    OceanModel= get_ECCO2(lat=lat,lon=lon,
        start_datetime=locstart,end_datetime=locend)

print(OceanModel)

.
.
.
```

2.1.1 CMEMS family

The CMEMS is a component of the European Union’s Copernicus Programme, dedicated to monitoring and forecasting the state of the global ocean. It provides free and open access to a wide range of high-quality marine data (*Copernicus Programme [online]* 2025).

One of CMEMS’s key products is the Global Ocean Ensemble Physics Re-analysis, which offers a comprehensive, multi-decade dataset of oceanic variables

like temperature, salinity, and velocity. The dataset spans from January 1993 to December 2023, with a spatial resolution of 0.25° and 75 vertical levels (spanning 6 km of the ocean from its surface). The data can be accessed on the Global Ocean Ensemble Physics Reanalysis page (Copernicus Marine, 2025).

The preprocessing is done using Python’s `xarray` library (*Xarray documentation [online]* 2025), which in principle allows working with data stored remotely. However, this approach proves unreliable due to occasional random data gaps, so the entire dataset has to be downloaded locally. For reference, a 10-year series of daily values occupies approximately 3,5 TB.

Listing 2.2 Code snippet: Download of the data from CMEMS

```
def get_CMEMS_fromfiles(lat, lon, CMEMS_Path,
    dataset_id="cmems_mod_glo_phy-mnstd_my_0.25deg_P1D-m",
    start_datetime="2021-01-01T00:00:00",
    end_datetime="2022-12-31T23:59:59" ):
    .
    .
    .
    return wrap360(CMEMSorig.rename({"uo_mean": "uvel", "vo_mean":
    "vvel", "so_mean": "salt", "thetao_mean": "temp"}), lon='longitude')
    .fillna(0.0).interp(latitude=lat, longitude=lon).fillna(0.0)
```

The function `wrap360()` converts values from a -180° to 180° longitude range to a 0° to 360° range.

Other CMEMS models

The CMEMS dataset comprises individual model outputs along with their statistical processing, which includes a mean model (*CMEMS mean*) and associated statistical deviations. Since all data are represented on the same grid, this enables direct and efficient comparison of simulation results.

The *CMEMS mean* product combines outputs from four different ocean reanalysis systems to provide an ensemble mean that enhances reliability. The models used in this mean are: GLORYS2V4, ORAS5, GloSea5, and C-GLORSv7. Except the *CMEMS mean* model we also use the CMEMS-cglo (Storto et al., 2016), the CMEMS-glor (Lellouche et al., 2013), and the CMEMS-oras (Zuo et al., 2017) models in our investigation.

CMEMS-cglo

`cglo` refers to the CMCC GLoBal Ocean Reanalysis System (C-GLORS). CMCC stands for the Euro-Mediterranean Center on Climate Change, an Italian research institution dedicated to climate-related research.

CMEMS-glor

Mercator Ocean is the French center for ocean analysis and forecasting. GLORYS, which stands for GLoBal Ocean ReanalYsis and Simulation, is the ocean reanalysis developed by Mercator Ocean.

CMEMS-oras

Based on ECMWF, 2025, the Ocean ReAnalysis System (ORAS) is a part of the operational system OCEAN. The OCEAN is derived from a prototype system ORAP (Ocean ReAnalysis Pilot). The CMEMS-oras model is developed by the European Centre for Medium-Range Weather Forecasts (ECMWF), an independent intergovernmental organisation supported by most countries in Europe.

Comparison

Global ocean reanalyses provide consistent, three-dimensional, gridded representations of the ocean’s physical state over multiple decades. They are generated using numerical ocean models combined with data assimilation techniques that incorporate both satellite and *in situ* observations. Based on Gounou et al. (2025), the three additional reanalyses used by us are all based on the NEMO (Nucleus for European Modelling of the Ocean) model. Each is forced at the surface by ERA interim and assimilates sea surface temperature (SST), sea level anomalies (SLA), sea ice concentration (SIC), and *in situ* temperature and salinity profiles. These reanalyses are designed to be as realistic as possible, closely matching observational data while remaining consistent with the physical constraints of the ocean model. The depth-wise grid consists of 75 levels: the thickness of the topmost layer is approximately 1 m; is close to 10 m at the depth of 100 m; and increases up to 200 m at the bottom (near 6000m). The concrete depths of the layers can be seen below:

0.50, 1.55, 2.66, 3.85, 5.14, 6.54, 8.09, 9.82, 11.77, 13.99, 16.52, 19.42, 22.75, 26.55, 30.87, 35.74, 41.18, 47.21, 53.85, 61.11, 69.02, 77.61, 86.92, 97.04, 108.0, 120.00, 133.07, 147.40, 163.16, 180.54, 199.79, 221.14, 244.89, 271.35, 300.88, 333.86, 370.68, 411.79, 457.62, 508.63, 565.29, 628.02, 697.25, 773.36, 856.67, 947.44, 1045.85, 1151.99, 1265.86, 1387.37, 1516.36, 1652.56, 1795.67, 1945.29, 2101.02, 2262.42, 2429.02, 2600.38, 2776.03, 2955.57, 3138.56, 3324.64, 3513.44, 3704.65, 3897.98, 4093.15, 4289.95, 4488.15, 4687.58, 4888.07, 5089.47, 5291.68, 5494.57, 5698.06, 5902.05

The three models share many similarities. Their differences are illustrated in Table 2.1.

Reanalysis	Production centre	COMMON	Model version	Surface Forcing	ASSIMILATION
GLORYS2V4	Mercator Océan	NEMO, ORCA1/4° 75 vertical levels TKE Altimetry ERA : 1993-2015 ERAinterim and bulk formulae Observations : SST, SLA, T/S profiles, SIC Multivariate assimilation, monovariate for the SIC	NEMO3.1 LIM2	No surface nudging precipitation, flux correction Climatological runoff + ice shelf and iceberg melting	SAM2 (SEEK) Large scale bias correction 7-day assimilation window Merge MDT (obs+model) Reynolds SST, CORA
C-GLORS	CMCC		NEMO3.4 LIM2	SST, SSS, SIC surface nudging	OceanVar (3Dvar) Large scale bias correction 7-day assimilation window Model MDT Reynolds SST, EN4
ORAS5	ECMWF		NEMO3.4.1 LIM2	Surface waves SST, SSS surface nudging	NEMOVAR (3Dvar) 5-day assimilation window HadISSTv2 SST, EN4

Table 2.1 CMEMS model family comparison. Taken from PRODUCT USER MANUAL Global Ocean Reanalysis Product (Gounou et al., 2025).

Table 2.1 describes the reanalysis name; the production centre, where it is created; what they have in common; which version of NEMO is used to model it; the surface forcing; and the assimilation. Since Ocean Models are not the main focus of this thesis, we do not describe them in more depth.

2.1.2 ECCO family

In contrast to the CMEMS models, which originate from European institutions, the ECCO model family is developed under the auspices of NASA in the United States (*ECCO [online]* 2025) by the ECCO Consortium, which is made up of an international group of scientists from several institutions (*Teams [online]* 2025). Initiated in 1998 within the framework of the World Ocean Circulation Experiment, the ECCO project set out to merge general circulation modelling with multiple observational datasets to deliver a dynamically consistent, time-resolved reconstruction of the global ocean. Since these earliest efforts, several updated versions of this dataset have been released. For the purposes of our investigation, we focus on two specific versions: ECCO2 and ECCOv4r4. The ECCO family models have 50 vertical levels (spanning 6 km of the ocean from its surface).

ECCO2

Using a Green’s Function methodology (Menemenlis et al., 2005), the first Estimating the Circulation and Climate of the Ocean, Phase II (ECCO2) synthesis spanning 1992-2007 was produced by estimating initial ocean temperature and salinity fields, surface forcing conditions, and selected empirical parameters within ocean and sea ice models by Menemenlis et al. (2008). The analysis was constrained by multiple datasets, including satellite altimetry, gravity measurements, drifter data, hydrographic profiles, and sea ice observations. Over time, the ECCO2 dataset has been steadily expanded and currently extends through the end of 2023.

Listing 2.3 Code snippet: Download of the data from ECCO2

```
# Download ECCO2 data and store in xarray
def get_ECCO2(lat, lon,
    ECCO2_Path='/SfOD/ECCO2/ECCO2/cube92_latlon_quart_90S90N/daily/',
    start_datetime="2021-01-01T00:00:00",
    end_datetime="2021-01-03T23:59:59" ):
    .
    .
    .
    return ECCO2
```

ECCOv4r4

The ECCO Version 4 is based on the fundamental paper by Forget et al. (2015), which uses more advanced methods for assimilating observational data (adjoint methods) in comparison to ECCO2. The following papers came out afterwards, and are improvements to the initial paper.

- Version 4 Release 2 - (Forget et al., 2016)
- Version 4 Release 3 - (Fukumori et al., 2017)

- Version 4 Release 4 (v4r4) - (Wang et al., 2020)

The ECCOv4r4 spans from the end of 1992 until the end of 2017. Unfortunately, there is only a partial overlap between the ECCOv4r4 model (2013-2017) and the Swarm satellite mission (2013-present).

2.2 Local seawater conductivity (TEOS-10)

The ocean conductivity is dependent on various factors. The most prominent contributors are salinity and temperature, which vary over time. To some extent, pressure (implied by the weight of the water = depth) also plays a role (Tyler et al., 2017). Salts in water break down to cations and anions that move due to electric fields. This movement creates electric currents. The dependence of electrical conductivity on salinity and temperature for the Earth’s oceans is described by the Thermodynamic Equation Of Seawater - 2010 (TEOS-10).

TEOS-10 is an equation based on a Gibbs function formulation from which all thermodynamic properties of seawater can be derived (*TEOS-10 [online]* 2025). To perform calculations, we use the Python implementation of this equation provided by the *Gibbs-SeaWater (GSW) Oceanographic Toolbox* software package (*TEOS-10 software [online]* 2025). The following code, Listing 2.4, illustrates the calculation of conductivity with the help of the `cond_from_model(model)` function, which takes the Ocean Model as the input, denoted by (`model`). As a prerequisite, the pressure, `press`, is first calculated.

Listing 2.4 Code snippet: Conductivity calculation with the GSW package

```
import gsw_xarray as gsw

# Equation of state TEOS-10/GSW
# Calculate conductivity from salinity/temperature at a coordinate
# as CMEMS or ECCO model

def cond_from_model(model):
    # calculate press for given depth and lat (depth must be negative!)
    pres= gsw.p_from_z(z=-model["depth"],lat=model["latitude"])
    # calculate conductivity, convert to S/m
    cond= 0.1*gsw.C_from_SP(SP=model["salt"],t=model["temp"],p=pres)
    cond.attrs["units"]="S/m"
    return cond
```

As output from the TEOS-10 equation, we get a gridded 4-D information for the ocean conductivity (1 temporal and 3 spatial dimensions). Therefore, the ocean conductivity is consistent with the Ocean Model, and its distribution reflects the advection of temperature and salinity by ocean flow, seasonal variations, or salinity reduction by freshwater inflow in the deltas of large rivers.

2.2.1 Electrical conductivity models of solid Earth

To solve the EMI equation (1.7), we require conductivity values throughout the entire spherical computational domain. This necessitates complementation of the seawater conductivity by conductivity models for the sediments, mantle, and core before using the `elmgTD` solver, which are sourced from existing literature. This is, however, not straightforward, because the different components have

different grids; therefore, we have to interpolate. At the end, the result is a single coherent data structure implemented as an `xarray`. Numerically consistent implementation of the sediment conductivity is in particular difficult, due to the laterally varying depth of the sea bottom and sediment thickness. The radial discretization provided by each ocean model is reused in our simulations to avoid interpolation of velocities. Instead, the sediment conductivity is mapped into the ocean grid just below local sea bottom depth, using an interpolation scheme preserving the total sediment conductance (vertically integrated conductivity).

Unlike in the oceans, the solid Earth conductivity does not change in time, at least not on the time scales considered here.

An overview of the Earth's structure by depth, along with descriptions and our modelling approach applied to each region, is presented in Figure 2.1.

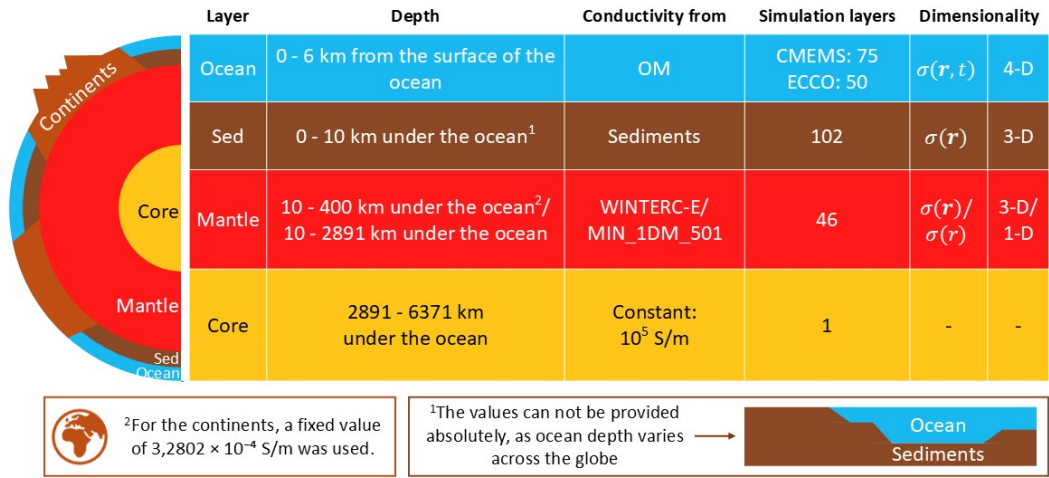


Figure 2.1 The Earth's depth profile and our modelling approach.

2.2.2 Sediments

Marine sediments are generally highly conductive. In certain regions, their thickness can exceed 10 km, and their vertical conductance may surpass that of the overlying ocean (Grayver, 2021). For this reason, it is essential to include marine sediments in our simulations.

We employ a sediment model developed by Grayver (2021), which also incorporates a physical model of sediment compaction, into which water infiltrates. Compaction is the compression of sediments due to the weight of overlying layers, leading to a reduction in porosity and water content. To represent ocean water properties, this model used the standard depth levels from the WOA18 dataset, which covers water depths up to 5,500 m across 102 vertical layers. These layers vary in thickness from 5 m near the surface to 100 m in the deep ocean. For regions deeper than 5,500 m, the values at the maximum depth are taken. The characteristics of this dataset include a global sediment thickness grid of 5"; and the sediment conductivity is modelled as 3-D.

2.2.3 Mantle

MIN_1DM_501

ESA’s MIN 1-Dimensional Mantle model, version 501 (MIN_1DM_501) is a conductivity model spanning the entire depth profile of the mantle. This 1-D model is a *Swarm Level 2 data product*, has 46 layers of irregular thicknesses increasing with depth, and is available on *Swarm Data Access [online]* (2025) under the SW_OPER_MIN_1DM_2__00000000T000000__99999999T999999_0501 ZIP file. This dataset is plotted below in Figure 2.2.

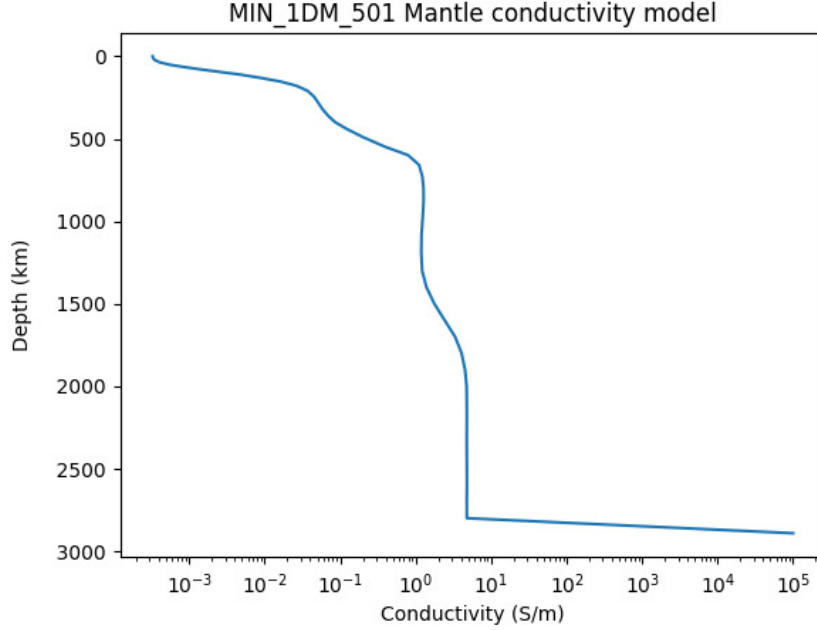


Figure 2.2 Dependence of the one-dimensional MIN_1DM_501 mantle conductivity model on depth. (logarithmic scale on the x-axis, depth increases downwards on the y-axis)

The theoretical background of the MIN_1DM_501 model is set in Section *Obtaining background 1-D conductivity model* in a paper by Kuvshinov et al. (2021), where the 1-D frequency domain inversion is described.

Two datasets from the Swarm mission are used to constrain this 1-D mantle conductivity model. The tidal M_2 signals are sensitive to the conductivity of the upper mantle, down to depths of approximately 400 km. Deeper mantle layers are constrained by their inductive response to magnetospheric variations occurring on timescales ranging from days to months.

WINTERC-emax

The WINTERC-emax three-dimensional model, abbreviated as WINTERC-e, is based on the work of Martinec et al. (2021). The development of the WINTERC-e model started with the WINTERC-G (Fullea et al., 2021) thermochemical model of the upper mantle derived through a joint inversion of seismic, gravity, topography, and heat flow data. As such, it is entirely independent of electromagnetic data. Based on this model, the electrical conductivity model

WINTERC-e was constructed by using laboratory measurements of the conductivity of individual minerals. These measurements were recalculated for the relevant pressure–temperature (P–T) conditions to estimate the bulk conductivity of the mantle composition. This process does not involve simple averaging but rather employs the Hashin-Shtrikman bounds to account for the composite nature of the mantle material. Additionally, the effects of partial melt and the presence of water, both of which significantly increase conductivity, were incorporated to produce an end-member model referred to as WINTERC-emax. Finally, the model was validated using M_2 tidal signals.

In one simulation run in our study, we employed this model to represent the 3-D conductivity structure down to a depth of 400 km, while the 1-D model MIN_1DM_501 is used for deeper regions.

2.2.4 Core

Since the core is highly conductive, a fixed value of 10^5 S/m is taken to model it (Kuvshinov et al., 2021). In principle, it could even be considered a perfect conductor.

2.2.5 Continents

For the continents, a fixed value of $3,2802 \times 10^{-4}$ S/m is used, which is consistent with the topmost value from the MIN_1DM_501 mantle model.

3 The elmgTD solver

To solve Equation (1.7), we need to tackle inputs coming from the forcing term: the ocean velocities, $\mathbf{u}(\mathbf{r}, t)$, and the main magnetic field, $\mathbf{B}_M(\mathbf{r}, t)$.

3.1 Velocities

The ocean velocities, $\mathbf{u}(\mathbf{r}, t)$, are taken from the Ocean Model, discussed in Section 2.1. The dataset of the ocean flows represents a four-dimensional entity (similarly to the ocean conductivities), with three spatial dimensions and one in the time domain.

3.2 Main Field

In our investigation, we employ three different models to describe the core geomagnetic field of the Earth, $\mathbf{B}_M(\mathbf{r}, t)$. As we see, this variable is again four-dimensional, meaning three spatial dimensions and one temporal. This section provides an overview of these models.

3.2.1 IGRF13

Based on the paper by Alken et al. (2021), the 13th generation of the International Geomagnetic Reference Field (IGRF13) is nowadays the standard model used to describe the main magnetic field of the Earth. The IGRF13 model parametrizes the magnetic field over time in piecewise linear segments with five-year intervals, so the secular variation is piecewise constant. The term secular variation is the standard term for the time derivative (change) of the main magnetic field. In other words the model IGRF13 calculates the instantaneous value of the main field for each day, including the linear approximation.

3.2.2 IGRF13 static

In order to explore the effect of main field secular variation on the OIMF, we carry out one run with time-independent IGRF13 model. The static IGRF13 values of $\mathbf{B}_M$ for epoch 2013.0 are used for this particular simulation.

3.2.3 CHAOS-7.18

Described in the paper by Finlay et al. (2020), the CHAOS-7.18 model, strongly relying on satellite magnetic observations, uses an even more refined time parametrization, in comparison to IGRF13, which is based on spline functions. This dataset can be downloaded from the DTU webpage (*Magnetic Field Models [online]* 2025). Note that newer versions of the CHAOS model are now available, but at the time of our investigation, we used version 7.18.

3.3 Solver description

With all required components now in place to solve Equation (1.7), we proceed to describe the numerical implementation of the problem.

Based on Velínský and Martinec (2005), Velínský (2013), and Šachl et al. (2019), the **elmgTD** solver is a parallelized, time-domain, spherical harmonic-finite element technique, which was tested and used to model the OIMF in previous efforts, like Finlay et al. (2024).

It is intended to solve the EMI problem in a sphere with internal, as presented by the right-hand side of Equation (1.7), and/or external forcing. The **elmgTD** solver models the full physics of the problem, including effects like vertical stratification of ocean flow, vertical stratification of ocean and mantle conductivities, electromagnetic self-induction, and galvanic ocean-mantle coupling. The resulting magnetic field is described with the help of vector spherical harmonic functions in angular directions.

3.4 Code implementation

Each simulation scenario, hereafter referred to as *run*, is not computed at once, but has multiple sub-runs, meaning the whole time span of e.g. 10 years is split up into multiple shorter intervals, which are then merged together at the end of the computation.

The **elmgTD** solver uses MPI parallelization in the time variable. For optimal use of supercomputer resources, the length of time series in days should be divisible by the number of nodes/MPI processes. Therefore, an initial short time segment is first calculated for each run, making sure that: a) the following segments will have optimal lengths; and b) all inputs are read and processed correctly. Once this is confirmed, a sequence of segments is calculated with typical fixed length of 512 days distributed between 16 nodes.

The code of the solver is written in Fortran and is initiated by a configuration file, e.g. **elmgTD_mpi_001_01.cfg**, where **_001_** describes the run number, and **_01** the sub-run number.

In Listing 3.1 we can see a code snippet from a configuration file, where the initial condition is set. In general,

- **neumann** = **T** is used for **run_XXX_01**, and
- **neumann** = **F** for all the other sub-runs.

The first sub-run starts from the Neumann boundary condition $\frac{\partial \mathbf{B}_{oi}}{\partial t} = 0$, as described in Section 1.2. The remaining sub-runs then follow on seamlessly by starting from a non-homogeneous Dirichlet initial condition (**neumann** = **F**), using the final time level of the previous sub-run as their own initial condition.

Listing 3.1 Code snippet from a configuration file: Initial condition setting

```
*** initial condition applied at t=0
    T for Neumann initial condition (homogeneous only: dB/dt=0)
    F for Dirichlet initial condition (either B=0 or B=B\_init,
        depending on InitFile)
neumann
T
```

The actual computations are executed on the IT4Innovations National Supercomputing Center by the Karolina supercomputer. The Karolina supercomputer with a peak performance of 15.7 PFlop/s was put in operation at IT4Innovations in 2021. In the year of its installation, Karolina became the most powerful supercomputer in the Czech Republic, and 69th in the world according to the TOP500 ranking (IT4Innovations, 2025). On average, a single simulation run in our setup required approximately 5100 node-hours.

4 Simulation runs

During our investigation, we perform a total of 12 simulation runs with the `elmTD` solver with varying parameters. The tables 4.1 and 4.2 show each one chronologically. We are always trying to keep the most parameters constant and change only one, so that we can compare the effects. The changed parameters are always highlighted in bold.

RUN	Ocean Model	Sediments	Mantle	Main Field
001	CMEMS mean	Grayver	MIN_1DM_501	IGRF13 static
002	ECCO2	Grayver	MIN_1DM_501	IGRF13
003	CMEMS-cglo	Grayver	MIN_1DM_501	IGRF13
004	CMEMS-glor	Grayver	MIN_1DM_501	IGRF13
005	CMEMS-oras	Grayver	MIN_1DM_501	IGRF13
006	CMEMS mean	Grayver	MIN_1DM_501	IGRF13
007	CMEMS mean	Grayver	MIN_1DM_501	CHAOS-7.18
008	CMEMS mean	Grayver	MIN_1DM_501	IGRF13
009	CMEMS mean	Grayver	MIN_1DM_501	IGRF13
010	CMEMS mean +SSS-CCI	Grayver	MIN_1DM_501	IGRF13
011	CMEMS mean	Grayver	MIN_1DM_501 + WINTERC-emax	IGRF13
012	ECCOv4r4	Grayver	MIN_1DM_501	IGRF13

Table 4.1 Simulation runs - configuration parameters.

RUN	inodth	inodph	jmax	kmax	kmaxl	kmax1d	dt	time span
001	360	720	240	177	103	52	1 day	2013.0-2023.0
002	360	720	240	125	76	52	1 day	2013.0-2024.0
003	360	720	240	177	103	52	1 day	2013.0-2023.0
004	360	720	240	177	103	52	1 day	2013.0-2023.0
005	360	720	240	177	103	52	1 day	2013.0-2023.0
006	360	720	240	177	103	52	1 day	2013.0-2023.0
007	360	720	240	177	103	52	1 day	2013.0-2023.0
008	720	1440	480	177	103	52	1 day	2013.0-2014.5
009	360	720	240	177	103	52	1 day	1993.0-2023.0
010	360	720	240	177	103	52	1 day	2013.0-2023.0
011	360	720	240	199	125	37	1 day	2013.0-2023.0
012	360	720	240	125	76	52	1 day	2008.0-2018.0

Table 4.2 Simulation runs - numerical parameters.

From Table 4.1, one can notice the folloing information:

- The **RUN** column is representing the number, which is given to a concrete simulation scenario for identification purposes.
- The **Ocean Model** is representing the input model as is discussed in Section 2.1.
- The **Sediments** denote which model is used for the modellig of the sediments. We can see that only the model *Grayver* is used.

- The **Mantle** shows which model is used to model the mantle, as outlined in Section 2.2.3. As we can see, except for one run, it is always the MIN_1DM_501.
- The **Main Field** describes a standard mathematical description of the Earth's main magnetic field, as explained in Section 3.2. We use it to reference the main geomagnetic field in our simulations. Most of the times, we use IGRF13.

Continuing with the numerical values in Table 4.2,

- the **inodth** (N_θ) column describes the north-south (latitudinal) resolution of the simulation, i.e., in the case of $inodth = 360$, the smallest latitudinal step is $\approx 0,5^\circ$. This is only an approximation, as the grid is not equidistant in the latitudinal direction. Instead, the gridpoints are placed at the roots of the Gauss–Legendre polynomials to facilitate numerical integration.
- the **inodph** (N_ϕ) is the east-west (longitudinal) resolution of the simulation, i.e., in the case of $inodph = 720$, the smallest longitudinal step is $0,5^\circ$. In the longitudinal direction the grid is equidistant.
- the **jmax** ($j_{\max}$) defines how many orders of the spherical harmonic degrees, j , are computed and subsequently plotted, e.g. $j = 1 = \text{monopole}$, $j = 2 = \text{dipole}$, $j = 3 = \text{quadrupole}$, $j = 4 = \text{octupole}$ and so on. In other words, it describes the spherical harmonic truncation degree.
- the **kmax** describes the total number of depth-wise layers in the simulation sorted from bottom to top. The following relations hold:

$$r(1) = 0$$

$$r(k_{\max}+1) = a = 6371,2 \text{ km, where } a \text{ is the sphere radius.}$$

- the **kmaxl** describes the index of the first layer with internal forcing, meaning for the layers $k < k_{\max l}$ the condition $u = 0$ holds.
- the **kmax1d** defines the index of last 1-D layer. Layers 1 to $k_{\max 1d}$ have laterally constant resistivity. The following relation holds: $k_{\max 1d} < k_{\max}$.
- the **dt** describes the smallest time step in the simulation. In our case it is always one day.
- the **time span** describes the input and output range of the simulation.

5 Results

In this section, we will present and compare the resulting outputs for different runs. The concrete simulation runs are presented not chronologically, but rather in a logical order of similarity. We begin by presenting the results of run No. 6 in detail, and then proceed with the comparison.

The spatial and Welch temporal power spectra (including filters) are generated using code developed by Rasmus Møller Blangsbøll from the Technical University of Denmark, available on GitHub (Blangsbøll, 2025), so let us briefly illustrate their mathematical background.

5.1 Spatial power spectrum

To describe the power of the resulting OIMF in the spatial dimension, we apply the Lowes-Mauersberger spherical harmonic power spectrum R_j , based on papers by Mauersberger (1956) and Lowes (1966), which can be mathematically described as

$$R_j = \frac{(j+1)(2j+1)}{4\pi} \left(\frac{a}{r}\right)^{2j+4} \sum_{j=1}^{\infty} \sum_{m=0}^j (|g_{jm}|^2 + |h_{jm}|^2). \quad (5.1)$$

The spatial power spectrum describes how the total magnetic field is distributed across different spatial scales, represented by spherical harmonic degrees j .

5.2 Filtering

The spatial spectra are presented either in unfiltered form, or a prescribed frequency range bandpass filter is applied to them. The bandpass filter is based on the paper by Gustafsson (1996). Applying different bandpass filters effectively decomposes the signal into distinct components corresponding to different period ranges.

The `filtfilt` function in SciPy (SciPy, 2025) is the implementation of so-called forward-backward filtering. It is used for analyzing batch-wise signal using a causal filter on the signal forward in time, and an anticausal filter backward in time on the signal. This is also known as a noncausal Infinite Impulse Response (IIR) filter.

5.2.1 Magnetic field coefficients

In the graphs in this chapter, we employ various filtering on the resulting OIMF signal, in order to aid the work of the Danish researchers. We will decompose the final oceanic magnetic field into components, which can be isolated from available observations.

For the time series of g_{jm} and h_{jm} coefficients, two filtering methods are used, e.i. *pad* and *gust*, which both have their implementation as a method of the `filtfilt` function in SciPy (SciPy, 2025). If the method *pad* is chosen,

the resulting signal is padded by zero at both ends of the time series; if *gust*, Gustafsson’s method is used (Gustafsson, 1996). This padding influences the resulting lines only at the ends of the signal. This can be seen, for example, in Figure 5.5, where between the years of 2015 and 2020 the resulting filtered signals are the same for both methods. These filters effectively highlight the annual variations, as they are specifically designed to do so.

5.3 Welch temporal power spectrum

The Welch temporal power spectrum is computed using Welch’s method (Welch, 2003), a specific approach to computing the power spectral density based on the Fourier transform. It can be viewed as a modified version of the standard Fourier analysis, aimed at reducing noise and improving spectral smoothness. In essence, Welch’s method segments the input signal into overlapping sections (or “snippets”), and then computes the Fourier transform of each one individually. The resulting spectra are then averaged, producing a smoother and more robust estimate of the power spectrum.

For these reasons, Welch’s method is chosen to compute the power spectral density $\hat{P}_{B_r}$ of the radial part of the magnetic field, which is chosen similarly as in Finlay et al. (2024) as

$$\hat{P}_{B_r}(f_n) = \text{Med} \left\{ \frac{L}{Q} \left[\frac{1}{L} \sum_{q=0}^{L-1} B_r(t_q) W(q) e^{-2\pi i q n / L} \right]^2 \right\}, \quad (5.2)$$

where $B_r(t_q)$ represents the input time series at each grid point, and f_n denotes the discrete set of frequencies used for estimating the power spectral density, since our signal is not continuous. Equation (5.2) applies this estimation approach. The parameter L corresponds to the length of the time series, chosen deliberately to capture variations on time scales comparable to the entire series duration. The weighing function $W(q) = \sin^2\left(\frac{\pi q}{L}\right)$ is a Hann window, and the normalization constant is given by $Q = \frac{1}{L} \sum_q W^2(q)$. The imaginary unit is defined as $i = \sqrt{-1}$. The operator Med computes the median of the power spectral density estimates across all spatial grid points on the sphere. For cases where the signal is derived from the OIMF, a spatial mask is applied so that only grid points located over oceanic or marine regions are included in the analysis.

5.4 Run_006 - Canonical run

Run_006 is, rather arbitrarily, defined as our *canonical run*. The influence of changing individual input parameters, *ceteris paribus*, on OIMF is then related to this run.

In Figure 5.1, one can see the Spatial power spectra, and in Figure 5.2, the Welch temporal power spectra for run_006.

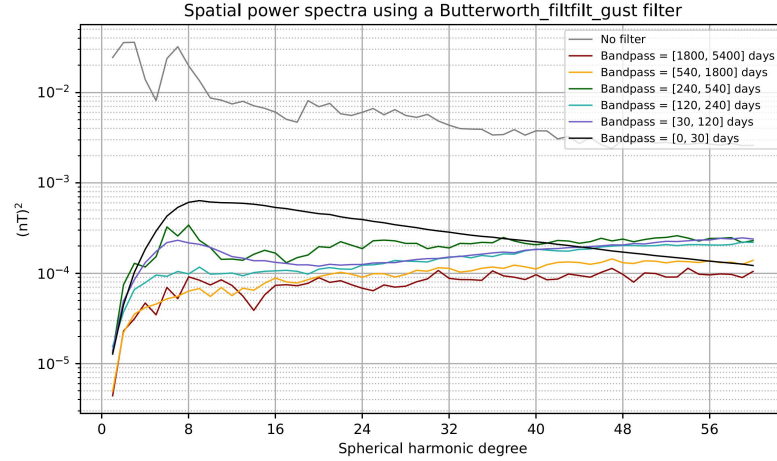


Figure 5.1 Spatial power spectra of run_006.

In Figure 5.1, the grey “No filter” signal represents our unprocessed data without any temporal filtering, the overall signal spectrum. The coloured bandpass filter lines, on the other hand, allow only signals within a specified period range to pass through. Applying different bandpass filters effectively decomposes the signal into distinct components corresponding to different period ranges. As can be seen, the bandpass in the range $[0, 30]$ days is, to some extent, the most dominant, indicating that high-frequency signals contribute significantly to the overall magnetic field. Unfortunately, signals with such short periods cannot currently be isolated from the Swarm magnetic data. At present, the signal resolution capability is limited to variations on the scale of annual and semi-annual periods (Finlay, *pers. comm.*).

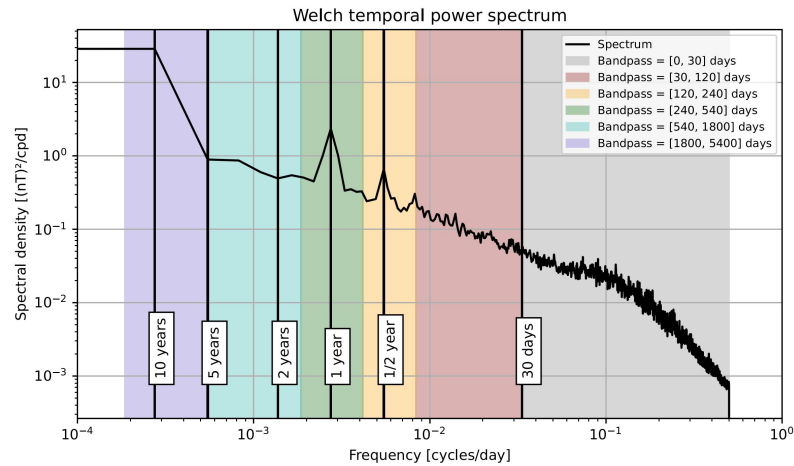


Figure 5.2 Temporal power spectrum of run_006.

In Figure 5.2, one can see that the annual and semi-annual periods are well-resolved.

Note that in Figure 5.2, the horizontal line on the far left of the graph does not carry any intrinsic physical meaning; it is merely a result of the graph’s plotting method, since the length of the time series is limited to 10 years.

In addition to the spectra, we have also generated plots of various other quantities for each simulation run. Due to space limitations, we will present here only a representative set of results from this run in Figure 5.3. While these plots are visually appealing and allow for some qualitative interpretation, they offer only limited quantitative insight into the magnetic field. For these reasons, the full set of plots, along with accompanying GIFs, is provided on a Zenodo page (Ucekajová, 2025).

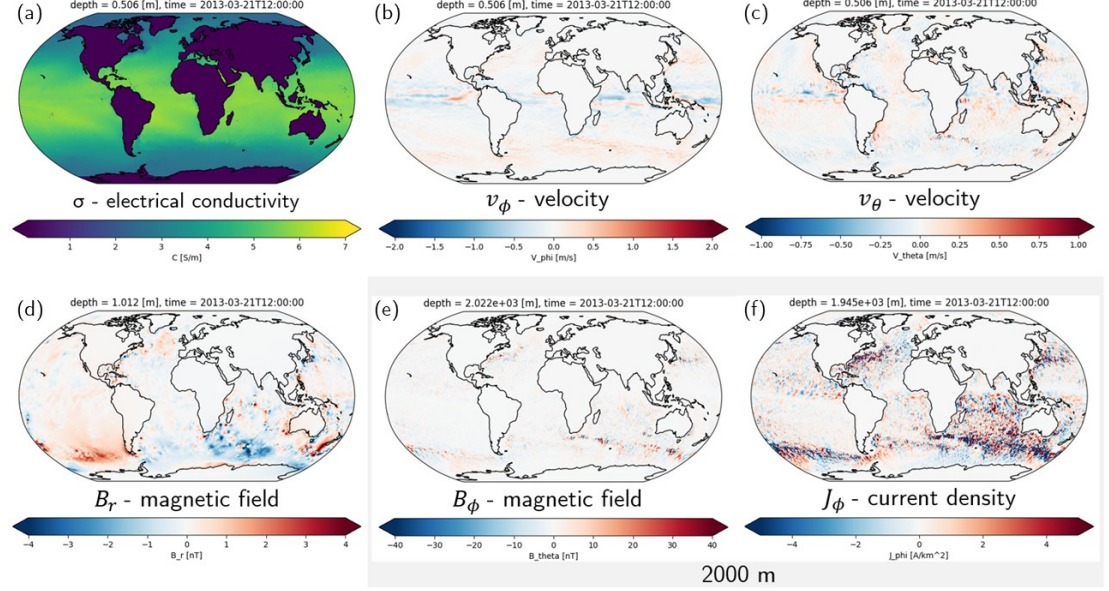


Figure 5.3 Snapshot plots during the spring equinox (2013-03-21) for run_006. The plots of the (a) electrical conductivity σ , (b) zonal velocity v_ϕ , and (c) meridional velocity v_θ come from the topmost layer of our simulation, which is approximately 0,5 m under the ocean surface level for these quantities. The (d) radial magnetic field B_r is also plotted at the topmost simulation layer, however the depth under the ocean is 1,0 m this time. Note that the (e) zonal magnetic field B_ϕ , and (f) zonal electrical current density J_ϕ highlighted in grey are plotted at the depth of 2000 m under the surface of the ocean.

The information we can deduce from Figure 5.3, is the following:

- The electrical conductivity plot (a) in Figure 5.3 demonstrates the effect of the temperature. Around the equator, the temperatures are higher; therefore, the conductivity is also higher in this region. The conductivity then decreases moving from the equator to the poles. The effect of reduced salinity, and consequently lower electrical conductivity, can also be observed at the mouths of major rivers, such as the Amazon, Saint Lawrence, Congo, or Yangtze, highlighted in Figure 5.4.

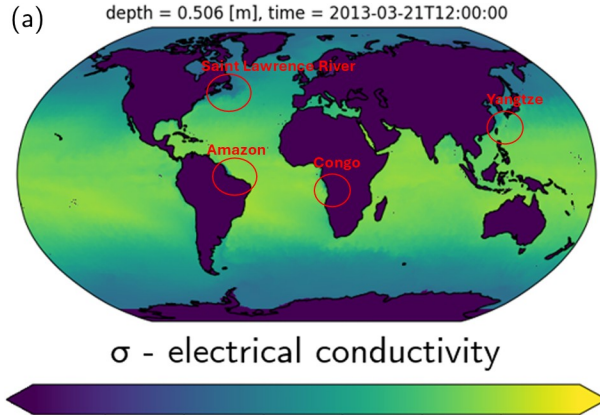


Figure 5.4 Magnified electrical conductivity plot with highlighted river deltas.

- In the velocity plots (b) and (c), we identify a clear trend of ocean flow around the equator. The eastward ACC is also prominent in the zonal velocity snapshot. This current is particularly distinct, since it can flow around the Earth without being impacted by a continental landmass.
- The magnetic field B_r on the surface shows well-documented behaviour, two big areas with opposite signs around Antarctica, a signature of the ACC.
- If we take a look at the variables in the depth, B_ϕ , and J_ϕ , we see significant structures, which fade away with going up towards the surface.

Also, the magnitude of the zonal magnetic field in the depths is roughly 10 times larger than on the surface, even 40 nT. This is mostly the toroidal component of the magnetic field, which is invisible on the surface (Velínský et al., 2019).

These figures are, however, only snapshots. If we plot the time series of the spherical harmonic coefficient g_{66} in Figure 5.5, we can see that the time variation of the magnetic field is high. There is significant, day-to-day variation of magnetic field present, but the annual period, related to seasonal variations of the ocean, is dominant. Application of a bandpass filter centered at the annual period clearly extracts this signal. The amplitude of the magnetic field oscillates over the course of the years between positive and negative values, which are well-known seasonal variations. The values of the OIMF are between 0,01 and -0,01 nT.

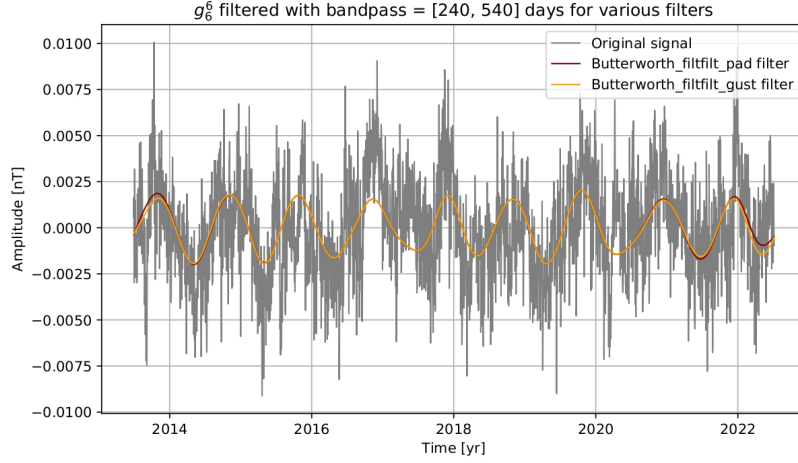


Figure 5.5 Time series g_{66} coefficients of run_006. The unfiltered signal is shown in grey, while the filtered signals in orange and red highlight the annual component of the original signal.

5.5 Effect of the Ocean Model on the OIMF

Runs 002, 003, 004, 005 & 006

The runs and Ocean Models used in this comparison are run_002—ECCO2; run_003—CMEMS-cglo; run_004—CMEMS-glor; run_005—CMEMS-oras; and run_006—CMEMS mean.

In Figure 5.6, we compare the spatial power spectra of these Ocean Models with and without bandpass filters.

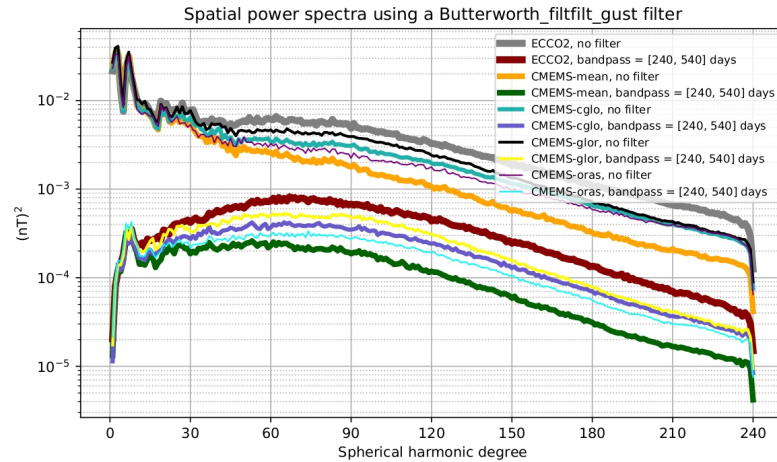


Figure 5.6 Multi-run comparison of spatial power spectra.

In Figure 5.7, we provide a zoomed plot of Figure 5.6, showing only the first 60 spatial harmonic degrees for better readability.

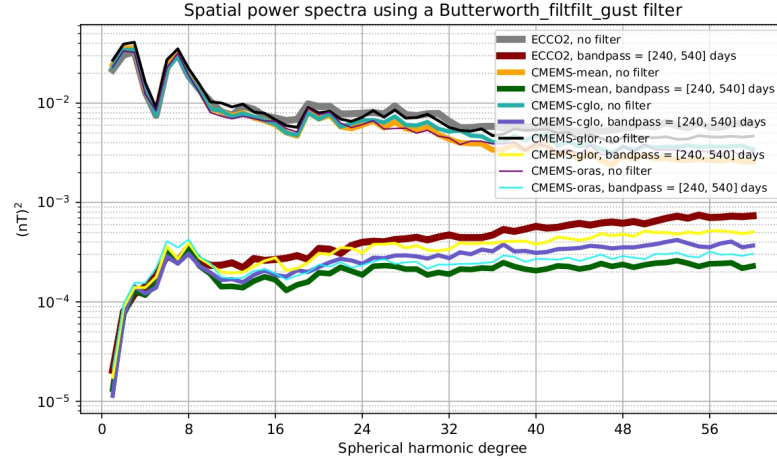


Figure 5.7 Multi-run comparison of spatial power spectra zoomed at lower spherical harmonic degrees.

What can be deduced from both figures is the high spread of the presented lines describing the different Ocean Models. We see that the input Ocean Models produce vastly different lines, especially in the higher spherical harmonic degree regions, meaning differences in small spatial structures between the models.

Up to spherical harmonic degree ≈ 60 all the lines start out more or less the same (only with small differences), which is why we plot Figure 5.7, to zoom in on this region. In Figure 5.7, we can see that roughly until $j = 10$, there are no differences between the lines. It means that all the Ocean Models describe the large-scale structures of the resulting OIMF power very similarly. The description then varies on small scales.

From both Figures 5.6 and 5.7, it is evident that the ECCO2 model predicts the largest magnitudes of the resulting power of the OIMF, which can be seen with the grey and red lines.

On the other hand, the most conservative estimates are obtained by the CMEMS-mean model presented by the orange and green lines. This is, however, not surprising, since the CMEMS-mean model is an averaged one, where small-scale structures get suppressed by the averaging. The rest of the models from the CMEMS family are distributed between these maxima and minima.

Ocean models differ due to variations in computational approaches, input data sources, and data assimilation techniques. This variability is actually encouraging, as the OIMF is sensitive to these parameters and therefore holds potential for complementing or constraining them.

To conclude the spectral analysis, the differences between models are most pronounced at higher spherical harmonic degrees, corresponding to smaller spatial scales. However, noticeable variations are also present in the mid-range, approximately between degrees 15 and 30. At the largest spatial scales, the models show good agreement.

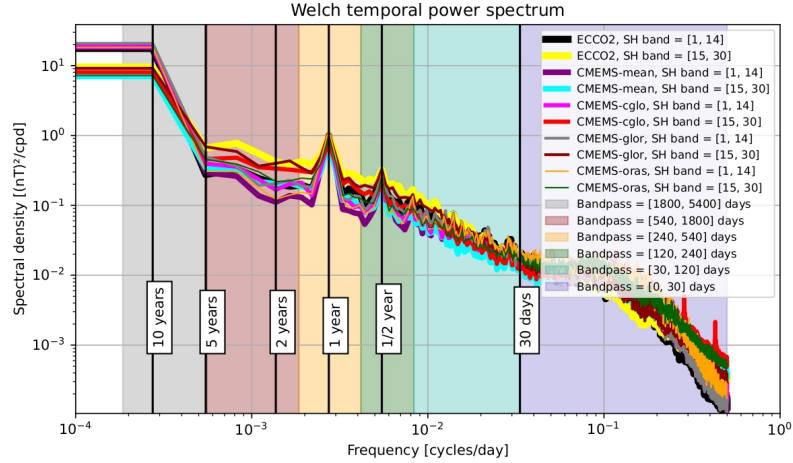


Figure 5.8 Multi-run comparison of temporal power spectra.

In Figure 5.8 we again see some spread, however, not as prominent as in Figure 5.6. Even though there are many lines in one plot and it is not so easy to read them, we can clearly deduce the annual and semi-annual periods, which are well-resolved for all the Ocean Models.

Note that in this plot, the bandpass of [1,14] is a large-scale one, and the [15,30] is a small-scale one, in which we are interested.

Even though the individual Ocean Models show different behaviour, the overall behaviour and trend of the lines is the same, which is true for both the spatial and temporal spectra. The spread can act as a confidence interval in which we expect to find the signals.

Runs 002 & 012 - ECCO family

Here we compare the ECCO2 and ECCOv4r4 models, which partially overlap only in the 2013-2017 time range. Therefore, when comparing these two simulation runs from the ECCO family, it does not make much sense to compare the temporal and spatial spectra. In Figure 5.9, we present a selected few g_{jm} coefficients. For convenience, we choose coefficients where $j = m$, but this choice has no effect on the informative value of the presented results.

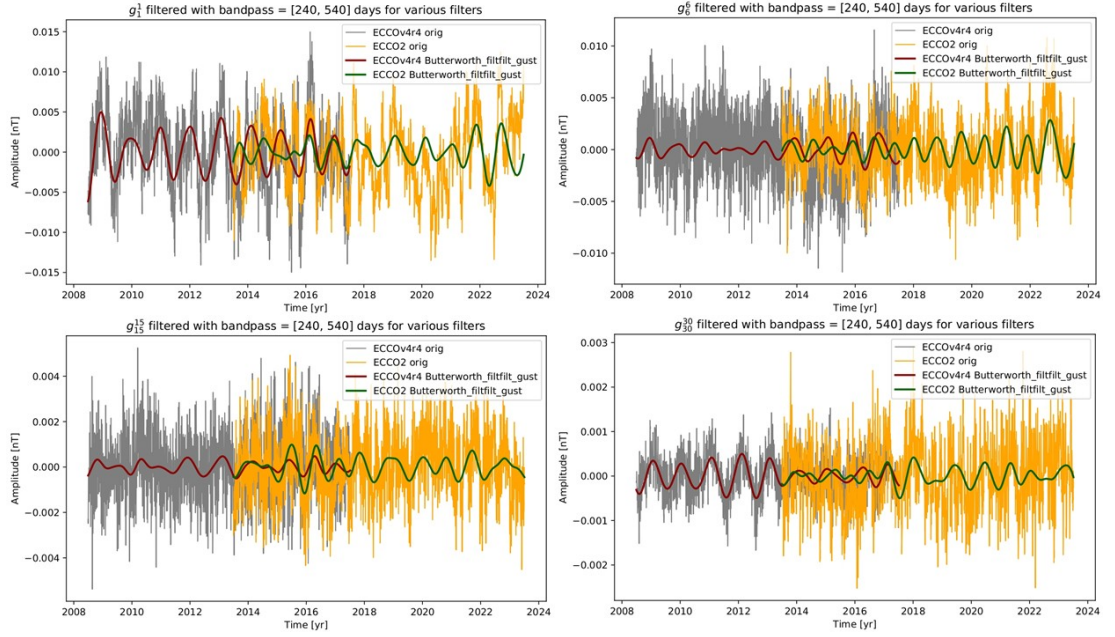


Figure 5.9 Time series coefficients g_{11} , g_{66} , $g_{15\ 15}$, and $g_{30\ 30}$ for run_002 & run_012.

In Figure 5.9, we can see the g_{11} , g_{66} , $g_{15\ 15}$, and $g_{30\ 30}$ time series coefficients for the runs 002 and 012, plotted in the same graphs providing direct visual comparison. However, the two runs overlap only on a limited time segment, five years, always in the middle of the graph.

From the graphs in Figure 5.9, we can see that on the overlapping segments, there is limited similarity in the lines. The resulting OIMF from the ECCO2 and ECCOv4r4 models shares limited overlap, again emphasizing the importance of the Ocean Model.

Overall, we conclude that the OM has a significant effect on the resulting OIMF. This should be a warning to a simple use of simulated OIMF for correction of magnetic measurements for other modelling purposes. On the other hand, it shows a general promise that the OIMF can in principle constrain the ocean model in the future.

5.6 Effect of the Main Field

Runs 001, 006 & 007

In this part of the study, we examine the impact of the choice of the model of the main geomagnetic field generated in the Earth's core. In Figures 5.10 and 5.11, we show spatial and temporal comparisons of the three different models used in our exploration, IGRF13 static, IGRF13, and CHAOS-7.18.

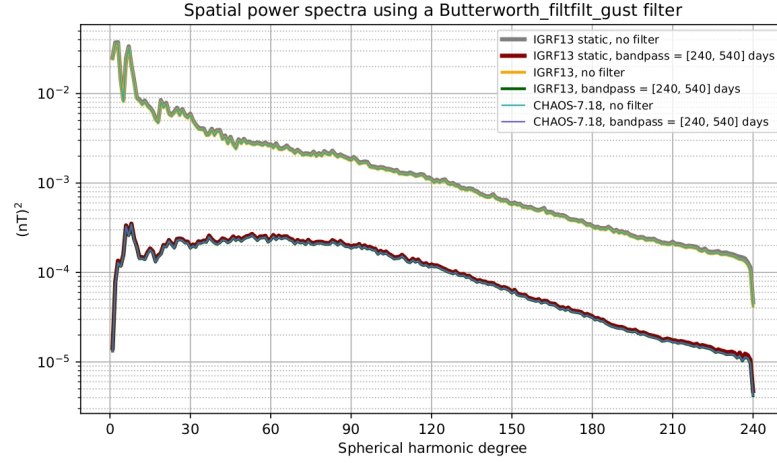


Figure 5.10 Spatial power spectra comparison of runs 001, 006 & 007.

In Figure 5.10, we can see that there is practically no difference between the different models. Let us look at Figure 5.11 for more insight.

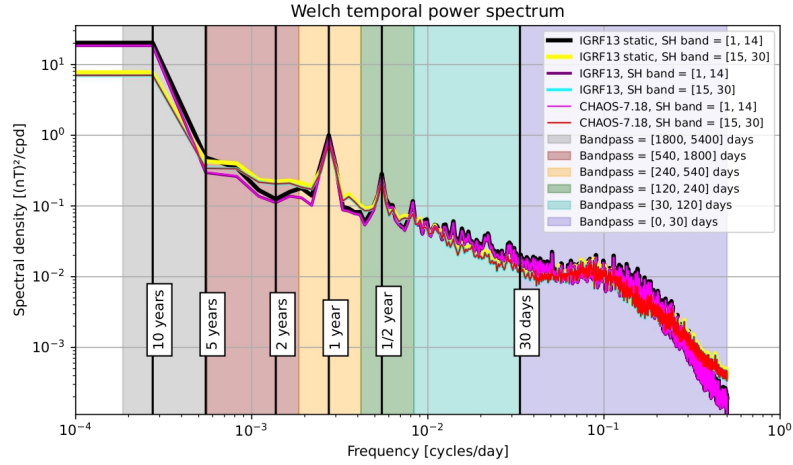


Figure 5.11 Temporal power spectra comparison of runs 001, 006 & 007.

From Figure 5.11 it is evident that the results obtained using IGRF13 and CHAOS-7.18 are here also practically identical. In contrast, the results based on the static IGRF13 model (epoch 2013.0) show slight differences.

We conclude that models incorporating at least a basic representation of secular variations of the main field are preferable. However, it makes little difference whether these variations are modeled using piecewise linear segments, as in IGRF, or splines, as in CHAOS-7.18.

5.7 Effect of the higher spatial resolution

Runs 006 & 008

The `run_008` is similar to our canonical `run_006`. The only difference is the resolution (*inodth*, *inodph*), which is doubled in this run, i.e., (720, 1440). Therefore, also the *jmax* is doubled compared to all the other runs as well, i.e., *jmax* = 480.

At this extremely high spatial resolution, only approximately 1.5 years of time series data are computed, because it is very expensive. This duration is insufficient for resolving annual periodicities. In Figure 5.12, we can see the relationship between the spherical harmonic degree and the magnetic power.

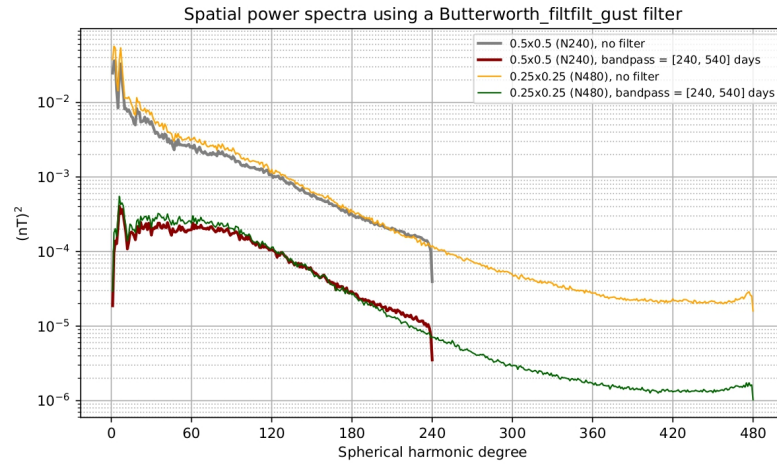


Figure 5.12 Spatial power spectra comparison of runs 006 & 008.

In Figure 5.12, we can see that both of the lines have a very similar behaviour. The big difference between the two resulting powers are the values at $j = 240$, where the power for the shorter run drops rapidly.

The main takeaway from Figure 5.12 is that increasing spatial resolution has some noticeable impact even on the spectra at lower spherical harmonic degrees. In general, the differences between resolutions are smaller than those caused by the choice of Ocean Model. It is also evident that for lower resolutions, the solution exhibits spectral convergence, as there are no significant jumps at the high-degree end of the spectrum.

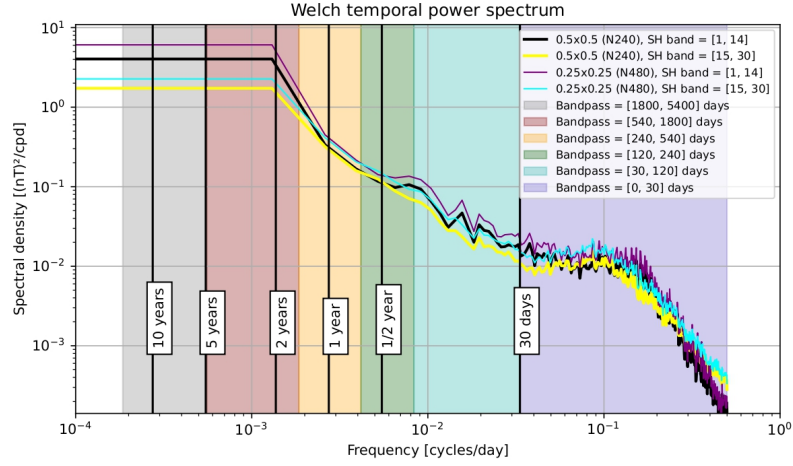


Figure 5.13 Temporal power spectra comparison of runs 006 & 008.

It is important to mention here that in order to create Figure 5.13, run_006 has to be truncated to match the same time range as run_008. As mentioned above, this range can however, not show annual nor semi-annual periods, since the given time period is not long enough. Therefore, we should focus here on the periodicities < 1 year.

5.8 Effect of the initial condition - long time run

Runs 006 & 009

In Figures 5.14 and 5.15, we show the spatial and temporal spectra solely for run_009, since the time ranges of run_006 and run_009 are different. Both runs are calculated using the homogeneous Neumann initial condition, but it is applied at time 2013.0 and 1993.0 in runs 006 and 009, respectively. In the interval 2013.0-2023.0, both runs are driven by identical CMEMS-mean velocities and conductivities, and IGRF13 main field.

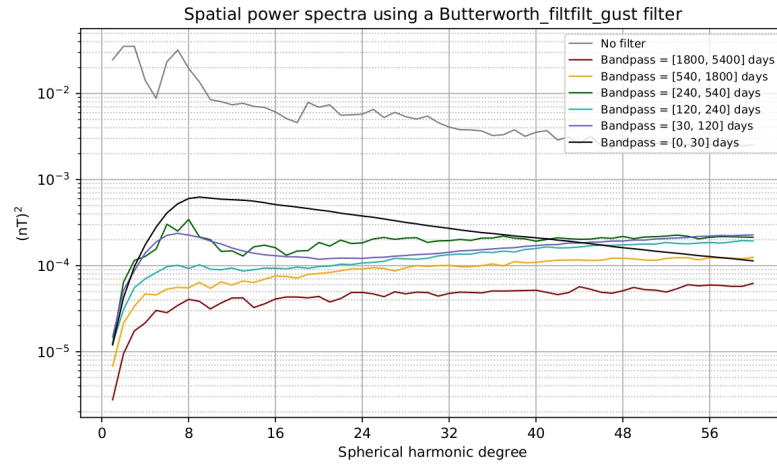


Figure 5.14 Spatial power spectra of run_009.

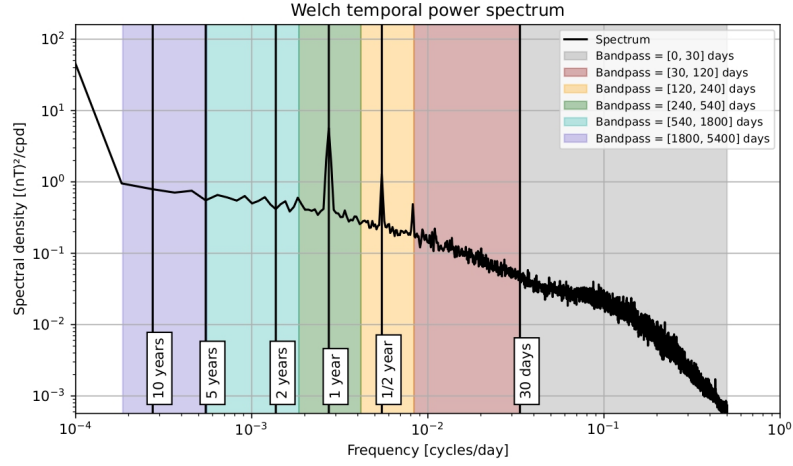


Figure 5.15 Temporal power spectrum of run_009.

The Figure 5.15 does not provide any more insight than the temporal spectrum of run_006. Still, the annual and semi-annual periods play the key role.

To provide a quantitative comparison, we created a comparison plot of the g_{jm} coefficients.

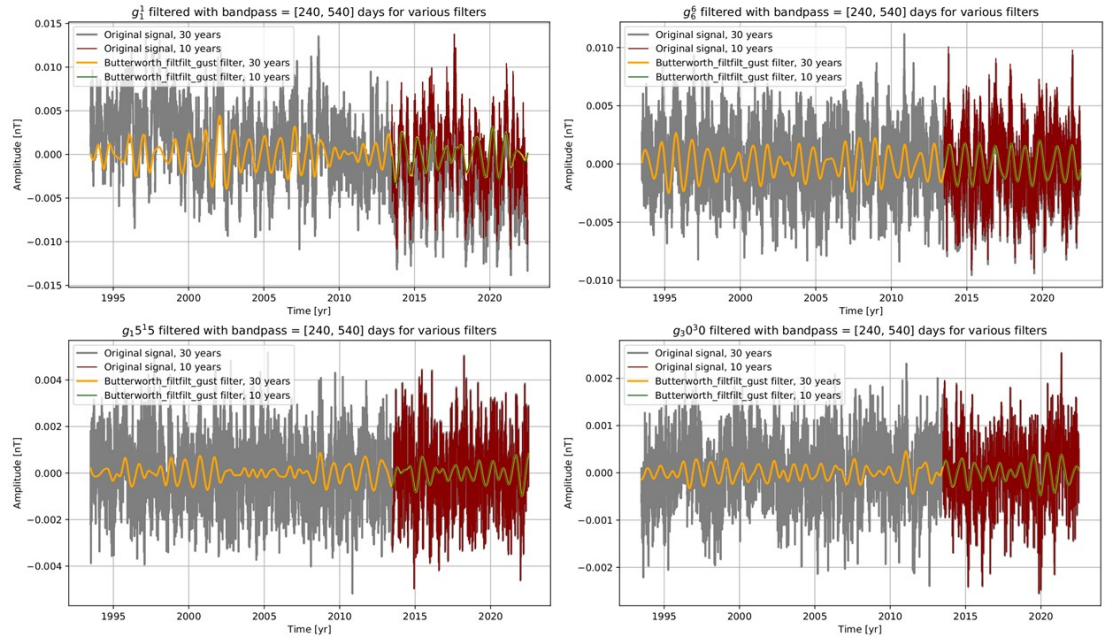


Figure 5.16 g_{11} , g_{66} , g_{1515} and g_{3030} coefficients for runs 006 and 009. Since run_009 is by 20 years longer than run_006, the overlap is only 10 years on the right-hand side of the plots shown in red, run_006. The grey line corresponds to run_009. The filtered signals are showed in green and orange, which are centered around zero.

In Figure 5.16, the effect of the initial condition is visible only in unfiltered, low-degree OIMF coefficients, clearly in g_{11} , and also weakly in g_{66} . It is expressed in the form of a static-like shift, and almost perfectly removed in the bandpass-filtered signal centered at the annual period. For higher degrees and smaller spatial scales, the differences are negligible even in unfiltered signals. These

properties point to large-scale, very-long period induction effects in the lowermost mantle and core, as the only viable explanation. In practice, this effect will be indistinguishable from the background of the core field.

The zero-centered nature of the filtered signal is evident in the g_{11} magnetic coefficient. While the original signal exhibits a prominent trend from the top left to the bottom right, the filtered signal remains vertically centered around zero.

5.9 Effect of the near-surface salinity

Runs 006 & 010

The simulation `run_010` incorporates a more advanced salinity model, specifically, the Sea Surface Salinity Climate Change Initiative (SSS-CCI) data model, which comes from an ESA project targeted to create improved calibrated worldwide Sea Surface Salinity models (*Sea Surface Salinity [online]* 2025). Data measured by ESA’s Soil Moisture and Ocean Salinity (SMOS), Aquarius, and Soil Moisture Active Passive (SMAP) missions are merged together to aid the creation of the SSS-CCI dataset, which in this simulation run replaces the CMEMS mean salinity values in the upper three model layers (5 metres), where available. Velocities stay unchanged. It is important to mention that the SSS-CCI dataset is available only up to October 2022; thus, for the final two months of the simulation, the salinity fields rely entirely on the CMEMS mean values. The data used in our simulation can be downloaded from *ESA Sea Surface Salinity Climate Change Initiative [online]* 2025.

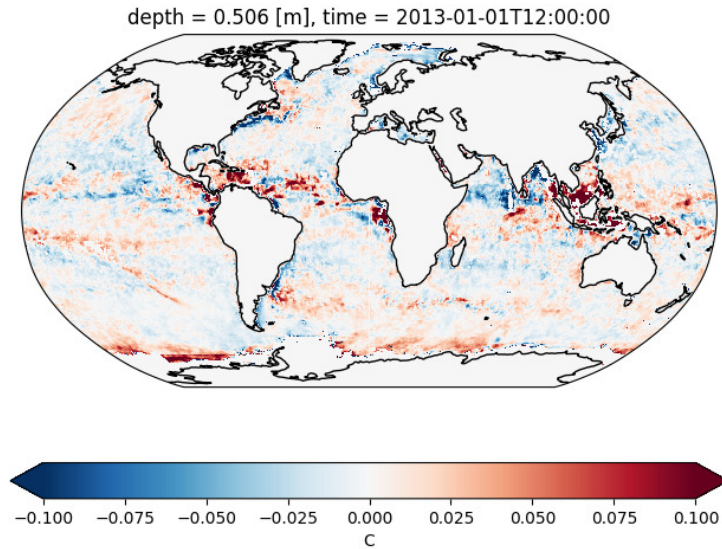


Figure 5.17 Differences in S/m of electrical conductivity in the upper layer between the CMEMS-mean model (`run_006`) and an alternative calculation (`run_009`), in which the salinity from CMEMS-mean is replaced with salinity from CCI-SSS satellite measurements.

In Figure 5.17, we can see that the differences in conductivities between `runs` 006 and 010 are approximately $\pm 0,1$ S/m, whereas from the electrical conductivity plot in Figure 5.3 we know that the lowest values of the conductivity in `run_006`

are around 2 S/m. Dividing 0,1 S/m by 2 S/m we get that the difference on the topmost oceanic layer between the two datasets is only about 5%.

If we take a look at the spatial power spectra comparison of the OIMF for runs 006 and 010 in Figure 5.18, we see absolutely no difference between the two lines. This result is, however, not so surprising, taking into consideration the similarity of the two datasets.

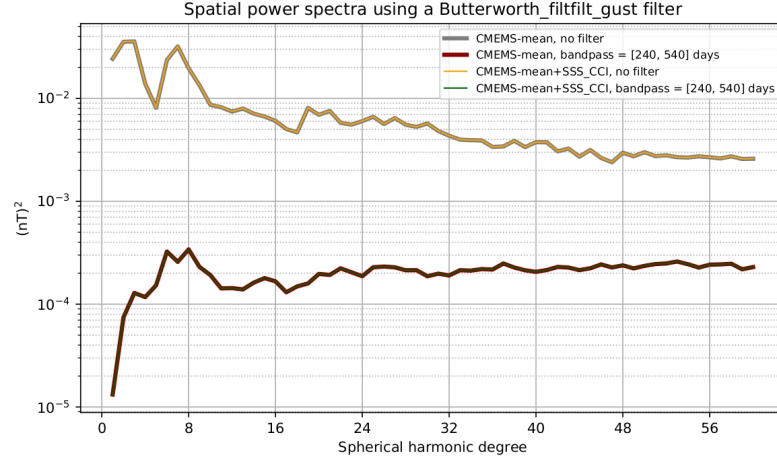


Figure 5.18 Spatial power spectra comparison of runs 006 & 010.

Let us take a look at the temporal power spectra comparison in Figure 5.19 for additional information.

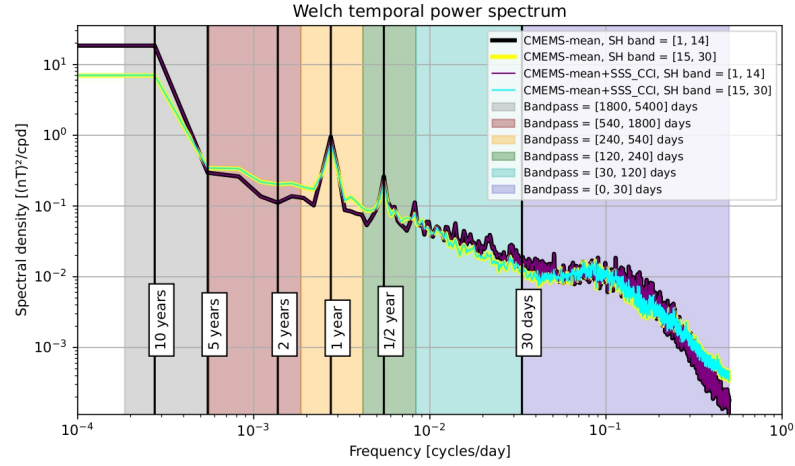


Figure 5.19 Temporal power spectra comparison of runs 006 & 010.

Again, we see that the lines match each other perfectly. Therefore, we conclude that the near-surface salinity has a negligible effect on the resulting OIMF. After double-checking the time-series of induced field coefficients g_{jm} and h_{jm} we find that the difference between the two runs is at the 4th or 5th significant digit. The effect of the surface layers is too small in comparison to the rest volume of the ocean. The OIMF exhibits low sensitivity to small-scale variations in surface ocean conductivity, rather, its sensitivity extends over the entire volume.

5.10 Effect of the mantle electrical conductivity

Runs 006 & 011

In this simulation run, we use a 3-D mantle model for the first 400 km instead of a 1-D one.

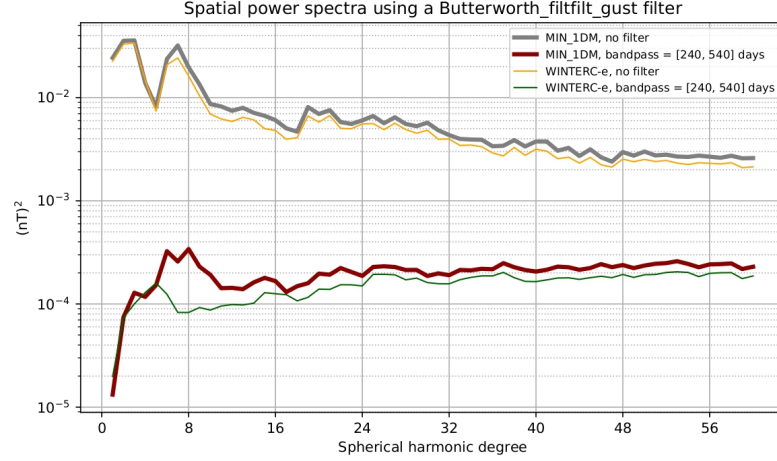


Figure 5.20 Spatial power spectra comparison of runs 006 & 011.

In Figure 5.20, we can see that the mantle conductivity can have some effect on the resulting OIMF, mainly on larger spatial scales. If we take a look at the WINTERC-e bandpass line, we see significant behaviour around the spherical harmonic degree $j = 8$.

However, from degree ~ 20 , the difference between the two spectra shows little 3-D effects and seems to produce a rather constant shift in the spectra. Unlike in the case of the effect of initial condition, here the shift is present even in the bandpass-filtered annual period. The effect of the mantle conductivity is therefore non-negligible, but a representation by a good 1-D mantle conductivity model should be adequate, without the overwhelming computational burden of a fully 3-D one.

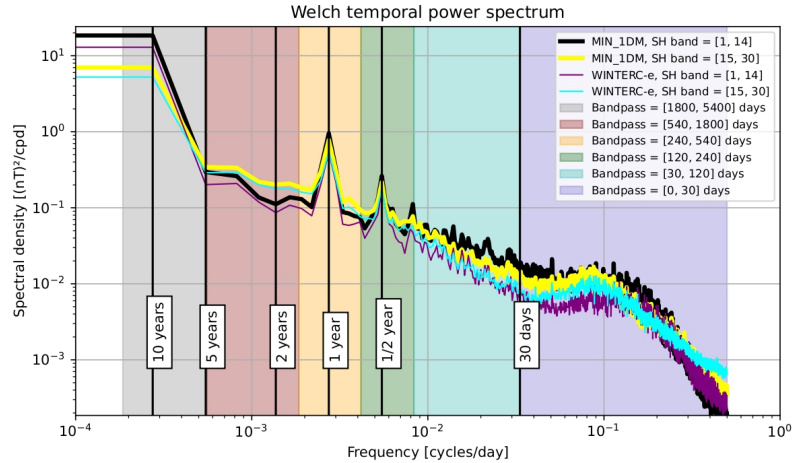


Figure 5.21 Temporal power spectra comparison of runs 006 & 011.

If we take a look at Figure 5.21, we can see small differences. The two sets of lines imitate each other's behaviour, with the thinner (partially 3-D) line always being under the thicker (fully 1-D) line.

We conclude that the dimensionality of the modelled mantle has some significant effect on the resulting OIMF, however, this effect is not so large as e.g. the effect of the OM.

Conclusion

In this thesis, we performed numerical modelling of the OIMF and investigated the sensitivity of the predicted signal to various inputs and model settings. Our results clearly show that the choice of Ocean Models is the dominant factor, exerting the strongest influence on the resulting OIMF. This is an encouraging outcome. Recent analyses of Swarm satellite data (Finlay et al., 2024; Finlay, *pers. comm.*) point that with current spatio-temporal resolution, and state-of-the-art corrections for the ionospheric field, the annual period band, and spherical harmonic range ~ 15 -30 offer the best chance to extract the OIMF from the complex geomagnetic signal. Therefore, clear differences between OIMF generated by individual models could in principle lead to a future exploitation of satellite magnetic measurements to constrain the state of the ocean. However, significant challenges remain in both numerical modelling and data analysis.

On the side of the numerical modelling, in Sections 5.8 and 5.10, we have demonstrated, that induction effects can be important, however, using a simple stationary approach is probably not justified for correct OIMF predictions. While the mantle conductivity model also influences the results, its effect is largely masked by the dominant core field at large scales and periods considered. Thus, while sensitivity to the 3-D mantle structure exists, its impact is clearly secondary to that of the Ocean Model.

The choice of main field model has only a minor influence on the resulting OIMF. Our comparison between IGRF13 and CHAOS-7.18 showed virtually no difference, indicating that the added complexity of CHAOS-7.18 does not translate into meaningful improvements in the results. A slightly larger deviation was observed when comparing IGRF13 to its static version, IGRF13 static, but even this difference remained small. Based on these findings, we conclude that the main field model has a limited impact on the final OIMF. Therefore, we recommend using IGRF13, as it offers sufficient accuracy while avoiding the increased computational demands of more complex alternatives like CHAOS-7.18.

The use of high spatial resolution has a limited effect on the final OIMF. Moreover, evaluating this scenario was challenging due to the short simulation duration, and given the high computational cost, we do not find it justified for typical applications.

Extended spin-up time of the OIMF model, using a 30-year long run demonstrates the importance of the induction effect on low-degree field, where a simple Neumann initial condition leads to a quasi-static shift of the field coefficients. However, when temporal filtering is applied, or one focuses on smaller spatial scales, the additional computational burden of the long spin-up is needless.

Additionally, the effect of near-surface salinity is negligible, as the CMEMS-mean model already provides a sufficiently accurate representation.

Overall, our findings highlight that the parameters with the most influence on the resulting OIMF are Ocean Models, 3-D mantle conductivity modelling, and spatial resolution.

Looking ahead, with advances in satellite-based magnetic field models, the OIMF forward modelling should now start to focus on the development of methods for efficient assimilation of satellite-observed OIMF data into the Ocean Models.

A mathematical apparatus of adjoint sensitivity kernels for the EMI problem with internal forcing needs to be developed for this purpose.

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