

Previously on Rotace Země

Terminology:

- Euler dynamic equations: 2nd order ODEs for Euler angles
- Balance of angular momentum in a co-rotating frame
- Liouville equation
- Liouville equation in the Tisserand frame



$$I_1 \dot{\Omega}_x - (I_2 - I_3) \Omega_y \Omega_z = M_x$$

$$I_2 \dot{\Omega}_y - (I_3 - I_1) \Omega_z \Omega_x = M_y$$

$$I_3 \dot{\Omega}_z - (I_1 - I_2) \Omega_x \Omega_y = M_z$$



$$\Omega_x = \dot{\varphi} \sin \vartheta \sin \psi + \dot{\vartheta} \cos \psi ,$$

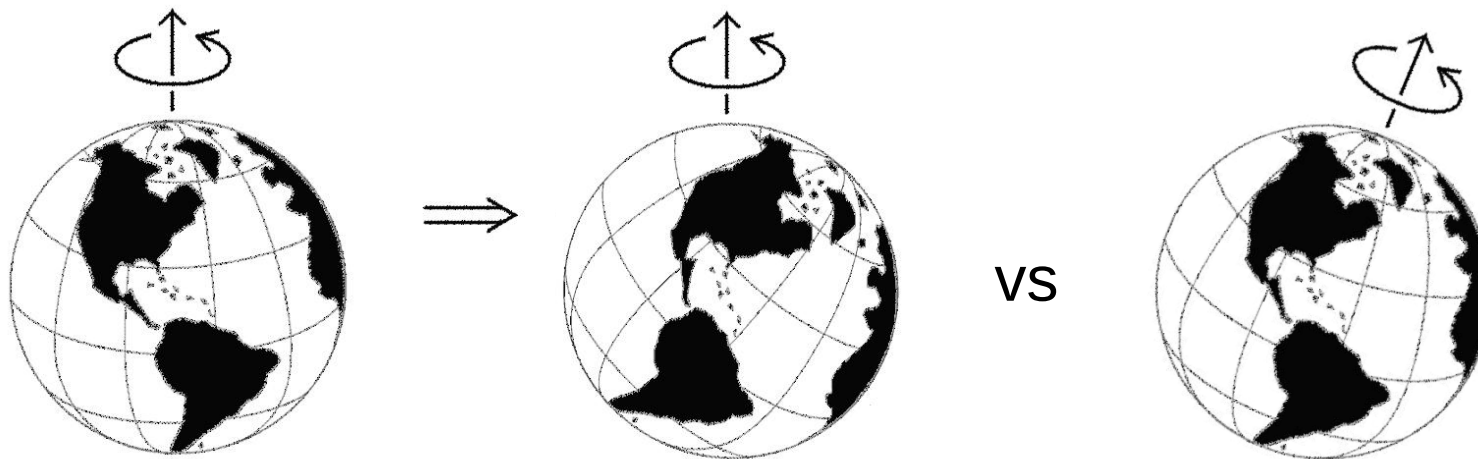
$$\Omega_y = \dot{\varphi} \sin \vartheta \cos \psi - \dot{\vartheta} \sin \psi ,$$

$$\Omega_z = \dot{\varphi} \cos \vartheta + \dot{\psi} ,$$

Previously on Rotace Země

Terminology and observational methods:

- Chandler wobble, polar motion (measurement of latitude)
- Precession, forced nutation (measurement of star inclinations)



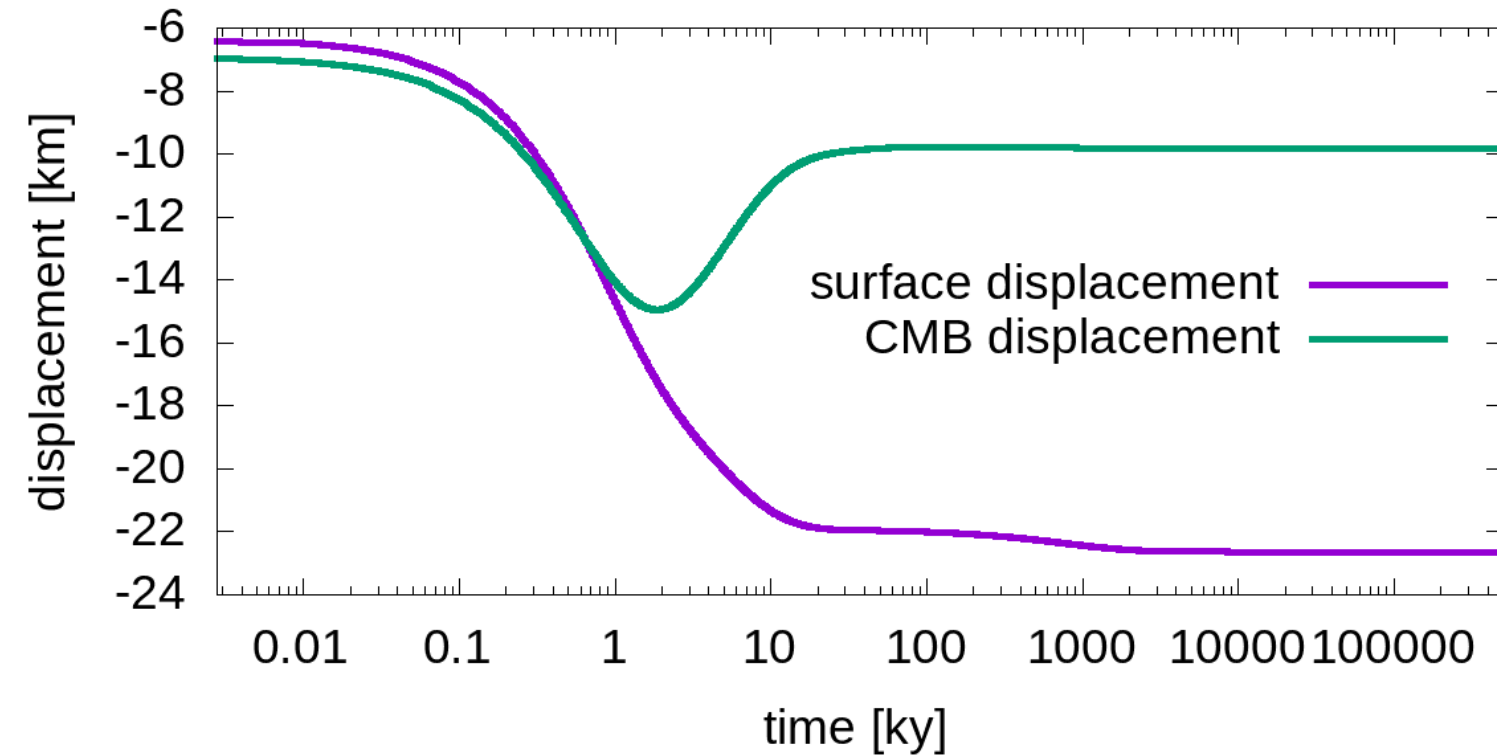
reality: superposition of both phenomena

Previously on Rotace Země

Frames of reference

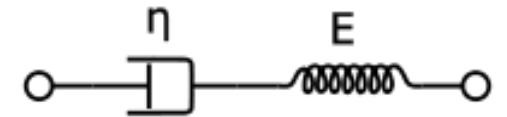
- Torque vs h : ball covered with ocean
- Net lithospheric rotation: Tisserand frame of the mantle vs geographic frame

Bulge formation / readjustment



$$\nabla \cdot \tau + f = 0,$$

$$\nabla \cdot u = 0,$$



$t = 0$: Elastic bulge

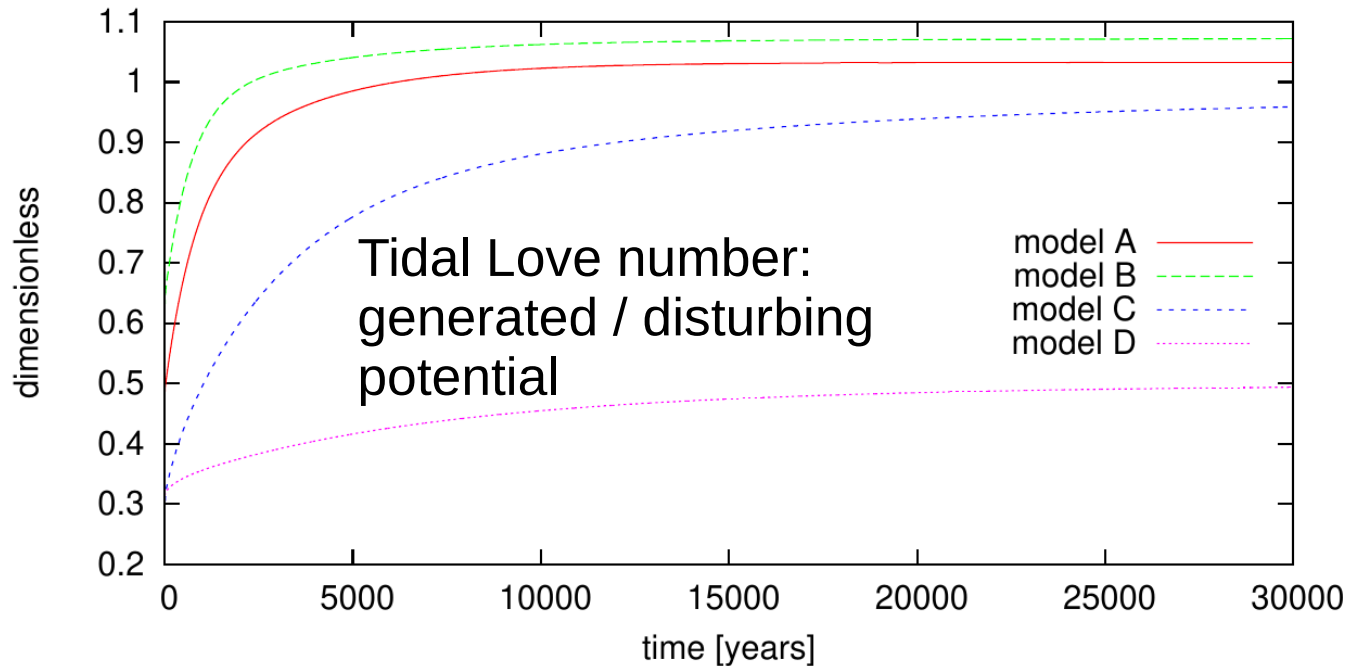
$t = \infty$: Fluid-limit / hydrostatic / long-term bulge

$0 < t < \infty$: Bulge readjustment

Depends on the viscosity and density structure of Earth

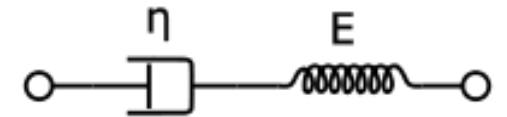
Rotational bulge: Love numbers

Figure 3.1: Evolution of $\frac{\Phi_{20}(t)}{\varphi_{20}(t)}$ for models A,B,C,D



$$\nabla \cdot \tau + f = 0,$$

$$\nabla \cdot u = 0,$$



$$k^T(s) = k_e^T + \sum_{i=1}^M \frac{k_i^T}{s - s_i}$$

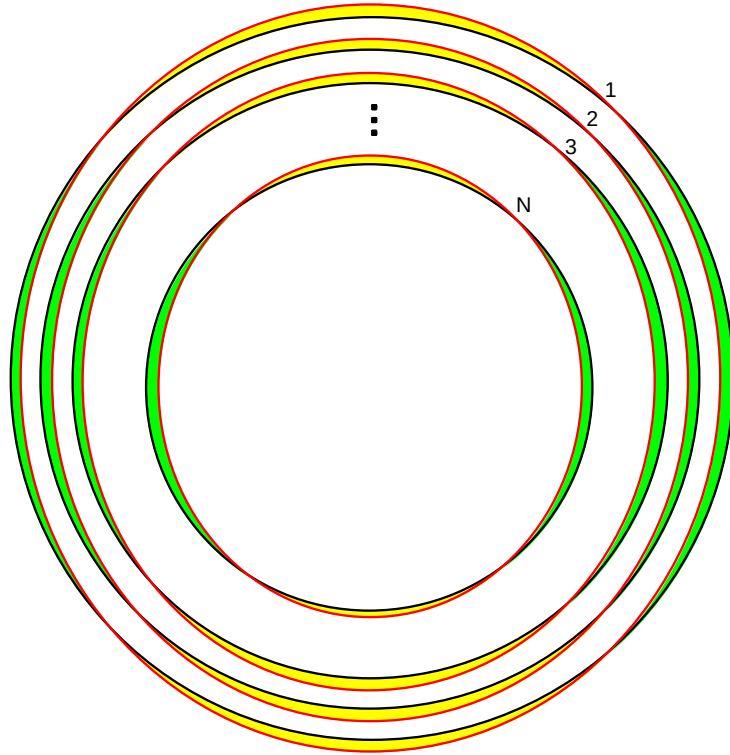
Normal / relaxation
modes

Temporal vs Laplace domain:

Most of the previous theory is written in the Laplace domain, some relaxation modes are typically neglected

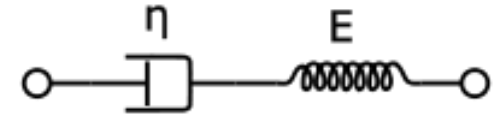
LIOSHELL

open access GitHub repository



- $\delta\rho = [\rho_0]_i$
- $\delta\rho = -[\rho_0]_i$
- $\delta\rho = 0$
- deformed body
- reference sphere

Smooth-profiled or layered bodies
Maxwell viscoelastic rheology
Spherical harmonic decomposition



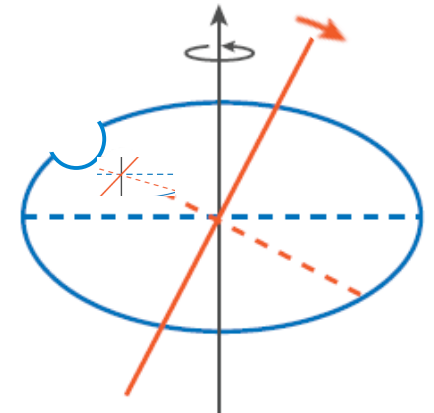
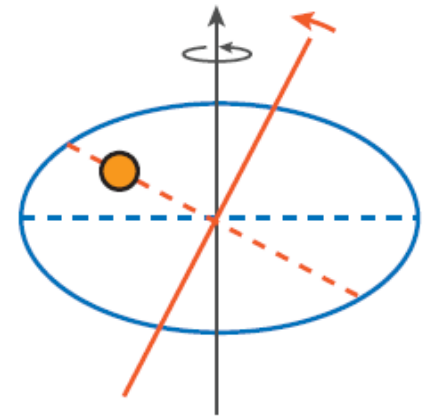
$$\nabla \cdot \boldsymbol{\tau} + \mathbf{f} = 0,$$

$$\nabla \cdot \mathbf{u} = 0,$$

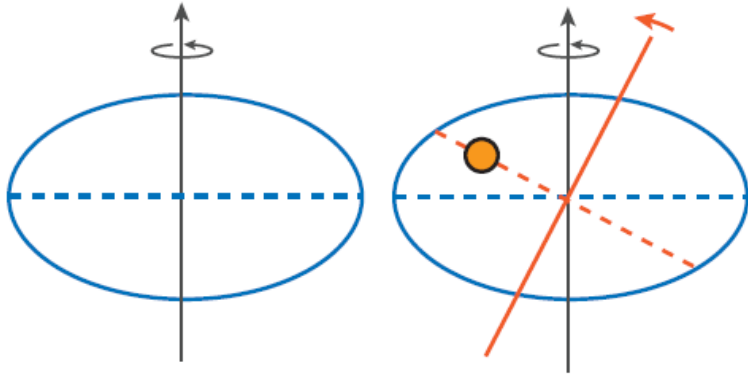
$$\boldsymbol{\tau}^d - \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) = -\frac{\mu}{\eta} \int_0^t \boldsymbol{\tau}^d dt'$$

$$\frac{d\boldsymbol{\omega}}{dt} = -\mathbf{I}^{-1} \cdot \left(\frac{d\mathbf{I}}{dt} \cdot \boldsymbol{\omega} + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) \right) \quad \text{OR} \quad \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) = 0$$

Mechanism: basic concept

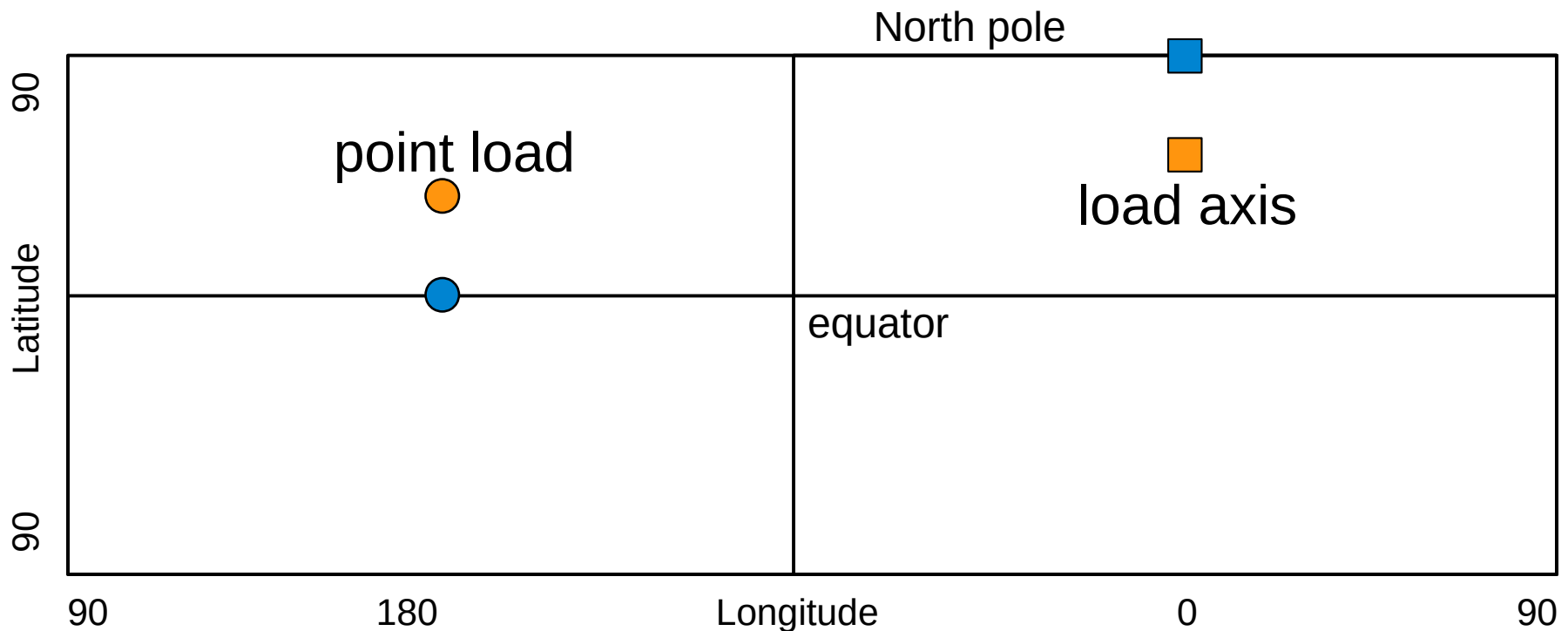


Mechanism: more detailed



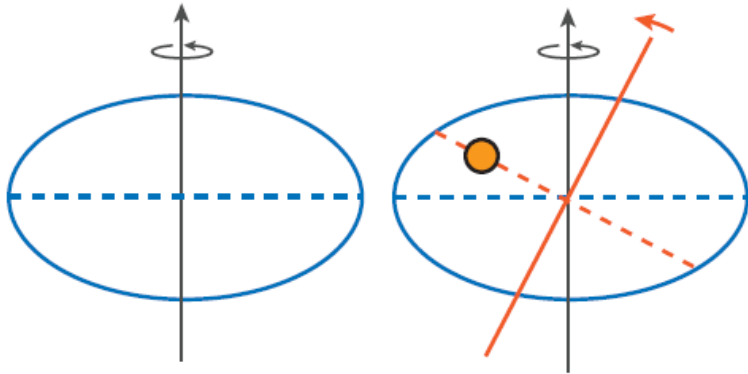
$$0 = \frac{d}{dt}(\mathbf{I} \cdot \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega})$$

Ann. Rev. Earth Planet. Sci.: Matsuyama I., Nimmo F., and Mitrovica J.X. (2014)



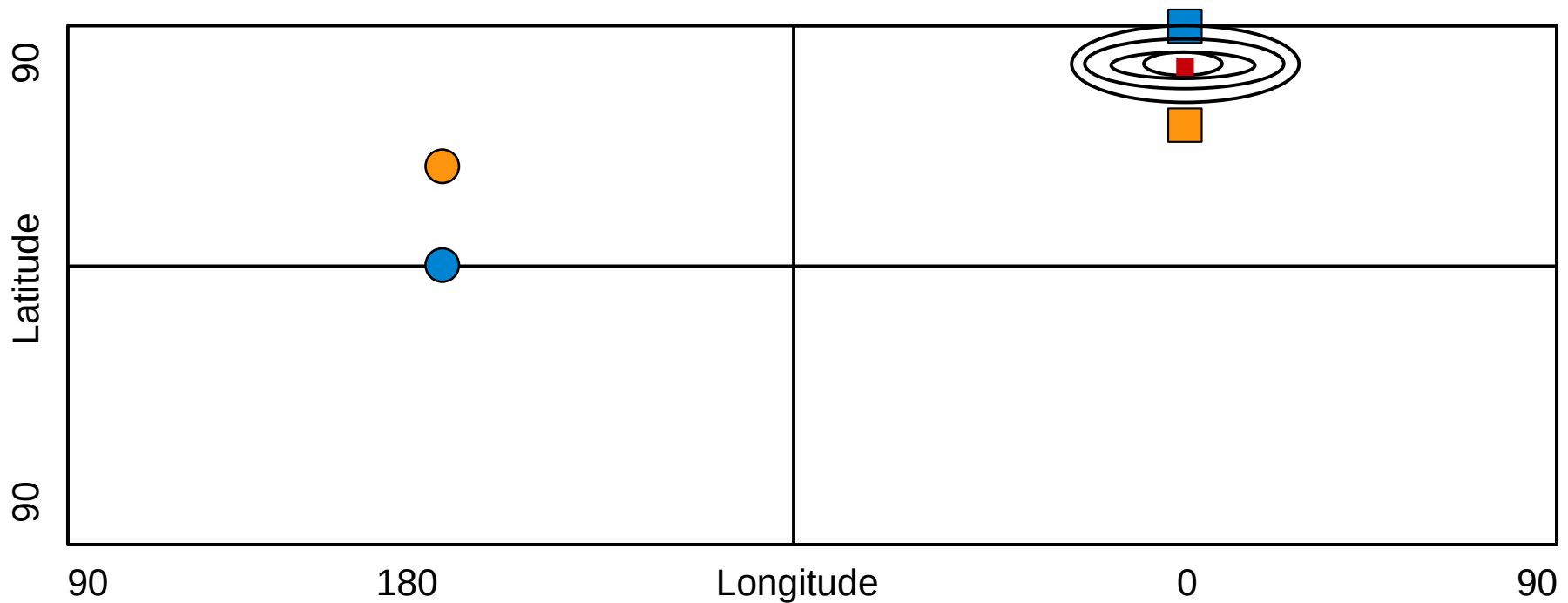
Mechanism: wobble

$\omega \parallel \text{MIA}$

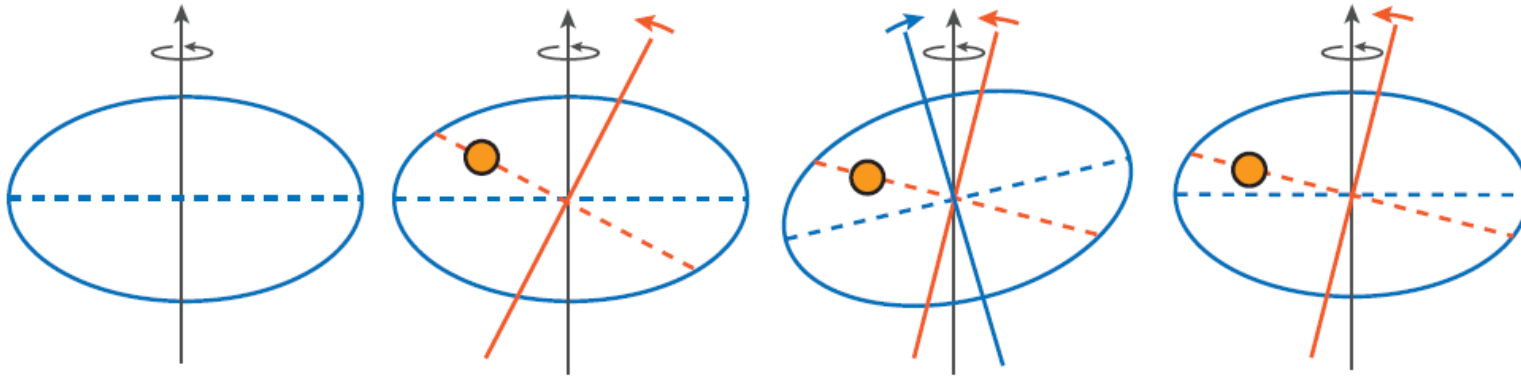


$$0 = \frac{d}{dt}(\mathbf{I} \cdot \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega})$$

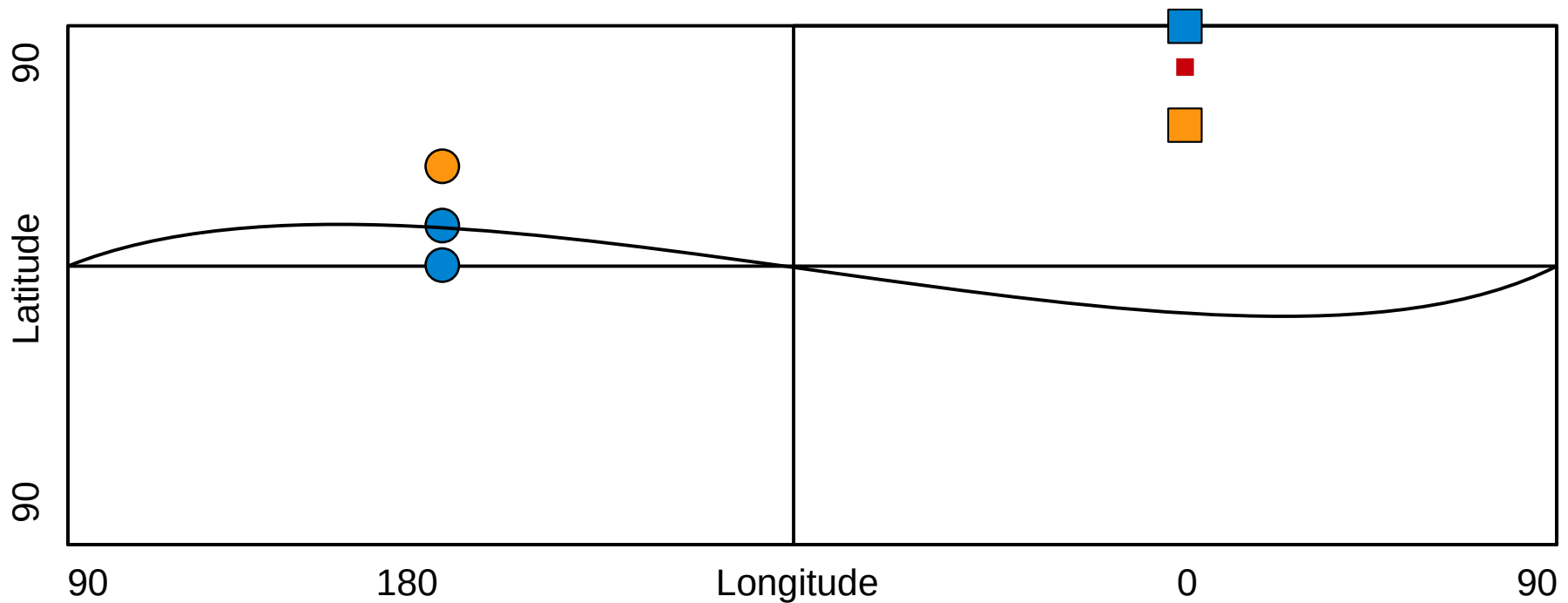
Ann. Rev. Earth Planet. Sci.: Matsuyama I., Nimmo F., and Mitrovica J.X. (2014)



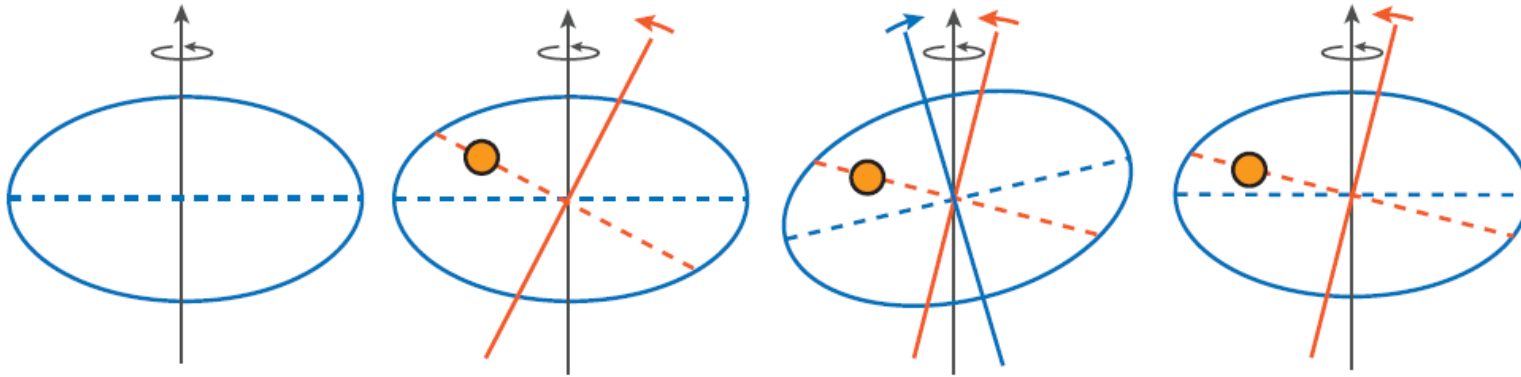
Mechanism: bulge readjustment



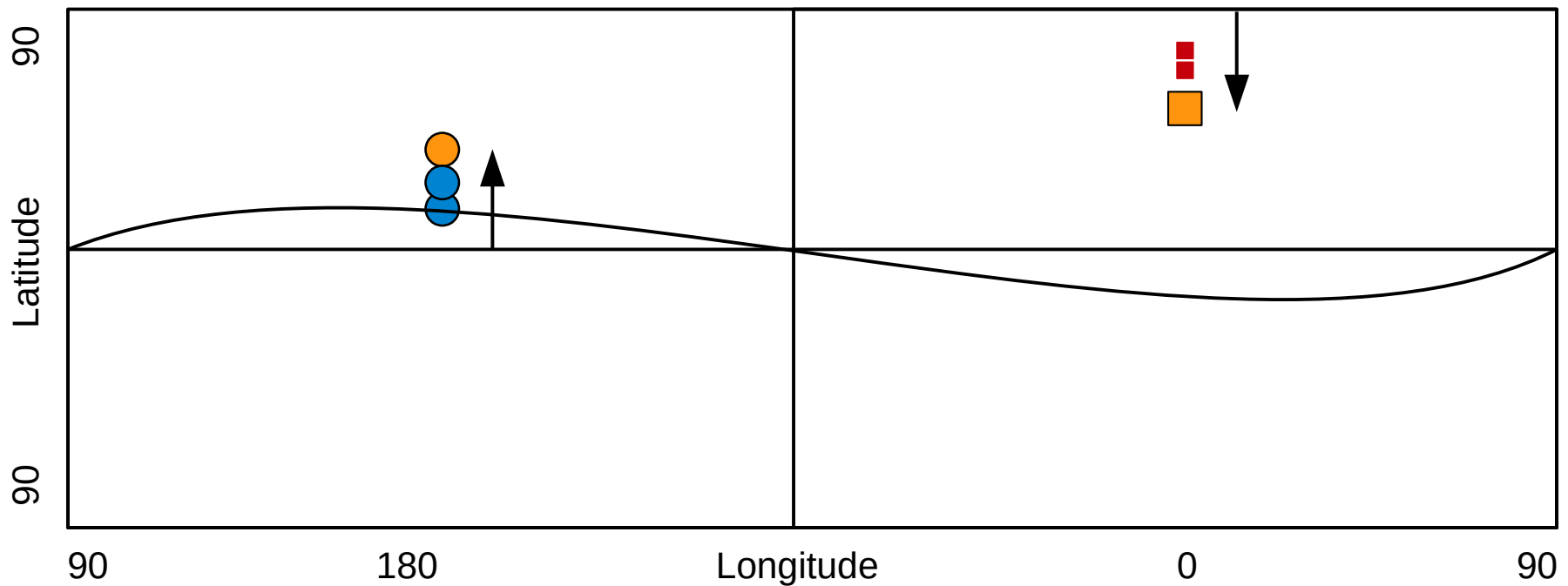
Ann. Rev. Earth Planet. Sci.: Matsuyama I., Nimmo F., and Mitrovica J.X. (2014)



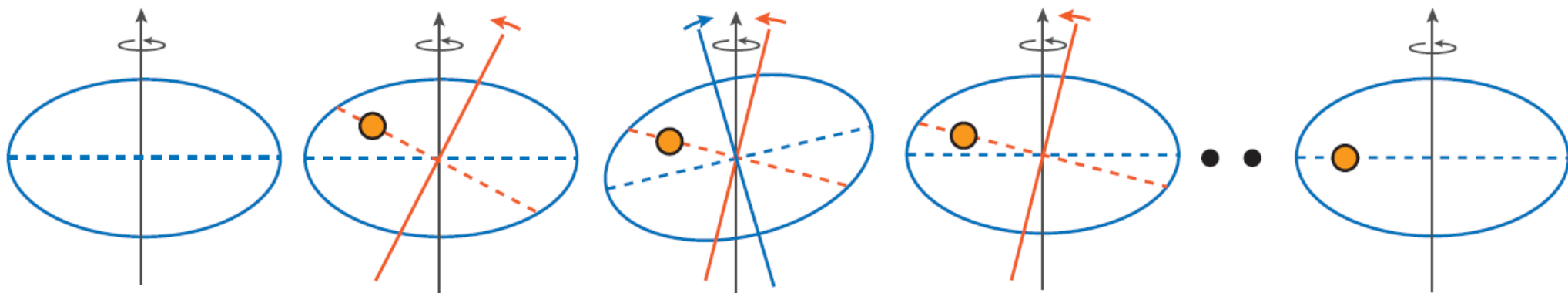
Mechanism: bulge readjustment



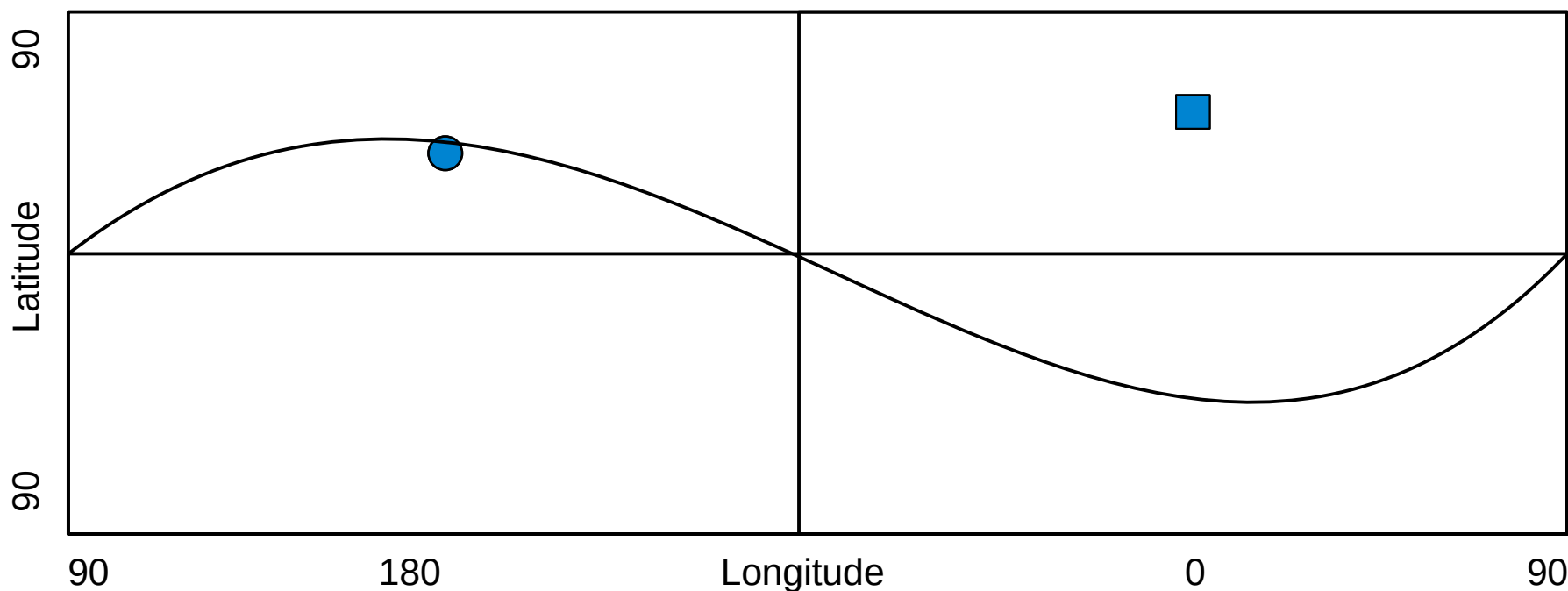
Ann. Rev. Earth Planet. Sci.: Matsuyama I., Nimmo F., and Mitrovica J.X. (2014)



Mechanism: load $\rightarrow$ equator scenario



Ann. Rev. Earth Planet. Sci.: Matsuyama I., Nimmo F., and Mitrovica J.X. (2014)



TPW: methods overview

Linearized Liouville Equation (LLE)

$$m_1(t) = \frac{\Delta I_{13}(t)}{C - A} + \frac{C \Delta \dot{I}_{23}(t)}{\Omega(C - A)(C - B)}$$

1)

$$m_2(t) = \frac{\Delta I_{23}(t)}{C - B} - \frac{C \Delta \dot{I}_{13}(t)}{\Omega(C - A)(C - B)}$$

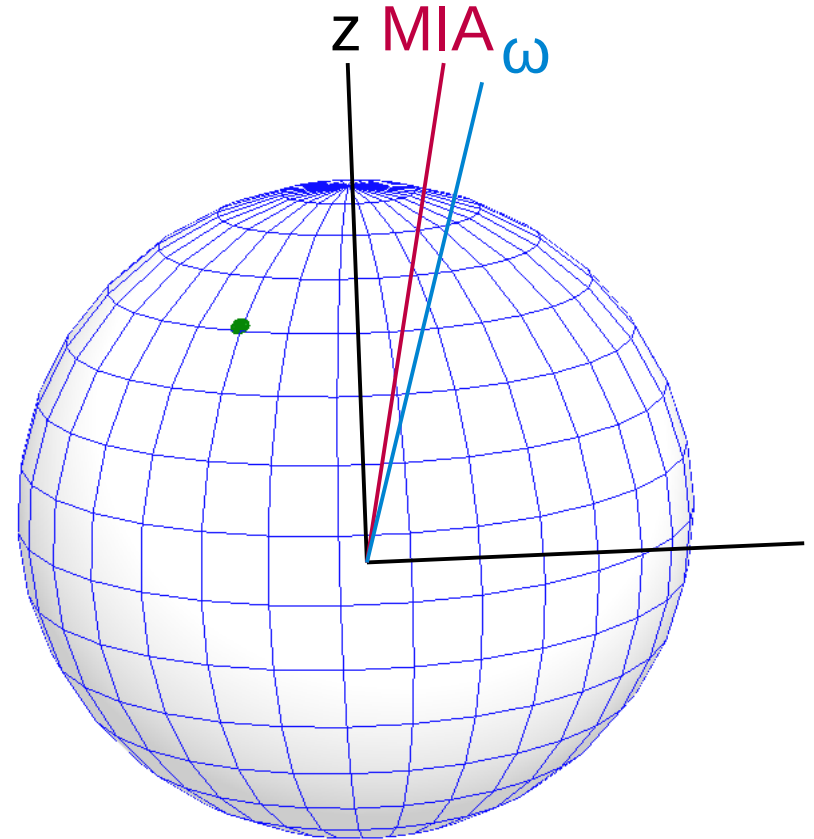
$$m_3(t) = -\frac{\Delta I_{33}}{C}$$

LE in the Tisserand frame

$$2) \quad 0 = \frac{d}{dt}(\mathbf{I} \cdot \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega})$$

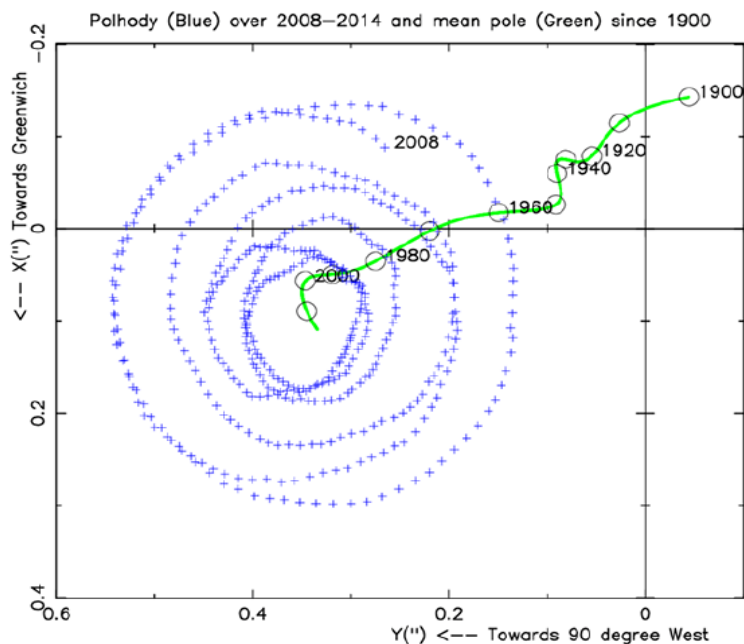
$\boldsymbol{\omega} \parallel \text{MIA}$ approximation

$$3) \quad \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) = 0$$

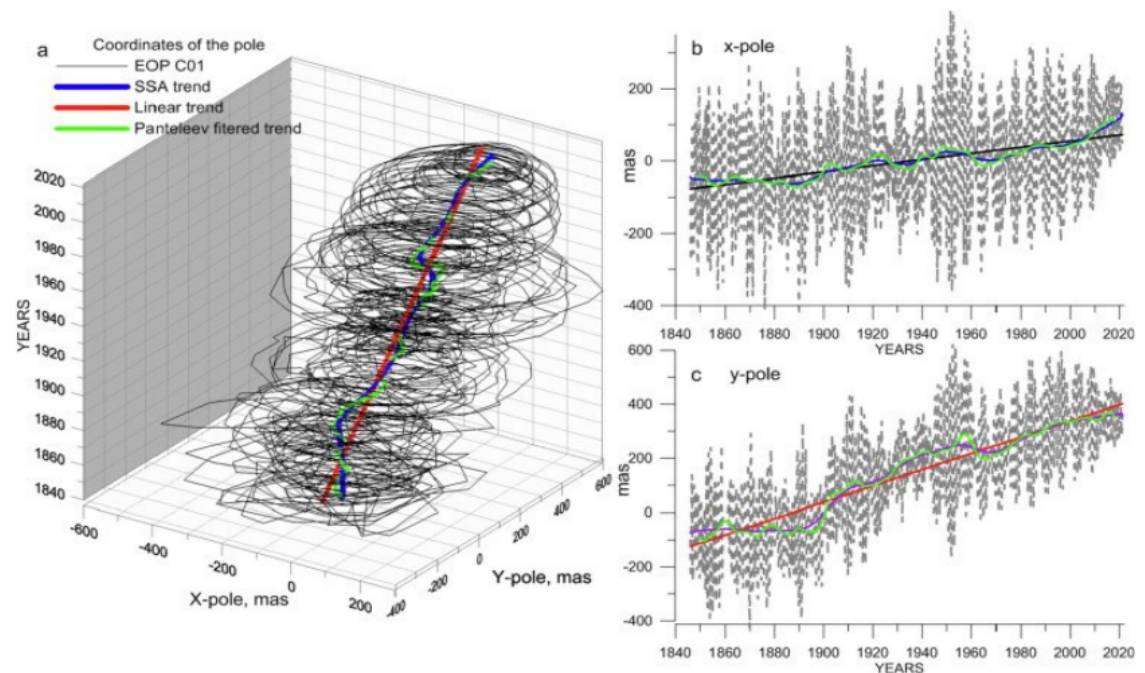


Observations: wobble + polar motion

- wobble can be triggered by any perturbation, but it is also damped
- polar motion / secular motion / true polar wander



credit: 100 Proofs that the Earth is a Globe

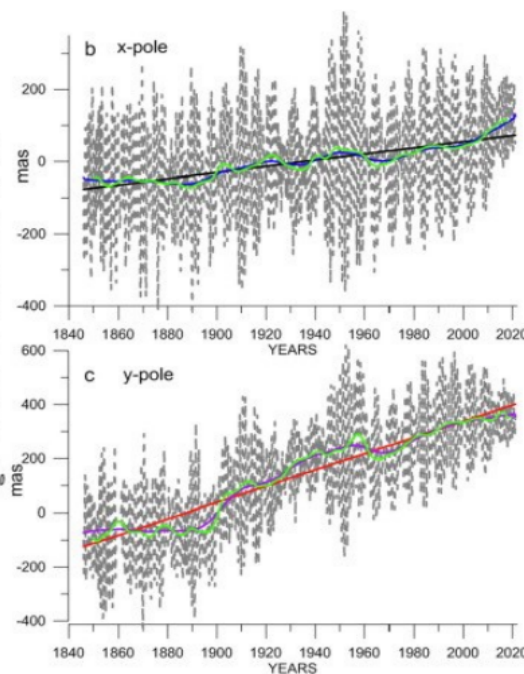
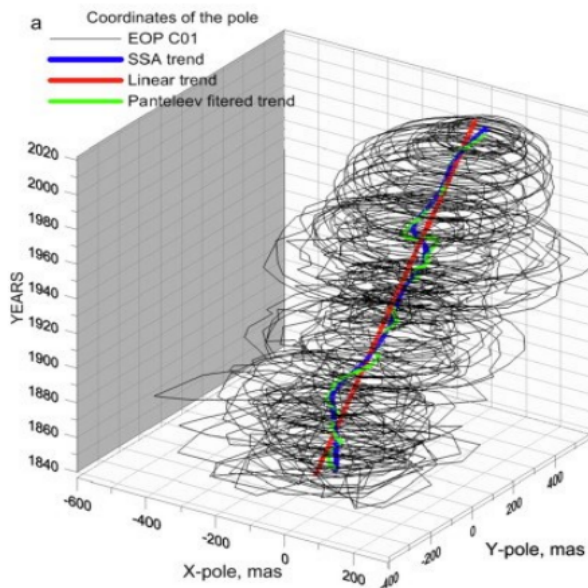


credit: Zotov et al., 2022

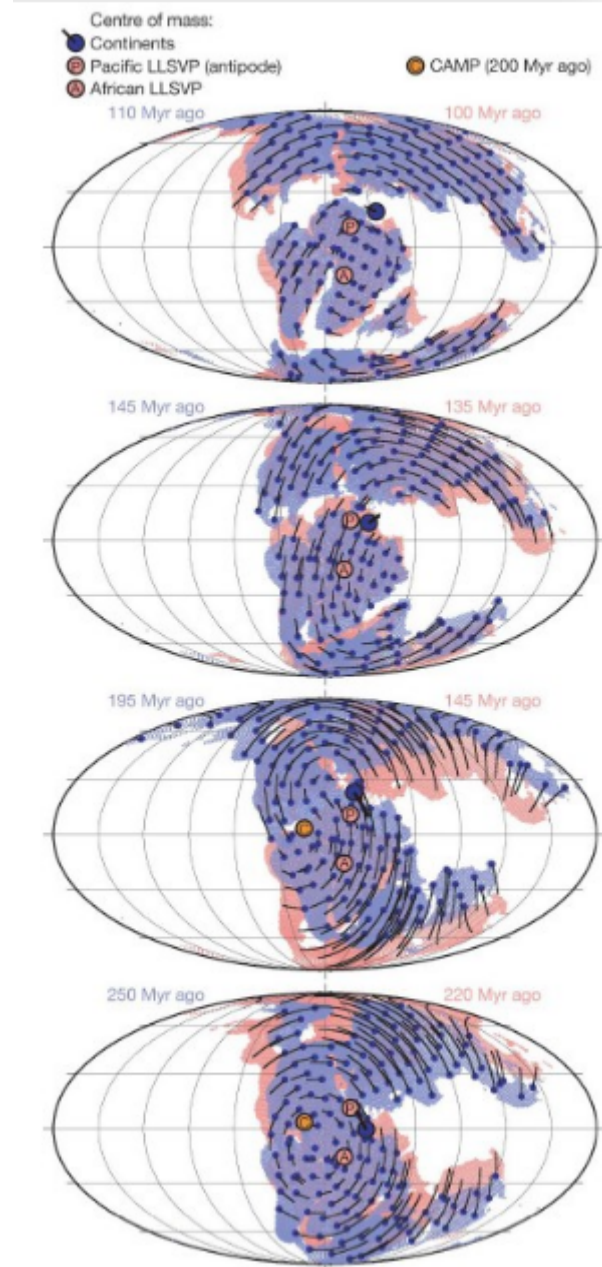
Frames of reference

Assumptions vs data

- When you fit the data, are your assumptions met?
- From the past century we have data from geographic frame
- For geological time scales we have “data” from hot spot frame



credit: Zotov et al., 2022



Comparison of solutions: loading by a glacier

LLE, excitation due to inertia changes only

$$\Omega^2(C - A)\phi_1 = \Omega^2 I_{13} + \Omega \dot{I}_{23} + \cancel{\Omega h_1} + \cancel{\dot{h}_2} - \cancel{L_2}$$

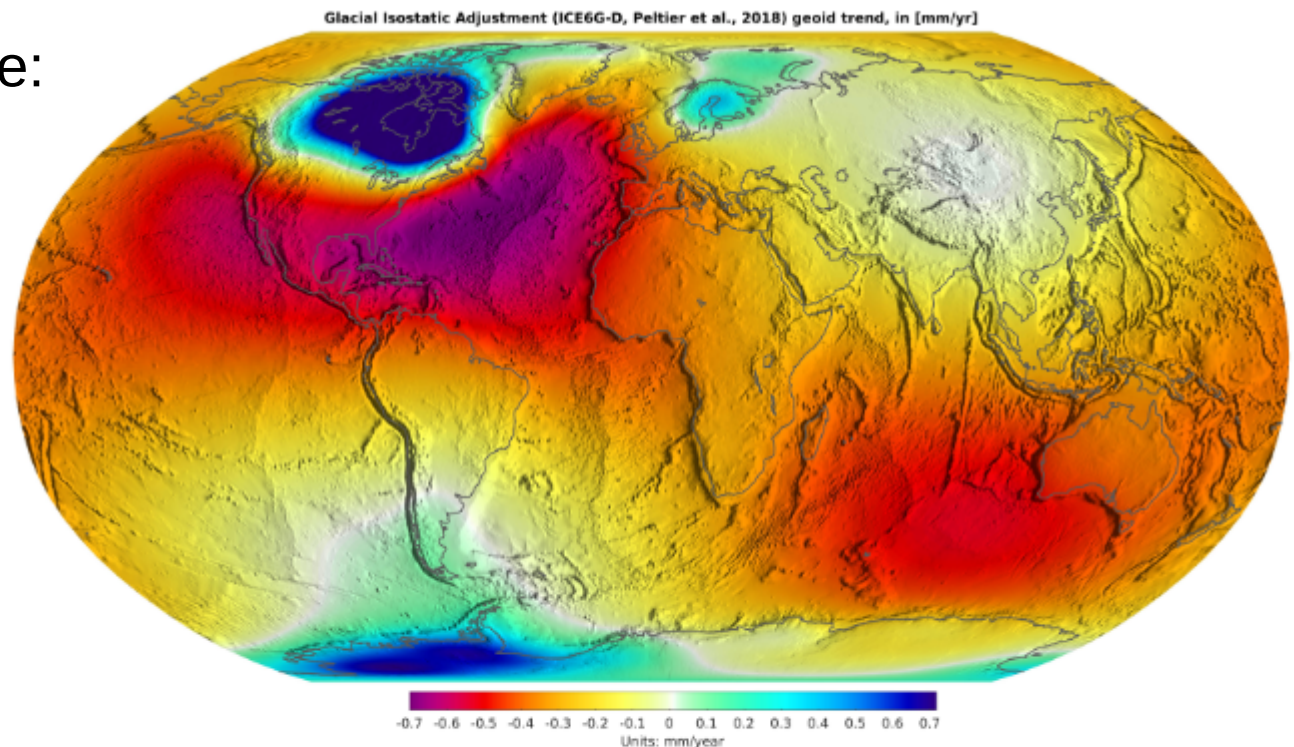
$$\Omega^2(C - A)\phi_2 = \Omega^2 I_{23} + \Omega \dot{I}_{13} + \Omega h_2 + \dot{h}_1 - L_1$$

$$\Omega^2 C \phi_3 = -\Omega^2 I_{33} - \Omega h_3 - \Omega \int_0^t L_3 dt$$

RHS: full source function

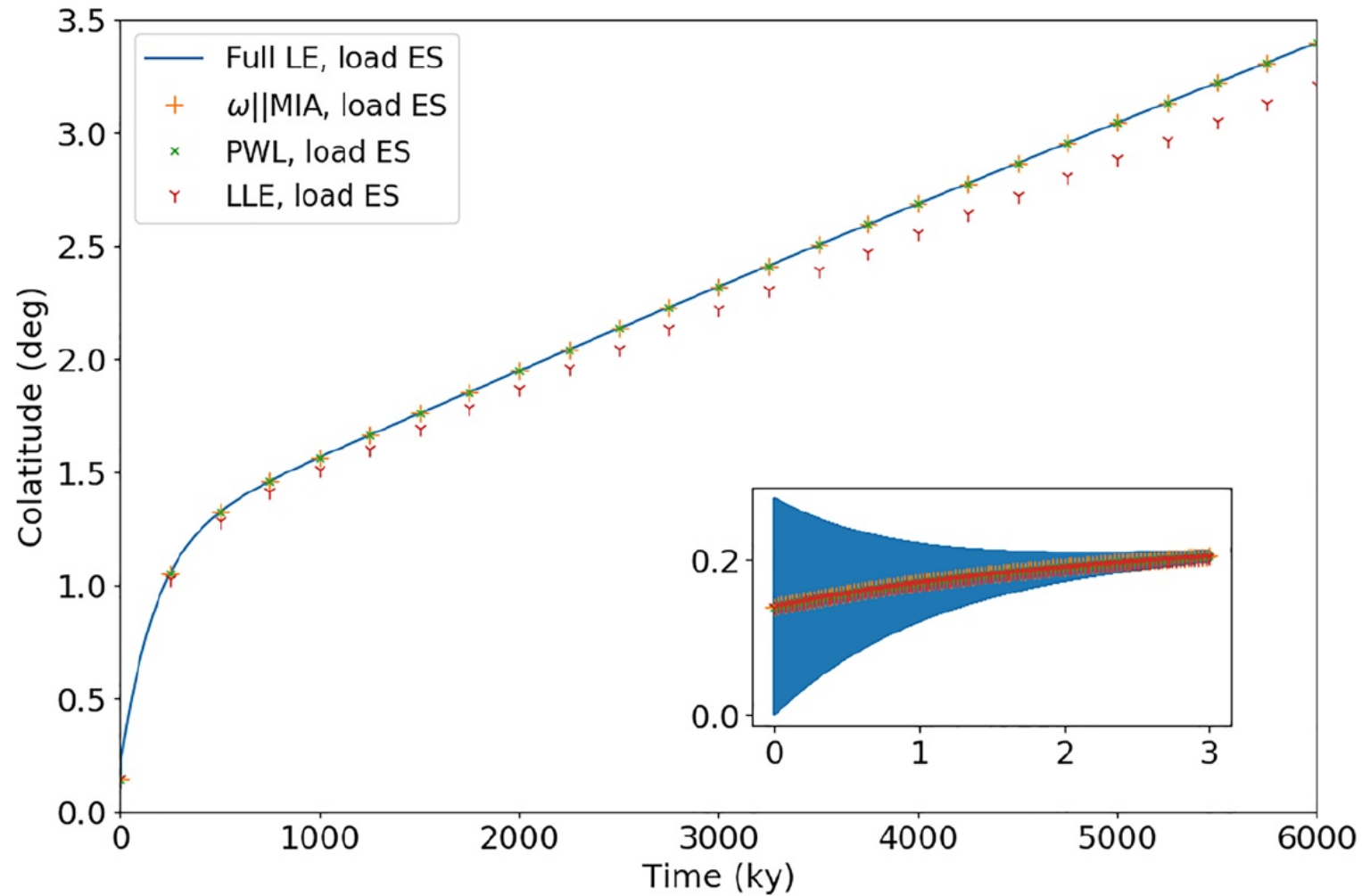
LE in the Tisserand frame:

$$0 = \frac{d}{dt}(\mathbf{I} \cdot \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega})$$

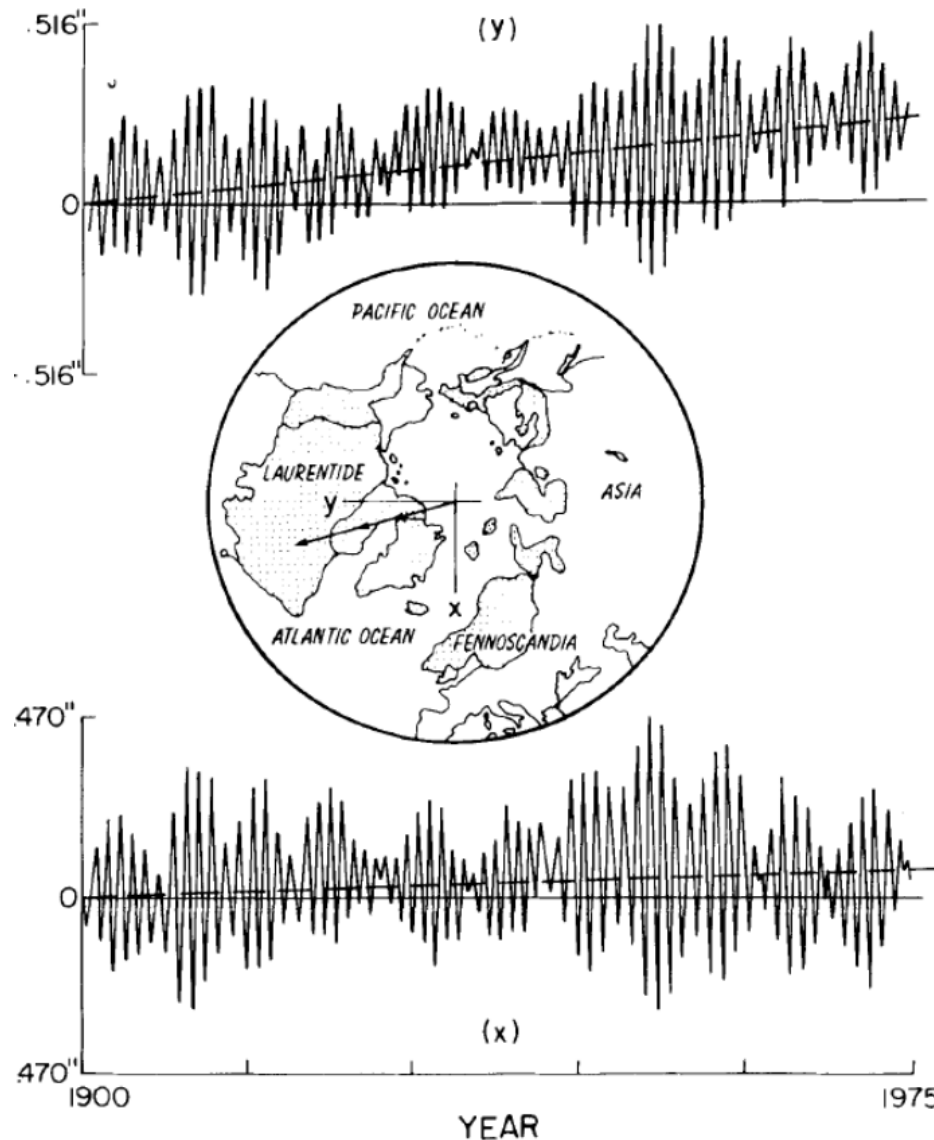


Contemporary geoid rates (in mm/yr) from GIA as predicted by the ICE6G-D model (Peltier et al., 2018). A 300km smoothing filter has been applied.

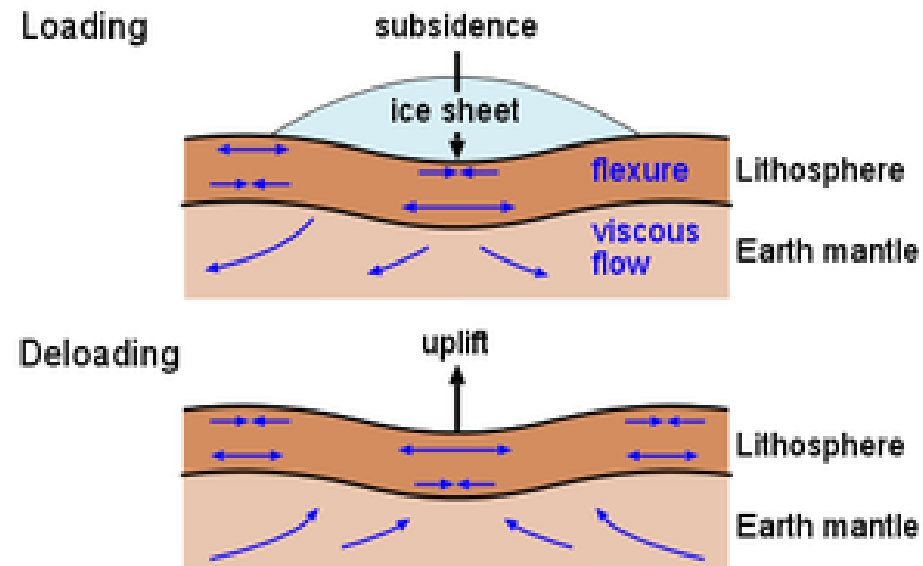
Comparison of solutions: loading by a glacier



TPW: Earth vs. planets and moons



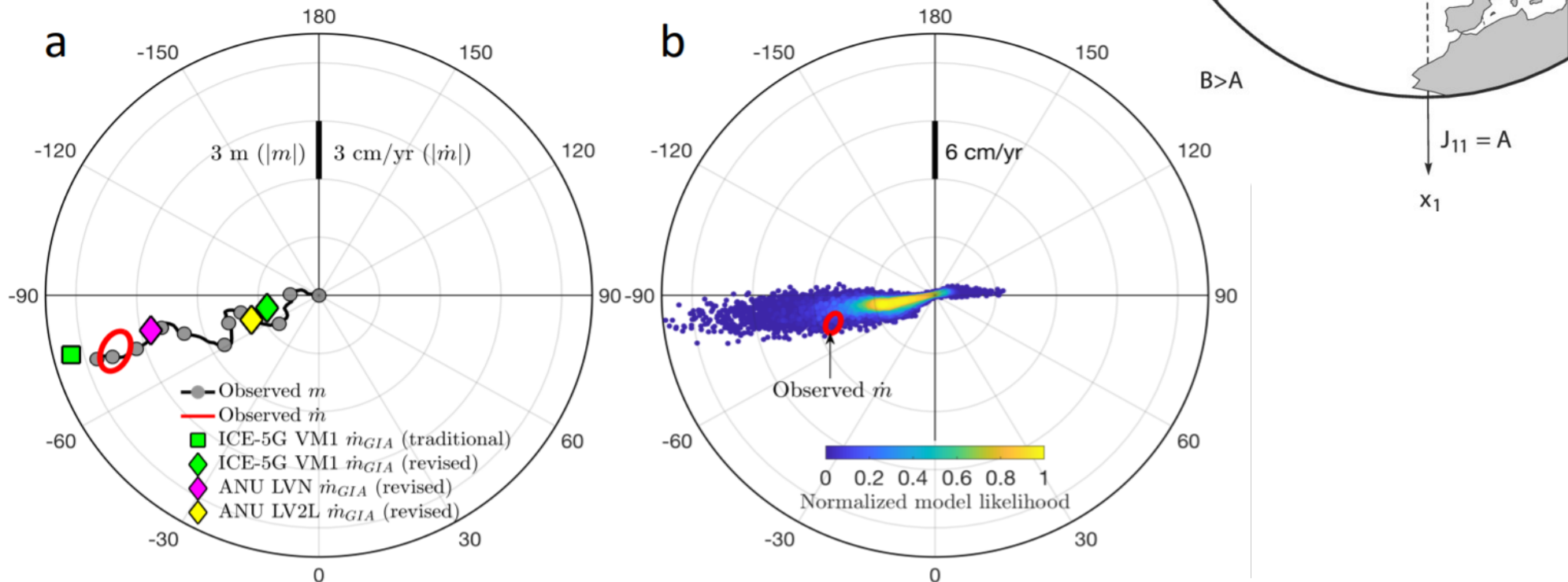
Wu and Peltier (1984):
Pleistocene deglaciation
and the Earth's rotation:
a new analysis



Polar wander on Earth: state of the art

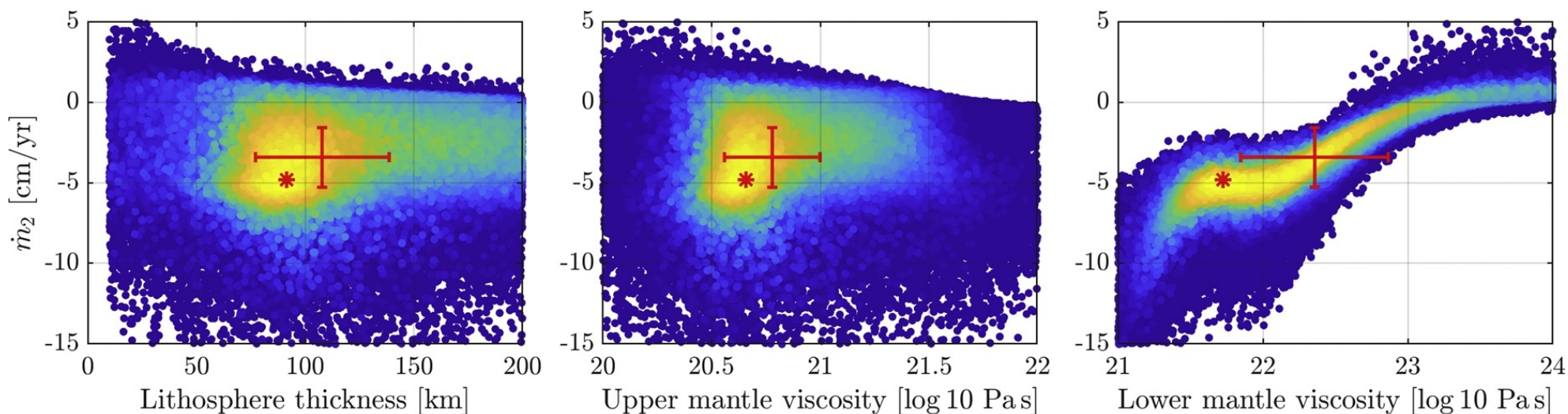
- Earth's (fossil) tri-axiality
- Land-ocean mass transfer
- Mantle convection as the missing source

Matsuyama et al. (2010)

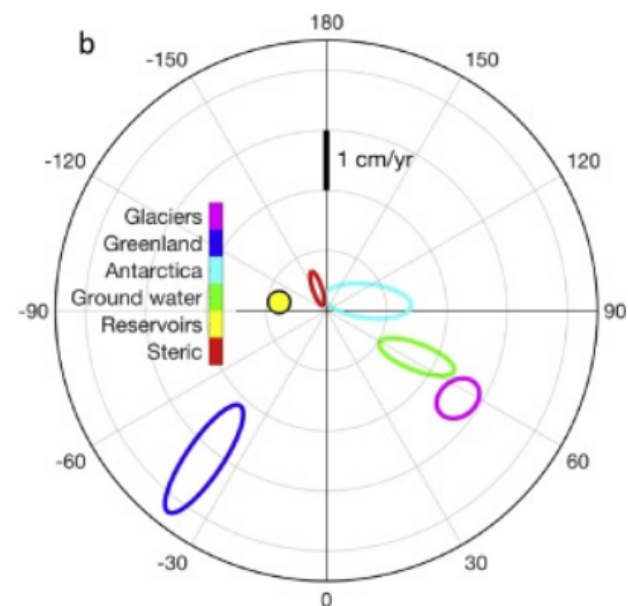


Adhikari et al. (2018, EPSL): What drives 20th century polar motion?

TPW: fine details on Earth

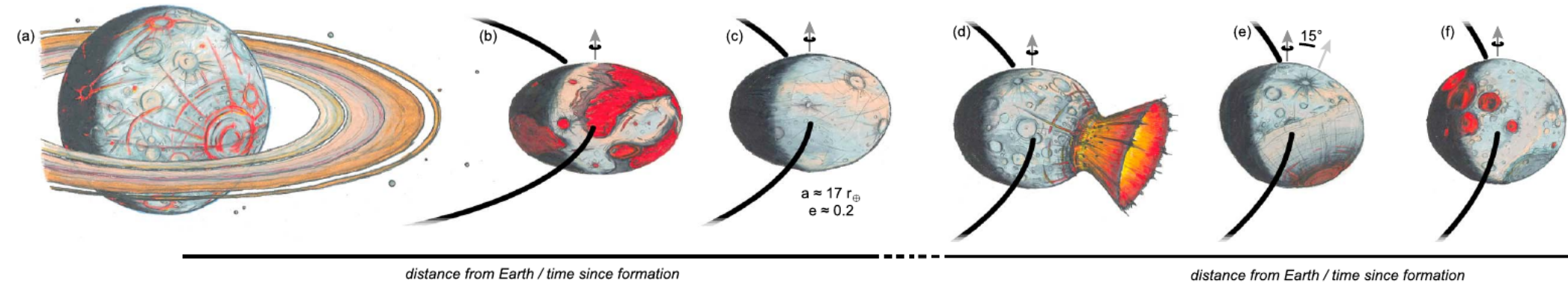


Source	GMSL rate (mm yr ⁻¹)	SPM rate $\dot{m}_1$ (cm yr ⁻¹)	SPM rate $\dot{m}_2$ (cm yr ⁻¹)	References
Groundwater	0.17 ± 0.04	0.77 ± 0.32	1.50 ± 0.62	Wada et al. (2012)
Dams/reservoirs	-0.29	-0.14	-0.78	Chao et al. (2008)

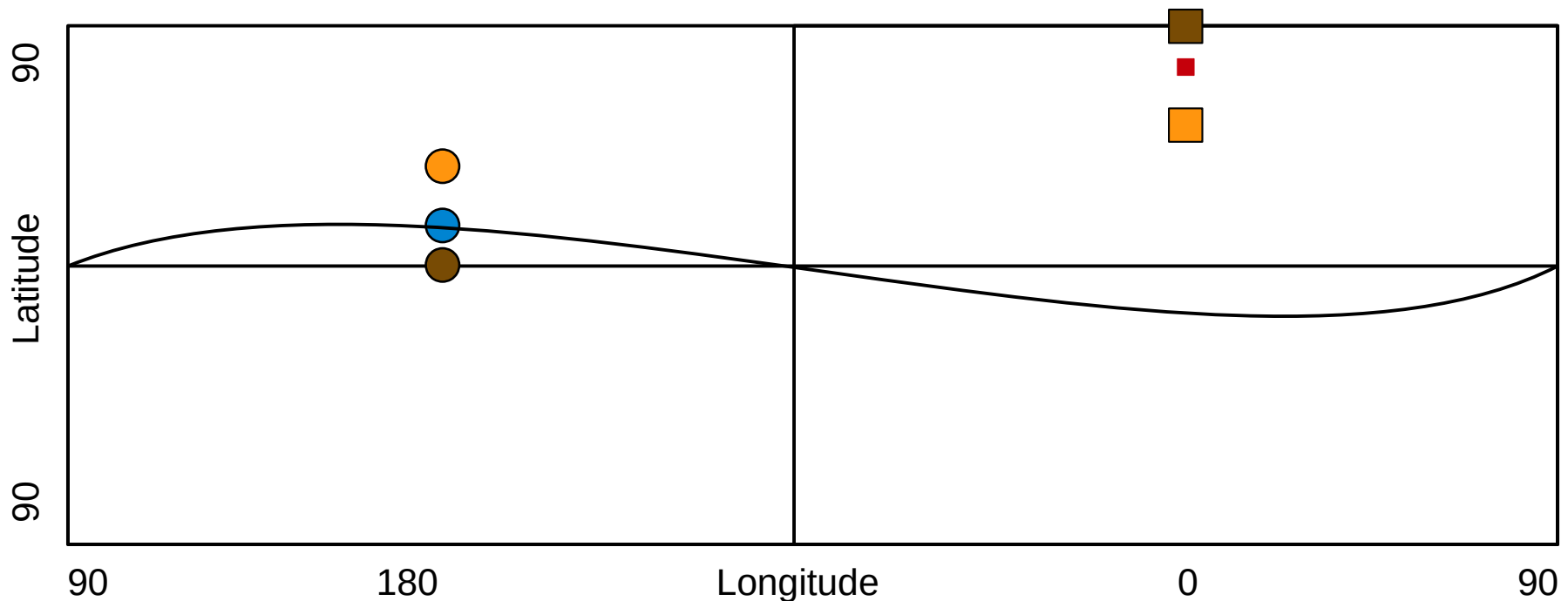


Adhikari et al. (2018, EPSL): What drives 20th century polar motion?

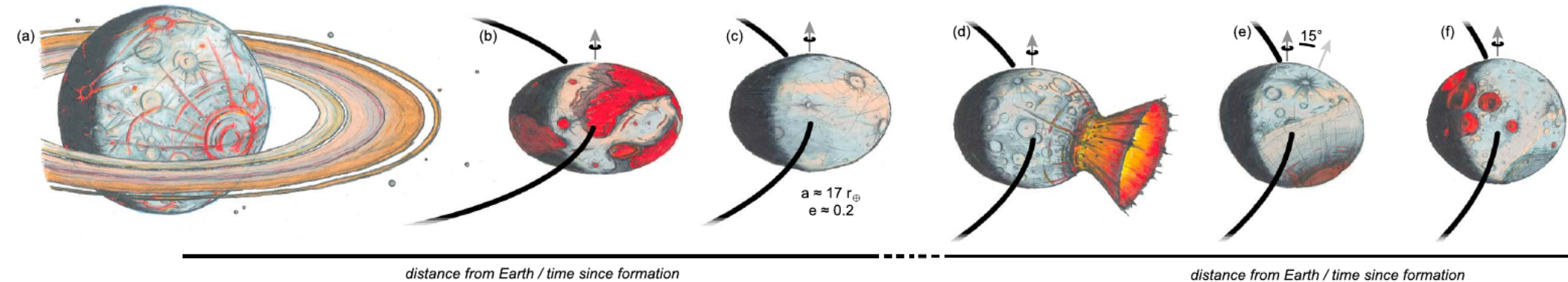
Mechanism: fossil (remnant) bulge



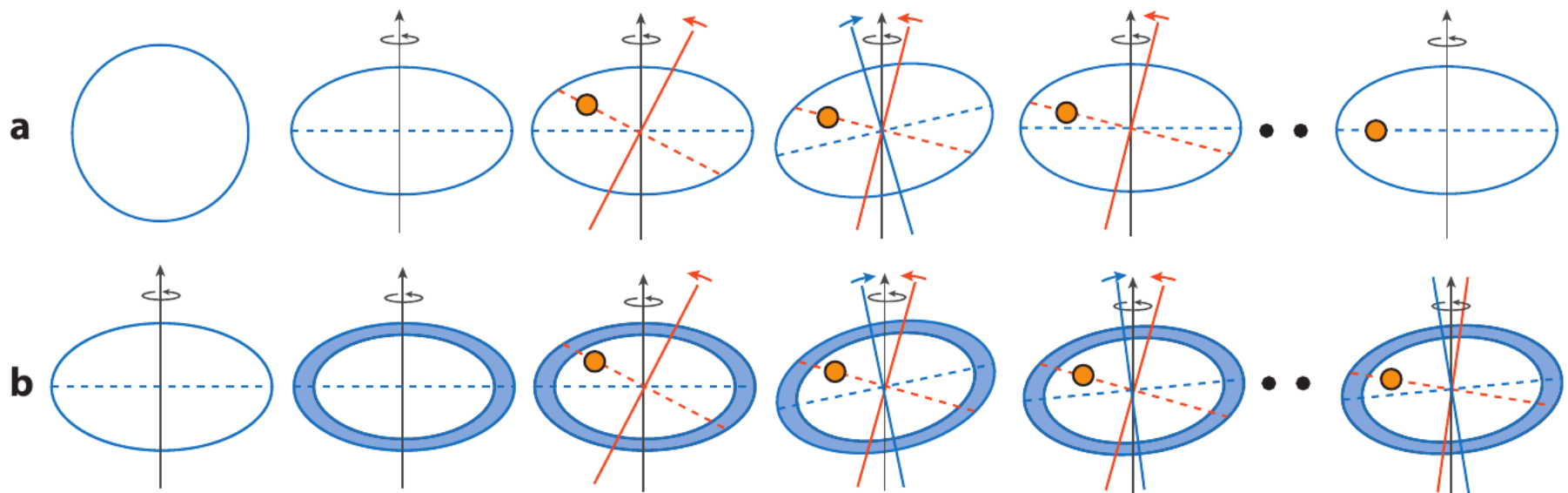
Keane and Matsuyama (2014): Evidence for lunar true polar wander and past low-eccentricity,...



Mechanism: fossil (remnant) bulge



Keane and Matsuyama (2014): Evidence for lunar true polar wander and past low-eccentricity,...



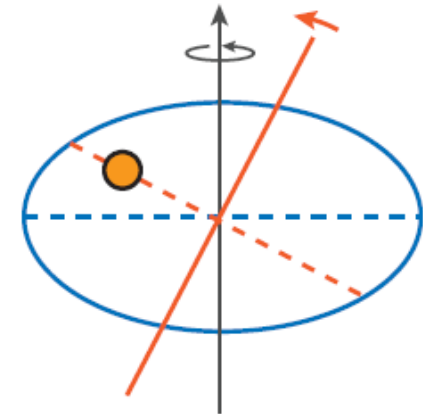
Ann. Rev. Earth Planet. Sci.: Matsuyama I., Nimmo F., and Mitrovica J.X. (2014)

Polar wander on tidally locked bodies

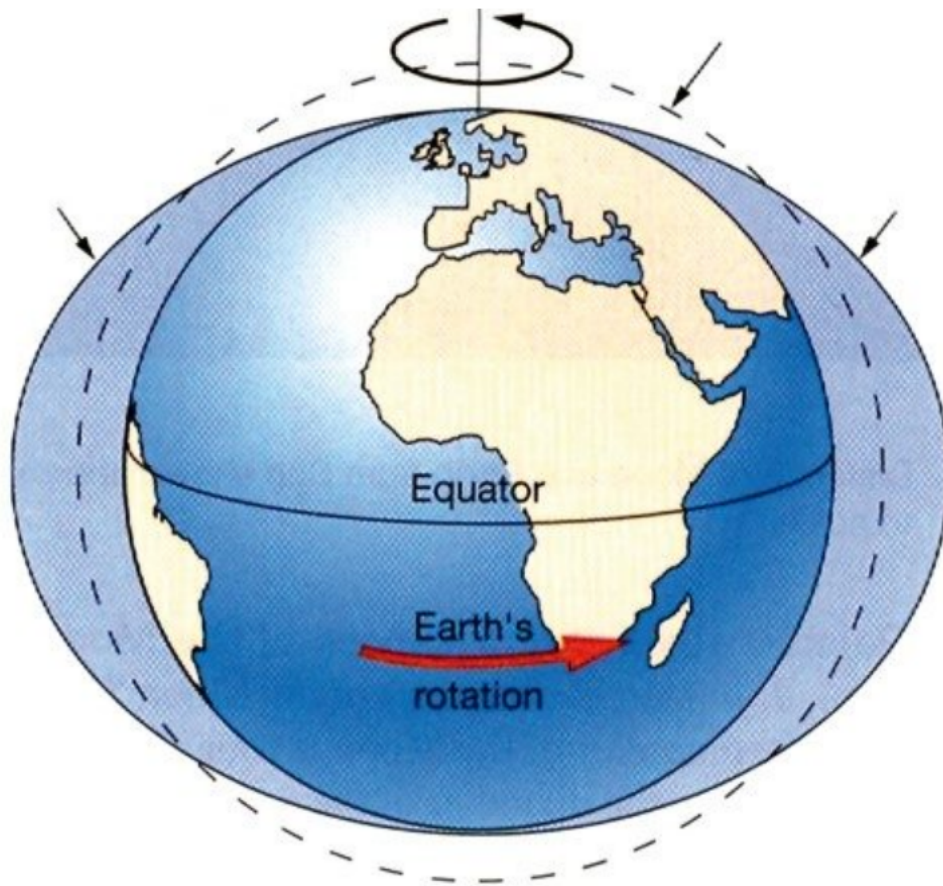
Rotation: bulge perpendicular to ω axis

Tides: bulge along the tidal axis

rotational axis

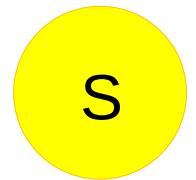


rotational axis

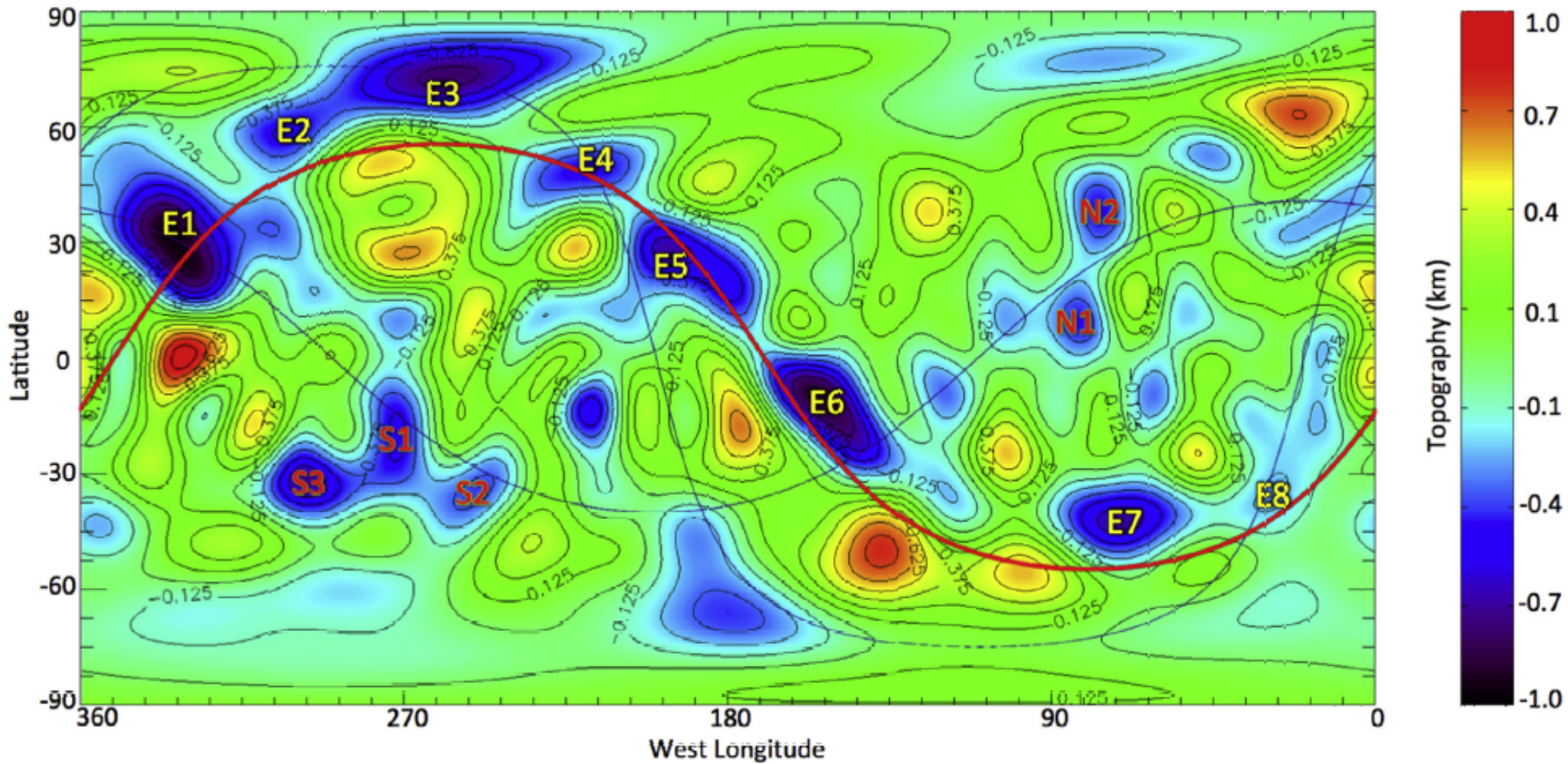


tidal axis

S

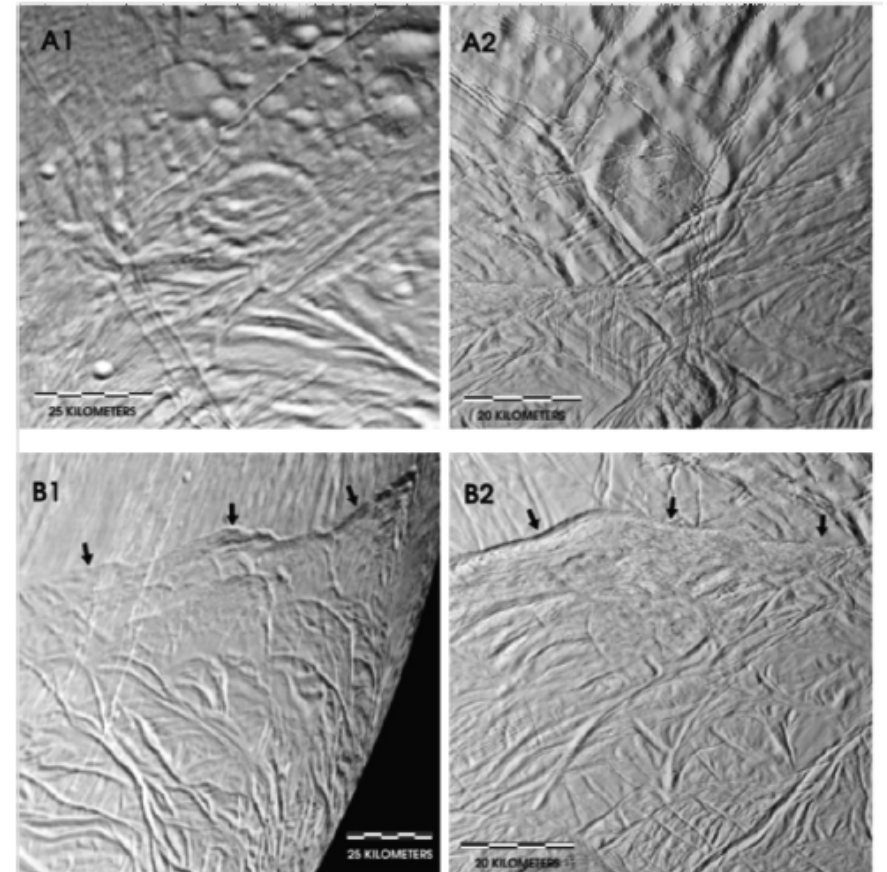
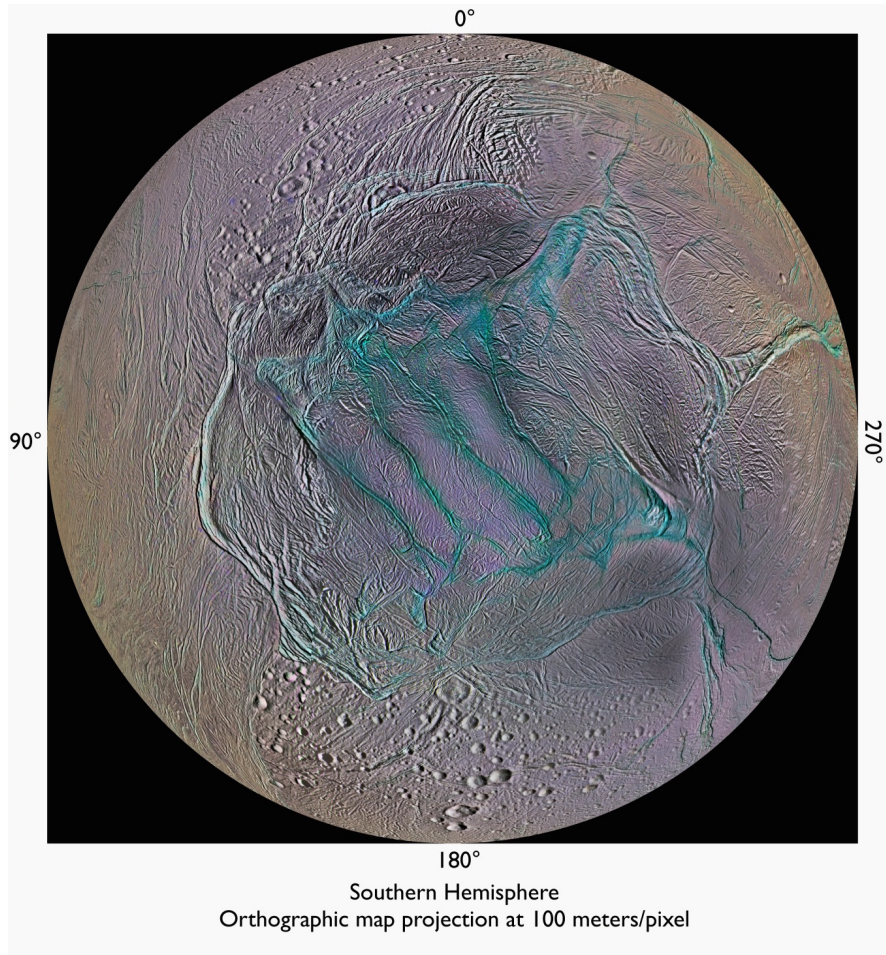


Enceladus



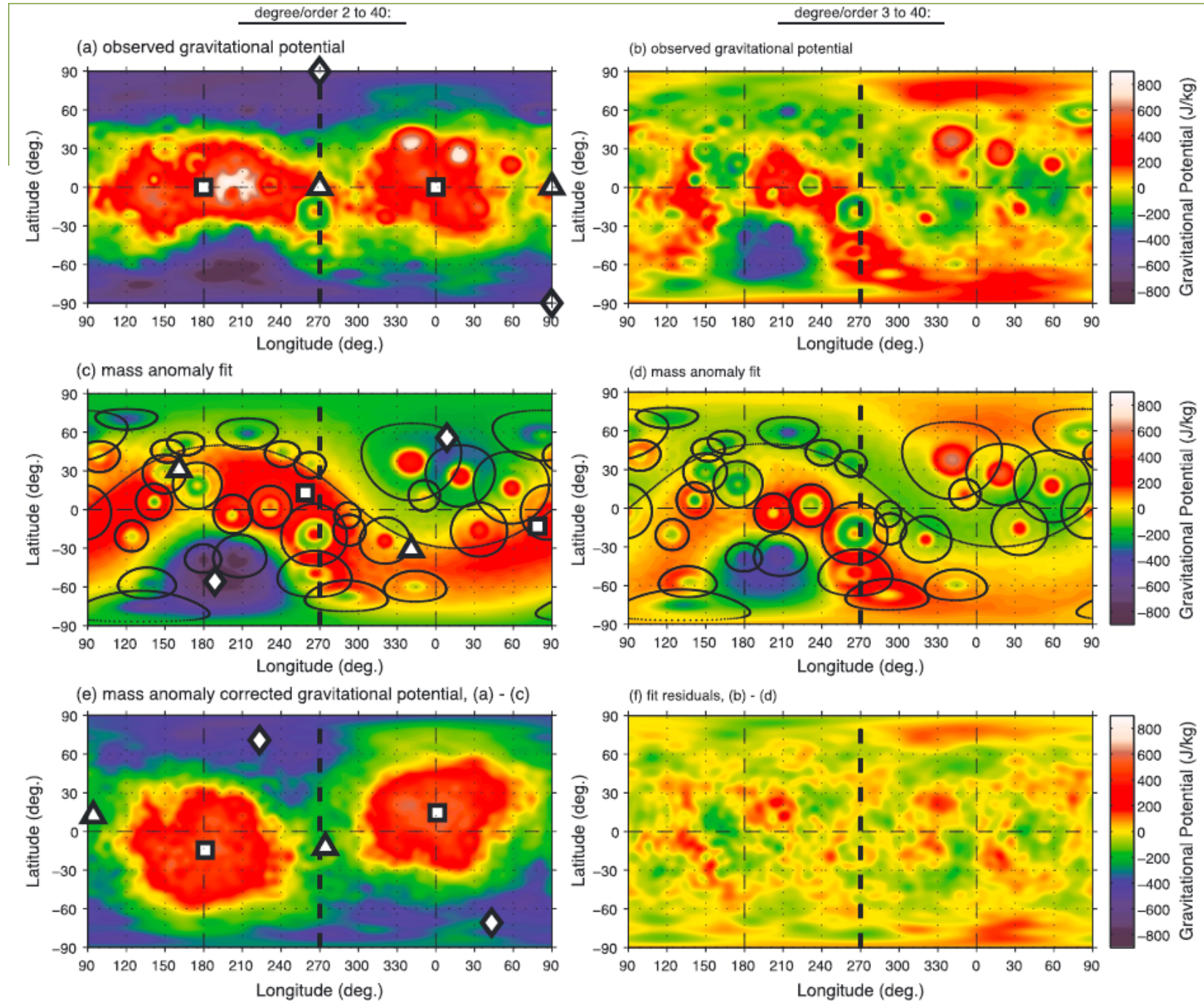
Tajeddine et al. (2017): True polar wander of Enceladus from topographic data

Enceladus

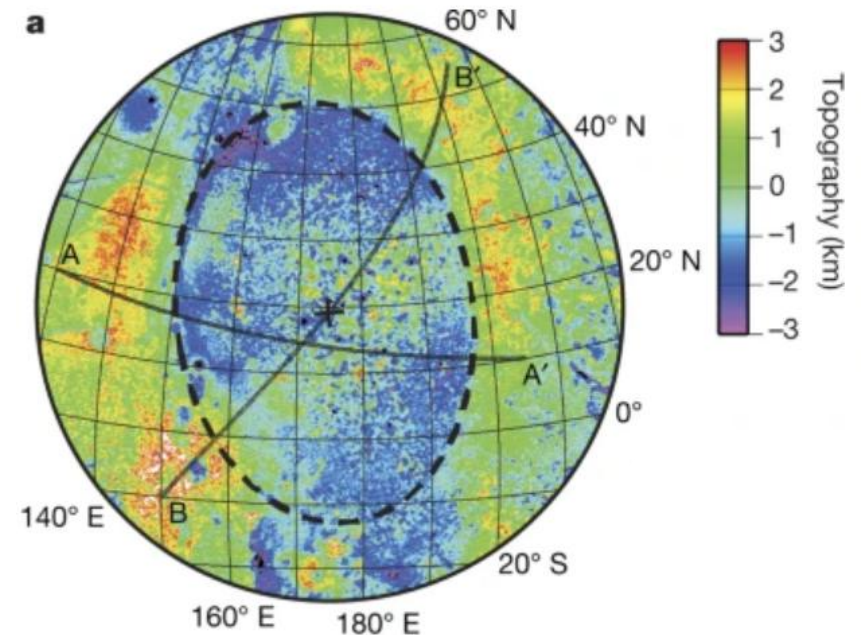
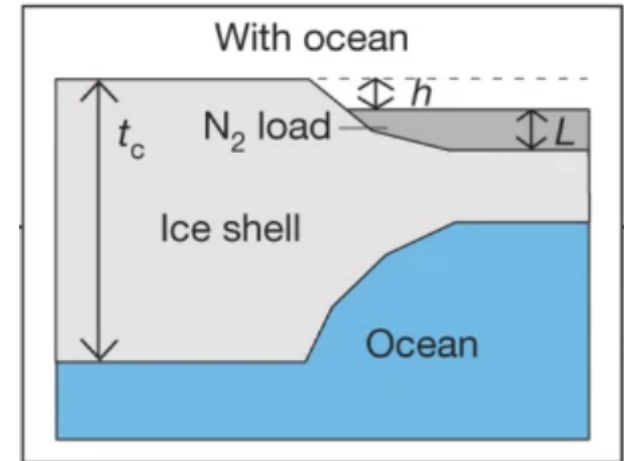
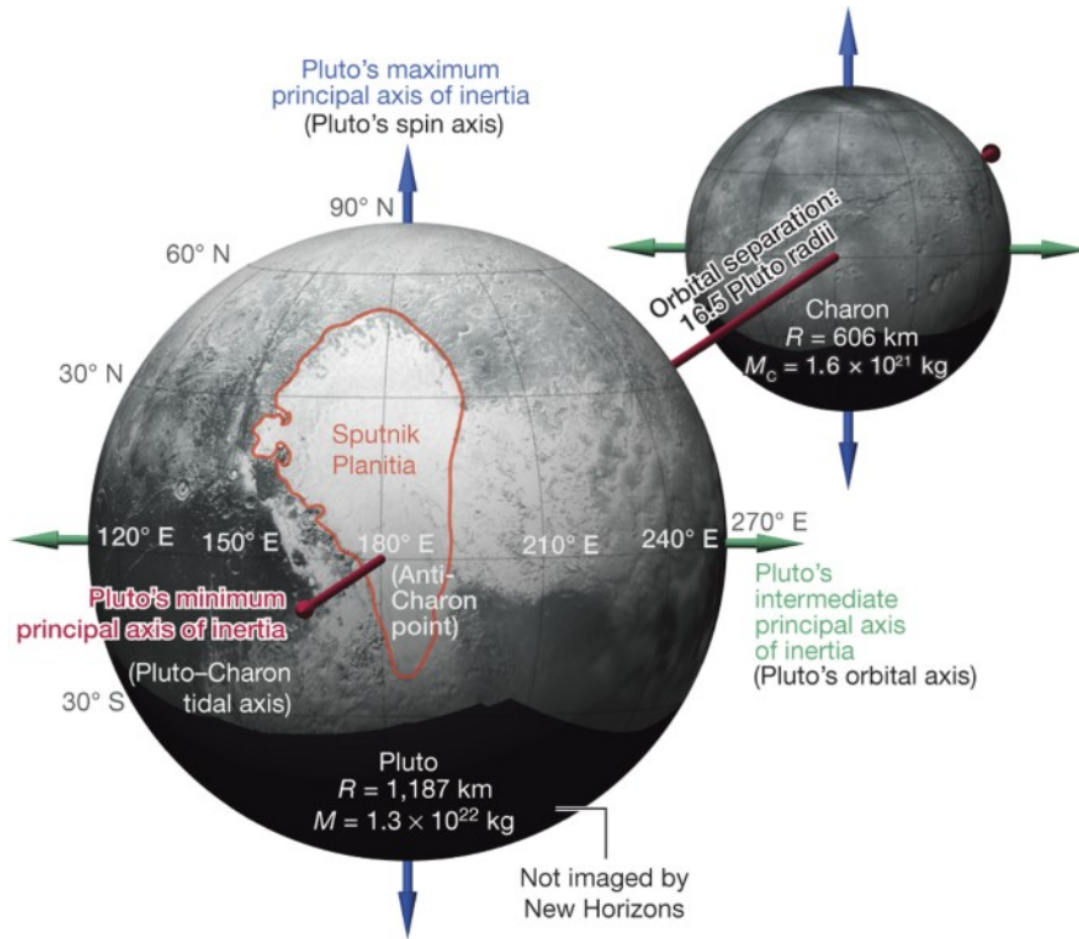


Tajeddine et al. (2017): True polar wander of Enceladus from topographic data

The Moon



Pluto

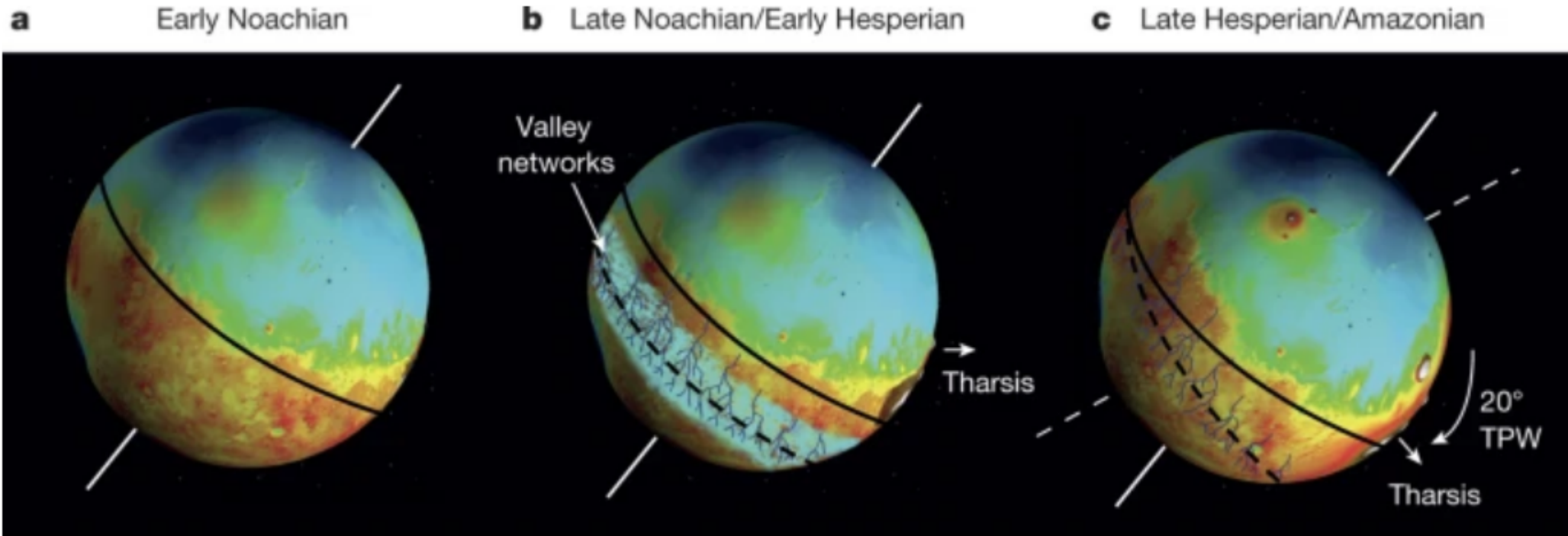
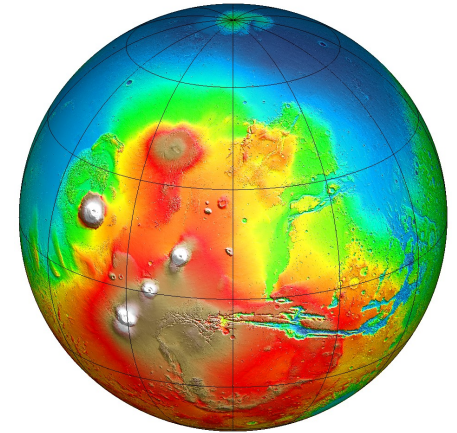


Keane et al., 2016: Reorientation and faulting of Pluto due to volatile loading within Sputnik Planitia (Nature)

Nimmo et al., 2016: Reorientation of Sputnik Planitia implies a subsurface ocean on Pluto (Nature)

Mars

Bouley et al. (2016, Nature): Late Tharsis formation and implications for early Mars



Formation of the dichotomy, heavy bombardment, dichotomy boundary in equatorial position

Tharsis dome formation, tropical precipitation with valley networks formation

Tharsis formation causes a TPW, valley networks on a small circle, Tharsis bulge on the equator

Large-wavelength convection (on exoplanets)

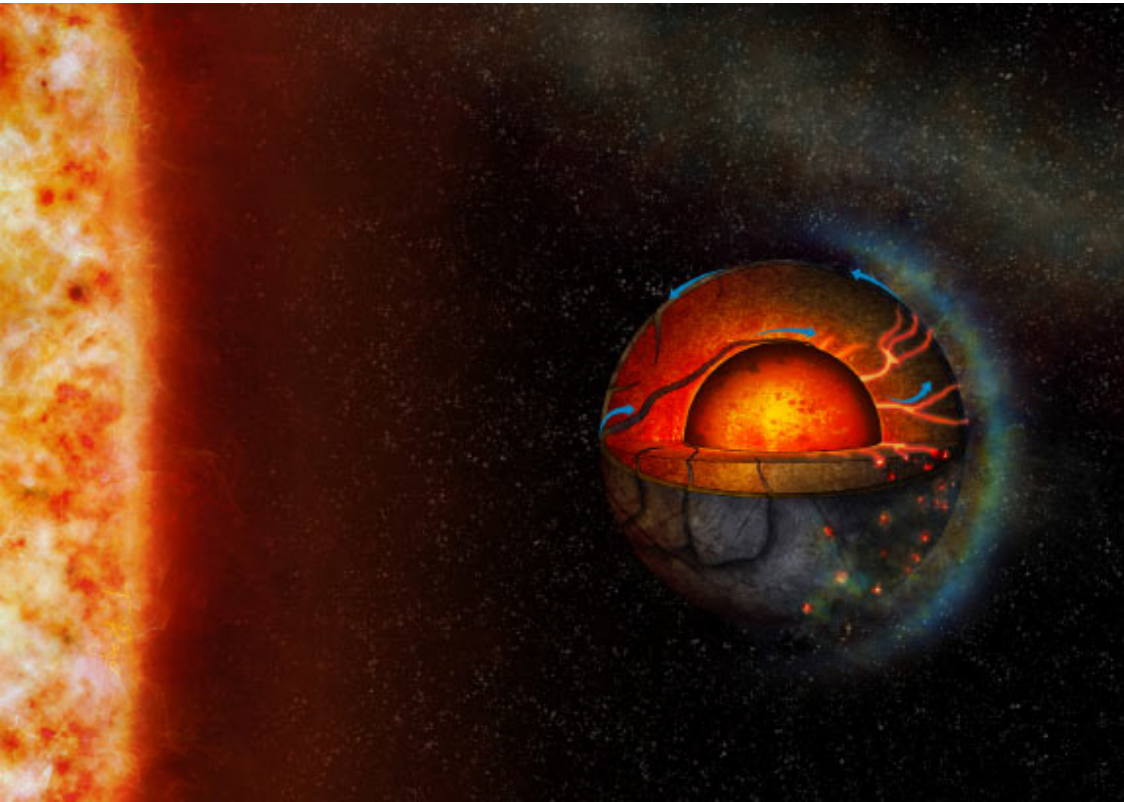
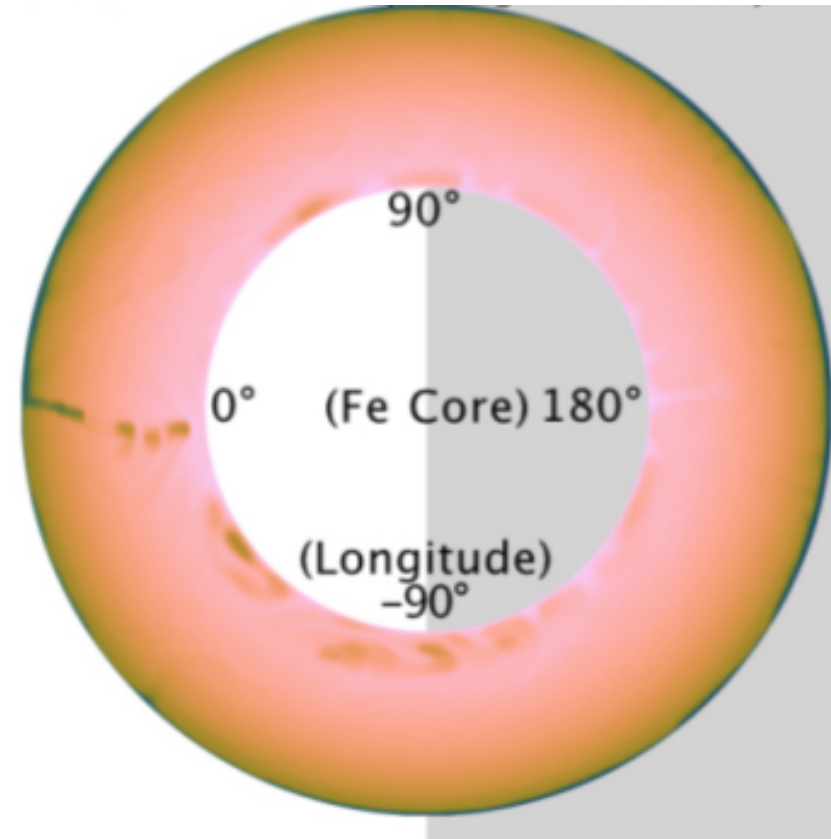


Image: Thibault Roger



Meier et al. (2021): Hemispheric tectonics

Slow rotating bodies: Venus

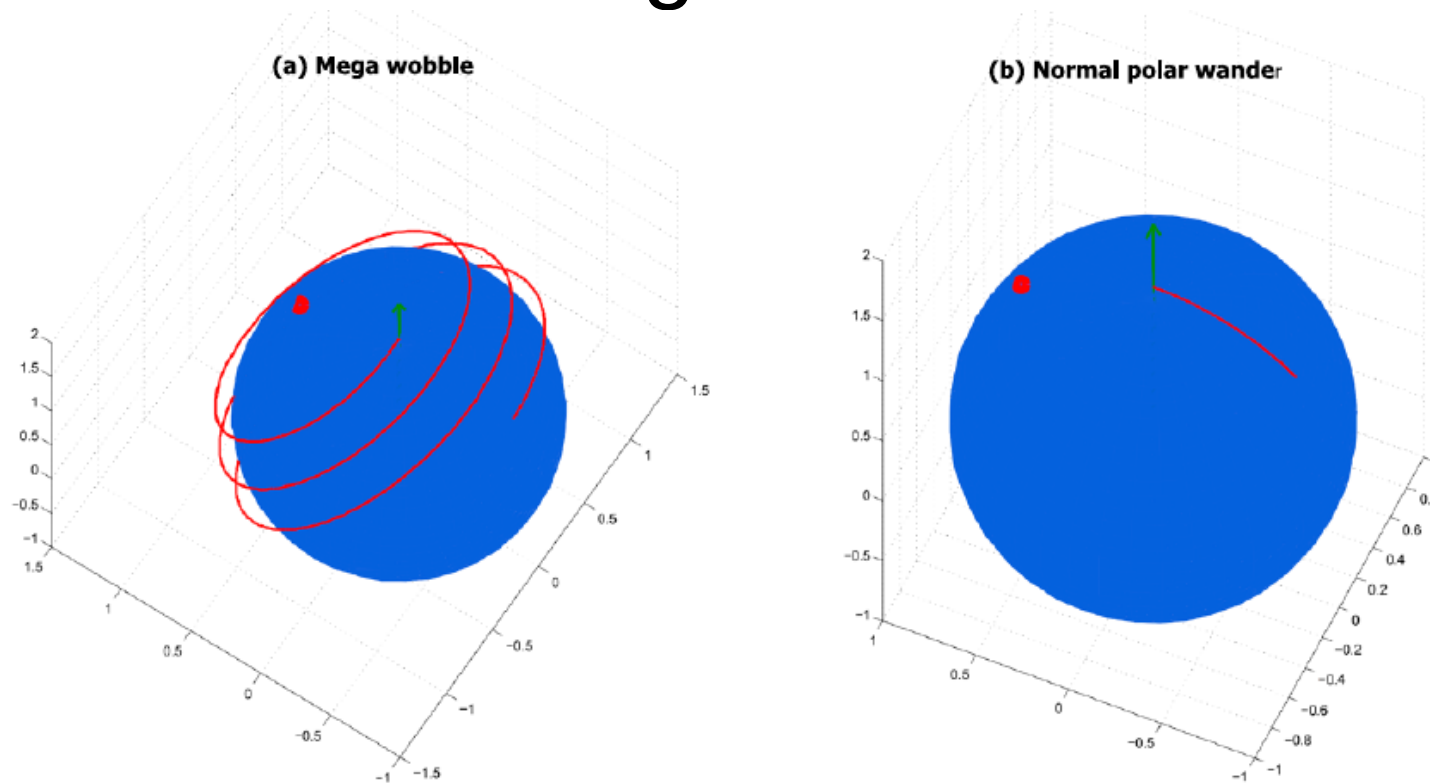
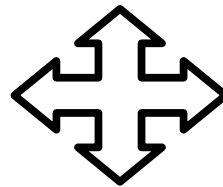
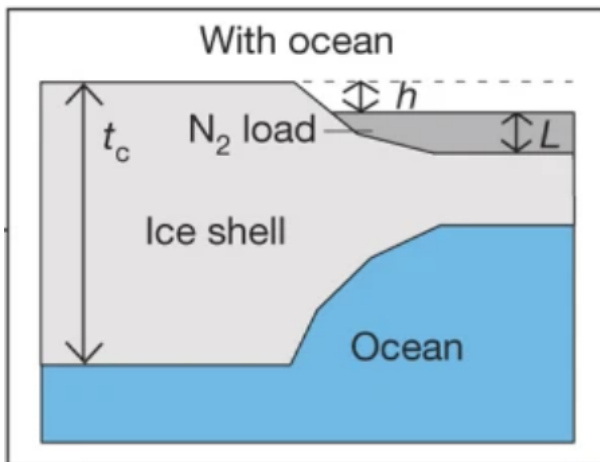
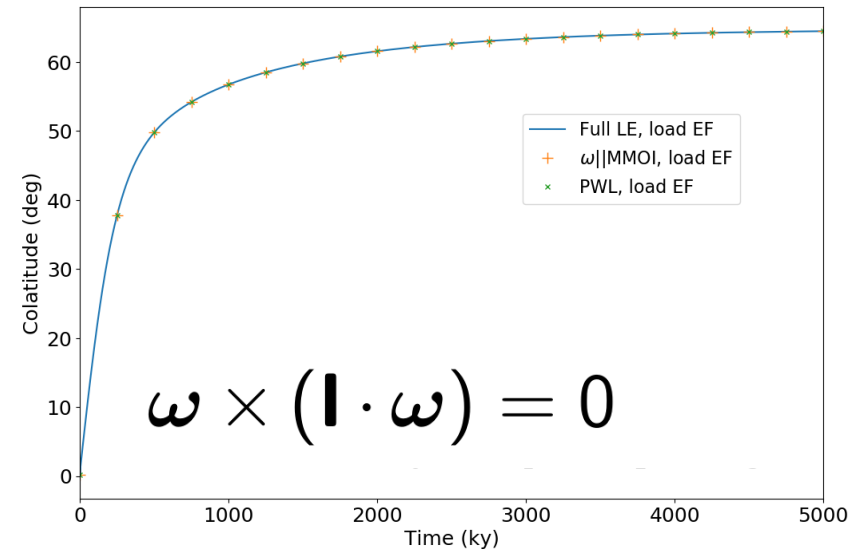
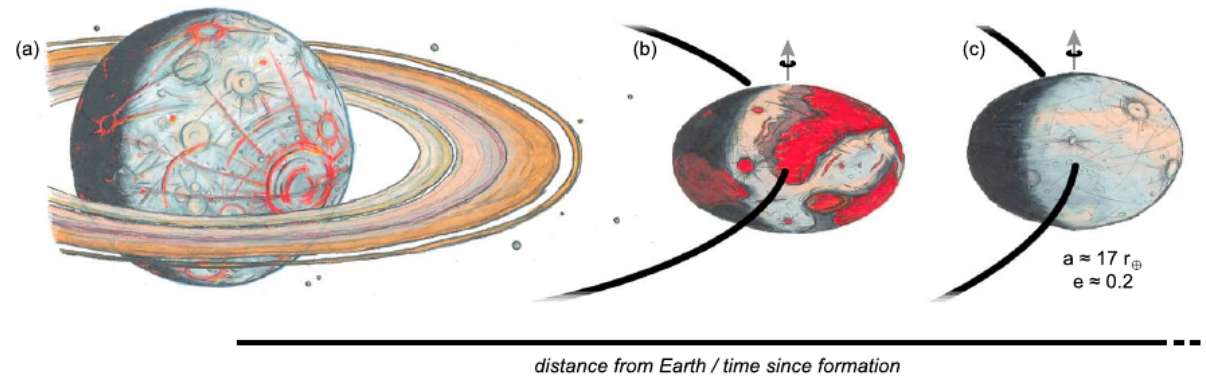


Figure 5. Two types of polar wander path. The green arrow is the initial position of the rotational axis and the red dot is the point mass load.

Research perspectives

- TPW rates can vary by orders of magnitude **and should be checked**



<-----/-----
 <-----xxx/-----
 <-----xoooVox-----
 <-----xooo~/~oooX-----
 <-----xo/oooX-----/ \-----
 <-----/xxx-----\o/-----
 <-----/-----^-----

Research perspectives