

JOINT PROBABILISTIC MULTI-ZONAL TRANS-DIMENSIONAL INVERSION ON PROPERTIES OF NEAR-SURFACE LAYERS FROM DISPERSION AND ELLIPTICITY CURVES

Miroslav Hallo¹, Walter Imperatori², Francesco Panzera³ and Donat Fäh⁴

¹ Researcher, Swiss Seismological Service, ETH Zurich, Zürich, Switzerland (miroslav.hallo@sed.ethz.ch)

² Researcher, Swiss Seismological Service, ETH Zurich, Zürich, Switzerland (walter.imperatori@sed.ethz.ch)

³ Researcher, Swiss Seismological Service, ETH Zurich, Zürich, Switzerland (francesco.panzera@sed.ethz.ch)

⁴ Head of Section, Swiss Seismological Service, ETH Zurich, Zürich, Switzerland (donat.fah@sed.ethz.ch)

ABSTRACT

This research focuses on the validation and application of a novel Bayesian approach to infer near-surface layered velocity models from dispersion and ellipticity curves. The inversion method relies on a varying number of layers, and it is formulated in the trans-dimensional Bayesian framework. The number of layers is treated as the model complexity governed by the law of parsimony. The model space is explored by a Markov chain Monte Carlo algorithm producing an ensemble of models drawn from the posterior probability of model parameters. The inversion method introduces also a multi-zonal formulation of the prior, allowing to include additional information to the inversion (e.g., from well logs). In this contribution, we focus on a validation of the inversion method using a synthetic test with a velocity gradient. We apply the approach to a Swiss site and show a comparison of the predicted theoretical and empirical amplification functions.

Keywords: inversion, Bayesian, dispersion curves, velocity profile, site characterization

INTRODUCTION

The physical properties of near-surface soil and rock layers play a fundamental role in the seismic site effects analysis, being an essential element for the local seismic hazard assessment. These properties (S- and P-wave velocities) can be inferred from observed surface-wave dispersion and ellipticity curves by non-linear inversion techniques. Nevertheless, results exhibit significant inherent non-uniqueness as different velocity models may provide a similar fit to observed data. Standard optimization inversion techniques explore the model space in order to minimize the data misfit, resulting in a single representative velocity model. Other models providing a similar misfit level are usually discarded in these approaches. Moreover, the observed data are subject to significant uncertainties that introduce bias-variance trade-offs; hence, the data uncertainties should be considered in the inversion.

The dispersion curves of the fundamental and higher modes of Rayleigh and Love waves together with the Rayleigh wave ellipticity, are the input data of the inverse modeling. These data are usually retrieved from the single-station and seismic array measurements of ambient vibrations or active seismic surveys (e.g., Foti et al., 2018). Single-station methods aim to retrieve the horizontal-to-vertical spectral ratio (H/V; Nogoshi and Igarashi, 1971; Nakamura, 1989; Fäh et al., 2001). As the ambient vibrations are a composition of multiple wave types, H/V ratio is not equal to the Rayleigh wave ellipticity. Hence, techniques to estimate Rayleigh wave ellipticity from the full wavefield have been developed (e.g., Hobiger et al., 2009). The array methods are aimed to identify the dispersion curves of the fundamental and higher modes (e.g., Aki, 1957; Okada, 2003). For example, Poggi and Fäh (2010) proposed an array frequency-wavenumber (f-k) processing method for joint estimation of Rayleigh wave ellipticity together with dispersion curves. Marano et al. (2012) used wave-field decomposition with factor graphs for the estimation of the Rayleigh wave ellipticity angles, and Rayleigh and Love wave dispersion curves.

An example of an active seismic array method is the multichannel analysis of surface waves (Park et al., 1999).

The inversion of dispersion and ellipticity curves to retrieve a 1D near-surface structure is a non-linear inverse problem. In the field of the near-source 1D imaging, Scherbaum et al. (2003) developed a joint inversion of dispersion and H/V curves constrained by a simple model, Fäh et al. (2003) proposed a non-linear inversion of H/V curves based on a genetic algorithm technique, the approach by Arai and Tokimatsu (2004) relies on the iterative singular value decomposition, and Wathelet et al. (2004) proposed an optimization inversion of dispersion curves based on the neighbourhood algorithm. Finally, Hallo et al. (2021) developed a joint inversion of multimodal Rayleigh and Love wave dispersion and Rayleigh wave ellipticity curves in the multi-zonal trans-dimensional Bayesian framework. In this contribution, we extend the validation of this inversion method using a detailed synthetic test with a velocity gradient, and we apply it to the SEPFL strong-motion station site (southwest Switzerland).

INVERSION METHOD

The Multi-zonal Trans-dimensional Inversion (MTI) technique has been introduced by Hallo et al. (2021). This method is a joint Bayesian inversion of Rayleigh and Love wave dispersion and ellipticity curves to retrieve near-surface 1D velocity models including their uncertainties. The key features of this approach are:

- Inference of both S- and P-wave velocity profiles (layered model of homogeneous layers)
- The number of layers is self-adapting and driven by the law of parsimony
- Joint inversion of multimodal surface wave dispersion and ellipticity curves
- Data uncertainties used as an inverse weight of the observed data
- Homogeneous prior on model parameters
- Multi-zonal formulation of the prior to include additional depth-specific information
- Exploration of the model space by a Markov chain Monte Carlo algorithm (MCMC)
- Parallel Tempering technique for effective exploration of model space
- Multiple parallel Markov chains with random and independent initial models
- It is suitable for problems with a non-unique solution
- It provides multiple indicators of the resultant velocity profile quality and reliability
- It provides reliable results for sites with low-velocity zones and velocity gradients
- It may be modulated for different type of waves or geophysical problems

The MTI method relies on a trans-dimensional Bayesian formulation of the model space (Green, 1995; Bodin et al., 2012). In such a framework, the posterior probability density function (posterior PDF) on the model space is proportional to the prior PDF and a likelihood function. This can be expressed by

$$p(\mathbf{m}|\mathbf{d}) \propto p(\mathbf{m}) p(\mathbf{d}|\mathbf{m}) \quad (1)$$

where $p(\mathbf{m}|\mathbf{d})$ is the posterior conditional PDF given observed data vector $\mathbf{d}$, $p(\mathbf{m})$ is the prior PDF of the model vector $\mathbf{m}$, and $p(\mathbf{d}|\mathbf{m})$ is the likelihood function. The latter quantifies how well model $\mathbf{m}$ fits the observed data $\mathbf{d}$ based on synthetic data modeled by a forward problem solver. As the forward solver, we use the Geopsy software package (Wathelet et al., 2020) that relies on the Thomson-Haskell matrix method. The prior PDF is assumed to be homogeneous within a range of physically plausible values, and these threshold values can be prescribed as depth-dependent using the multi-zonal formulation by Hallo et al. (2021). Green (1995) proved validity of Eq. 1 for model vectors $\mathbf{m}$ of variable length (i.e., models with varying number of layers in this case). That allows us to formulate a mathematically rigorous formulation, where the number of layers is treated as the model complexity and governed by the law of parsimony (Hallo and Gallovič, 2020).

To the implementation, the model space is explored by the Metropolis-Hastings algorithm (Hastings, 1970) with implemented Parallel Tempering technique (Sambridge, 2014). This technique deploys multiple parallel Markov chains with random and independent initial models that reduce a possible bias

caused by a selection of the initial model. The result is an ensemble of solutions that represents a discrete approximation of the posterior PDF. Such an ensemble of velocity models can be processed to obtain marginal histograms of properties of interest (e.g., seismic velocities, interface depths, v_{s30} , SH-wave amplification, etc.). To complete, some representative velocity profiles may be selected (e.g., the maximum likelihood ML model providing the lowest data misfit or the maximum a posteriori MAP model estimate).

VALIDATION BY A SYNTHETIC TEST

The MTI technique has been validated by inverting synthetic data referred to a “target model” in Hallo et al. (2021). Here, we extend the validation on a complex target model characterized by a continuous positive velocity gradient in detail. The synthetic data (i.e., data for the inversion test) were computed by Geopsy software using the velocity model from Hallo et al. (2021, table 2 therein). The vector of observed data consists of fundamental and first higher modes of Rayleigh waves, fundamental mode of Love waves, and ellipticity of Rayleigh waves (see Fig. 1). The synthetic data are noise-free.

We infer the ensemble of solutions (using a single-zone homogeneous prior) and apply no constraints on the velocity gradient. Hence, low-velocity zones are allowed to occur freely everywhere along the profile. The exploration of the model space was performed by 460 Markov chains of the Parallel Tempering (437 exploration and 23 sampling chains), and the ensemble of solutions consists of ~575,000 random model samples drawn from the posterior PDF. Since the synthetic data are noise-free, we expect to find a model with very low data misfit (i.e., a high data variance reduction). Indeed, the data variance reductions for ML and MAP models are 99% and 96%, respectively, and modeled data are close to the observations (Fig. 1). The histogram of all layered models in the ensemble of solutions shows a maximum probability of having four-to-five layers (Fig. 1f), which means that at least four layers (three layers and half-space) are necessary to fit the data. Less than four layers does not fit data sufficiently, while more than five layers does not provide significantly better data fitting.

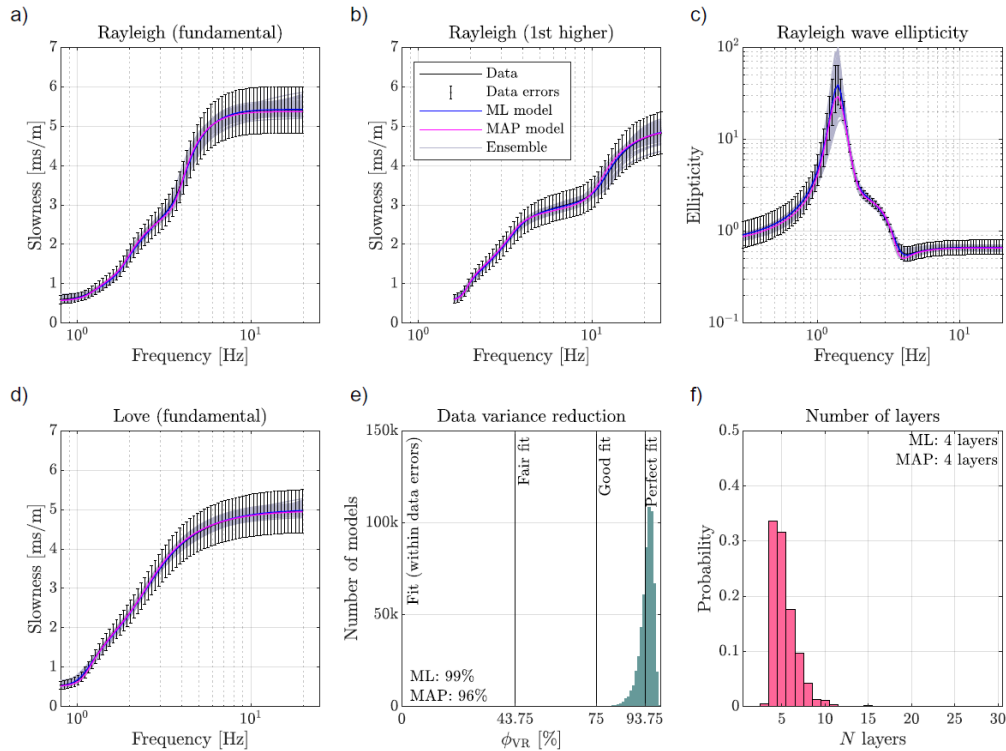


Figure 1. Fit of the observed (synthetic) and modeled data (ensemble of solutions). In panels a)-d) the observed data (black curves with errorbars) are overlaid by data modeled in velocity models from the ensemble of solutions (see legend) demonstrating almost perfect data fit. Panels e) and f) show histograms for data variance reduction and number of layers, respectively.

The posterior marginal PDFs of v_S and v_P (Fig. 2) are narrow in the uppermost (homogeneous) layers and rather broad at depth where the gradient is located. Note also an interface in ML and MAP models at ~ 80 m depth that does not correspond to the target model. This artificial interface compensates for the gradual velocity increase in the depth range 40–160 m, and it is characterized by a high uncertainty (Fig. 2c). This results from the self-adapting number of layers. The average profile (AM of PDF in Fig. 2f) provides a reliable estimate of the velocity gradient down to approximately 120 m depth, but it is rather smooth around the interface with significant velocity contrast at 160 m depth. Note that the inferred profiles do not possess artificial low-velocity zones, even though they were allowed freely in this test. The posterior marginal PDF of ρ (Fig. 2d) is completely flat, which means that this parameter cannot be resolved using the observed data. Inferred values of ρ for the ML and MAP models are therefore random and have no predictive value. Finally, the posterior marginal PDF of ν (Fig. 2e) shows a good prediction of the Poisson's ratio in the uppermost ~ 80 m of the profile, while it is uncertain below. It proves, that it is possible to infer P-wave velocity profiles from the standard site characterization measurements; however, uncertain profiles may exhibit trade-offs between S- and P-wave velocities.

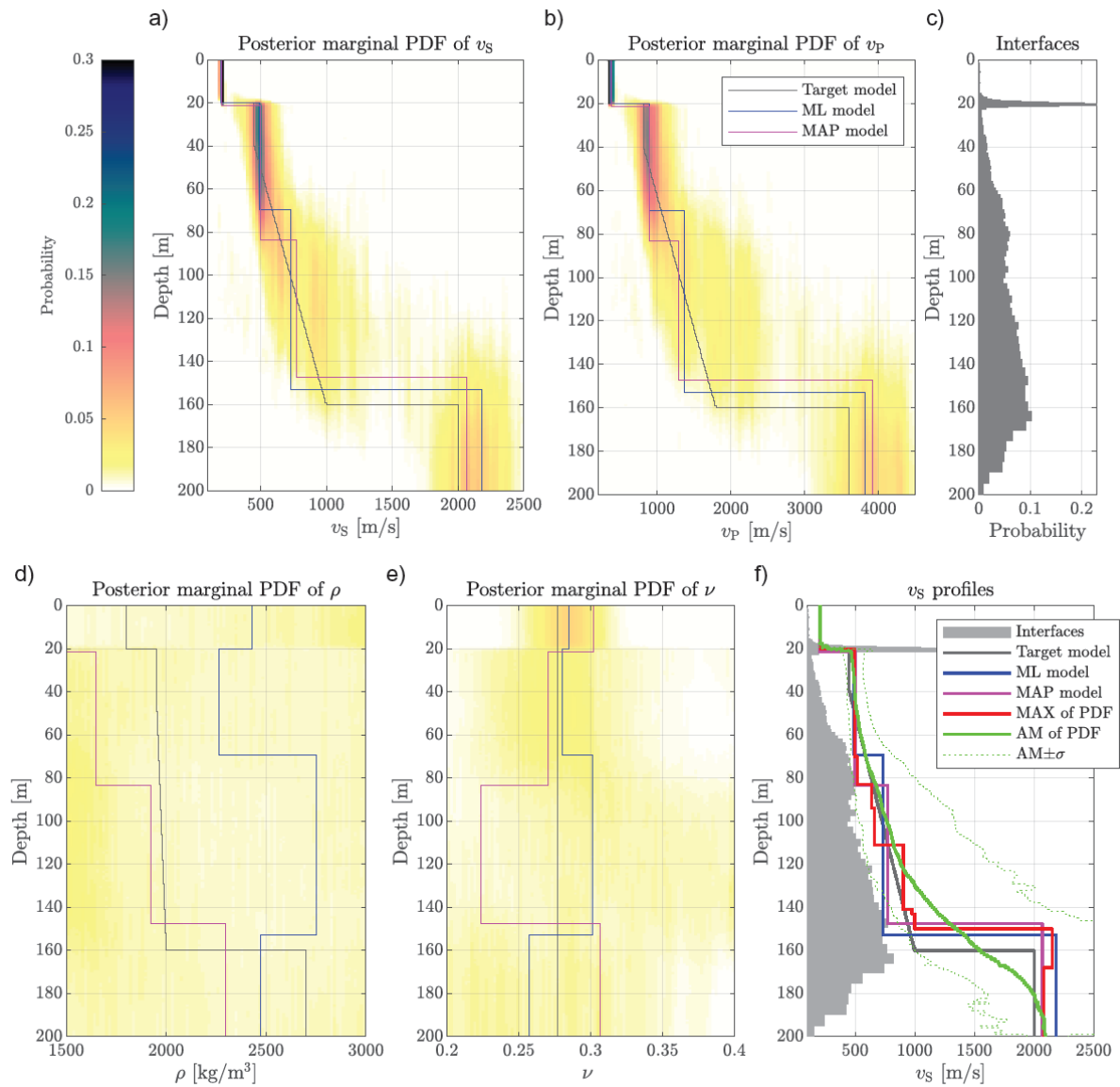


Figure 2. Inversion of the synthetic data. Panels a), b), d), and e) show posterior marginal PDFs overlaid by the ML, MAP, and target models (see legend). Panels c) and f) show the histogram for the interface depth and inferred S-wave velocity profiles.

APPLICATION TO A SWISS SITE

We applied the MTI method to real surface wave data measured in the vicinity of the SEPFL seismic station site. This seismic station, installed in 2011, is part of the Swiss Strong Motion Network (SSMNet, Hobiger et al., 2016) in the Canton Vaud. The site characterization measurements and analysis at this site are described in detail by Michel et al. (2013). The observed data consist of fundamental, first higher, and second higher modes of Rayleigh waves, and the ellipticity of Rayleigh waves retrieved by the f-k processing method (see Fig. 3d).

For the inversion of real data (Fig. 3), we use a simple prior (a single-zone homogeneous prior), and we apply constraints on the positive velocity gradient (low-velocity zones are prohibited). The exploration of the model space was performed by 620 Markov chains of the Parallel Tempering (589 exploration and 31 sampling chains), and the ensemble of solutions consists of $\sim 1,000,000$ random model samples drawn from the posterior PDF ($\sim 38,000$ steps of Markov chains). The data fit (Fig. 3d) is very good for a joint inversion of four different data types (data variance reduction up to 90%). Posterior marginal PDFs of v_S and v_P are sharp in the uppermost ~ 35 m of the profile, suggesting that S- and P-wave velocities are well resolved within this depth range. The bedrock can be estimated in the depth range of 33 to 40 m. Moreover, the posterior marginal PDFs of v_S and v_P are smoothed below the depth of ~ 35 m, meaning the seismic velocities of bedrock are subject to significant uncertainty.

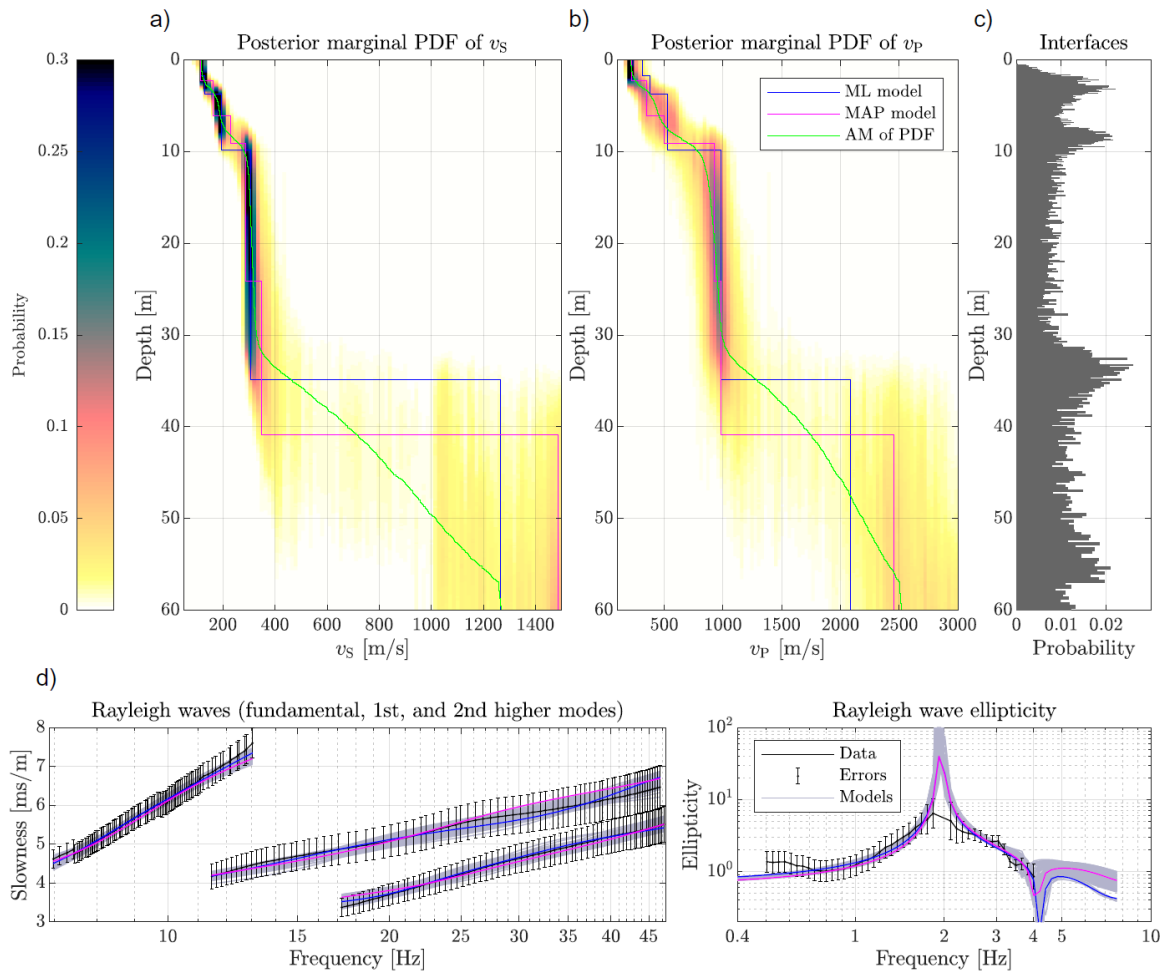


Figure 3. Inversion of real data of the SEPFL site. Panels a) and b) show posterior marginal PDFs overlayed by the ML and MAP models (see legend). Panel d) shows the fit of the real data (black curves with errorbars) and data modeled in velocity models from the ensemble of solutions.

The ensemble of solutions can be used for evaluating the uncertainty of any arbitrary property associated with velocity profiles. In Fig. 4, we show such evaluation in the quarter-wavelength (QWL) representation and theoretical (SH-wave) amplification referenced to the Swiss profile (Poggi et al., 2011). The amplification referenced to the Swiss profile predicts the site amplification that is required by an empirical spectral modeling (Edwards et al., 2013). It is of great importance especially for constructing seismic hazard maps. In Fig. 4d, the theoretical (our predicted) amplification at the SEPFL site is compared with the empirical amplification computed from earthquake recordings by the method of Edwards et al. (2013). The comparison shows a high similarity of the amplification values as well as frequencies of spectral peaks. It has to be noted, that the SEPFL is an example of a site with 1D resonance effects, while the theoretical amplification of sites with 2D/3D effects may be unpredictable (e.g., Chieppa et al., 2021). To complete, the statistic of inferred v_{s30} values at the SEPFL site shows a reliable interval of 232-251 m/s that corresponds to the soil class C in the EC8 classification (CEN 2004).

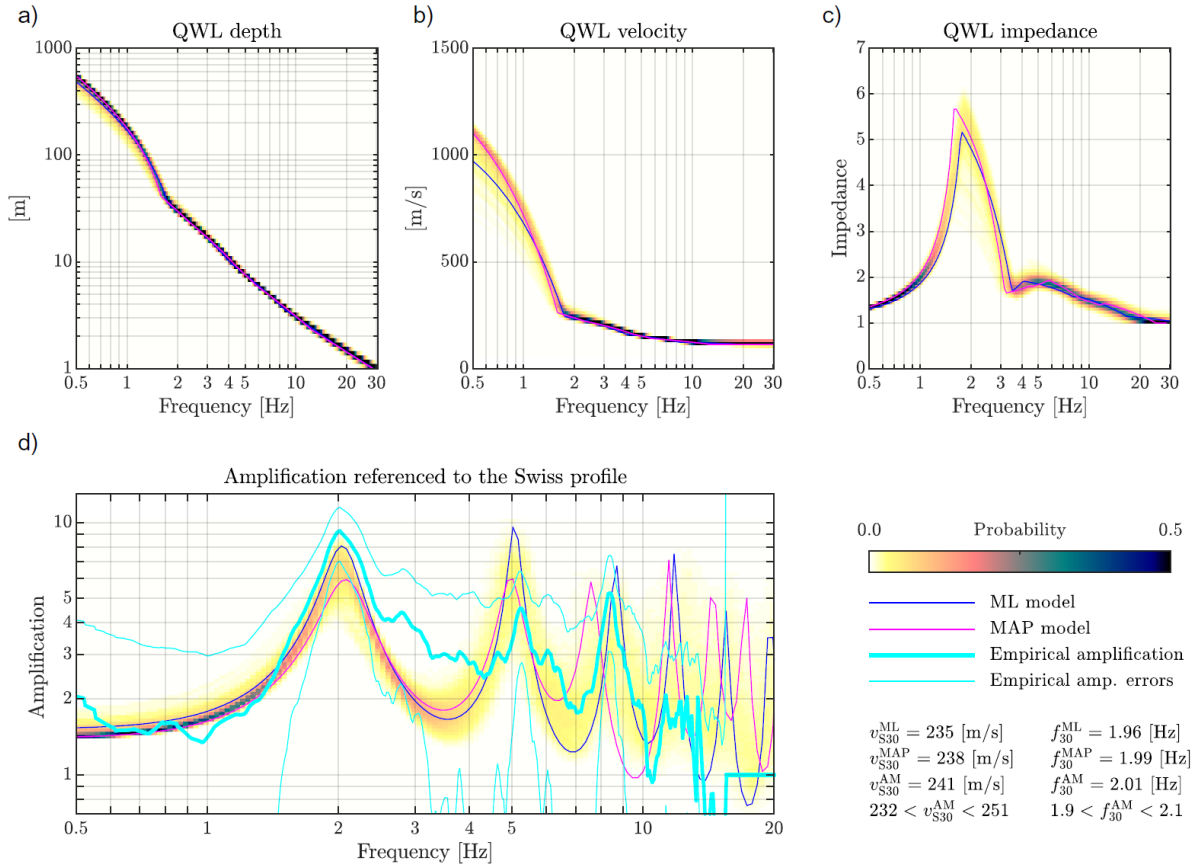


Figure 4. Quarter-wavelength representation and amplification at the SEPFL site (real data inversion). Panels a)-c) show the quarter-wavelength representation by means of posterior marginal PDFs overlaid by the ML and MAP models. Panel d) shows a comparison of the theoretical amplification referenced to the Swiss profile (Poggi et al., 2011) and the empirical amplification (Edwards et al., 2013) of the SEPFL station site.

CONCLUSIONS

In this contribution, we showed further validation and application of the joint multi-zonal trans-dimensional inversion technique introduced by Hallo et al. (2021). The synthetic test helped us to demonstrate the behavior of the algorithm with a subsurface having a velocity gradient. The inferred S- and P-wave velocity profiles of homogeneous layers satisfactorily matched the target model with a gradient. The solution uncertainty is pronounced where the gradient is located and it is compensated by an artificial interface. We also applied the approach to data of the SEPFL site, southwest Switzerland.

The inferred velocity profiles at this site are in agreement with previous site characterization studies (Michel et al., 2013). Finally, the local theoretical amplification referenced to the Swiss profile (predicted by our ensemble of models) shows a high similarity to the amplification computed from earthquake recordings by the method of Edwards et al. (2013).

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