

Resolution tests of three-dimensional convection models by traveltimes tomography: effects of Rayleigh number and regular versus irregular parametrization

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SUMMARY

We present here the results obtained from 3-D tomographic inversion of synthetic data in order to investigate the ability of tomography to reveal the detailed thermal structures created by mantle convective processes. The synthetic input velocity models are based on the density heterogeneities obtained from a 3-D spherical-shell model of thermal convection. P and pP arrival-times are used in the inversion. We study the effect of the length-scale of the input anomalies characterized by the Rayleigh number of convection simulations ($Ra = 3 \times 10^5$, 8×10^5 , 10^6) and assess the sensitivity of the inverse problem to parametrization and explicit regularization (damping). We concentrate mainly on the effect of projection error and we show that its role is substantial with increasing chaotic convective behaviour. We use both irregular cell parametrization and regular cell parametrization (equisurface area). Due to the uneven distribution of sources and receivers, we found substantial differences between inversion output for models with regular and irregular cells. For the irregular parametrization, the structures located in the well-covered regions are resolved quite successfully, although the resolving power decreases with depth. In the poorly covered regions, where the ray sampling demands large parametrization cells, the correspondence between the input and output is rather low due to high parametrization error. An explicit regularization is not needed if only the projection error is assumed. On the other hand, for the regular parametrization, the inverse problem is very unstable and oscillations occur, unless an explicit regularization is applied. However, if an optimum damping is applied, the global correlation between the input and output anomalies is substantially higher than in irregular case. Further, we compare the power spectra of input seismic velocity anomalies and the inversion output. In case of irregular parametrization, the spectra of input model and inversion output differ especially in the uppermost part of the lower mantle and in the lowermost mantle, again due to the high parametrization error associated with the large parametrization cells. Much better agreement between the spectra of input and output is obtained in regular parametrization model, provided that a proper damping is applied. Therefore, we conclude that on a global scale, the regular parametrization gives better results than the irregular one, while the inversion with the irregular parametrization is able to resolve much more detailed structures in well-covered regions with the same number of model parameters and thus with the same computational costs.

Key words: resolution tests, synthetic inversion, tomography, traveltimes.

1 INTRODUCTION

An understanding of lateral heterogeneity in seismic wave velocities in the mantle is one of the most important issues in the geodynamical application of seismic tomography. In particular, distinguishing thermal from chemical origins of the heterogeneities is critical be-

cause the dynamical significance of the heterogeneity has different implications on the Earth's mode of convective heat transfer. The use of traveltimes data for deducing the lateral variations of seismic velocities in the mantle has a long history, going back to Aki *et al.* (1977). This technique has been called traveltimes tomography (Aki & Richards 1980) in contrast to tomography based on waveform

analysis (e.g. Woodhouse & Dziewonski 1984; Li & Romanowicz 1995). A comparison of the two approaches can be found in Dziewonski (2000). It has been well known from the early asymptotic analysis (Turcotte & Oxburgh 1967) that convection for high Rayleigh number is characterized by thin thermal anomalies (horizontal boundary layers, upwellings, as well as downwellings). Thus, it is very important for the inversion procedure to determine their thickness, how they are deflected by phase transitions, whether potential layering of mantle convection and/or the existence of small mantle plumes can be determined, what are characteristic wavelengths of temperature anomalies at different depths.

To answer these questions, it is very important to know what the resolution of particular tomographic techniques is. The resolution is best shown by the resolution matrix (e.g. Lévêque *et al.* 1993; Vasco *et al.* 2003; Soldati & Boschi 2005). However, the computation of the resolution matrix is time-consuming (Boschi *et al.* 2007). This is one of the reasons why only the standard resolution tests, like checker-board test (e.g. Inoue *et al.* 1990; Su *et al.* 1994; Vasco *et al.* 1995; Káráson & van der Hilst 2001; Fukao *et al.* 2003) or layer-cake test (e.g. Bijwaard *et al.* 1998) are used. In these tests, the artificial structures are often constructed by means of the particular parametrization used to obtain the synthetic data and then this same parametrization is employed in the inversion of the synthetic data. It is clear that this approach can reveal only a part of the resolution problems, as it neglects mainly the projection error (see the next section for a detailed explanation).

Detailed resolution tests should thus start from models of seismic velocity structures containing a broader variety of wavelengths than those yielded by tomography parametrization and, simultaneously, these models should be in agreement with the physics of mantle dynamics. In this paper, we will take the view that the heterogeneities in the mantle are generated only by thermal convection. The ability of traveltime tomography to resolve thermal anomalies developed in mantle convection simulations has been investigated by ‘seismic tomographic’ filtering (Johnson *et al.* 1993; Mégin *et al.* 1997; Tackley 2002). However, it can be shown that the short wavelength anomalies can leak into long wavelength (Trampert & Snieder 1996), if the wavelength of the anomalies is underestimated. Thus, the tomographic inversion should be employed (Honda 1996; Bunge & Davies 2001; Běhounková *et al.* 2005). Following the work by Bunge & Davies (2001), we will use the thermal anomalies from 3-D spherical-shell convection to construct a synthetic input model of seismic velocity anomalies and compute the differential travel-times for a series of synthetic earthquake events and an array of stations positioned on the spherical model.

One of the fundamental problems in retrieving the 3-D Earth structure is an uneven ray sampling. For example, it was shown by Davies & Bunge (2001) that inhomogeneous ray coverage results in obtaining a 3-D model which is faster than its underlying 1-D reference model. Further, it produces an ill-conditioned inverse problem and some regularization has to be applied. Here, we will use both irregular and regular parametrization and assess the sensitivity of the synthetic tomographic inversion to the explicit regularization for various values of the Rayleigh number Ra .

We will use the traveltime tomography for P and pP waves. The addition of the other phases would improve the resolution in some regions (see Lei & Zhao 2006). However, the total number of P waves used for the real tomographic inversion strongly exceeds the total number of the other phases. Therefore, the characteristics of the traveltime inversion are not changed essentially. Employing spectral analysis, we will also compare properties of regular and irregular parametrization, which was demonstrated to be capable of capturing

thermal anomalies caused by 2-D mantle convection (Běhounková *et al.* 2005). This adaptive scheme based on Spakman & Bijwaard (2001), partly reveals thin plumes and improves the estimate of amplitude anomalies. In this work, we will demonstrate that in 3-D spherical-shell convection, this technique is even more effective than in a 2-D situation.

2 METHOD

2.1 Inversion technique and parametrization

In our synthetic traveltime inversion, we use arrival times of P and pP waves. The delay d_i for the i th ray is defined as (for derivation see e.g. Nolet 1987)

$$d_i = T_i - T_i^0 + \varepsilon = \int_{L_i^0} \Delta s(\mathbf{r}) dL_i^0 + \varepsilon_i + \xi_i + \eta_i, \quad (1)$$

where 0 denotes reference model quantities and i is the index of the i th ray, $i = 1, \dots, N$. Δs is the difference between slowness s and reference slowness s^0 , ε_i is the picking error, ξ_i represents error arising from approximating (linearizing) the path L_i by the reference path L_i^0 and η_i is the error arising from mis-determination of hypocenters, origin times and errors of station corrections. Non-linear problem (1) should be solved iteratively (reference path in each step should be traced in seismic velocity model resulting from previous step—e.g. Bijwaard & Spakman 2000). Here, we restrict ourselves to a linear approximation, where $\xi_i = 0$, $\forall i$ and only one iteration is carried out.

To solve the eq. (1), we employ both irregular (Spakman & Bijwaard 2001) and regular cell parametrization:

$$\Delta s(\mathbf{r}) = \sum_{j=1}^M s_j c_j(\mathbf{r}) + \zeta(\mathbf{r}), \quad (2)$$

where $c_j(\mathbf{r})$, $j = 1, \dots, M$ is either regular or irregular orthonormal basis in L_2 ($c_j(\mathbf{r}) = C_j^{-1/2}$ if $\mathbf{r}$ is in the j th cell, $c_j(\mathbf{r}) = 0$ elsewhere; C_j is the volume of the j th cell), M is number of parameters, s_j are unknown parametrization coefficients and $\zeta(\mathbf{r})$ denotes a projection error. By substituting (2) into (1), we get

$$\mathbf{d} = \mathbf{G}\mathbf{m} + \mathbf{e}. \quad (3)$$

Here, $\mathbf{d}$ denotes the vector of delays, element of matrix $\mathbf{G}$ represents an arc lengths of the i th ray in the j th cell and elements of the model vector $\mathbf{m}$ give slowness difference in each cell: ($m_j = C_j^{-1/2} s_j$) and $\mathbf{e}$ is the vector of errors $e_i = \varepsilon_i + \xi_i + \eta_i + \bar{\zeta}_i$, where $\bar{\zeta}_i$ denotes integral of projection error $\zeta(\mathbf{r})$ along the i th reference ray. We study a linear problem here ($\xi_i = 0$) and we neglect both picking error ($\varepsilon_i = 0$) and error arising from mis-determination of sources ($\eta_i = 0$). We concentrate on the role played by the projection error $\bar{\zeta}_i$ only and our obtained resolution should be considered as an upper limit. In reality, where both ε_i , therefore, η_i are non-zero, the resolution would be even worse. Since the picking, mis-location, origin time, station corrections and linearization errors are neglected, the entire unpredictable part of the problem is caused by an insufficient resolution of the input model due to the adopted parametrization (projection error).

Large parametrization cells in some parts of the mantle caused by poor ray coverage are unable to resolve relatively small-scale convection features, thus producing high projection error. In order to estimate its amplitude and spatial distribution, we calculate the projection of an input model onto the parametrization basis. In the case of cell parametrization, the projection of the input model is

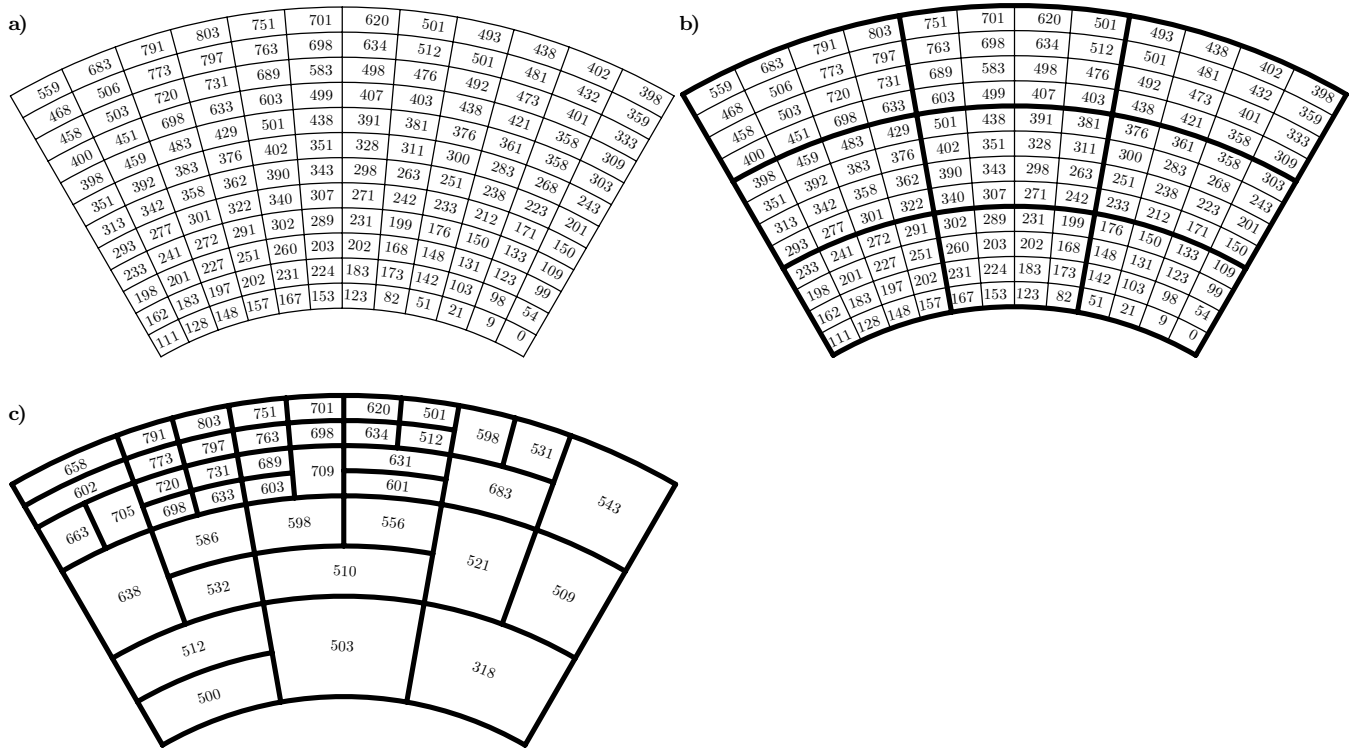


Figure 1. Two-dimensional sketch of the algorithm for irregular parametrization, (a) basic cells and hit counts, (b) basic cells (thin line) and largest possible target cells (thick line), (c) irregular cell parametrization and hit counts, $L_r = 3$, $L_\varphi = 3$, $M_E = 2$, a minimum hit count of 500 rays per cell.

equal to an average of input model over each cell— $\bar{s}_j$ is the average of the input model in the j th cell. The projection error $\zeta(\mathbf{r})$ then yields:

$$\zeta(\mathbf{r}) = s(\mathbf{r}) - \bar{s}(\mathbf{r}), \quad (4)$$

where $\bar{s}(\mathbf{r}) = \bar{s}_j$, if $\mathbf{r}$ is in the j th cell.

We solve an overdetermined inverse problem ($N > M$) and look for the solution by minimizing the misfit function S :

$$S = \|\mathbf{G}\mathbf{m} - \mathbf{d}\|_2^2 + \lambda^2 \|\mathbf{m}\|_2^2. \quad (5)$$

The damping coefficient λ specifies the weight between the minimization of the model vector $\mathbf{m}$ and the minimization of the difference between the observed and the predicted data. We use the LSQR algorithm by Paige & Saunders (1982) and its numerical implementation (<http://www.stanford.edu/group/SOL/software/lsqr.html>) to minimize the functional S .

In the synthetic problem, we know both input and output seismic structures. Consequently, we can compare the input synthetic model and inversion results and thus assess the resolving power of the inversion. For this purpose, we use either the correlation coefficient ρ or comparison of the power spectra of the input and the output.

As another characteristic of the inversion resolution, we define the percentage fit r as:

$$r = \left(1 - \frac{\|\mathbf{d}^{\text{pred}} - \mathbf{d}\|_2^2}{\|\mathbf{d}\|_2^2}\right) \cdot 100 \text{ per cent}, \quad (6)$$

where $\mathbf{d}^{\text{pred}} = \mathbf{G}\mathbf{m}$.

To compare the input and output seismic velocity structures, we further use their power spectra and root mean square (RMS). For this purpose, spherical harmonic coefficients are evaluated by integrating the seismic velocity anomalies. The integrational step for irregular parametrization is equal to 1.125° , which is the size of

the equiangular basic cell (paragraph 2.2). The vertical resolution corresponds to the vertical dimension of the basic cells (60 km).

2.2 Construction of parametrization cells

To construct the irregular basis, we use scheme by Spakman & Bijwaard (2001) and the hit-equalizing algorithm based on van der Hilst *et al.* (2004). The principle of this algorithm is demonstrated in a schematic 2-D case in Fig. 1. First, we divide the domain into small equiangular basic cells (see Fig. 1a; number in each cell gives corresponding hit count—number of rays crossing given cell). Then we have to choose the two parameters characterizing our target irregular parametrization: (a) minimum number of rays $n_{\min}$ demanded in each resulting parametrization cell and (b) the size of largest admissible parametrization cell (in terms of small basic cells). In the example shown in Fig. 1, $n_{\min}$ is 500 and largest irregular parametrization cell can contain maximum 4×4 basic cells (see Fig. 1b). The number of basic cells in each direction (r , θ , φ) has to satisfy the condition: $K_\bullet = L_\bullet \cdot 2^{M_E}$, where $\bullet$ denotes directions r , θ or φ . $K_\bullet$ defines the number of basic cells and $L_\bullet$ is the number of the largest cells in each direction and 2^{M_E} is number of basic cells in the each largest cell in one direction. If the number of rays in the largest cell is higher than $n_{\min}$, this cell is a candidate for recursive splitting. That is: we try to divide each cell containing at least $n_{\min}$ rays into halves. If both halves still contain at least $n_{\min}$ rays and the ratio of the sides does not exceed 2:1, the cell is divided. If there is more than one option how to make division, the one with the lowest hit count difference between the halves is chosen. The loop ends, either when we reach the basic cell or when the cells cannot be divided anymore, because the new cells after the division would not contain enough ($n_{\min}$) rays. In Fig. 1(c), resulting parametrization is shown. While in the basic basis cells, Fig. 1(a), the hit count varies from zero in some

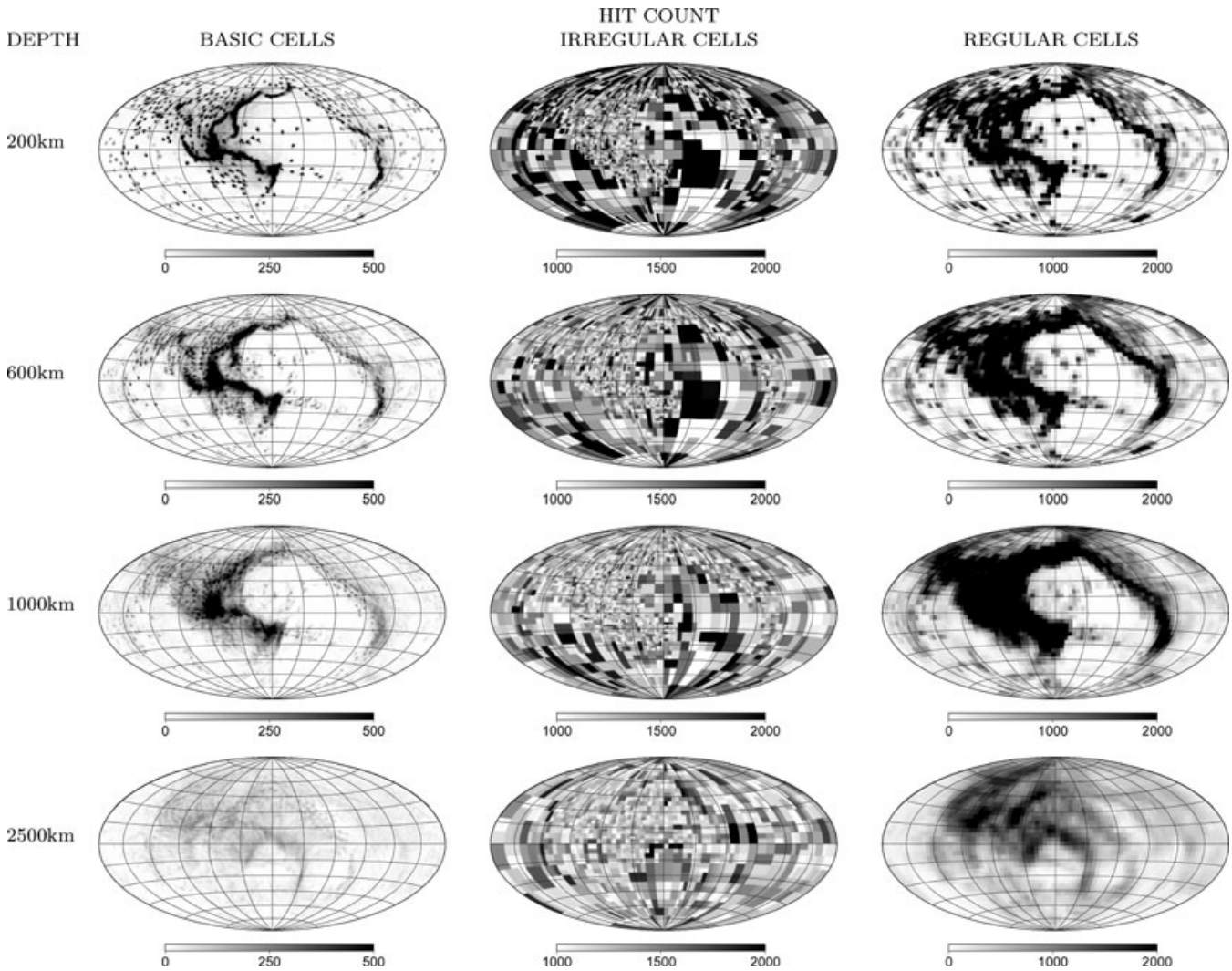


Figure 2. Hit count for basic cells (first column), irregular (second column) and regular (equal surface area cells, third column) cell parametrization for depth 200 km (first row), 600 km (second row), 1000 km (third row) and 2500 km (fourth row).

parts of the domain to 800 rays per cell in other parts, after applying the hit-equalizing algorithm, the number of rays in cells ranges from about 300 rays to 800 rays. Therefore, each parametrization cell is constrained by approximately the same amount of data. It should be noted, that due to the a prior choice of the largest irregular cell, there may remain some cells, which contain less than the required 500 rays (see the cell in the lower right corner in Fig. 1c).

In the irregular parametrization model, we define the size of largest cell by $L_r = 3$, $L_\theta = 10$, $L_\varphi = 20$, $M_E = 4$, which yields $48 \times 160 \times 320$ basic cells with size $60 \text{ km} \times 1.125^\circ \times 1.125^\circ$. After applying the hit-equalizing algorithm with a minimum of 1000 rays per cell, we get 35 886 irregular cells. In our regular parametrization model, we employ 36 428 equal surface area cells ($207 \text{ km} \times 4^\circ \times 4^\circ$ cell size on the equator). The resolution of the regular parametrization model was chosen to produce approximately the same number of parameters as the irregular one. That allows us to compare these two methods with similar computer costs.

2.3 Seismic velocity model and ray tracing

Synthetic seismic velocity anomalies are derived from the models of basally-heated thermal convection of Zhang & Yuen (1996) for

the Rayleigh number $Ra = 3 \times 10^5$, $Ra = 8 \times 10^5$ and $Ra = 10^6$ with constant viscosity and thermal expansivity. The cut-off degree of the spherical harmonic expansion of the model is 256 and thus the horizontal resolution of the model is 0.7° . The vertical resolution has 128 points. We suppose that density linearly depends on temperature,

$$\Delta\rho(\mathbf{r}) = -\rho_M\alpha\Delta T(\mathbf{r}), \quad (7)$$

where $\rho_M = 4,500 \text{ kg/m}^3$ is the reference density of the convection model and $\alpha = 1.4 \cdot 10^{-5} \text{ K}^{-1}$. Relative density and seismic velocity anomalies are then related by

$$\frac{\Delta\rho(\mathbf{r})}{\rho_0(r)} = p_k(r) \frac{\Delta v(\mathbf{r})}{v_0(r)}, \quad (8)$$

where $\rho_0(r)$ and $v_0(r)$ are the reference density and velocity from the PREM (Dziewonski & Anderson 1981) and p_k is the proportionality factor (Karato 1993) which depends on r .

The distribution of sources and receivers is chosen from the International Seismological Centre (ISC)¹ database (years 1964–2001).

¹International Seismological Centre Online Bulletin <http://www.isc.ac.uk/Bull>.

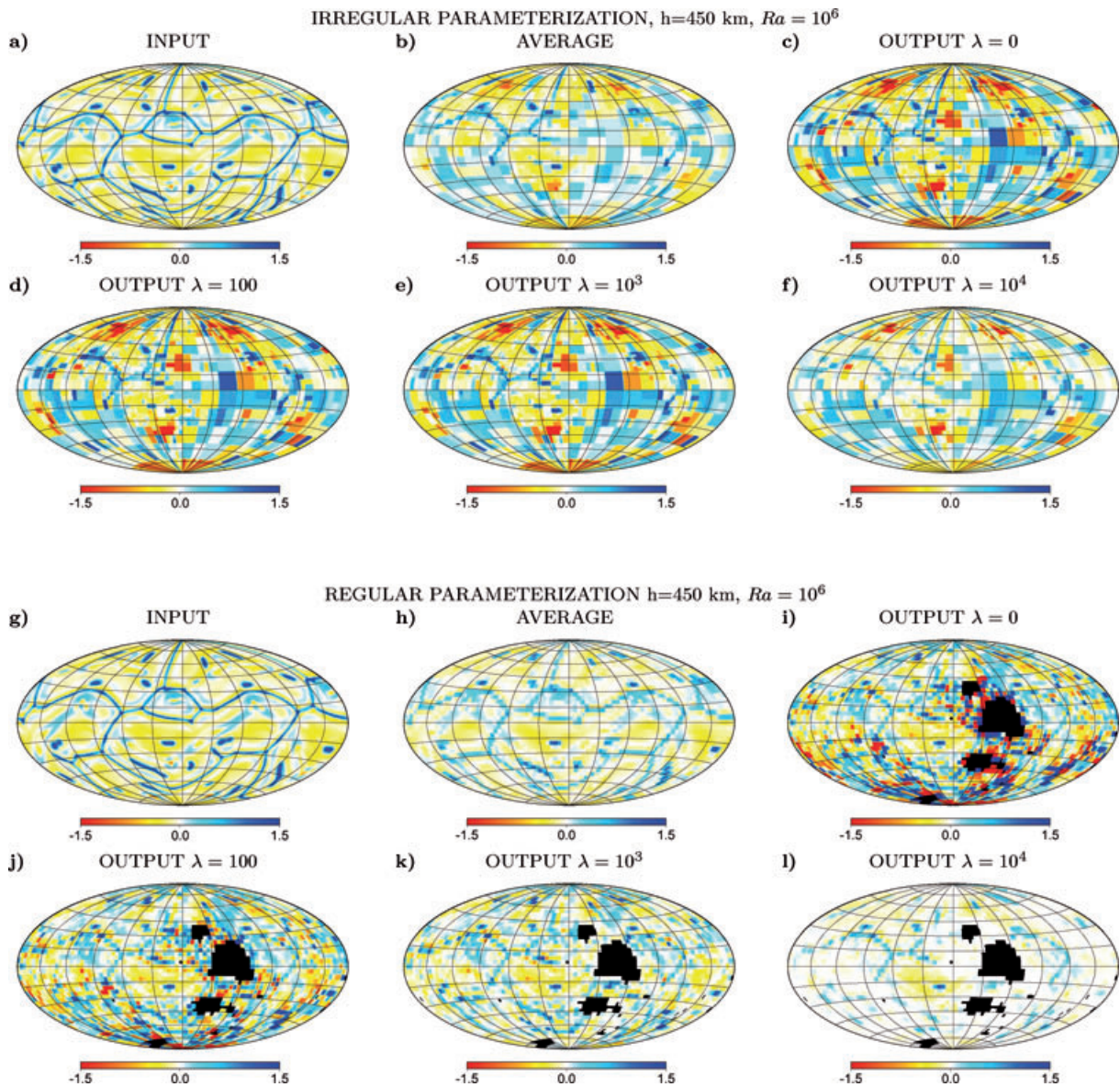


Figure 3. Results of the inversion for the irregular (a)–(f) and regular (g)–(l) parametrization and synthetic velocity model for a Rayleigh number $Ra = 10^6$. Relative seismic velocity anomalies (in per cents of reference velocity) are shown. Figures are plotted at a depth of 450 km. Plates (a) and (g)—input model, (b) and (h) average through irregular and regular cell parametrization, respectively, (c)–(f) results for irregular parametrization and the damping coefficient $\lambda = 0, 100, 10^3$ and 10^4 , (i)–(l) results for regular parametrization and the damping coefficient $\lambda = 0, 100, 10^3$ and 10^4 , black colour means no information (no rays) in the cell.

We use 2500 locations of events with $m_b > 5.5$ and 462 stations. Chosen stations are not closer than 4° to avoid linearly dependent rows in the matrix $\mathbf{G}$. The program CRT (Červený *et al.* 1988) is employed for the ray tracing. Since we assume a linear problem, we calculate rays through the depth-dependent PREM model (Dziewonski & Anderson 1981) where the ocean layer is omitted for simplicity. We take into account only teleseismic ($\Delta > 25^\circ$) P and pP waves. The total number of rays is 925, 054.

3 RESULTS

3.1 Irregular vs. regular parametrization

The efficiency of the hit-equalizing algorithm is demonstrated in Fig. 2, where the hit count is plotted at four depths (200, 600, 1000

and 2500 km). The left column shows the number of rays per basic cell. It ranges from 0 to about 4000 with the areas having a very low hit count, especially below the Pacific. The middle column shows the number of rays per irregular cell after application of hit-equalizing algorithm with minimum of 1000 rays. It produces small cells in the well-covered regions and huge cells in the poorly-covered regions. The right column shows the hit count for a regular (equal surface area) parametrization with a high ray density in some areas but an extremely low density in the other areas.

Results of the inversion for input anomalies based on temperature model with $Ra = 10^6$ are given in Fig. 3. The upper two rows are for an irregular parametrization, the lower two rows for regular parametrization. Panel (a) shows the input model at the depth of 450 km. In panel (b), there is an average of input anomalies over parametrization cells. Some small-scale features (narrow cold

Table 1. Correlations ρ and percentage fit r .

λ	$Ra = 3 \times 10^5$ $\rho_{\text{INPUT} \times \text{AVERAGE}} = 0.82$			irregular parametrization $Ra = 8 \times 10^5$ $\rho_{\text{INPUT} \times \text{AVERAGE}} = 0.73$			$Ra = 10^6$ $\rho_{\text{INPUT} \times \text{AVERAGE}} = 0.60$		
	r [per cent]	$\rho_{\text{IN} \times \text{OUT}}$	$\rho_{\text{OUT} \times \text{AV}}$	r [per cent]	$\rho_{\text{IN} \times \text{OUT}}$	$\rho_{\text{OUT} \times \text{AV}}$	r [per cent]	$\rho_{\text{IN} \times \text{OUT}}$	$\rho_{\text{OUT} \times \text{AV}}$
0	97.92	0.74	0.91	97.11	0.61	0.83	93.41	0.45	0.76
100	97.92	0.74	0.91	97.11	0.61	0.83	93.41	0.46	0.76
10^3	97.86	0.75	0.92	97.06	0.62	0.84	93.32	0.46	0.78
10^4	88.58	0.68	0.83	85.30	0.58	0.79	79.07	0.43	0.72
10^5	11.66	0.48	0.59	8.66	0.38	0.52	7.75	0.26	0.44

λ	$Ra = 3 \times 10^5$ $\rho_{\text{INPUT} \times \text{AVERAGE}} = 0.92$			regular parametrization $Ra = 8 \times 10^5$ $\rho_{\text{INPUT} \times \text{AVERAGE}} = 0.88$			$Ra = 10^6$ $\rho_{\text{INPUT} \times \text{AVERAGE}} = 0.77$		
	r [per cent]	$\rho_{\text{IN} \times \text{OUT}}$	$\rho_{\text{OUT} \times \text{AV}}$	r [per cent]	$\rho_{\text{IN} \times \text{OUT}}$	$\rho_{\text{OUT} \times \text{AV}}$	r [per cent]	$\rho_{\text{IN} \times \text{OUT}}$	$\rho_{\text{OUT} \times \text{AV}}$
0	96.80	0.11	0.12	97.25	0.14	0.16	93.60	0.11	0.15
100	96.80	0.65	0.71	97.25	0.65	0.74	93.59	0.47	0.61
10^3	96.74	0.80	0.87	97.17	0.75	0.85	93.46	0.63	0.82
10^4	93.20	0.64	0.70	90.97	0.60	0.68	85.33	0.54	0.71
10^5	29.00	0.38	0.42	22.54	0.34	0.38	14.88	0.43	0.44

downwellings) disappeared—were smeared over large parametrization cells and, on the other hand, some seismically slow (red) anomalies appeared due to ‘leaking’ from the shallower depths through large parametrization cells. Output of the inversion for four values of damping coefficient λ is shown in panels (c), (d), (e) and (f). Results do not differ significantly between the model without damping and models with damping factors of 100 and 1000. Higher damping (10 000) already reduces the amplitude of resulting velocity anomalies considerably. In the case of regular parametrization (last two rows in Fig. 3), the average of input anomalies over parametrization cells (panel h) is much closer to the input anomalies than in the case of irregular parametrization (*cf.* Fig. 3b), thus indicating a considerably lower parametrization error. The result of inversion without damping (panel i), however shows that the low parametrization error is balanced by a strongly oscillating solution in poorly-covered regions. An increase of the damping factor suppresses the oscillations which finally disappear when damping $\lambda = 1000$ is employed (panel k). With further increase of λ (panel l), the amplitude of inversion result is already too low and information about the input structure is lost in most of the mantle. The correlation ρ and data fit r are shown in the Table 1.

The behaviour of inversion is further demonstrated on spectra of input and output anomalies. Fig. 4 shows the decadic logarithm of power spectra as a function of depth for the models from Fig. 3. First, let us have a look at the results for an irregular parametrization (panels 4a–f). The reduction of power of averaged anomalies (panel b) compared to the input structure (panel a) clearly shows that the adopted parametrization is generally not capable of resolving narrow input features. Spectra of the output do not differ too much for the damping factors 0, 100 and 1000 (panels c, d and e). For a very high damping ($\lambda = 10\,000$, panel f), the spectrum is much shorter, which indicates that considerable part of the power of small-scale anomalies is already lost. We observe relatively high power on low degrees (degrees 1–25) at the depths range 450–900 km in the power of the output anomalies (panel c), while there is hardly any power at this spectral and depth interval in the input anomalies (panel a). This ‘leakage’ is caused by the large and deep parametrization cells which appear in poorly-covered regions in the upper mantle. To a lesser extent, a similar effect is observed in the lowermost 400 km of

the mantle. The parametrization is more uniform in the lowermost mantle—cells in well-covered regions are larger than at the shallow depths, but in poorly-covered regions, the maximum size of cells is smaller than huge cells at the surface area. Inversion using the regular parametrization without damping (panel i) produces extremely high power on short wavelength due to small-scale oscillations and damping is necessary to produce reasonable spectra. From a spectral point of view, an optimum inversion result (with the spectrum closest to the input spectrum) is obtained for regular parametrization with the damping factor $\lambda = 1000$. For the same model we also get maximum correlation between the input and output seismic velocity structures (Table 1). On the other hand, the advantage of the irregular parametrization is that explicit damping only weakly influences the results. Moreover, the solution in well-covered regions is very close to the input and more detailed than in the regular case with the same computational costs. However, we should be extremely careful in interpreting the inversion results in poorly-covered regions, since their correspondence with the input structure is rather low.

3.2 Results of inversion for different wavelengths of input anomalies

Further, we will concentrate on the efficiency of the inversion for the inputs based on different Rayleigh number convection models. We will discuss the results obtained for $Ra = 3 \times 10^5$ and for $Ra = 10^6$. We will further restrict ourselves to the model with irregular parametrization without explicit damping. Fig. 5 shows the model for $Ra = 3 \times 10^5$ at four depths: 200 km (first column), 600 km (second column), 1000 km (third column) and 2500 km (fourth column). In the first row, there are input seismic velocity anomalies. The second row shows the average over the parametrization cells. Since the size of input anomalies is relatively large, the adopted parametrization is able to fit them. Therefore, the projection of the input anomalies to the parametrization cells is relatively close to the distribution of input anomalies and projection error is rather low. The only exception is the depth 200 km, where the average distribution has significantly lower amplitudes than the input. This is due to the fact that the input temperature anomalies have their maximum

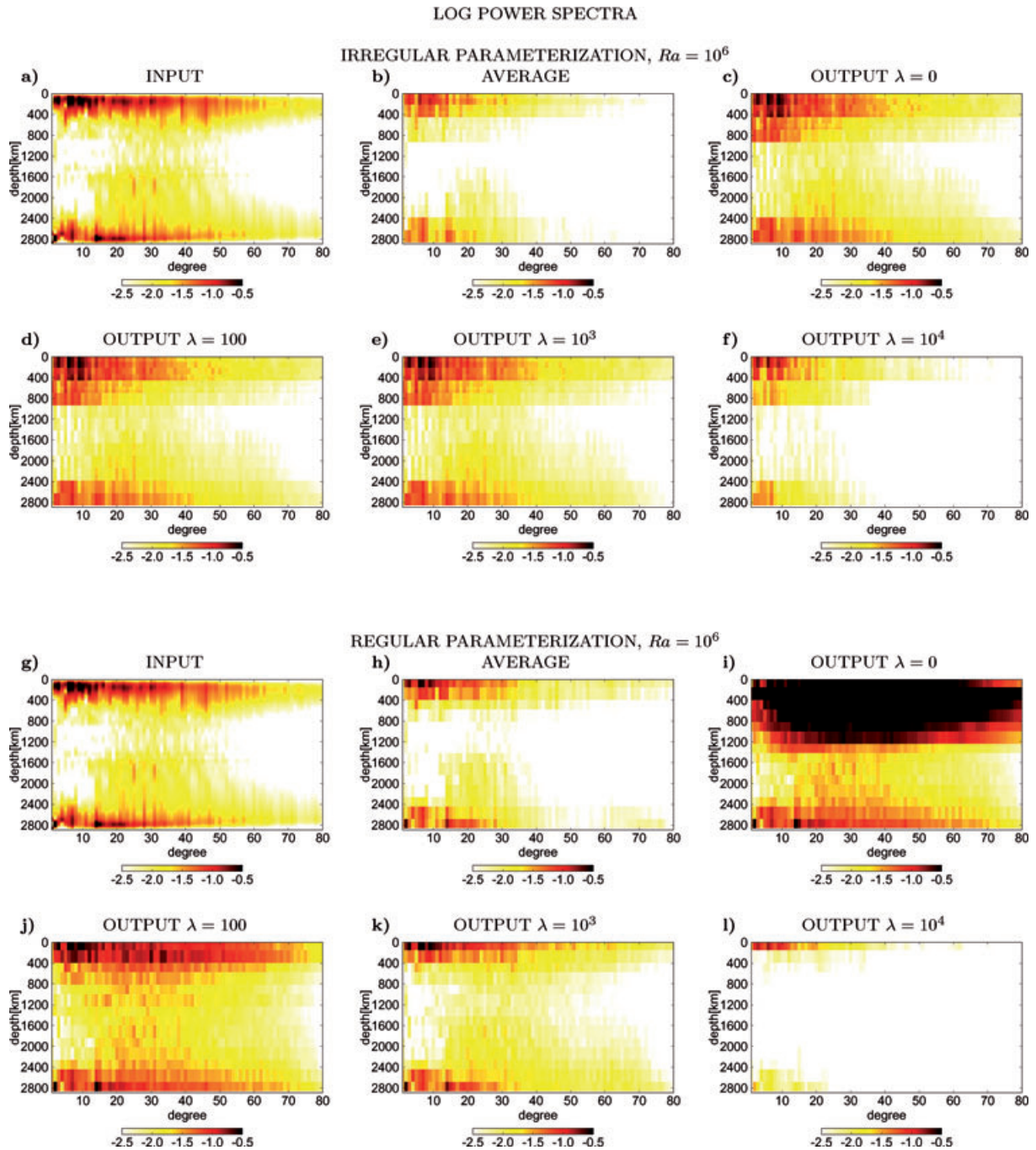


Figure 4. Decadic logarithm of the power spectra of results (as a function of the spherical harmonic degree and the depth) for both regular and irregular parametrization. Plate (a)–(f)—spectra for the input, average and output models ($\lambda = 0, 100, 10^3, 10^4$) for irregular parametrization. Plate (g)–(l)—spectra of the input, average and output models ($\lambda = 0, 100, 10^3, 10^4$) for regular parametrization.

amplitude just at this depth and their amplitude decreases strongly towards the surface. Therefore, the radial averaging performed over relatively small cells, considerably reduces the amplitude. A similar effect, although to lesser extent, is observed also at 2500 km depth. The results of the inversion for zero damping are shown in Fig. 5, the third row. Apparently, the inversion is able to resolve most features of the input model quite successfully (except of a few really poorly-covered regions) in the upper mantle. The fourth row in Fig. 5 shows the absolute value of the difference between the input and average structure, normalized by RMS of the input anomalies at a given depth. This demonstrates the spatial variability of parametrization

error showing that this error has maxima in the areas of strong input anomalies. Similarly, the last row in Fig. 5 depicts the difference between the input and output anomalies, that is, the total error of the inversion. The difference between the input and the output reflects the parametrization error, but its general level is, of course, higher, especially in the poorly-covered regions.

The RMS of the input, average and output anomalies is given in Fig. 6(a). The black line is for the input anomalies, the green line for the average over parametrization cells and the blue line for the RMS of the output. The input curve differs from the average and the output especially close to the surface and the core–mantle boundary. The

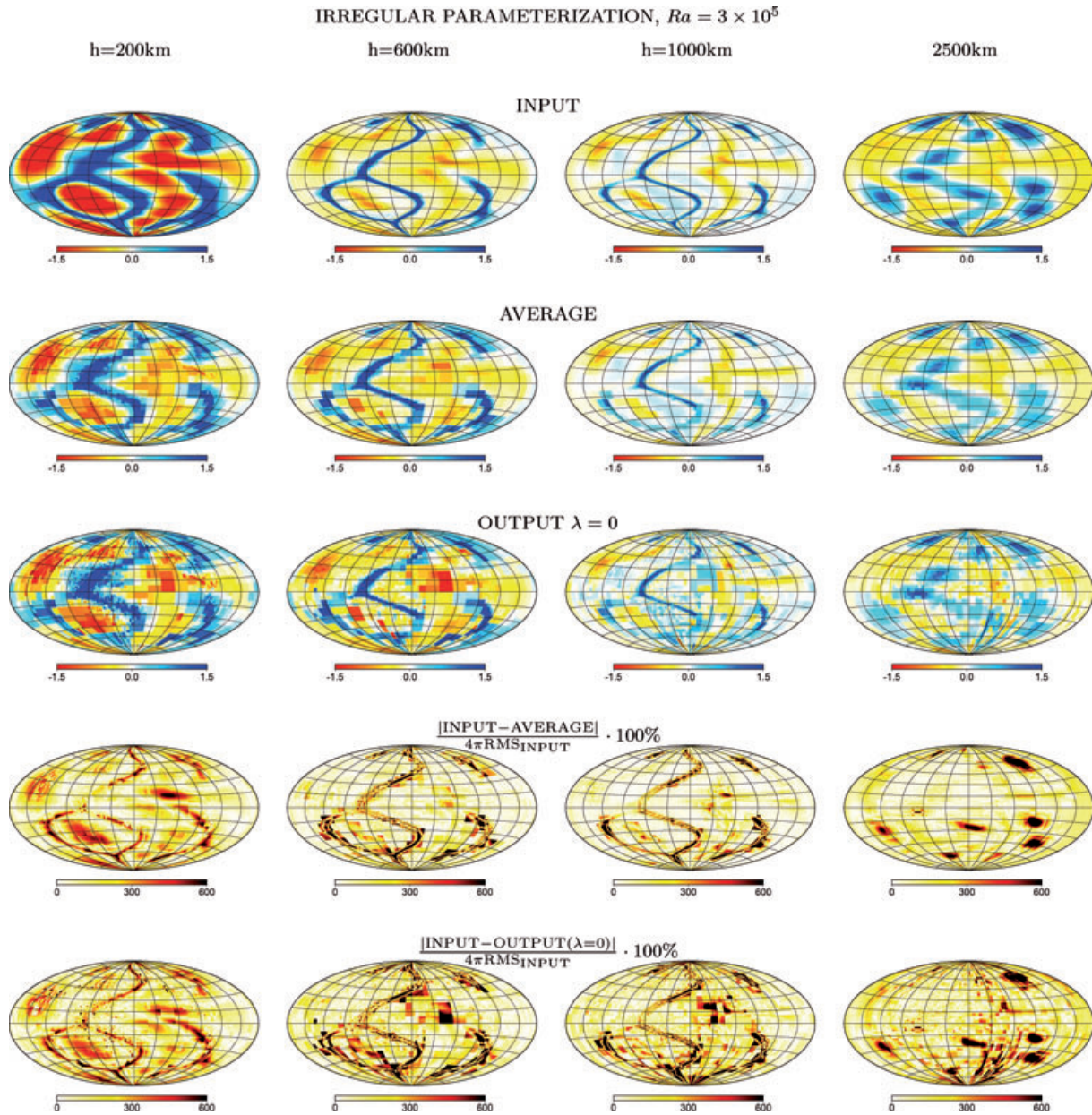


Figure 5. Results for the irregular parametrization for a Rayleigh number $Ra = 3 \times 10^5$. Relative seismic velocity anomalies (in per cents of reference velocity) are shown. Horizontal cross-section at the depth of 200 km (first column), 600 km (second column), 1000 km (third column) and 2500 km (fourth column). The first row—cross-section of input model, the second row—average through irregular cells, the third row—results of the inversion for $\lambda = 0$, the fourth row—absolute value of difference between input model and average normalized by $4\pi RMS$ of the input model at given depth, the fifth row—the normalized difference between input and output ($\lambda = 0$) model.

input anomalies vary strongly with depth here and the ray coverage is not dense enough to support sufficiently fine depth resolution. Step decrease in the RMS of average and output at a depth of about 900 km is caused by the large parametrization cells, which extend up to this depth. Fig. 6(b) shows the RMS of the differences between the input and the average (black line), input and the output (green line) and the average and the output (blue line).

Results of the inversion for the input based on the convection model with $Ra = 10^6$ are shown in Fig. 7. There are again four columns corresponding to the depths of 200, 600, 1000 and 2500 km. The input anomalies (first row) are of smaller scales than those in Fig. 5. Since the ray coverage (and, therefore, the parametrization as well) is the same as in the case with

lower Rayleigh number, it is not surprising, that the average over parametrization cells fits the input anomalies less than in the case shown in Fig. 5. Results of the inversion are shown in the third row of Fig. 7. While in the well-covered regions (*cf.* Fig. 2), anomalies are resolved quite successfully, in the sparsely covered areas, the inversion output may be completely wrong. The difference between the input and output anomalies (fifth row of Fig. 7) clearly shows that the resolution power of the inversion is for the higher Rayleigh number convection anomalies, considerably lower than in the case of lower Rayleigh number model (*cf.* Fig. 5, bottom row). The error is strikingly high especially in the mid-mantle (600 km depth and 1000 km depth). Correlation ρ between input, average and output and percentage fit r for all above discussed models are summarized

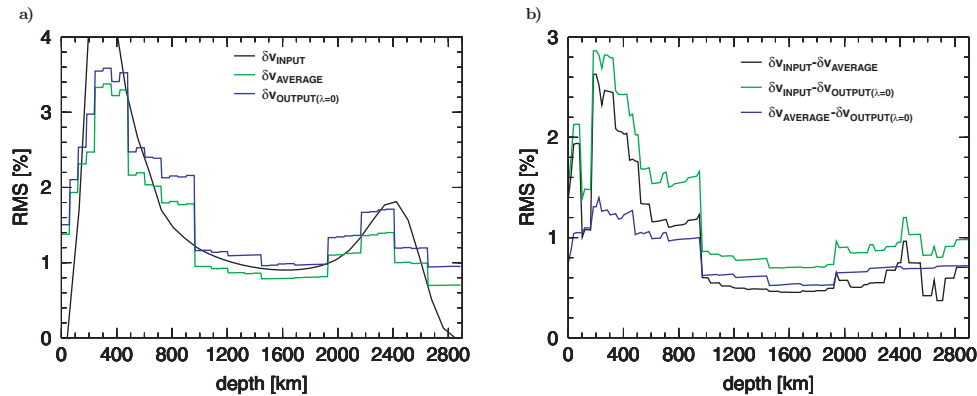
IRREGULAR PARAMETERIZATION, $Ra = 3 \times 10^5$ 

Figure 6. Root mean square for the input model ($Ra = 3 \times 10^5$), average and inversion result ($\lambda = 0$) (plate a) and its differences (plate b).

in Table 1. For comparison, there are also listed the correlations and percentage fit for the models with regular parametrization and three different values of Rayleigh number. Fig. 8 shows the RMS of the the input, average and output anomalies for the $Ra = 10^6$ and for the respective differences between the input and the average and the input and the output.

The amplitudes of parametrization error and the total error of the inversion are summarized in Table 2. By the parametrization error, we mean the RMS of the difference between the input and the average seismic velocity normalized by $4\pi \text{RMS}_{\text{INPUT}}$ at given depths. We evaluate it at depths of 200, 600, 1000 and 2500 km. Similarly, the total error is a normalized RMS of the difference between the input and the output seismic velocity structures. The values given in Table 2, therefore, correspond to the RMS of the quantities plotted in last two rows of Figs 5 and 7. Parametrization error measured by the normalized difference between the input and the average distribution for $Ra = 3 \times 10^5$ (second column of Table 2) varies with depth. It has a maximum at the depth of 200 km and a minimum at the depth of 1000 km. The total error of the inversion for the low Ra results (difference between the input and the output; the third column of Table 2) is quite uniform at the depths shown (it varies between 2027 and 2118). For the model based on a higher Rayleigh number (10^6), the parametrization error is only slightly higher (by about 10 per cent) at shallow depth (200 km) and in the deep mantle (2500 km), but it is considerably higher (by about 100 per cent) in the mid-mantle (600 and 1000 km depths). The total error of the inversion is higher by 25 per cent than corresponding value for the low Ra at the depth of 200 km, greater by about 50 per cent at 2500 km and more than twice higher than corresponding value for the low Ra in the mid-mantle (600 and 1000 km).

The resolving power of the inversion is further demonstrated on the two vertical cross-sections for the model with the low Ra (3×10^5). Panel 9(a) shows the position of the first cross-sections, taken through a relatively poorly-covered area (*cf.* hit count, panel 9b). Input anomaly is given in panel 9(c), panel 9(d) shows the average over parametrization cells and panel 9(e) the inversion output. Clearly, the parametrization is not fine enough and is not able to fit either the shape or the amplitudes of the input correctly. Therefore, it is not surprising that the outcome of the inversion is rather poorly correlated with the input structure. On the contrary, in the well-covered regions panel 9(f), hit count in panel 9(g) the input structure (9h) is nicely fitted by the chosen parametrization (*cf.* average distribution, panel 9i) and well resolved by the inversion (panel 9j).

The two vertical cross-sections for the high Rayleigh number $Ra = 10^6$ are shown in Fig. 10. One of them is again taken through a seismically slow upwelling structure in a poorly-covered region (panel 10a) and the other one through the seismically fast downwelling in a well-covered region (panel 10f). While the plume (panel 10e) is almost not visible, the downwelling structure (panel 10j) is resolved with some success.

4 DISCUSSION

Compared to the synthetic 2-D inversion (Běhounková *et al.* 2005) where the resolution of the narrow vertical anomalies decreases substantially below the depth of 1000 km, the resolution of the 3-D inversion with irregular parametrization is rather good for the entire mantle. The smearing of the input anomalies along the rays observed in the lower mantle in the 2-D inversion is also much weaker in the 3-D case.

We found substantial differences between the inversion output for regular and irregular parametrization. For the irregular parametrization, the input structures are resolved successfully in the well-covered regions. In the poorly-covered areas, where the distribution of rays is very sparse, rather large parametrization cells can occur and the projection error can be significant there. However, explicit regularization is not needed in our inversion, where only projection error is taken into account. The normalized power spectra of the inversion output decay reasonably even if no damping is applied. However, they differ from the spectra of the input anomalies especially in the upper part of the lower mantle and in the lowermost mantle, where the inversion results are negatively influenced by the large parametrization cells in the poorly covered regions. For the regular parametrization, the inverse problem is unstable and oscillations occur unless explicit regularization is applied. Without damping, the spectra of inversion output are flat due to high degree oscillations, especially in the upper mantle. On the other hand, if a proper damping is used, the output spectrum is much closer to the input one than in any irregular parametrization model. The best fit of the input and output spectra is obtained for the damping parameter $\lambda \sim 10^3$.

The regular parametrization produces much lower parametrization error in the badly covered regions than the irregular one, but at the cost of necessary explicit regularization. When comparing the input synthetic structures with the inversion output, we can state that the best result is obtained for the damping coefficient λ about 10^3 .

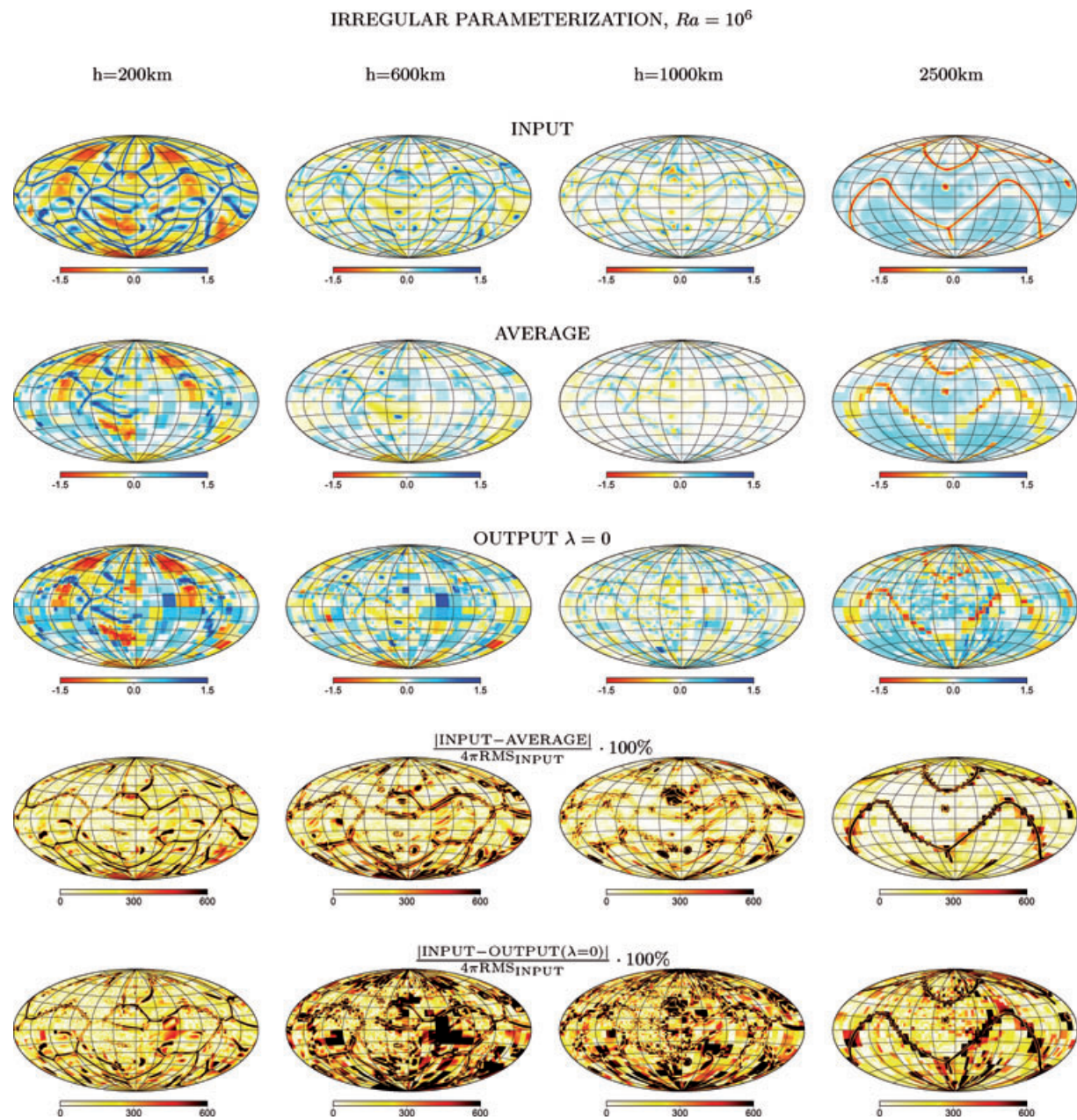


Figure 7. Results for the irregular parametrization for a Rayleigh number $Ra = 10^6$. Relative seismic velocity anomalies (in per cents of reference velocity) are shown. Horizontal cross-section at the depth of 200 km (first column), 600 km (second column), 1000 km (third column) and 2500 km (fourth column). The first row—cross-section of input model, the second row—average through irregular cells, the third row—results of the inversion for $\lambda = 0$, the fourth row—absolute value of difference between input model and average normalized by $4\pi RMS$ of the input model at given depth, the fifth row—the normalized difference between input and output ($\lambda = 0$) model.

Table 2. RMS of the differences between the input and average, and between the input and result of the inversion (for $\lambda = 0$) at four depths. Each value is normalized by the factor $4\pi RMS_{INPUT}$, where RMS_{INPUT} is RMS of input anomalies at a given depth.

depth [km]	$Ra = 3 \times 10^5$		$Ra = 10^6$	
	IN-AVERAGE	IN-OUT($\lambda = 0$)	IN-AVERAGE	IN-OUT($\lambda = 0$)
200	1903.89	2089.59	2234.42	2538.79
600	1435.35	2103.70	2654.58	4912.10
1000	1138.68	2027.65	2264.90	4477.63
2500	1435.30	2118.16	1832.22	3339.17

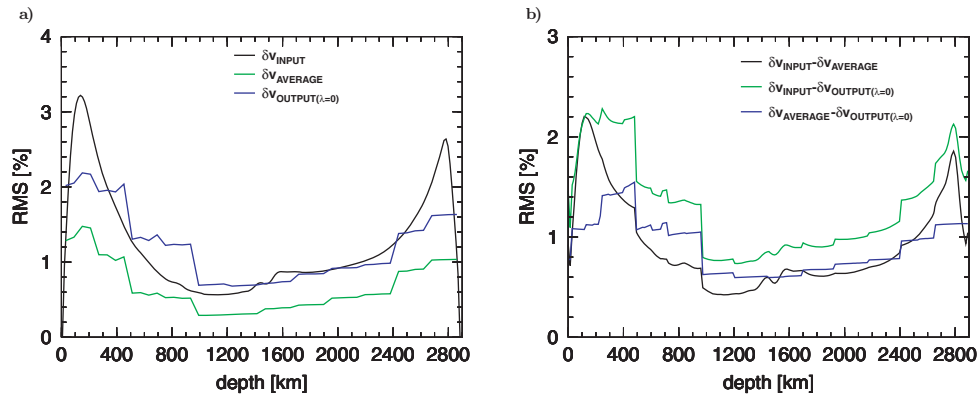
IRREGULAR PARAMETERIZATION, $Ra = 10^6$ 

Figure 8. Root mean square for the input model ($Ra = 10^6$), average and inversion result ($\lambda = 0$) (plate a) and its differences (plate b).

In the real data inversion, however, we do not know the input structure and therefore, some strategy how to choose a proper damping factor is needed. In the real data inversions, an analysis of L-curves is often used for this purpose (Hansen 1992). We will show here the relation between the damping factor, correlation of the input and output structure (which is not known in real data inversion) and the percentage fit, that is the difference between the measured and predicted arrival times (which is known in real data inversion). Further, we will analyse the curvature of L-curve and examine the correspondence of its maximum with the damping factor $\lambda_{\max}$ that maximizes the correlation between the input and output seismic velocity structure. These relations are demonstrated in Fig. 11 for the models with the Rayleigh number 10^6 . Fig. 11(a) is for irregular parametrization. The upper panel shows the percentage fit as a function of damping factor λ (in logarithmic scale). The middle panel shows the correlation between the input and output seismic velocity structures and the lower panel displays the L_2 norm of the model vector. For the low values of λ (up to about 2000), we get high percentage fit r (over 90 per cent). Further increase of λ suppresses the model norm too much, useful information is lost and the data cannot be successfully fitted any more. The correlation between the input and output does not change substantially for λ between 0 and 5000 and then it decreases rapidly. This demonstrates that explicit regularization is not necessary in this case—correlation of the input and output seismic velocity structures is relatively high already for $\lambda = 0$. L_2 norm of the model vector is more or less constant for the irregular parametrization up to λ 1000 and for higher λ decreases. The situation in regular parametrization model is different (Fig. 11b). The model norm (lower panel) is higher than in irregular case for low λ due to oscillations and decreases over the whole range of damping factor. Correlation (middle panel) is rather low for $\lambda = 0$, but it increases with increasing damping factor as oscillations of the output model are being suppressed. After reaching the maximum for $\lambda \sim 2000$ it decreases again as the model norm is too much suppressed, similarly to previously discussed irregular case. In Fig. 11(c) the L-curve is shown (the solution norm $\|\mathbf{m}\|_{L_2}$ plotted as a function of residual norm $\|\mathbf{Gm} - \mathbf{d}\|_{L_2}$ in log-log scale). The curvature of this curve is shown in Fig. 11(d) as a function of damping coefficient in log-scale. Maximum curvature is found for $\lambda \sim 2000$, that is for the same damping which yields maximum correlation between the input and output structure. We thus conclude that the analysis of the L-curve can be used to choose proper damping factor in real data inversion without knowing the input seismic velocity structure.

An important question is of course, how tomography can recover geodynamic models (Méglin & Romanowicz 2000; Becker & Boschi 2002). The ability of our kinematic tomographic inversion to retrieve geodynamic (convection) models is illustrated in Fig. 12. Here, the correlation between the inversion input and output seismic velocity distributions is shown as a function of depth and spherical harmonic degree for $Ra = 10^6$. In the case of irregular parametrization (panels a and b), the input and output models correlate in the uppermost mantle and then in the lower mantle below the depth of about 1200 km on low harmonic degrees. Between the depths of about 400 and 1200 km there is a gap, where the correlation is poor due to the large parametrization error in poorly covered areas and small amplitudes of input anomalies. When regular parametrization with optimum damping ($\lambda = 1000$, panel d) is used, the correlation between the input and output models is generally higher. This means that on global scale the regular parametrization is more successful if an optimum damping is applied. In irregular parametrization model, the global correlation is lower because of the large parametrization cells in poorly covered areas. On the other hand, significantly finer resolution can be obtained here in well-covered regions with the same total amount of model parameters as in regular model.

The upper limit of the resolution of the irregular parametrization model is given by the size of basic cells which is 1.125° in our case. In order to have approximately the same number of parameters (and comparable computer demands), the used regular parametrization grid is coarser—its best possible resolution is 4° . Therefore, we cannot reach the best resolution of the irregular parametrization model in well-covered regions under the same computational cost. On the other hand, in the poorly covered regions, where the low hit count demands large irregular parametrization cells, the regular model has better resolution (provided we use proper damping). By using finer regular grid, we could reach the resolution comparable to the resolution in well-covered regions of irregular model, but at the cost of considerably higher number of models parameters and, therefore, more memory-demanding and time-consuming requirements.

In this study, we have restricted ourselves purely on the effect of parametrization error. Therefore, we get the best possible resolution for adopted parametrization. If the picking error and error arising from mis-determination of sources would be included, the resolution would be even worse. Hence, some extra damping would be necessary for both types of parametrization to eliminate the effect of these errors.

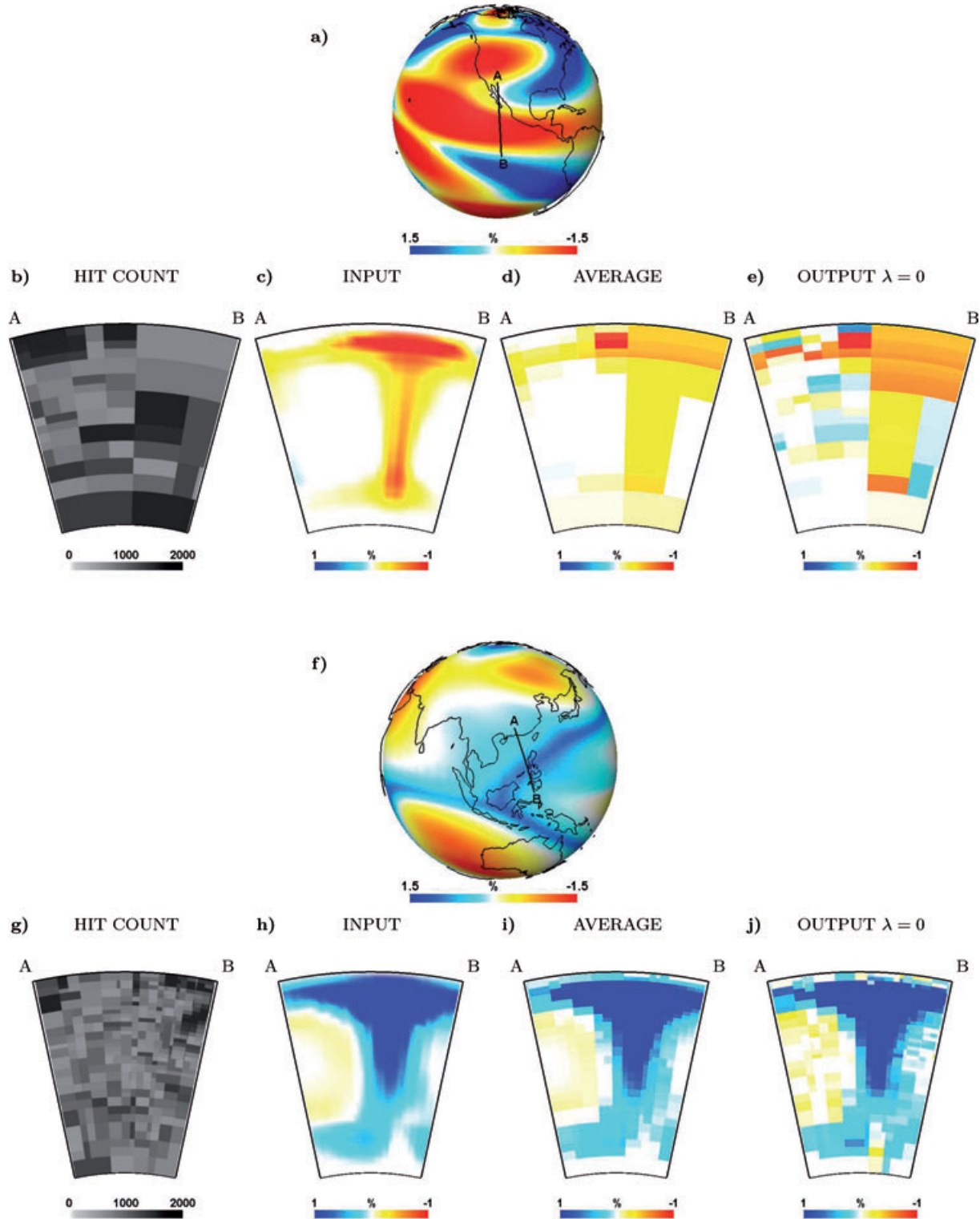
IRREGULAR PARAMETERIZATION, $Ra = 3 \times 10^5$ 

Figure 9. Two vertical cross-section in the model with $Ra = 3 \times 10^5$ and irregular parametrization. Vertical cross-section (a)–(e) through upwelling, (a) location of the cross-section, (b) hit count, (c) input, (d) average, (e) result of the inversion for $\lambda = 0$. Vertical cross-section (f)–(j) through downwelling, (f) location of the cross-section, (g) hit count, (h) input, (i) average, (j) result of the inversion for $\lambda = 0$. Relative seismic velocity anomalies (in per cents of reference velocity) are shown.

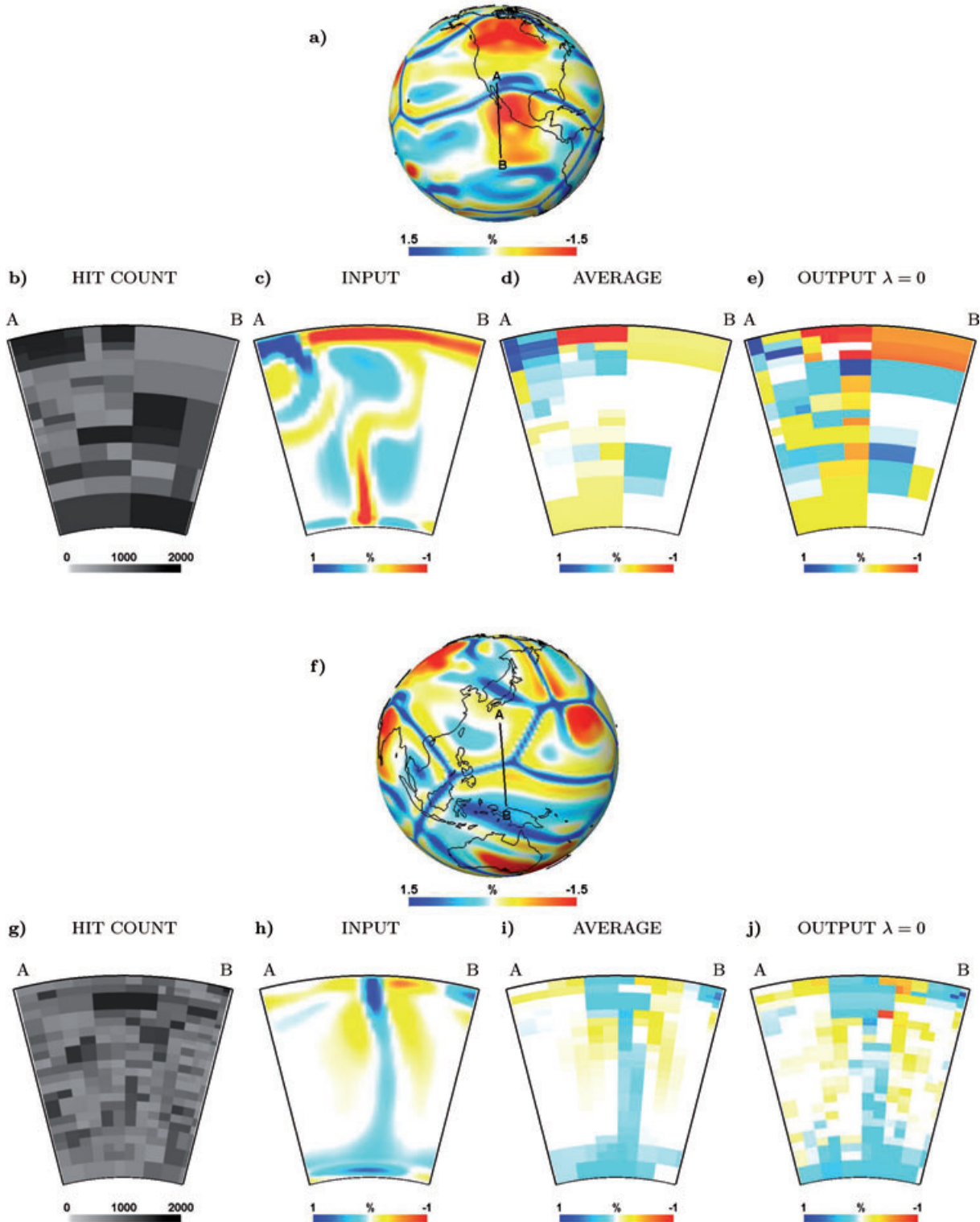
IRREGULAR PARAMETERIZATION, $Ra = 10^6$ 

Figure 10. Two vertical cross-section in the model with $Ra = 10^6$ and irregular parameterization. Vertical cross-section (a)–(e) through upwelling, (a) location of the cross-section, (b) hit count, (c) input, (d) average, (e) result of the inversion for $\lambda = 0$. Vertical cross-section (f)–(j) through downwelling, (f) location of the cross-section, (g) hit count, (h) input, (i) average, (j) result of the inversion for $\lambda = 0$. Relative seismic velocity anomalies (in per cents of reference velocity) are shown.

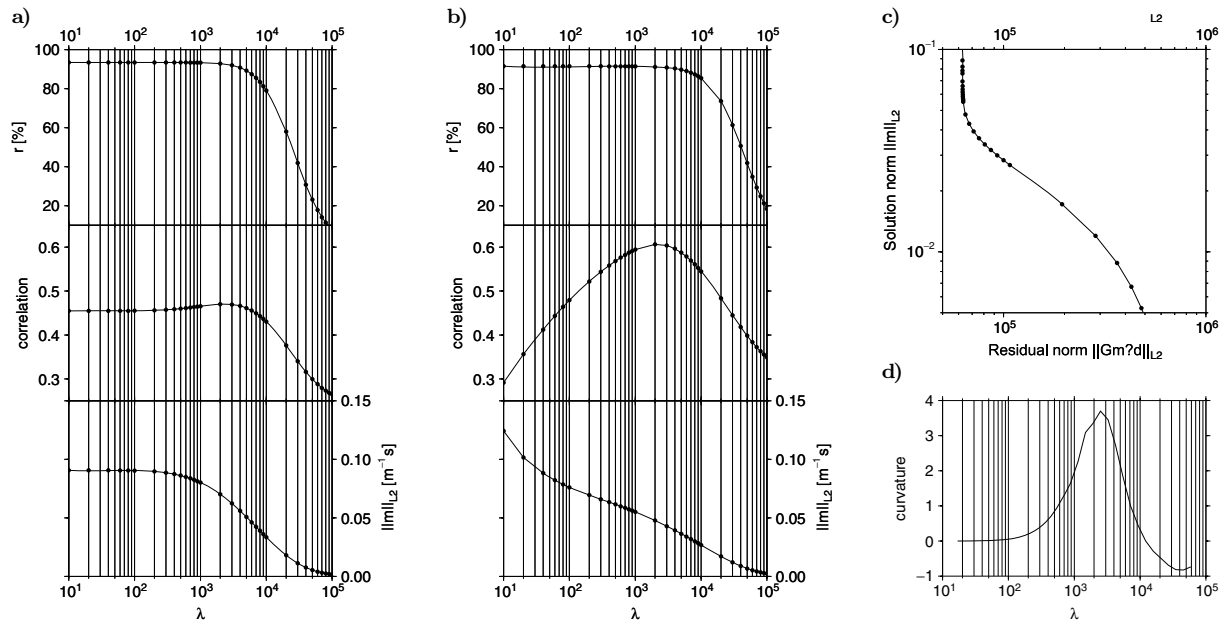


Figure 11. Results for Rayleigh number $Ra = 10^6$, plate (a) irregular parametrization and plate (b) regular parametrization—percentage fit r (upper panel), correlation ρ (middle panel) and L_2 norm of the model vector $\mathbf{m}$ (lower panel) as a function of a damping coefficient λ ; (c) L-curve in log–log scale (Hansen 1992) and (d) curvature as a function of λ in log-scale for the regular parametrization.

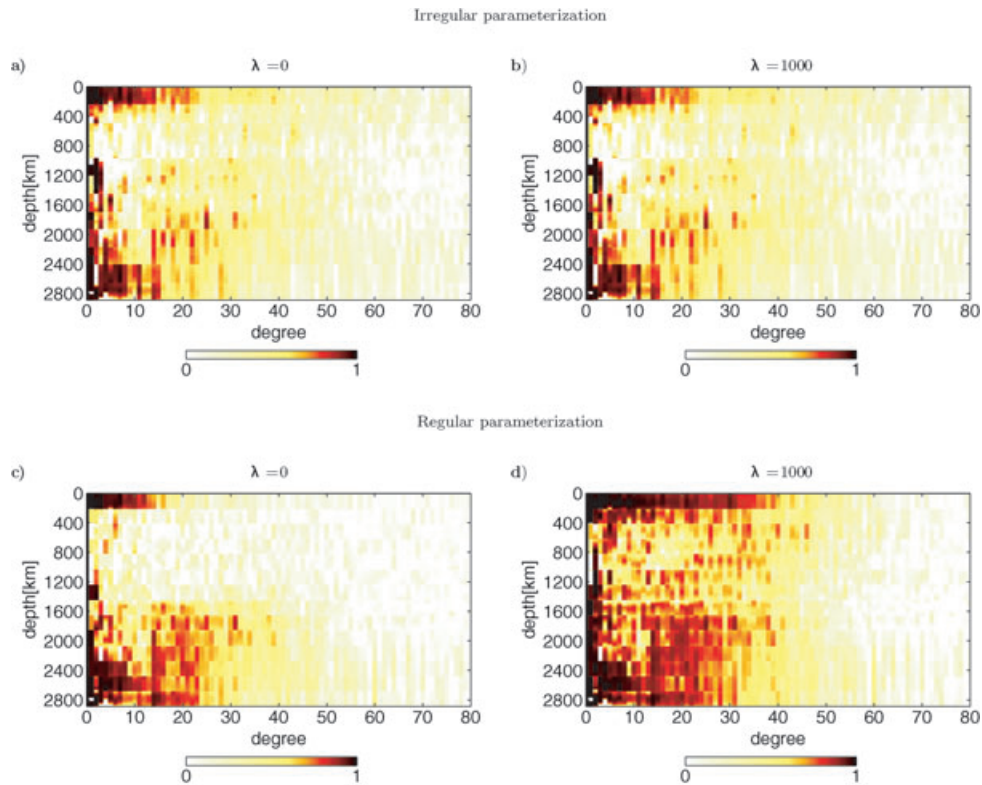


Figure 12. Correlation between the input and output models as a function of depth and spherical harmonic degree. Results for the Rayleigh number $Ra = 10^6$. Panel (a) irregular parametrization and damping $\lambda = 0$, panel (b) irregular parametrization and damping $\lambda = 1000$, panel (c) regular parametrization and damping $\lambda = 0$, panel (d) regular parametrization and damping $\lambda = 1000$.

Synthetic tests discussed in this paper were carried out for Rayleigh numbers in the range 3×10^5 – 10^6 . Although the Rayleigh number estimates based on uppermost mantle values of viscosity, thermal expansivity and thermal conductivity are higher than $Ra = 10^6$ (e.g. Turcotte & Schubert 2002) for whole mantle con-

vection and may reach even $Ra = 10^8$ (Yanagisawa & Yamagishi 2005), an effective Rayleigh number is much lower due to the material changes throughout the mantle. An increase of viscosity in the lower mantle can be more than one order of magnitude (e.g. Čadež & Fleitout 2003; Mitrović & Forte 2004), decrease of thermal

expansivity may reach one order of magnitude (Katsura *et al.* 2005) and thermal conductivity can be substantially increased by its component corresponding to the radiative transfer (e.g. Badro *et al.* 2004; Hofmeister 2005). Thus, the convection simulations with Rayleigh number ranging from 3×10^5 to 10^6 should provide a reasonable approximation of the mantle structure wavelengths except for the shallow upper mantle, where the plate-like behaviour is hard to simulate unless complex rheological description is used.

5 CONCLUSIONS

While the 2-D synthetic tomographic inversion (Běhounková *et al.* 2005) shows a rather poor resolution of narrow vertical structures at the depths below 1000 km and strong effect of explicit regularization, the resolution of the 3-D inversion (with comparable wavelengths of the input anomalies) in the lower mantle is much better. Also, the effect of explicit regularization is much weaker in 3-D than in 2-D. Increasing the Rayleigh number reduces the wavelength of the input anomalies. Therefore, the detection of these thinner anomalies (trying to explain the data with the same parametrization) becomes increasingly difficult, since the projection error could be rather high even in the well-covered regions.

We found substantial differences between the inversion output for regular and irregular parametrization due to the uneven distribution of rays. If the irregular parametrization is used, no explicit regularization is necessary unless the data error is included (Montelli *et al.* 2004). The inversion is able to resolve relatively fine structures in well-covered areas—on the other hand, in the poorly covered regions the large projection error causes extremely bad resolution. Huge parametrization cells negatively influence the overall resolution and power spectra of the output model. Therefore, on a global-scale inversion with regular parametrization is generally more successful in resolving the input seismic velocity structure, provided that a proper damping is chosen to suppress the oscillations caused by the ill-conditioned inverse problem.

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