

$$\text{Grad } \vec{V} = \mathbf{F}^T \cdot \text{grad } \vec{v}$$

Skalární pole $\Theta(X) = \theta(x)$

$$\text{Grad } \Theta \cdot \vec{s} = \vec{s} \cdot \text{Grad } \Theta = \vec{s} \cdot (\mathbf{F}^T \cdot \text{grad } \theta) = (\mathbf{F} \cdot \vec{s}) \cdot \text{grad } \theta = \text{grad } \theta \cdot (\mathbf{F} \cdot \vec{s})$$

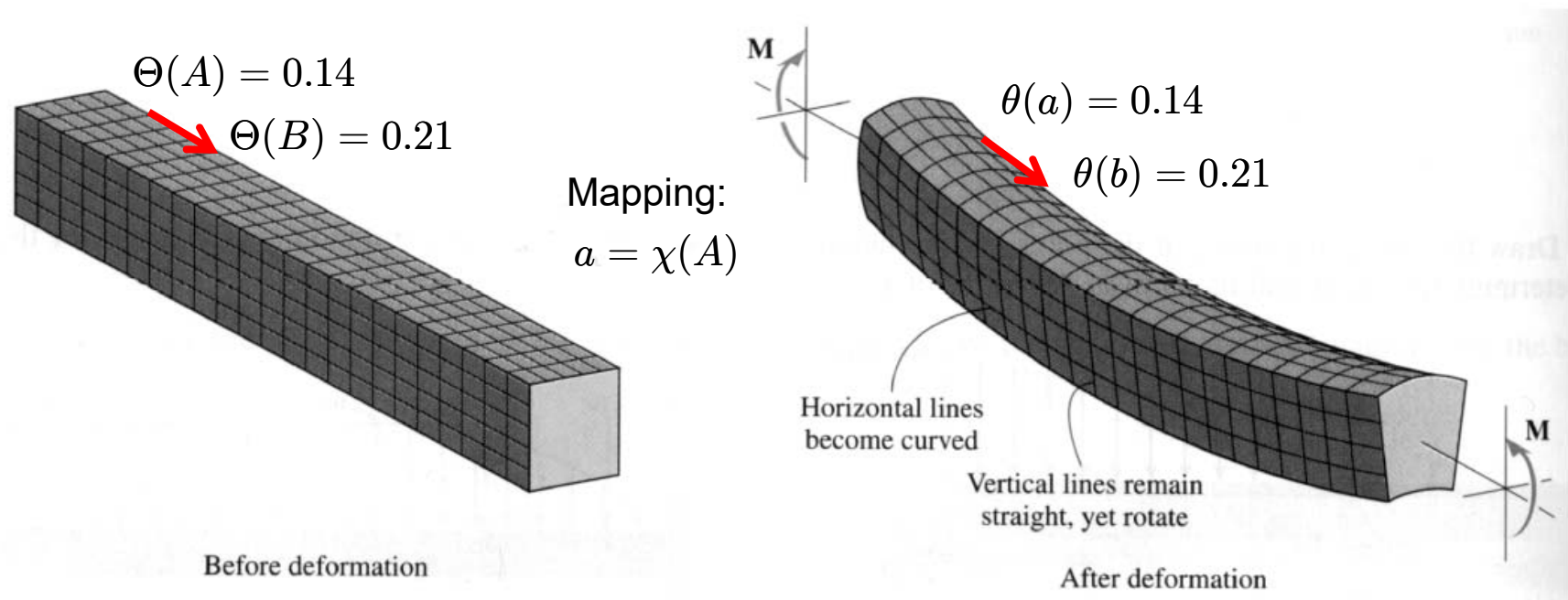
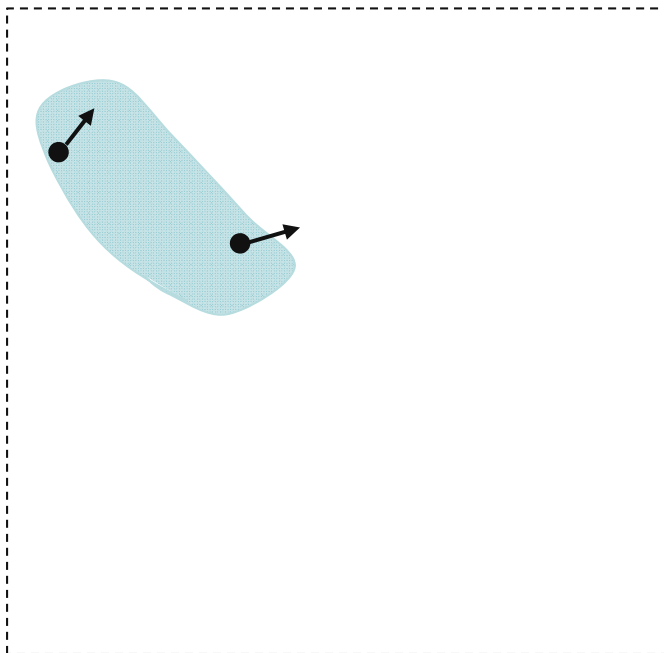


Figure 3.4 Initially straight beam and the deformed bent beam

Množiny stejných vektorů (soubor stejných hodnot), „žijících“ ale na jiné doméně
Prostorovou diferenciaci lagrangeovských a eulerovských polí je nutno rozlišovat

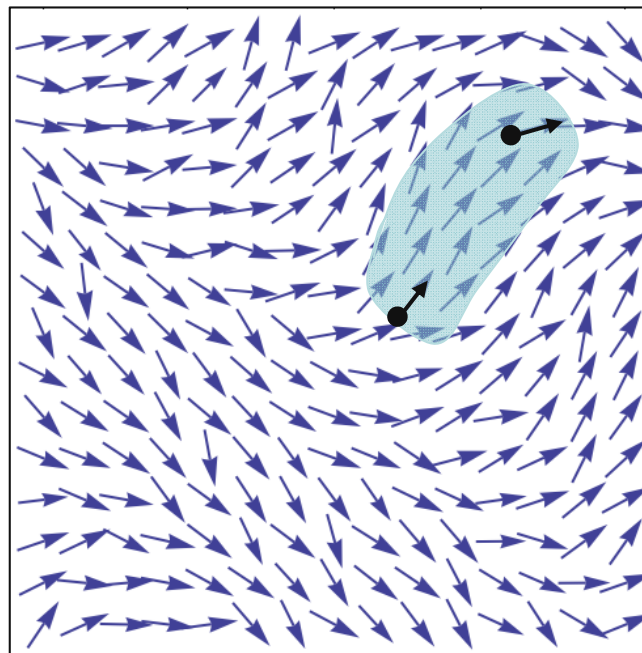
Diferenciální operátory v okamžité a referenční konfiguraci

$$t_0 = 0$$



???

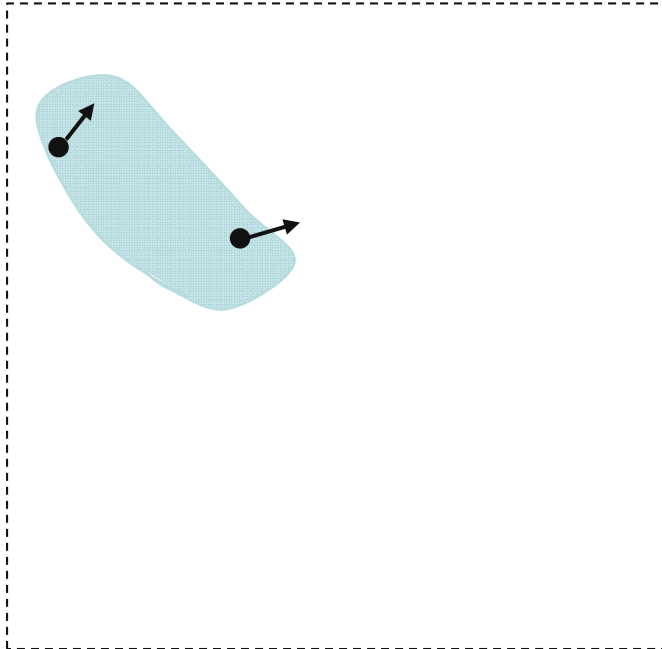
$$t > t_0$$



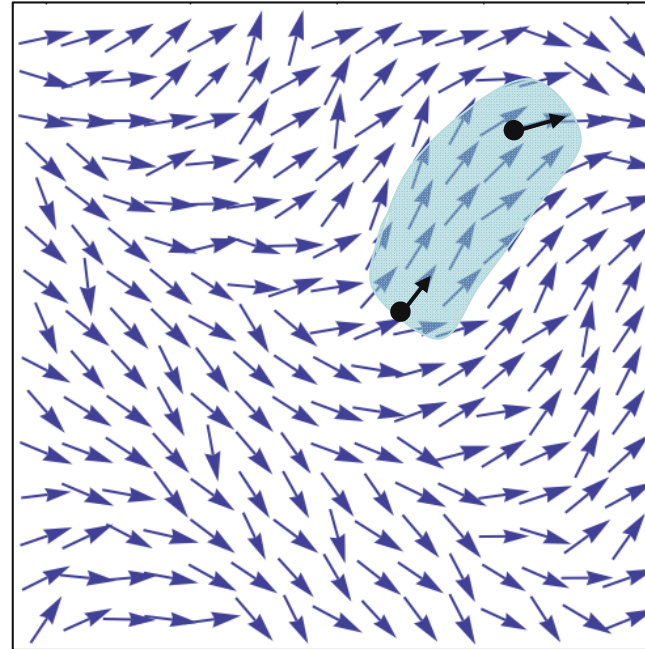
$$\operatorname{div} \vec{v} = 0$$

Diferenciální operátory v okamžité a referenční konfiguraci

$t_0 = 0$



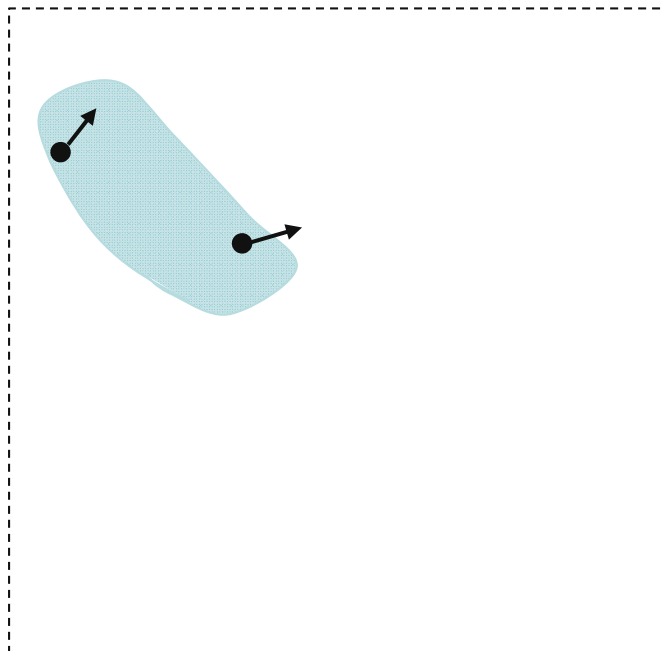
$t > t_0$



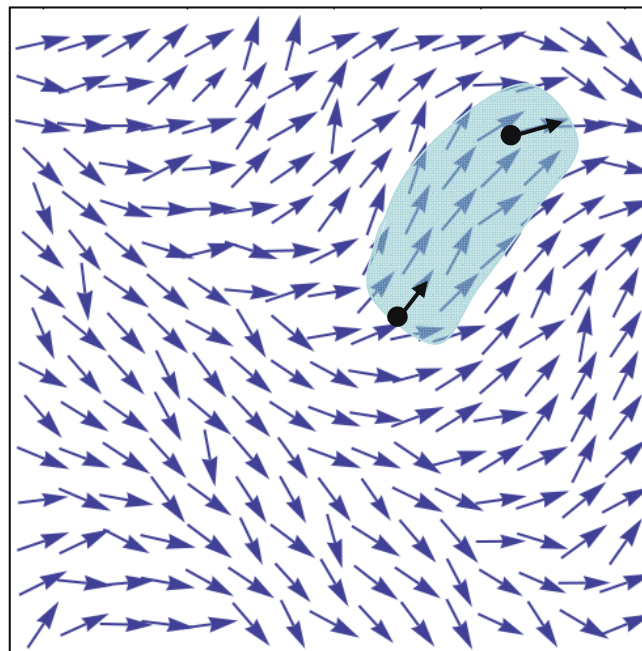
$$\operatorname{div} \vec{v} = \vec{e}_k \frac{\partial}{\partial x_k} \cdot v_l \vec{e}_l$$

Diferenciální operátory v okamžité a referenční konfiguraci

$t_0 = 0$



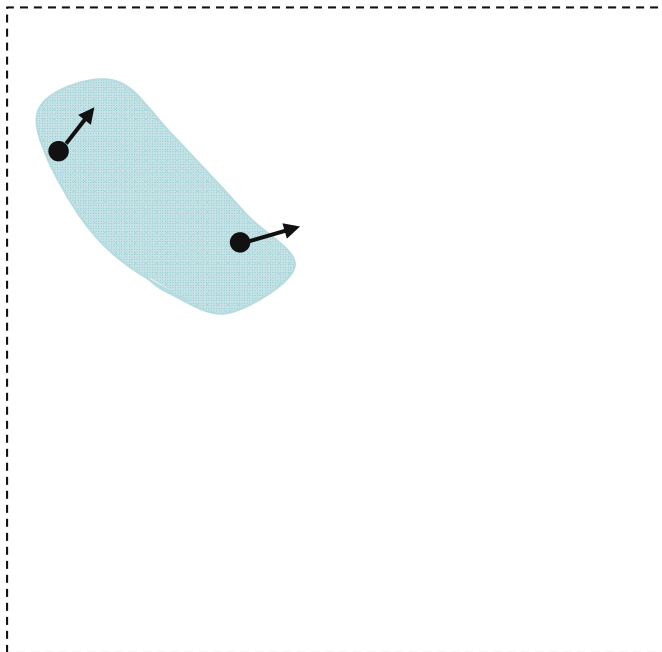
$t > t_0$



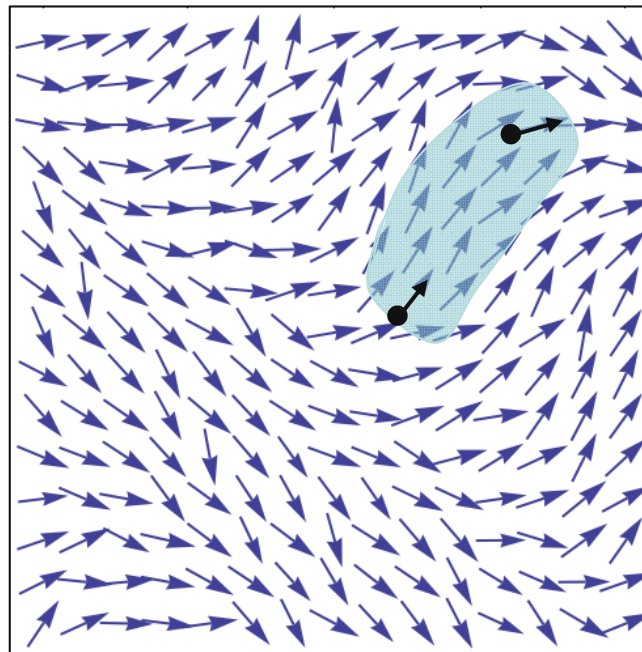
$$\operatorname{div} \vec{v} = \vec{e}_k \frac{\partial}{\partial x_k} \cdot v_l \vec{e}_l = \frac{\partial v_k}{\partial x_k}$$

Diferenciální operátory v okamžité a referenční konfiguraci

$t_0 = 0$



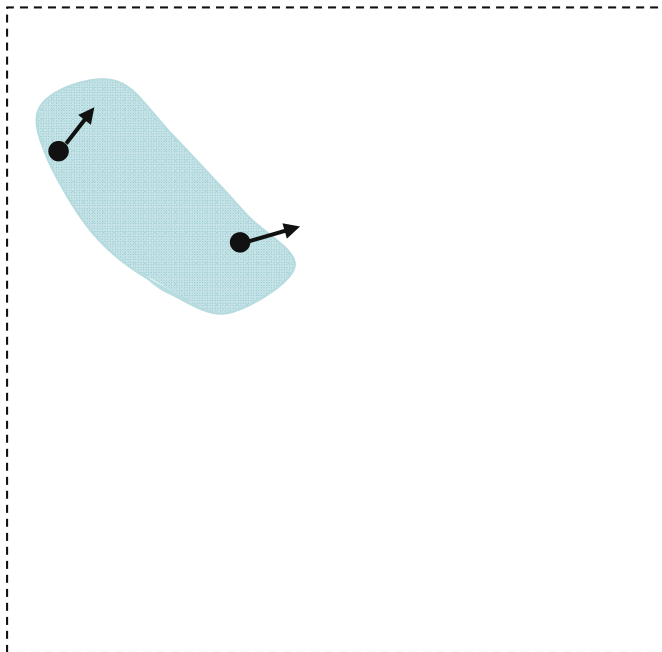
$t > t_0$



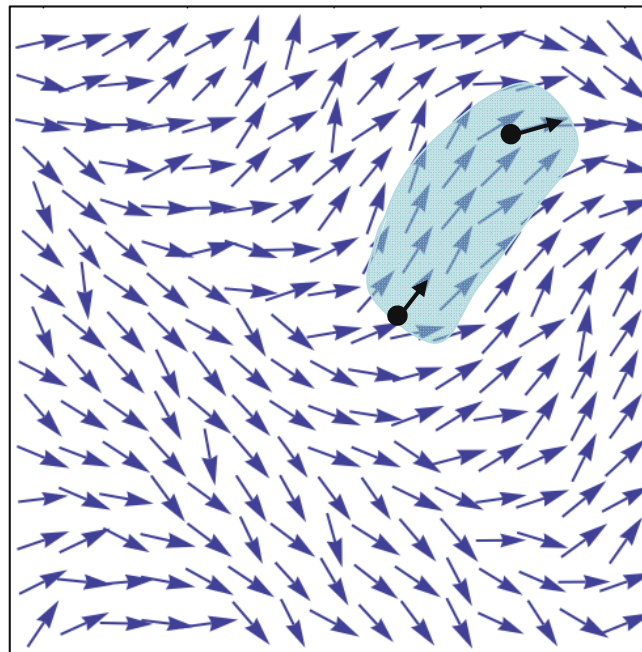
$$\operatorname{div} \vec{v} = \vec{e}_k \frac{\partial}{\partial x_k} \cdot v_l \vec{e}_l = \frac{\partial v_k}{\partial x_k} = \frac{\partial v_k}{\partial X_K} \frac{\partial X_K}{\partial x_k}$$

Diferenciální operátory v okamžité a referenční konfiguraci

$t_0 = 0$



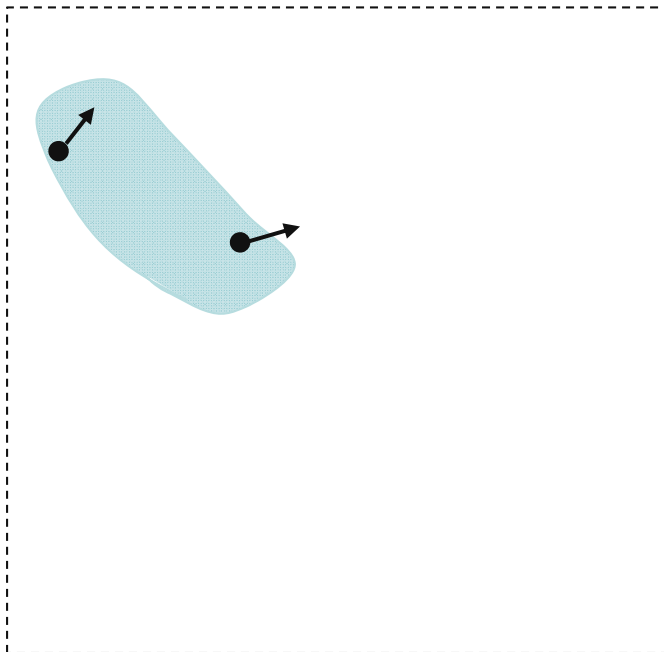
$t > t_0$



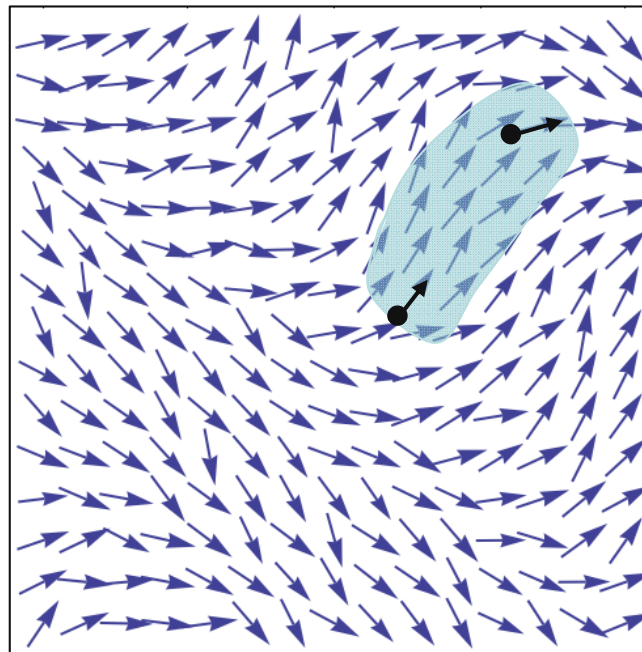
$$\operatorname{div} \vec{v} = \vec{e}_k \frac{\partial}{\partial x_k} \cdot v_l \vec{e}_l = \frac{\partial v_k}{\partial x_k} = \frac{\partial V_k}{\partial X_K} \frac{\partial X_K}{\partial x_k} = \frac{\partial V_k}{\partial X_K} F_{Kk}^{-1}$$

Diferenciální operátory v okamžité a referenční konfiguraci

$t_0 = 0$



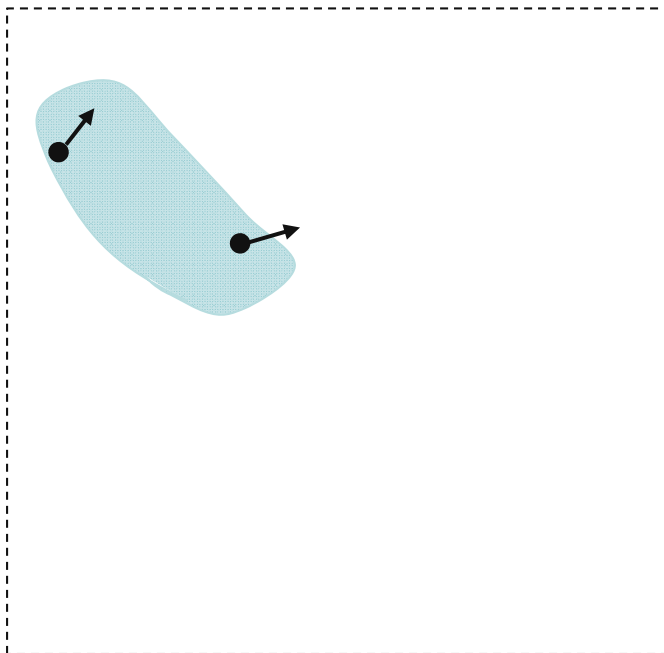
$t > t_0$



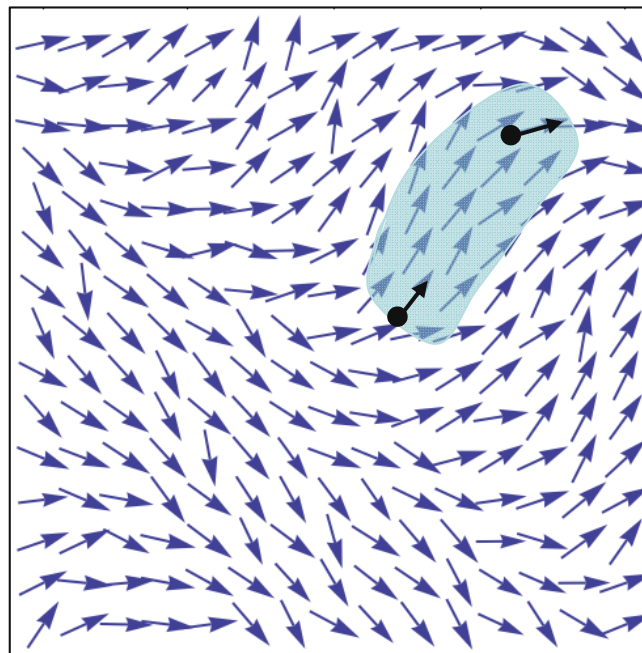
$$\operatorname{div} \vec{v} = \vec{e}_k \frac{\partial}{\partial x_k} \cdot v_l \vec{e}_l = \frac{\partial v_k}{\partial x_k} = \frac{\partial V_k}{\partial X_K} \frac{\partial X_K}{\partial x_k} = \frac{\partial V_k}{\partial X_K} F_{Kk}^{-1} = F_{Kk}^{-1} (\operatorname{Grad} \vec{V})_{Kk} =$$

Diferenciální operátory v okamžité a referenční konfiguraci

$t_0 = 0$



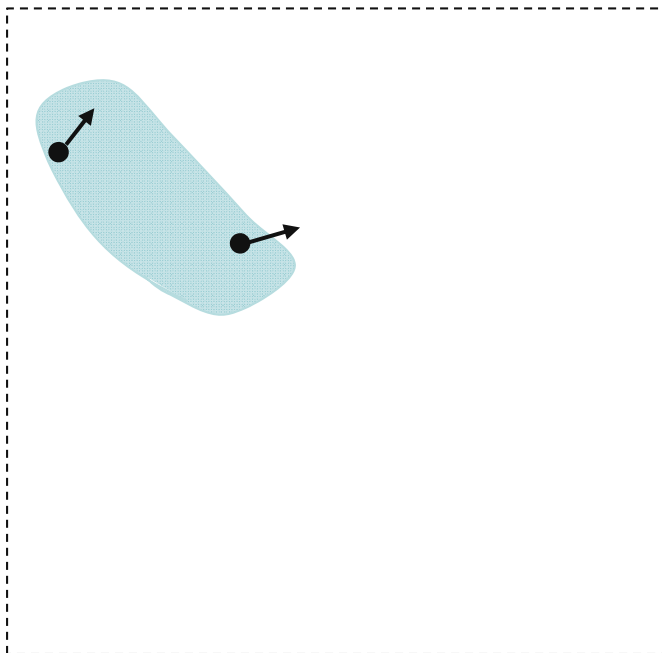
$t > t_0$



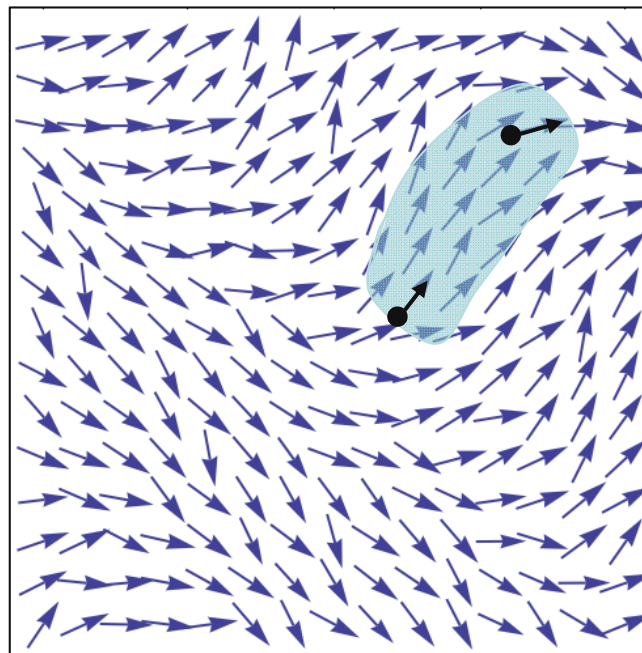
$$\begin{aligned} \operatorname{div} \vec{v} &= \vec{e}_k \frac{\partial}{\partial x_k} \cdot v_l \vec{e}_l = \frac{\partial v_k}{\partial x_k} = \frac{\partial V_k}{\partial X_K} \frac{\partial X_K}{\partial x_k} = \frac{\partial V_k}{\partial X_K} F_{Kk}^{-1} = F_{Kk}^{-1} (\operatorname{Grad} \vec{V})_{Kk} = \\ &= F_{kK}^{-T} (\operatorname{Grad} \vec{V})_{Kk} \end{aligned}$$

Diferenciální operátory v okamžité a referenční konfiguraci

$t_0 = 0$



$t > t_0$



$$\begin{aligned} \operatorname{div} \vec{v} &= \vec{e}_k \frac{\partial}{\partial x_k} \cdot v_l \vec{e}_l = \frac{\partial v_k}{\partial x_k} = \frac{\partial V_k}{\partial X_K} \frac{\partial X_K}{\partial x_k} = \frac{\partial V_k}{\partial X_K} F_{Kk}^{-1} = F_{Kk}^{-1} (\operatorname{Grad} \vec{V})_{Kk} = \\ &= F_{kK}^{-T} (\operatorname{Grad} \vec{V})_{Kk} = \mathbf{F}^{-T} : \operatorname{Grad} \vec{V} \end{aligned}$$

Cvičení 2-3 – Geometrie deformace

Linearizace

$$\mathbf{V} = ??? + O(|\mathbf{H}|^2)$$

$$\begin{aligned}\mathbf{V} &= \sqrt{\mathbf{F} \cdot \mathbf{F}^T} = \sqrt{(\mathbf{I} + \mathbf{H}^T) \cdot (\mathbf{I} + \mathbf{H})} = \sqrt{\mathbf{I} + \mathbf{H}^T + \mathbf{H} + \mathbf{H}^T \cdot \mathbf{H}} = \\ &= \mathbf{I} + \frac{1}{2} (\mathbf{H}^T + \mathbf{H} + \mathbf{H}^T \cdot \mathbf{H}) + O(|\mathbf{H}|^2) = \mathbf{I} + \frac{1}{2} (\mathbf{H}^T + \mathbf{H}) + O(|\mathbf{H}|^2)\end{aligned}$$

$$\mathbf{V} = \mathbf{I} + \frac{1}{2} (\mathbf{H}^T + \mathbf{H}) + O(|\mathbf{H}|^2)$$

$$\mathbf{R} = ??? + O(|\mathbf{H}|^2)$$

$$\mathbf{F} = \mathbf{V} \cdot \mathbf{R}$$

$$\mathbf{F} \cdot \mathbf{R}^T = \mathbf{V} \cdot \mathbf{R} \cdot \mathbf{R}^T = \mathbf{V}$$

$$\mathbf{R}^T = \mathbf{F}^{-1} \cdot \mathbf{V}$$

$$\mathbf{R} = \mathbf{V} \cdot \mathbf{F}^{-T}$$

$$\mathbf{F}^{-1} = \mathbf{I} - \mathbf{h}^T = \mathbf{I} - \mathbf{H}^T + O(|\mathbf{H}|^2)$$

$$\mathbf{R} = \mathbf{V} \cdot \mathbf{F}^{-T} = \left(\mathbf{I} + \frac{1}{2} (\mathbf{H}^T + \mathbf{H}) \right) \cdot (\mathbf{I} - \mathbf{H}) + O(|\mathbf{H}|^2)$$

$$\mathbf{R} = \mathbf{I} + \frac{1}{2} (\mathbf{H}^T - \mathbf{H}) + O(|\mathbf{H}|^2)$$

Vyjádřete vektor $d\vec{x}$ jako superpozici translace, rotace a deformace.

$$d\vec{x} = \mathbf{F} \cdot d\vec{X}$$

Tento výraz lze obecně vyjádřit pomocí polárního rozkladu jako součin dvou matic, které popisují rotaci a deformaci:

$$d\vec{x} = \mathbf{F} \cdot d\vec{X} = \mathbf{R} \cdot \mathbf{U} \cdot d\vec{X} = \mathbf{V} \cdot \mathbf{R} \cdot d\vec{X}$$

Nyní chceme $d\vec{x}$ vyjádřit jako superpozici, tj. součet příspěvků od translace, rotace a deformace.

Připomeňme, že

$$\mathbf{F} = \mathbf{I} + \mathbf{H}^T = \mathbf{I} + \frac{1}{2}(\mathbf{H}^T - \mathbf{H}) + \frac{1}{2}(\mathbf{H} + \mathbf{H}^T) = \mathbf{I} + \tilde{\mathbf{R}} + \tilde{\mathbf{E}}$$

$\uparrow$

linearizovaný
lagrangeovský
tenzor
rotace

$\uparrow$

linearizovaný
lagrangeovský
tenzor
deformace

A tedy

$$d\vec{x} = \mathbf{F} \cdot d\vec{X} = (\mathbf{I} + \tilde{\mathbf{R}} + \tilde{\mathbf{E}}) \cdot d\vec{X} = \underbrace{d\vec{X}}_{\text{translace}} + \underbrace{\tilde{\mathbf{R}} \cdot d\vec{X}}_{\text{rotace}} + \underbrace{\tilde{\mathbf{E}} \cdot d\vec{X}}_{\text{deformace}}$$

Platí jen pro malé deformace!