

HEATING IN THE D''-LAYER AND THE STYLE OF MANTLE CONVECTION

Dedicated to the Memory of K. Pěš

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Summary: We have investigated the influence of heating in the D''-layer on the convection dynamics for the Rayleigh number $Ra = 10^6$. Strong heating, which may represent a local small-scale heat transfer in the D''-layer, results in an increase of lateral heterogeneities near the upper and lower boundaries, the blob-like structure of upwellings and the stabilization of the convection pattern. The influence of the electromagnetic heating was found to be too weak to produce any substantial effect which is in contrast with the idea of Braginskii and Meitlis [1].

1. INTRODUCTION

The influence of internal heating due to decay of U, Th and K has been taken into account in several simulations of mantle convection as a constant heat source distributed throughout the whole mantle (e.g. [2-7]). These simulations have shown that homogeneously distributed internal heat sources increase convection instabilities. However, a heterogeneous distribution of heat producing elements can lead to a decrease of the amplitude of convection [8] and/or to a stabilization of large plumes via interactions with smaller plumes [9].

Another heating mechanism was considered by Braginskii and Meitlis [1]. They dealt with the electromagnetic (elmg) heating which could be substantial in the D''-layer because of the high electrical conductivity and a strong elmg field present in the layer. They hypothesized that elmg heating controls the creation of plumes, as it increases the instability of the boundary layer.

Christensen [10] demonstrated the possibility of the presence of small-scale convection at the base of the mantle. In the first approximation, the influence of small-scale convection on mantle processes may be viewed as another local heat source.

The aim of this study is to consider local heating in a simple numerical simulation of mantle convection. We have concentrated on the influence of heating in the D''-layer on the global style of convection.

2. MODEL DESCRIPTION

We have used a 2-D rectangular box with an aspect ratio of 4 to simulate mantle convection. The momentum and energy equations for an incompressible, Boussinesq fluid with an infinite Prandtl number limit and a constant viscosity are

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$$\nabla \cdot \mathbf{v} = 0 \quad , \quad (1)$$

$$\nabla^2 \mathbf{v} + Ra T \mathbf{e}_z - \nabla p = 0 \quad , \quad (2)$$

$$\frac{\partial T}{\partial t} = \nabla^2 T - \mathbf{v} \cdot \nabla T + R \quad , \quad (3)$$

where $\mathbf{v}$ and T are, respectively, dimensionless velocity and temperature, $Ra = \alpha g d^3 \Delta T / \kappa \nu$ is the Rayleigh number for purely base heated convection (α is the thermal expansivity, g is the gravity acceleration, d is the layer thickness, ΔT represents the temperature difference between the bottom and the top, κ is the thermal diffusivity, and ν denotes the kinematic viscosity), $R = d^2 Q / k \Delta T$ is the dimensionless internal heating (Q represents the volume heat sources and k is the thermal conductivity), p is pressure, t is dimensionless time, and $\mathbf{e}_z$ is the unit vector pointing upward (i.e. against gravity acceleration).

Eqs. (1)–(3) are solved by imposing the following boundary conditions: all boundaries are impermeable and shear stress-free. The temperature is fixed at the top ($T=0$) and at the bottom ($T=1$) and the reflecting conditions are applied to the side walls. For spatial variables, we have used Fourier spectral decomposition to solve system (1)–(2) analytically, and to perform the differential operators in (3) in the spectral domain; the time-stepping has been carried out by a second-order explicit Runge-Kutta scheme in the spatial domain.

3. ELMG HEATING IN THE D'' -LAYER

The presence of high electric conductivity in the D'' -layer leads to additional internal heating at the base of the mantle [1]. Elmg (Joule) heating in a material with electrical conductivity σ is expressed as $J = \sigma E^2$, where E is the intensity of the electrical field. The dependence of σ on temperature can generally be expressed as $\sigma = \sigma_0 \exp(-\beta/T)$ where σ_0 and β are constants characterizing the material [11] and T is the absolute temperature. In the simplest model, where intensity E is kept constant, the dimensionless elmg heating can be expressed in the form:

$$R = R_0 \exp(-\beta/(T + T_0)) \quad , \quad (4)$$

thus representing the additional nonlinearity in system (1)–(3) (T_0 stands for the dimensionless upper surface temperature).

The magnitude of the heating can be deduced from the possible decay of the energy density of the elmg field in the core. The strong model of the geomagnetic field in the core with density of 10 Jm^{-3} [12] and the time decay of the elmg field during reversals of $O(10^5 \text{ yr})$ [13] yield the mean value of $O(10^{-11} \text{ Wm}^{-3})$ for local heating which corresponds to R of $O(10^{-2})$. Since the typical radioactive heating for chondrites

expressed in the dimensionless form is $O(10^1)$ [4], we conclude that elmg heating produces only *weak heating*.

4. RESULTS

All results presented here were computed in a box with the aspect ratio of 4 for $Ra = 10^6$, which is the lower limit for the lower-mantle convection [14]. The vigor of convection is high enough for $Ra = 10^6$ to study the effect of heating on the stability of convection. The convection for aspect ratio of 4 is still weakly controlled by the boundary conditions at the side walls of the box, but a large aspect ratio would be computationally too expensive.

We started with a purely basally heated system. Fig. 1 consists of typical frames of the temperature field. Both boundary layers are unstable which results in a more or less chaotic creation of new plumes and cold downwellings. Moreover, a strong background mean flow, due to which the plumes and downwellings drift away, can appear as visible in the right part of the box. This leads to plume-plume and downwelling-downwelling interactions. These features were studied in detail by Hansen et al. [15].

We then tested the influence of *weak* temperature-dependent heating in the D'' -layer (represented by the bottom part of the box occupying 1/16 of the area of the whole box) with maxima of R of $O(10^0)$ at the bottom of the box to be sure that the real value of the elmg heating lies below the tested case. We started our simulations from the same state as in the case with $R = 0$ and compared the differences between the two cases. During several overturns no substantial effect due to internal heating was found. This suggests that the style of mantle convection can hardly be affected by the heat production of the elmg field in the lower mantle.

Fig. 2 displays the temperature field with *strong* uniform heating $R = 100$ in the D'' -layer. The strong heating can serve as a representation of local heat transfer in the D'' -layer - e.g. small-scale convection [10], radiative and excitation heat transfer [16,17]. The lower boundary layer becomes more overheated, and detached blobs are generated by the boundary layer instabilities. The upward motion of these blobs is fast, and they finally disappear after interaction with the cold upper boundary layer. "Diapiric plumes" can also be generated but the flow of material inside them is extremely fast as one can see in the plume forming at the left side of the box. Since the vigor of convection is enhanced by the overheated material, the downward flow is also faster, thus enabling cold material to penetrate into the lower boundary layer. The location of plumes and downwellings is stable. The described scenario results in an increase of lateral heterogeneities at both the upper and the lower boundaries. This is consistent with the recent results of lower mantle seismic tomography [18–20] revealing the largest seismic velocity anomalies near the core mantle boundary whereas the middle of the mantle exhibits lower amplitudes. Another mechanism was considered by Tackley et al. [21] who showed numerically that the anomalies observed can be caused by the time-dependent nature of convection with an endothermic phase transition.

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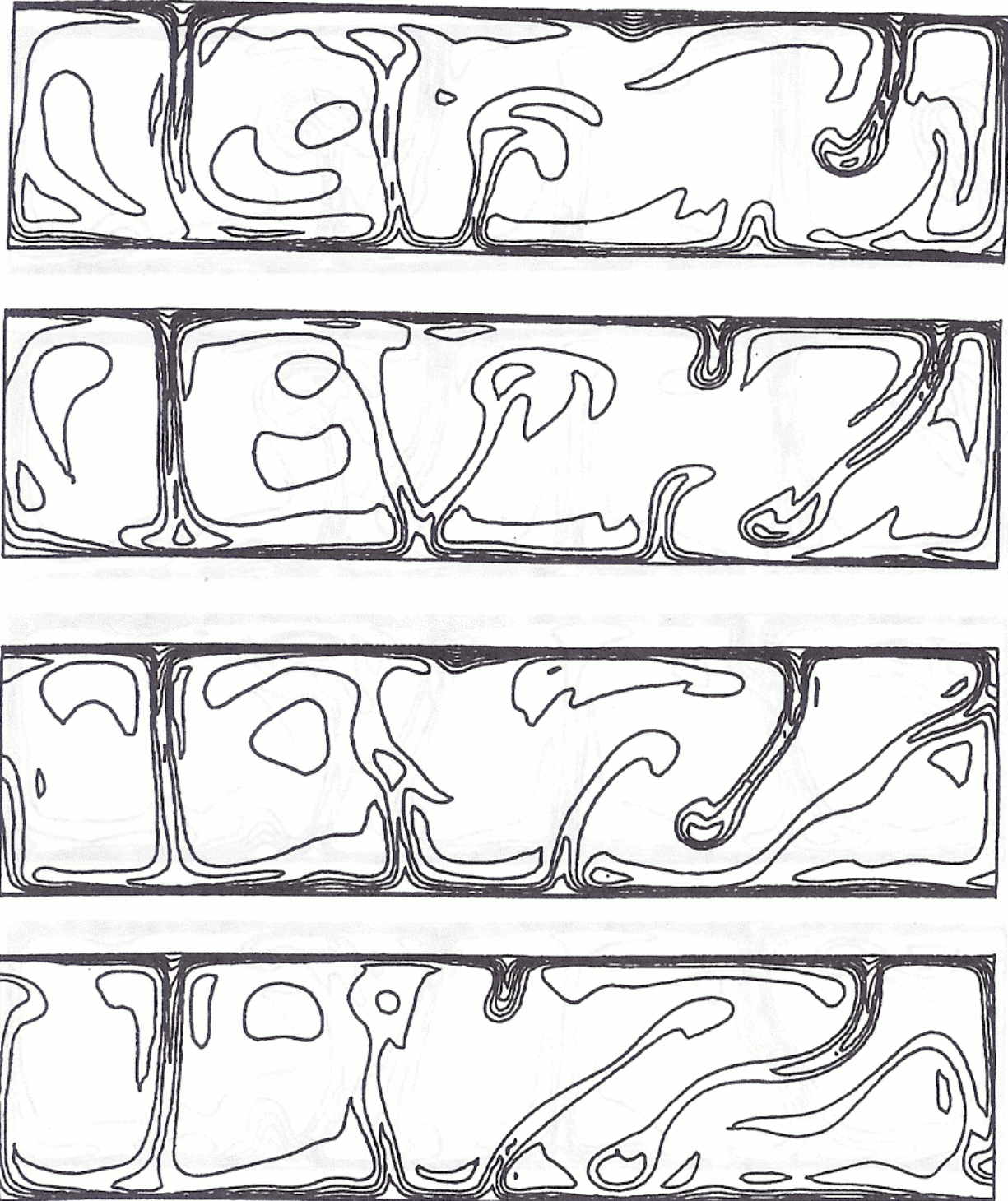


Fig. 1. Typical temperature fields for $Ra = 10^6$ and $R = 0$. $T = 0$ at the top and $T = 1$ at the bottom. Contour interval is 0.1. The time step between the snapshots is approximately 1/6 of the overturn.

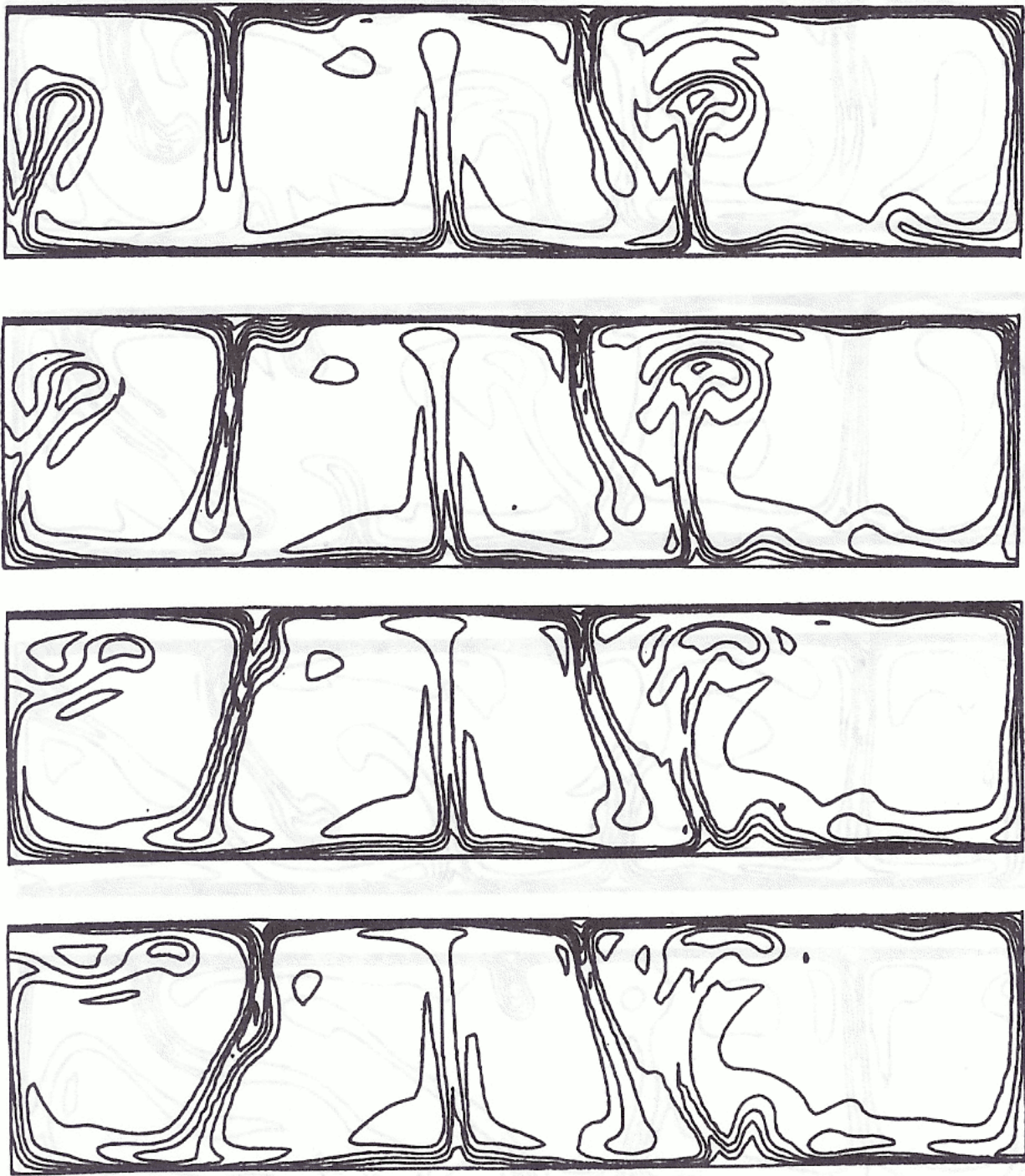


Fig. 2. Temperature fields stabilized by the internal heating $R = 100$ in the D'' -layer which occupies $1/16$ of the bottom part of the box. The Rayleigh number Ra , the boundary conditions and the contour interval are the same as in Fig. 1 but the time step between the snapshots is $1/12$ of the overturn.

5. CONCLUSIONS

We have studied numerically the effect of internal heating in the D'' -layer on mantle circulation for $Ra = 10^6$, which is the lower limit for mantle material properties [14]. The hypothesis of Braginskii and Meitlis [1] has not been confirmed since the internal heating in the D'' -layer is too weak to influence global circulation. Strong heating gives rise to larger lateral heterogeneities near both the upper and lower boundaries, which is consistent with seismic tomography results, and stabilizes the convection pattern. The symmetry between cold and hot regions is broken by overheating the lower boundary layer. The increase of the lower boundary instability gives rise to the creation of isolated hot detached blobs whereas cold blobs can be created only after downwelling-downwelling interactions. This kind of heating could represent an approximation of the internal heat transfer in the D'' -layer but this hypothesis is to be subject of further studies.

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References

- [1] S. I. Braginskii, V. P. Meitlis: O peregreivnoi neustoičivosti v niznei mantii u granici s jadrom. *Fizika Zemli*, 8 (1987), 31–35 (in Russian).
- [2] J. R. Baumgardner: Application of supercomputers to 3-D mantle convection. In: S.K. Runcorn (Ed.), *The Physics of the Planets*, John Wiley & Sons, 1988, pp. 199–231.
- [3] J. R. Baumgardner: Three-dimensional treatment of convective flow in the Earth's mantle. *J. Stat. Phys.*, 39 (1985), 501–511.
- [4] A. M. Leitch, D. A. Yuen: Internal heating and thermal constraints on the mantle. *Geophys. Res. Lett.*, 16 (1989), 1407–1410.
- [5] A. M. Leitch, D. A. Yuen, G. F. Sewell: Mantle convection with internal heating and pressure-dependent thermal expansivity. *Earth Planet. Sci. Lett.*, 102 (1991), 213–232.

- [6] D. A. Yuen, A. M. Leitch, U. Hansen: Dynamical influences of pressure-dependent thermal expansivity on mantle convection. In: R. Sabadini, K. Lambeck and E. Boschi (Eds.), *Glacial isostasy, sea-level and mantle rheology*, Kluwer, Dordrecht 1991, pp. 663–702.
- [7] A. V. Malevsky, D. A. Yuen, L. M. Weyer: Viscosity and thermal fields associated with strongly chaotic non-Newtonian thermal convection. *Geophys. Res. Lett.*, **19** (1992), 127–130.
- [8] S. Honda: A preliminary analysis of convection in a mantle with a heterogeneous distribution of heat-producing elements. *Phys. Earth Planet. Inter.*, **34** (1984), 68–76.
- [9] S. Honda, D. A. Yuen: Mantle convection with moving heat-source anomalies: geophysical and geophysical implications. *Earth Planet. Sci. Lett.*, **96** (1990), 349–366.
- [10] U. R. Christensen: Mantle rheology, constitution, and convection. In: W.R. Peltier (Ed.), *Mantle Convection: Plate tectonics and global dynamics*, Gordon and Breach, New York 1989, pp. 595–655.
- [11] W. D. Parkinson: *Introduction to geomagnetism*. Scottish Academic Press, 1983.
- [12] R. T. Merrill, M. W. McElhinny: *The Earth's Magnetic Field*. Academic Press, London 1983.
- [13] J. A. Jacobs (Ed.): *Geomagnetism*, Vol. 2, Academic Press, London 1987.
- [14] G. T. Jarvis, W. R. Peltier: Convection models and geophysical observations. In: W.R. Peltier (Ed.), *Mantle Convection: Plate tectonics and global dynamics*, Gordon and Breach, New York 1989, pp. 479–594.
- [15] U. Hansen, D. A. Yuen, S. E. Kroening: Transition to hard turbulence in thermal convection at infinite Prandtl number. *Phys. Fluids A*, **2** (1990), 2157–2163.
- [16] E. A. Lubimova: Theory of thermal state of the Earth's mantle. In: T. F. Gaskell (Ed.), *The Earth's mantle*, Academic Press, London and New York 1967, pp. 231–323.
- [17] J.-P. Poirier: *Introduction to the physics of the Earth's interior*. Cambridge Univ. Press, New York 1991.
- [18] I. Inoue, Y. Fukao, K. Tanabe, Y. Ogata: Whole mantle *P*-wave travel time tomography, *Phys. Earth Planet. Inter.*, **59** (1990), 294–328.
- [19] W. J. Su, A. M. Dziewonski: Predominance of long-wavelength heterogeneity in the mantle. *Nature*, **352** (1991), 121–126.
- [20] R. L. Woodward, A. M. Forte, W. J. Su, A. M. Dziewonski: Constraints on the largescale structure of the Earth's mantle. In: E. Takeuchi et al. (Eds.), *Evolution of the Earth and the Planets*, AGU, 1993, pp. 89–109.
- [21] P. J. Tackley, D. J. Stevenson, G. A. Glatzmeier, G. Schubert: Effects of an endothermic phase transition at 670 km depth on spherical mantle convection. *Nature*, **361** (1993), 699–704.