



Eigenoscillations and Stability of Rocking Stones: The Case Study of “The Hus Pulpit” in The Central Bohemian Pluton

ELIŠKA ZÁBRANOVÁ,¹ CTIRAD MATYSKA,² JAKUB STEMBERK,^{1,3} and JIŘÍ MÁLEK¹

Abstract—Stability of precariously balanced rocks is employed in seismic hazard estimates. Standard toppling models of these rocks are based on a rigid body motion, i.e., this approximation assumes that their eigenfrequencies are infinite in any position of a rock. This assumption is, however, questionable in the case of rocking stones oscillating with lowest eigenfrequencies of only several hertz when resonance effects facilitating rolling/wobbling due to a seismic signal cannot be a priori excluded. An example demonstrating existence of such low frequencies is “the Hus Pulpit” rocking stone located in the Central Bohemian Pluton (49.568N, 14.363E). The observed lowest frequencies recorded by a Guralp CMG-40T seismometer located on the top of this rocking stone are 3, 7 and 9 Hz with the quality factors of 60, 55 and 77, respectively. Using the COMSOL Multiphysics software package we model the oscillations of this rocking stone numerically and demonstrate that these observed low frequencies and their dampings can be explained by a force interaction between the stone base and the bedrock that can formally be interpreted as the damped elastic response of the bedrock to its surface deformation by the stone. We also numerically study stability of the stone under the presence of such an interaction and conclude that beginning of toppling requires very high horizontal PGV (PGA) for input seismic frequencies even at the resonance frequencies. However, different numerical models, where the seismic displacements are considered as the direct input at the rock base, can result in rock instabilities under a presence of an $M = 7$ earthquake at a distance of about 150 km.

Key words: Precariously balanced rocks, the Hus Pulpit, eigenfrequencies, data and modeling, seismic hazard.

1. Introduction

Precariously balanced rocks can be used to provide constraints on estimates of strong ground motions. Standard methodology deals with rigid-body rocking motion, where a rigid block is free to rotate about a supporting point and forcing due to horizontal ground motions is considered (e.g. Shi et al. 1996; Anooshehpour et al. 2004; Veeraraghavan et al. 2017). The problem is solved numerically by a non-linear ordinary differential equation corresponding to inverted pendulum (e.g. Housner 1963; Weichert 1994; Shi et al. 1996). This methodology is employed in seismic hazard models (Anderson and Brune 1999; Schurch and Becker 2005; Brune et al. 2006; Purvance et al. 2008). However, toppling and overturning of precariously balanced rocks represent strongly non-linear problem and thus the results are dependent on a considered ground motion input (Apostolou et al. 2007). A rigid-body approximation is justified in cases, where elastic eigenfrequencies of the precariously balanced rocks are much higher than ground-motion frequencies, and thus resonances need not to be taken into account.

There are, however, cases when the lowest eigenfrequencies of the studied bodies close to collapse or breaking can be only several hertz or lower, falling thus into a typical seismic-frequency band and, therefore, resonances may occur. The examples are, e.g., natural rock columns in the Chiricahua Mountains, Arizona, where the heights of columns may be limited by a fundamental vibrational mode that matches the seismic shaking of large and moderate prehistoric earthquakes occurred nearby (Hall 1996), the Minaret of *Jam* in Afghanistan with fundamental period longer than one second (Menon

¹ Institute of Rock Structure and Mechanics, Czech Academy of Sciences, Prague, Czech Republic. E-mail: eliskazabranova@centrum.cz

² Department of Geophysics, Faculty of Mathematics and Physics, Charles University, Prague, Czech Republic.

³ Department of Physical Geography and Geoecology, Faculty of Science, Charles University, Prague, Czech Republic.

et al. 2004) or speleothems (Lacave et al. 2004; Becker et al. 2006), where eigenfrequencies can be measured in situ, calculated theoretically and modelled numerically by finite element method (Gribovski et al. 2017, 2018). Precariously balanced rocks represent another class of bodies as they are not fixed to their bedrocks and thus their wobbling is possible. Their dynamics is, therefore, more complicated and the question arises whether small-amplitude resonances result in a resonance-like behavior during wobbling and toppling.

The main aim of this paper is to present a model that is able to explain low eigenfrequencies of rocking stones with relatively low heights and small fundamentals and to study its oscillations and stability when resonances during quasi harmonic seismic excitations can be generated. We present the case study for “the Hus Pulpit” rocking stone (Fig. 1) located in the Central Bohemian Pluton (49.568N,14.363E) representing a suitable natural example.

2. Eigenoscillations of “The Hus Pulpit” and Their Physical Explanation

2.1. Observation of Low-Frequency Modes

In order to record frequencies of “the Hus Pulpit” rocking stone, we located a broadband three-component seismometer Guralp CMG-40T, with the frequency range 30 s–40 Hz covering the seismic band from potential earthquakes at local and regional distances, at the top of the stone and hit the stone repeatedly at several different places with a rubber hammer (Fig. 1). The signal generated in this way was well above natural noise level and we thus obtained a series of velocigrams enabling to identify not only the frequencies but also the quality factors of the three low-frequency modes. Note that the mass of the seismometer is 5 kg whereas the mass of the stone is 18.1 tonnes (see Sect. 2.2 below) and thus the influence of the seismometer load to the stone oscillations is negligible. Typical unfiltered velocigrams of all components and their spectra are in Fig. 2.

The observed fundamental frequencies f are 3, 7 and 9 Hz, with the quality factors Q equal to 60, 55



Figure 1

“The Hus Pulpit” located in the Central Bohemian Pluton (49.568N,14.363E)—the rocking stone suitable for testing theoretical mode calculations by observations of the stone eigenfrequencies and their dampings. Oscillations of the stone were generated by a rubber-hammer hit (top snap) and recorded by a Guralp CMG-40T seismometer (bottom snap)

and 77, respectively, that were obtained by finding best-fitting exponential decay $\sim \exp(-\pi ft/Q)$, t is the time, of the mode amplitudes in the time domain. The eigenoscillations of the free stone with no forces acting at its whole boundary are much higher (hundreds of Hz), see the calculations in Sect. 2.2 below. The lowest spurious vibration mode of the seismometer is 440 Hz according to the device manual, adjustment of the seismometer to the stone was standard and the amplitudes of oscillations were so small that a slip of the instrument along the stone surface was excluded. We thus conclude that the observed low frequencies are a result of an interaction between the stone and the bedrock and we will try to explain them by a simple model describing

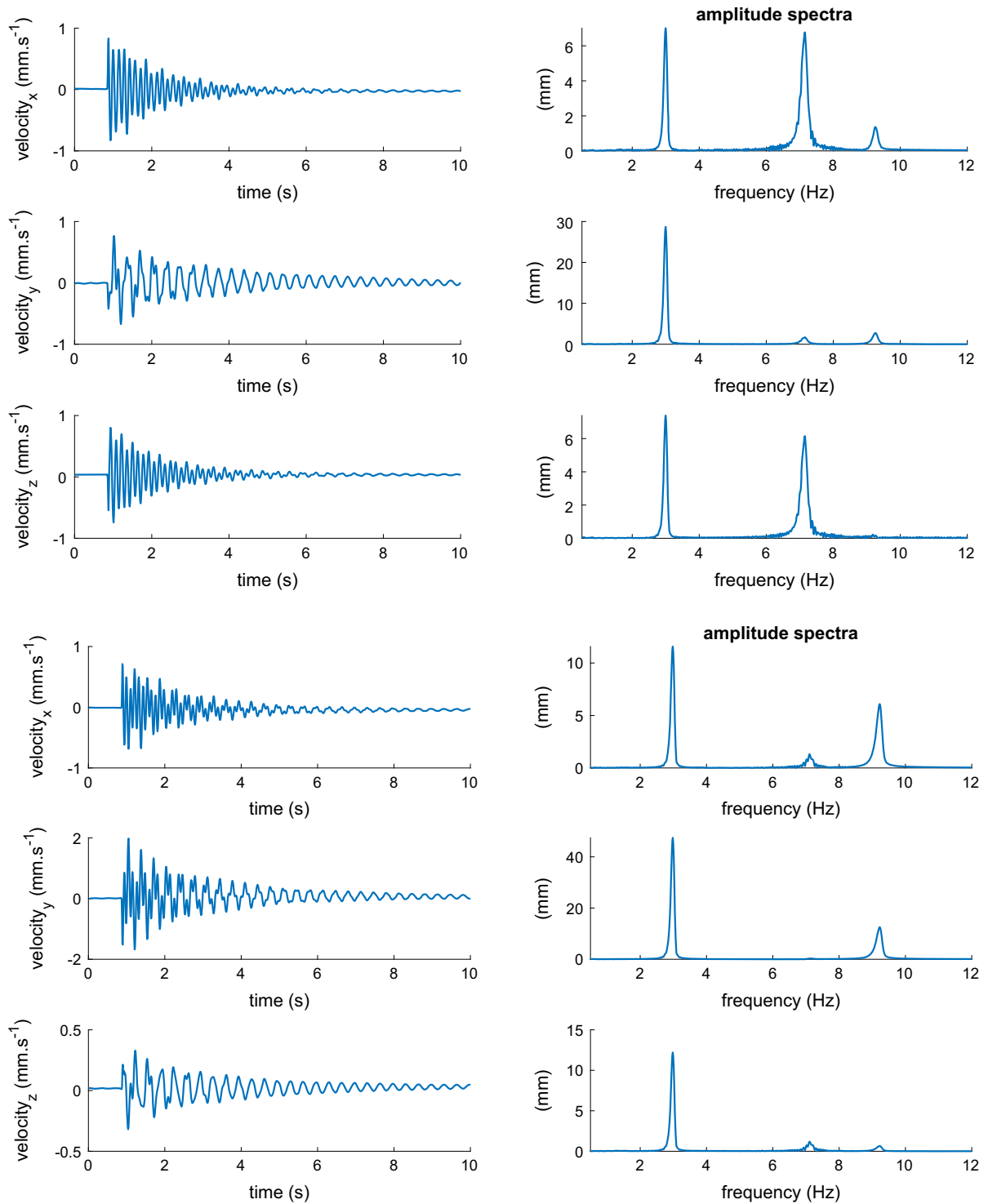


Figure 2

Two examples of unfiltered velocigrams of the horizontal (x, y) and the vertical (z) components and their amplitude Fourier spectra after applying the Hann windows

damped elastic response of the bedrock to the stone motions. We mention here that we also identified another mode at the frequency of about 30 Hz on the records of the vertical component of oscillations but this mode was so weak that we were not able to determine its quality factor. Particle motion analysis (Fig. 3) reveals that the 3 Hz and 7 Hz modes are characterized by particles oscillating in vertical planes which are mutually perpendicular. In the case of the 9 Hz mode the particle motions are not planar. We think that this fact reflects a complicated 3-D dynamics of the system.

2.2. Calculations of the Stone Eigenfrequencies

In order to model oscillations of “the Hus Pulpit”, it was necessary to prepare the 3-D digital model of the stone. The geometry of “the Hus Pulpit” is rather complicated that requires an increased effort in preparing the mesh covering the shape of the studied body. We started from 163 snaps that were processed by Agisoft PhotoScan software. The obtained mesh is illustrated in Fig. 4. The body (in .stl format) was implemented into the COMSOL Multiphysics software package that enables to calculate eigenfrequencies as well as the transient dynamics of the studied body for various material properties. The highest point of the stone is about 1.9 m above

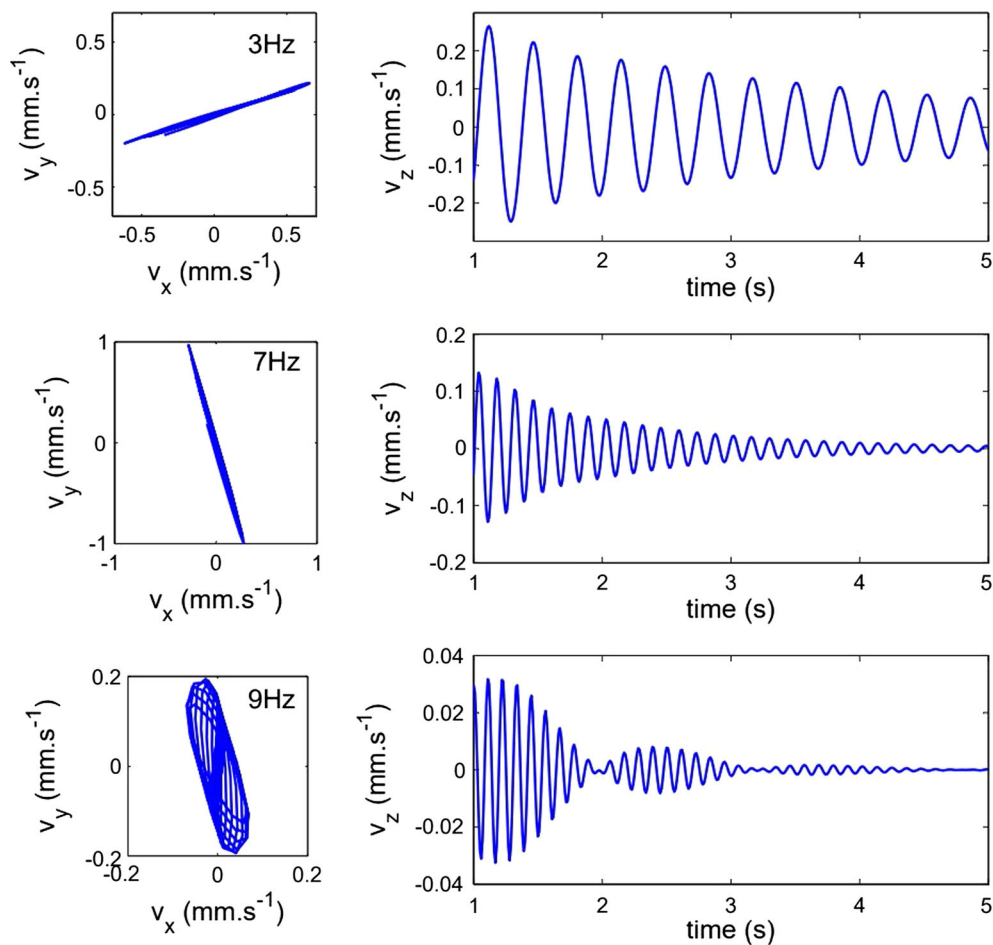


Figure 3

Horizontal (left panels) and vertical (right panels) particle velocities of the modes 3 Hz (top panels), 7 Hz (middle panels) and 9 Hz (bottom panels) obtained by applying a spectral window ± 1 Hz (3 Hz and 7 Hz) and ± 0.5 Hz (9 Hz)

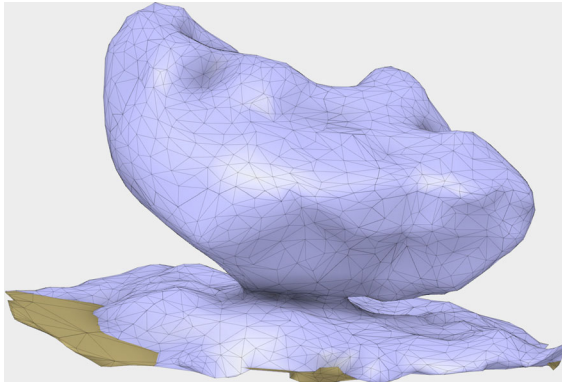


Figure 4

Geometry of “the Hus Pulpit” and its bedrock obtained from the Agisoft PhotoScan software

the base and the largest horizontal diameter is about 3 m; however, the geometry of the stone is strongly irregular and its volume is 6.72 m^3 . The area of the stone base is 0.43 m^2 , the center of mass is 83 cm above the base but the vertical projection of the center of mass to the base is shifted towards one of the vertices (Fig. 5). The granite is characterized by Young’s modulus $E = 50 \text{ GPa}$, Poisson’s ratio $\nu = 0.2$ and density $\rho = 2700 \text{ kg/m}^3$, therefore the mass of the stone is 18.1 tonnes and its weight is 177 kN. Since the stone is a complicated 3-D geometrical structure and the contact area resembles a triangle with round edges, the rocking point may change its position during strong rock motions and thus full 3-D calculations of the eigenoscillations and stability are needed.

The eigenfrequencies f of an elastic body yielded by COMSOL are obtained by solving the common momentum equation in the frequency domain,

$$\nabla \cdot \boldsymbol{\tau} = -4\pi^2 \rho f^2 \mathbf{u}, \quad \text{where } \boldsymbol{\tau} = \lambda \nabla \cdot \mathbf{u} \mathbf{I} + \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T),$$

where $\boldsymbol{\tau}$ is the stress tensor, $\mathbf{u}$ is the displacement of the body particles, ∇ denotes the nabla operator (i.e. $\nabla \cdot$ is the divergence operator), $\mathbf{I}$ represents the identity tensor, ^{T} denotes the tensor transposition, $\lambda = E\nu/((1+\nu)(1-2\nu))$ and $\mu = E/(2(1+\nu))$ are the Lamé elastic coefficients. Moreover, suitable boundary conditions representing additional constraints to stone motions must be added to this equations to obtain a unique solution. They play very

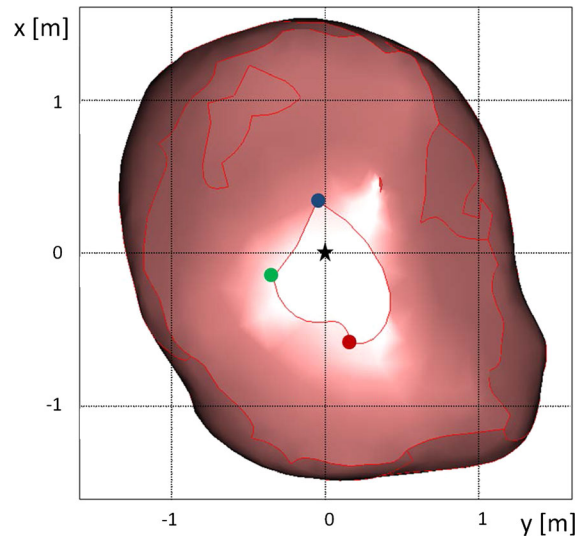


Figure 5

The surface of “the Hus Pulpit” digital model that consists of a system of vertices connected by curved edges. The white area inside the edges connecting the colour points is the base of the stone; its vertices are coloured to recognise their motions in Figs. 8 and 9. The black star represents the vertical projection of the center of mass; the assignment of horizontal axes direction does not have any special physical meaning; it was employed by COMSOL and only roughly corresponds to the sensor axes. The darkness of the colour corresponds to the vertical distance of the stone surface from the base plane

important role in the physics of oscillations and substantially influence eigenfrequencies of the stone.

2.2.1 Eigenoscillations of the Model with a Free or Fixed Base

No low eigenfrequencies would exist if the stone were completely free, i.e., $\boldsymbol{\tau} \cdot \mathbf{n} = 0$ on the whole surface of the stone including its base, where $\mathbf{n}$ is the normal unit vector to the boundary. For the studied stone with such a free boundary the lowest eigenfrequencies are 416, 490 and 617 Hz. However, the physics of eigenoscillations changes under the presence of different constraints applied to its base.

If the bedrock were ideally stiff and no slip of the stone base were possible, the constraint at the base would be simply $\mathbf{u} = 0$. In such a case, suitable for modeling of natural rock columns (Hall 1996) or hoodoos (Anooshehpour et al. 2013), there would be the three fundamental kinds of oscillation eigenmodes: transverse modes with particles moving in

vertical planes when the body is subjected to horizontal bending, rotational modes generated by torsions with respect to a vertical axis and longitudinal modes describing vertical extensional motions. The lowest eigenfrequencies of the stone eigenmodes calculated for the fixed base are 65 and 72 Hz (transverse modes), 104 Hz (torsional mode) and 272 Hz (longitudinal mode).

2.2.2 Eigenoscillations of the Model with an Elastic Response of the Bedrock

Clearly, the model of the stiff bedrock with no slip between the bedrock and the stone base cannot explain the observed frequencies. This is the reason why we decided to deal primarily with the forces acting at the base of the stone and try to explain the observed damped eigenoscillations by means of a relation between these forces and the base displacement. In order to obtain harmonic damped oscillations, we test whether the observations can be explained if the force (local traction) $\mathbf{F}$ acting at the base of the stone is considered in the simple form,

$$F_i = k_i \left(-u_i - \delta_i \frac{\partial u_i}{\partial t} \right), \quad i = x, y, z, \quad (1)$$

where x and y are the horizontal coordinates (see Fig. 5), z is the vertical coordinate, k_i is the i -th “stiffness” coefficient, δ_i is the i -th damping coefficient and t denotes the time. This approach is motivated by the way how an elastic response of soil to loading is modelled in civil engineering (e.g. Verruijt 2010). We consider the studied granitic stone to be perfectly elastic and thus the only damping arises through the interaction (1).

The tested traction acting at the base of the stone is thus determined by a choice of the three stiffness coefficients and the three damping coefficients. The model traction can formally be interpreted as the elastic response of the bedrock to its surface deformation by the stone if there is no slip between the stone and the bedrock. These stiffness and damping coefficients are a priori unknown and one of the modeling goals is to determine them from numerical simulations. By means of an trial and error method we found that the choice $k_x = 1.5 \times 10^9$ N/m, $k_y = 8.6 \times 10^6$ N/m, $k_z = 9.7 \times 10^8$ N/m, $\delta_x =$

3.3×10^{-4} s, $\delta_y = 1.9 \times 10^{-3}$ s, $\delta_z = 1.0 \times 10^{-3}$ s, yields the lowest eigenfrequencies of the model to be: 3, 6, 9, 10 and 36 Hz that are close to the observed ones. The quality factors of the three fundamental eigenmodes also correspond to the observed values mentioned above.

The oscillations are much more complicated than in the previous case with no motion ($\mathbf{u} = 0$) at the base. We have found from animations in COMSOL that the 3 Hz and 6 Hz modes are quasi-rotational modes around horizontal axes that are mutually perpendicular. Whereas rotation axis of the 6 Hz mode lies approximately at the base plane, that of the 3 Hz mode lies deeply. All these motions are calculated for small amplitudes when the rock is in full contact with the bedrock, i.e., they represent motion of the stone due to the deformation of the bedrock and not rolling of the stone with respect to a bedrock point. However, the doublet 9 and 10 Hz oscillates in a rather complicated 3-D manner. Existence of such a doublet could be explanation of the fact that observed particle motion at 9 Hz is not planar (Fig. 3). The simplest is the 36 Hz mode with up and down oscillations, i.e., it corresponds to the longitudinal mode from the example with the fixed base.

The question arises how to interpret the force described by Eq. (1). In vertical direction it is clearly the response of the bedrock to vertical deformation. When only static volume force due to the gravity is considered, the model results in the static vertical displacement u_z of the stone at the base to be $-0.37 \text{ mm} < u_z < -0.03 \text{ mm}$ (orientation of the z -axis is opposite to the gravity acceleration). However, explanation of the horizontal stone displacement may be more complicated because the base edges are not sharp and thus an additional motion by small rolling/wobbling cannot be excluded. Moreover there could also be gaps, where the stone base is not in contact with the bedrock due to the surface irregularities; they would represent weakening of the mechanical contact between the stone and the bedrock and might explain why we need substantially different horizontal stiffnesses to explain the observations by means of our model, where no such gaps are taken into account.

3. Stability of Oscillating Stone

3.1. Linear Models

The question arises, which quantity should act as a threshold of “critical situation”, when vibrations of the stone in response to ground motion are just before reaching the minimum horizontal acceleration required for rocking motion. Motions of the stone are associated with the changes of the traction at its base. The weight of the stone is 177 kN and thus we assume that the total vertical traction T_z defined as the integral of the vertical component of local traction F_z over the stone base, i.e., $T_z = \int_{\text{base}} F_z dx dy$, indicates reaching of the critical threshold, if magnitudes of its oscillations are close to 177 kN. We study dependencies of the traction on seismic input in both *the displacement model* (seismic displacement s is directly prescribed at the base) and *the force model*, where the seismic displacement s is incorporated to the model by (1) the traction acting at the base in the form:

$$F_i = k_i \left(s_i - u_i + \delta_i \frac{\partial(s_i - u_i)}{\partial t} \right), \quad i = x, y, z. \quad (2)$$

These models are linear, i.e., T_z depends on s_i linearly after subtracting the static weight of the stone. For a fixed direction of seismic input displacement, it is, therefore, sufficient to evaluate maximal $T_z^{\max}$ for one arbitrarily chosen amplitude of the input displacement s^a . Critical value s^c of the seismic input amplitude then follows simply from the relation $s^c/s^a = 177 \text{ kN}/T_z^{\max}$.

3.1.1 Displacement Model

In order to obtain estimates of critical seismic input amplitudes for the displacement model, we tested response of the T_z oscillations to harmonic horizontal input in the frequency range $0.1 \text{ Hz} < f < 10 \text{ Hz}$ such that $s_x = s_y = a \sin(2\pi ft)$, $a = \sqrt{2}/2 \text{ mm}$ acting at times $t \geq 0$, i.e., the total horizontal amplitude of the input is 1 mm. The static force due to the gravity was subtracted. Initially (for $t < 0$) the stone was in rest and $s_x = s_y = 0$. Therefore, the second time derivative of the input displacement (i.e., acceleration) is a delta-function at $t = 0$ and thus the output is, in principle,

generated for all frequencies. This is the reason why the prevailing output is dominated by high frequencies around 70 Hz (Fig. 6) that corresponds to the lowest eigenfrequencies of the displacement model although the input displacement for $t > 0$ was in the low-frequency range. We calculated the response of T_z to such horizontal motions at several frequencies. We found that the ratio between the maximal amplitudes of $T_z^{\max}$ and the amplitude of the input velocity is almost frequency-independent and is about 10 kN/(mm/s). This means that the threshold value of $T_z^{\max}$ is reached for velocity amplitudes of about 18 mm/s. Note that the response of $T_z^{\max}$ to vertical seismic motions s_z is about 1.5 times higher, i.e., the threshold vertical velocity amplitude is about 12 mm/s.

3.1.2 Force Model

The response of the base traction to harmonic input s_i in the force model described by Eq. (2) is more complicated. The response to horizontal loading is almost three orders of magnitude lower than in the displacement case, i.e., the corresponding threshold velocities are of the order of several meters per second. An exception are *inputs at the resonant frequencies* that provide lower threshold values: the 3Hz input yields the threshold velocity (acceleration) equal to 780 mm/s (1.50 g), the 6 Hz input exhibits this threshold for 230 mm/s (0.88 g) and the 9Hz inputs reaches the threshold for 1200 mm/s (6.9 g). Although the resonance at 6Hz lowers the threshold magnitude, the velocity (acceleration) of 230 mm/s

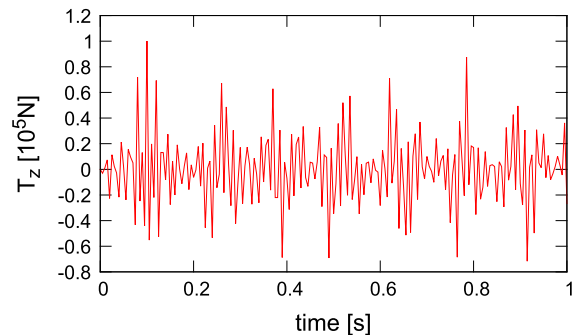


Figure 6

Displacement model: response of the vertical component of the total traction $T_z = \int_{\text{base}} F_z dx dy$ of the studied stone to 1 Hz harmonic horizontal displacement $s_x = s_y = a \sin(2\pi t)$, $a = \sqrt{2}/2 \text{ mm}$ acting at times $t \geq 0$. The static force due to the gravity is subtracted and the stone is initially at rest

(0.88 g) is still so high that it can hardly be achieved by any seismic source in the vicinity of the Bohemian Central Pluton. These results point to a high stability of “the Hus Pulpit” even at the resonant frequencies of the fundamental eigenmodes but, on the other side, they clearly lower threshold velocity values, see the time-behavior of T_z for 6Hz input signal that exhibits typical resonance behavior (Fig. 7).

3.2. Non-Linear Force Model Under a Presence of Wobbling/Toppling

In order to study wobbling and potentially toppling of the stone, we must take into account the fact that a part of the base becomes free during strong motions of the stone and need not be in the contact with the bedrock. The gravity force must be incorporated into modeling in this case because deformation of the bedrock by the weight of the stone is substantial in determining whether or not a part of the stone base is in contact with the bedrock. The static response of the stone at rest to gravity force results in negative vertical displacement u_z of the base but during stone motions displacement of the base changes and it becomes free at the points, where $u_z > 0$. This condition can be incorporated into the Eq. (2) simply in the form

$$F_i = k_i H(-u_z) \left(s_i - u_i + \delta_i \frac{\partial(s_i - u_i)}{\partial t} \right),$$

$$i = x, y, z,$$

$$H(-u_z) = 0 \text{ if } u_z > 0 \text{ and } H(-u_z) = 1 \text{ if } u_z \leq 0,$$
(3)

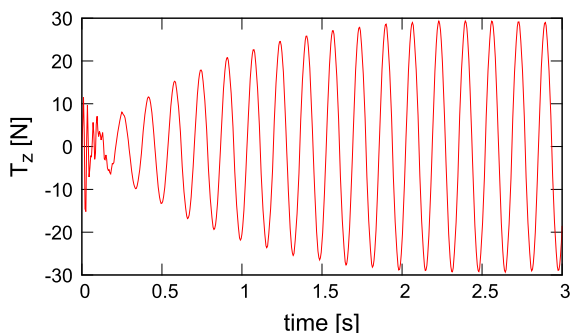


Figure 7

Linear force model: the vertical component of the total traction at the base of the studied stone generated by the 6 Hz harmonic horizontal displacement $s_x = s_y = a \sin(2\pi 6t)$, $a = \sqrt{2}/2$ mm acting at times $t \geq 0$. The static force due to the gravity is subtracted and the stone is initially at rest

H thus denotes the Heaviside function. The term $k_i H(-u_z)$ may be viewed as the stiffness coefficient depending on u_z and this is the reason why the force F is strongly non-linear when a part of the stone base becomes free.

First, we study rocking due to a constant horizontal acceleration, i.e. the quasi-static case. The ratio between the height of the center of mass above the base and the smallest distance of its projection to the base edge is approximately 4:1. Therefore, constant horizontal acceleration of about 0.25 g or slightly less should initiate rocking of the stone according to a 2-D rigid-body approximation, see the explanation in Apostolou et al. (2007, Eq. 1 and Fig. 1). However, the condition (3) at the base facilitates rotation of the stone around a part of an edge because adjacent part of the bedrock can deform and move deeper. Figure 8 shows that a horizontal constant acceleration of only 0.17 g is sufficient to start rocking motion in this complicated 3-D elastic case.

In order to demonstrate complicated dynamics of the stone at a critical state just before toppling occurs, we again calculate the response to harmonic bedrock input at 6Hz (Fig. 9). Wobbling starts when one vertex of the base exceeds the threshold limit $u_z = 0$ and non-linearity in Eq. 3 activates. It can be followed by pivoting (only one vertex and an adjacent part of the base are in the contact with the bedrock) and/or even by small jumps where all

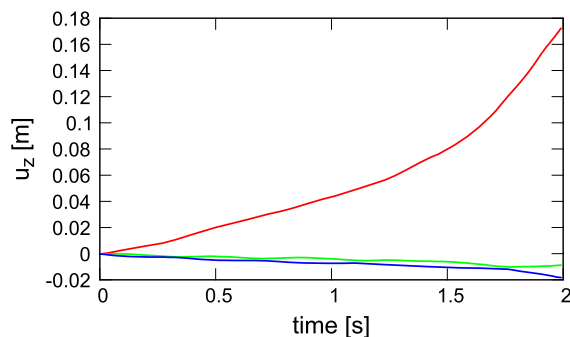


Figure 8

Non-linear force model: toppling of the stone model if the constant horizontal acceleration of the magnitude 0.17 g is acting in the same direction as in the dynamic examples shown previously. The curves show vertical displacement of the base vertices, denoted as green, blue and red points in Fig. 5

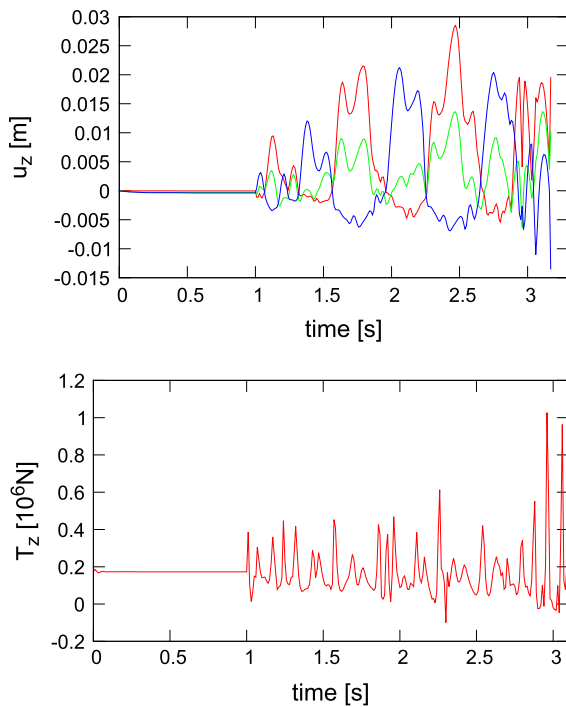


Figure 9

Non-linear force model: vertical motions of the base vertices denoted as green, blue and red points in Fig. 5. and the vertical component of the total traction at the base of the stone generated by the 6 Hz harmonic horizontal displacement $s_x = s_y = a \sin(2\pi 6t)$, $a = 10$ mm, starting at $t = 1$ s

vertices simultaneously satisfy the condition $u_z > 0$. For comparison with linear models, total vertical traction at the base is also shown.

One can clearly see that during pivoting, non-linearity in Eq. (3) is very strong and energy easily leaks from 6 Hz to other modes. When toppling occurs, the character of the response thus substantially differs from the linear models. Note that the displacement amplitude of the input was 14.1 mm, which corresponds to the velocity (acceleration) amplitude of 533 mm/s (2.0 g). It is more than two times higher than the threshold limit found in the linear force model that is another confirmation of the strong energy leakage out of the 6 Hz.

4. Concluding Remarks

In standard calculations of precariously balanced rocks toppling, the rocks are treated as 2-D rigid

bodies (e.g. Weichert 1994; Shi et al. 1996; Anooshehpour et al. 2004; Apostolou et al. 2007; Veeraraghavan et al. 2017). The advantage of this approach is that the problem is described by an ordinary differential equation and continuum mechanics of (visco)elastic bodies need not be taken into account. Such an approach is justified when eigenfrequencies of the body oscillations are much higher than a frequency range of a toppling source. We present the case study of the stability of “the Hus Pulpit” rocking stone located in the Central Bohemian Pluton, to demonstrate that the situation may be very complicated under the presence of the low frequencies falling into a seismic range—the lowest observed eigenfrequencies of the stone are 3, 7 and 9 Hz.

First, we applied seismic horizontal input directly to the base of the stone ignoring thus these low eigenfrequencies (lowest eigenfrequencies of the model were about 70 Hz). We demonstrated that critical amplitude of input bedrock velocity (when amplitude of the vertical component of total traction at the base of the stone reaches 177 kN) might be of the order of 1 cm/s (vertical bedrock motion) to 2 cm/s (horizontal bedrock motions) at the frequencies up to 10 Hz. Note that a PGV of this order can be generated by an $M = 7$ earthquake at a distance of about 150 km (Boore et al. 2014; Stewart et al. 2016) that is a typical distance of the studied stone from the nearest seismically active zones.

In order to explain the observed low frequencies of stone oscillations, we also present the model of the force interaction between the stone base and the bedrock. The rocking of the stone due to a seismic forcing is then almost impossible although the resonance effects are present. However, the force model presented in this study need not to be the only explanation of the observed low eigenfrequencies of “the Hus Pulpit” rocking stone as most of the contact area between the stone and its bedrock is invisible and the fundamental eigenmodes are influenced by the shape and character of this area, which may be topologically very complicated (Domokos et al. 2012). This is the reason, why, e.g., low-frequency rolling/wobbling due to non-planar effects and/or mechanical weakening of the system stone-bedrock due to gaps in the contact between the stone and the

bedrock cannot be excluded. Further research dealing also with local irregularities in the contact area is just needed.

Acknowledgements

We thank Ladislav Hanyk for his comments, Miroslav Coubal for technical help, Rasool Anooshehpour and an anonymous reviewer for the helpful and constructive reviews. This research was supported by the program “Support of Perspective Human Resources – Postdoctoral Research at the Institutes of the Czech Academy of Sciences” and the project “Strategy AV21 - Natural Hazards” funded by the Czech Academy of Sciences.

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