

Settling of inertial particles in turbulent Rayleigh-Bénard convection

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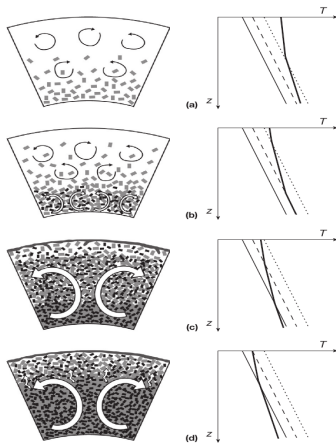
Early Earth

The Earth's mantle initially formed a global magma ocean



- Accretion (no radiation: $\sim 10^5$ K, full radiation: $\sim 10^2$ K)
- Core segregation: $\Delta T \sim 1700$ K
- Moon forming impact: $T_{\text{final}} \sim 7000$ K
- Short lived isotopes (Al, K, ...)

Crystallization process



From Solomatov (2015), ToG

Onset of crystallization is given by intersection of the **adiabat** with the **liquidus**. **Lateral variations** matter.

Rheological transition separates liquid and solid-state convection.

Fractional vs equilibrium crystallization?

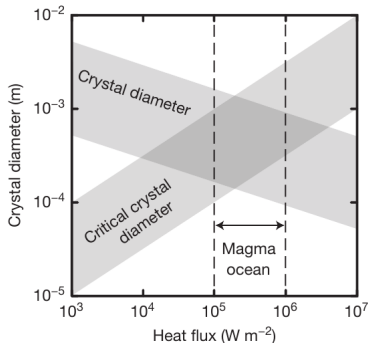
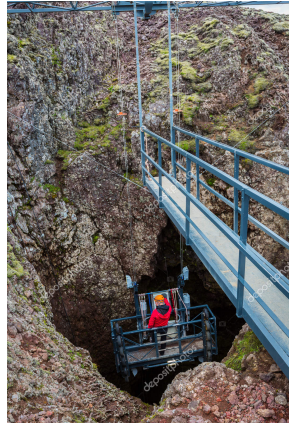
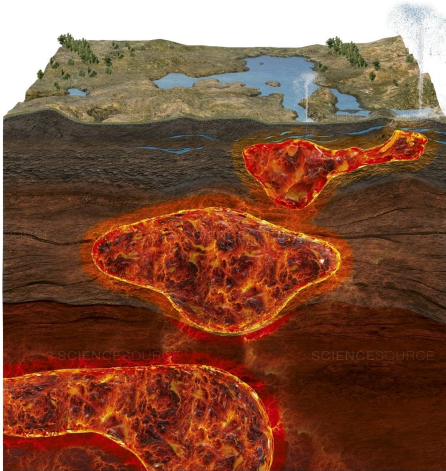


Figure 8 The crystal size during the early stages of crystallization of the magma ocean (assuming that it is controlled by nucleation in the downwelling flow) and the critical crystal size for suspension are shown as functions of the heat flux. The width of the curves (gray bands) represents the uncertainty range of one order of magnitude.

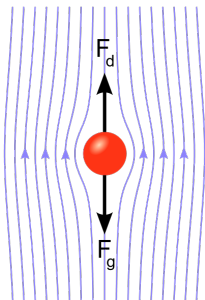
Key question:
cooling time vs
residence time

← Based on the
assumption of
re-entrainment:
part of the available
convective energy
lifts the sediments

Magma chambers



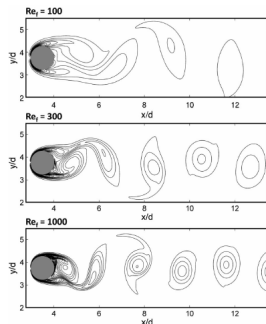
Particle settling - overview



Stokes' law

Depending on the particle Reynolds number, $Re_p := v_t r_p / \nu$, sinking is in either laminar or turbulent regime

← **laminar**, turbulent →

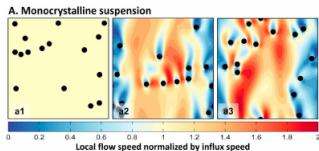


Suckale et al., 2012

Existing methods

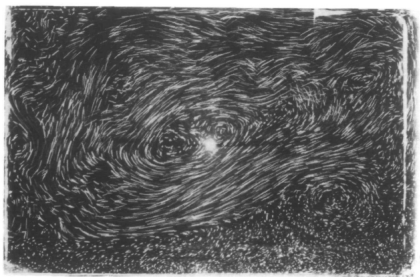
- o Macroscopic methods that incorporate particles via Lagrangian tracers
- o **Semi-analytical and numerical methods based on Basset-Boussinesq-Oseen equation**
- o Direct numerical simulations of fluid-solid systems

Verhoeven & Hansen, 2009, Suckale et al., 2012, Maas & Hansen, 2015, 2019

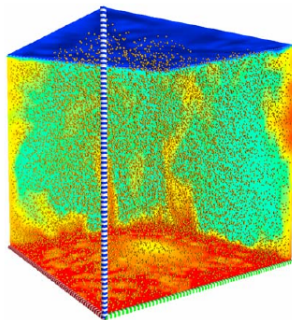


What happens with small particles?

They behave like tracers and are vigorously mixed, but do they settle? M & N (1989): $N(t) = N_0 \exp(-v_t t/H)$ vs $N(t) = N_0$



Martin & Nokes, 1989



Verhoeven et al. (2009)
Maas & Hansen (2015, 2019)

Governing equations: fluid

In terms of non-dimensional quantities, NVS read:

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \sqrt{\frac{Pr}{Ra}} \nabla^2 \mathbf{u} + \theta \hat{\mathbf{z}} \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (2)$$

$$\partial_\tau \theta + (\mathbf{u} \cdot \nabla) \theta = \frac{1}{\sqrt{PrRa}} \nabla^2 \theta, \quad (3)$$

where Ra and Pr are the Rayleigh and Prandtl number that control the flow characteristics:

$$Ra := \frac{\alpha g \Delta T H^3}{\nu \kappa}, \quad Pr := \frac{\nu}{\kappa}. \quad (4)$$

Ra of magma ocean

For the Rayleigh number one gets roughly ($\eta = 0.1$ Pa s)

$$Ra = \frac{\alpha g \Delta T H^3}{\kappa \nu} = \frac{5 \cdot 10^{-5} \cdot 9.8 \cdot 1 \cdot 2.89^3 \cdot 10^{18}}{5 \cdot 10^{-7} \cdot 2.5 \cdot 10^{-5}} \simeq 10^{27}.$$

and the Prandtl number $Pr = \nu/\kappa \simeq < 5, 50 >$.

A similar problem, i.e. parameters that cannot be reached in direct numerical simulations, is faced in MHD community when dealing with core dynamics.

Governing equations: particles

Simplified Maxey-Riley equation (Basset-Boussinesq-Oseen eq.):

$$\frac{d\mathbf{v}}{dt} = \beta \frac{D\mathbf{u}}{Dt} + \frac{1}{St}(\mathbf{u} - \mathbf{v}) + \Lambda \hat{\mathbf{z}}, \quad (5)$$

Non-dimensional parameters in control of the particle dynamics:

$$\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}; \quad St = \frac{r_c^2 \sqrt{\alpha g \Delta T}}{3\nu\beta\sqrt{H}}; \quad \Lambda = \frac{\beta-1}{\alpha\Delta T}. \quad (6)$$

Governing equations: particles

Simplified Maxey-Riley equation:

$$\frac{d\mathbf{v}}{dt} = \beta \frac{D\mathbf{u}}{Dt} + \frac{1}{St}(\mathbf{u} - \mathbf{v}) + \Lambda \hat{\mathbf{z}}, \quad (7)$$

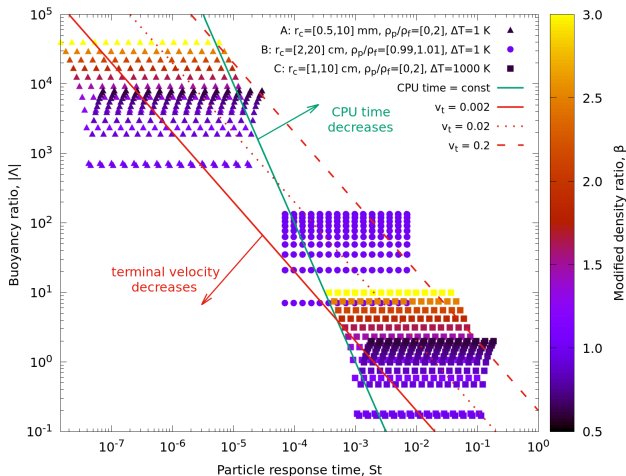
Non-dimensional parameters in control of the particle dynamics:

$$\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}; \quad St = \frac{r_c^2 \sqrt{\alpha g \Delta T}}{3\nu \beta \sqrt{H}}; \quad \Lambda = \frac{\beta - 1}{\alpha \Delta T}. \quad (8)$$

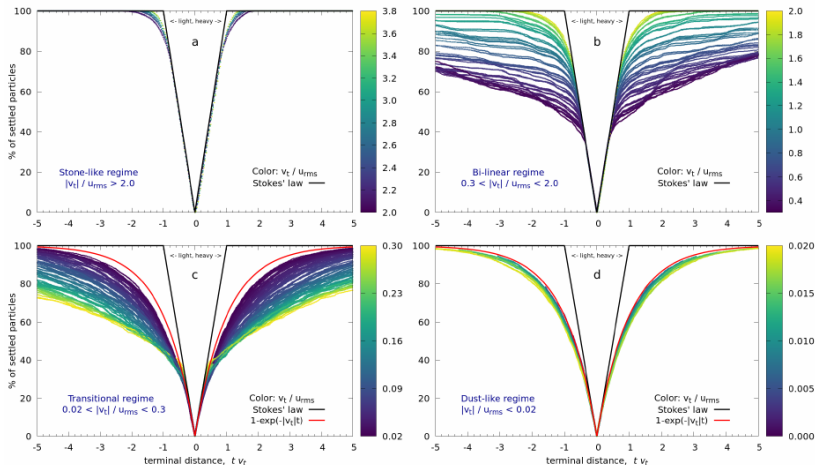
Rest of the presentation: Application of macroscopic model of thermal convection with BBO (resp. MR) approach to modelling the solid phase <https://arxiv.org/pdf/2005.05448.pdf>.

Model parameters: the 5D jungle

Only two control parameters are related to the particles: r_p and ρ_p



Results: Settling curves, colour = v_t / u_{rms}

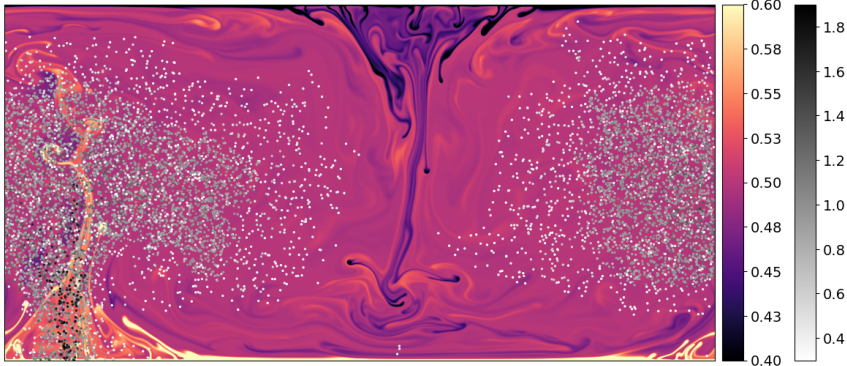


x-axis: Terminal distance: $v_t t$

Regimes: Slow belt

(Settling curves that fall below the analytical models)

Temperature field and suspended particles. Time $t=12.9$, $Ra = 10^{10}$, $Pr = 50$.



Our model: Poisson process?

$$\text{M\&N(1989)} : \frac{dN}{dt} = -A v_t c(0), \quad c(0) = N/AH \quad \Rightarrow \quad N(t) = N_0 \exp\left(\frac{-v_t t}{H}\right)$$

$$dN = -\lambda N(t) dt.$$



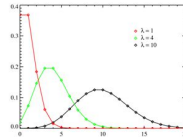
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$$dN = -(1 - \mathcal{P}_e) dN_c \stackrel{?}{=} -N v_t dt.$$

Poisson distribution

- Distribution for: how many light bulbs will burn out in a month?
 - Large building.
 - Maintenance department immediately replaces burned out bulb.
 - Choose λ = the average number that burn out in a month



How many mathematicians does it take to change a light bulb?

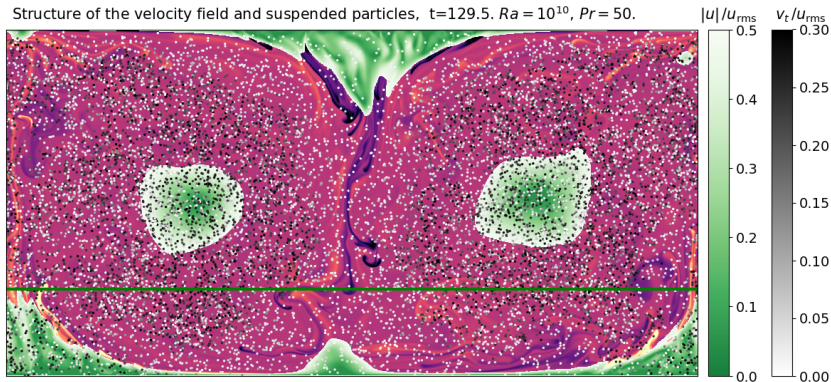
In earlier work, Haynor[1] has shown that one mathematician can change a light bulb.



$$dN_c = N dt/t_c$$

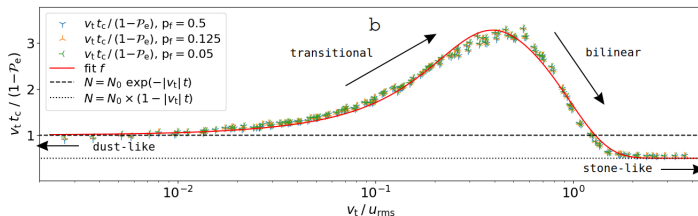
$$dN = -(1-\mathcal{P}_e) dN_c \stackrel{?}{=} -N v_t dt$$

Structure of the velocity field and suspended particles, $t=129.5$, $Ra = 10^{10}$, $Pr = 50$.



Our model: probabilistic approach

$$-dN = (1 - \mathcal{P}_e) dN_c = (1 - \mathcal{P}_e) \frac{N dt}{t_c} = \frac{(1 - \mathcal{P}_e)}{v_t t_c} N v_t dt$$

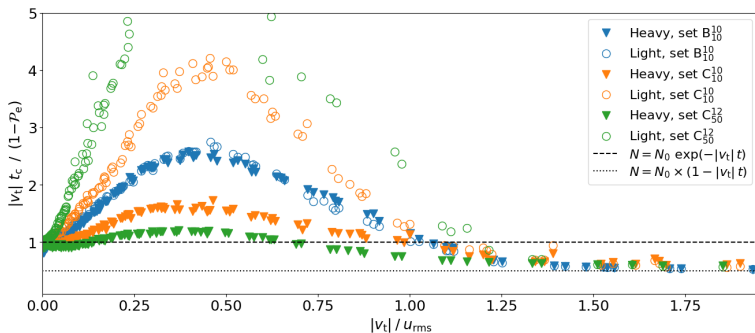


$$\frac{t_c |v_t|}{1 - \mathcal{P}_e} = f(|v_t|/u_{rms}) := 0.5 + \frac{A}{\sigma\sqrt{2\pi}} \left[1 + \operatorname{erf} \left(\frac{\lambda(x - \mu)}{\sigma\sqrt{2}} \right) \right] \exp \frac{-(x - \mu)^2}{2\sigma^2}$$

$$\frac{dN}{dt} = \frac{-|v_t| N}{f(|v_t|/u_{rms})} \Rightarrow N(t) = N_0 \exp \left(\frac{-|v_t| t}{f(|v_t|/u_{rms})} \right).$$

Distribution f

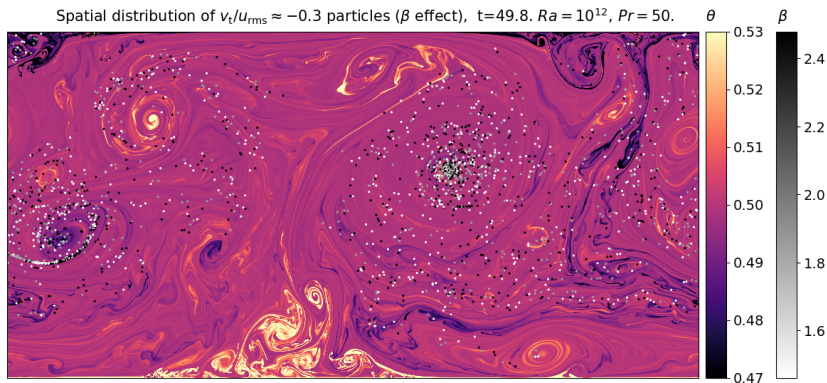
The f function depends on Ra , Pr and is different for light and heavy particles



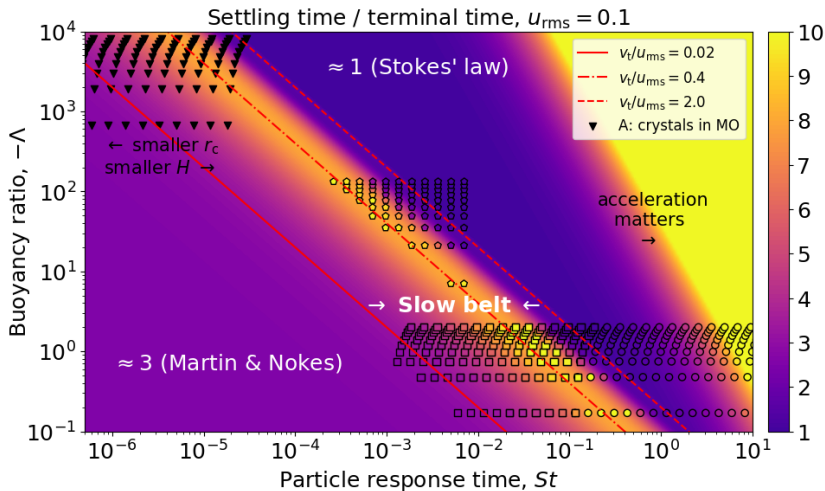
Light particles get trapped

The f distribution is larger for light particles

Spatial distribution of $v_t/u_{rms} \approx -0.3$ particles (β effect), $t=49.8$, $Ra = 10^{12}$, $Pr = 50$.



Residence time



How to use our model?

- 1 Compute the Stokes velocity of your particle
- 2 Estimate mean velocity of your thermal flow
- 3 Compute Ra and Pr of your flow
- 4 Look for parameters of $f(v_t/u_{\text{rms}})$
- 5 Average settling: $N(t) = N_0 \exp [(-v_t t)/f]$

Long story short

Results: residence time $\sim$ terminal time

We neglect: rotation and re-entrainment

► Watch movies