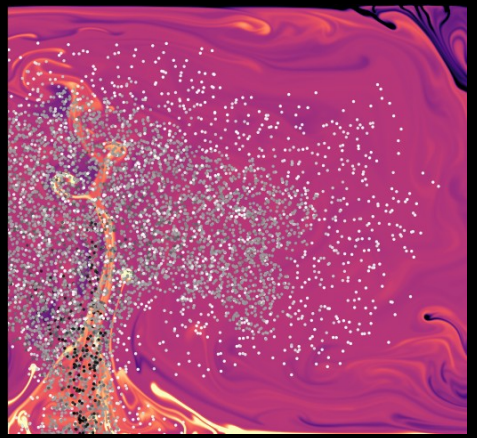
Inertial particles in a convecting fluid: Implications for magma reservoirs

Vojtěch Patočka
(patocka.vojtech@gmail.com)

Nicola Tosi and Enrico Calzavarini

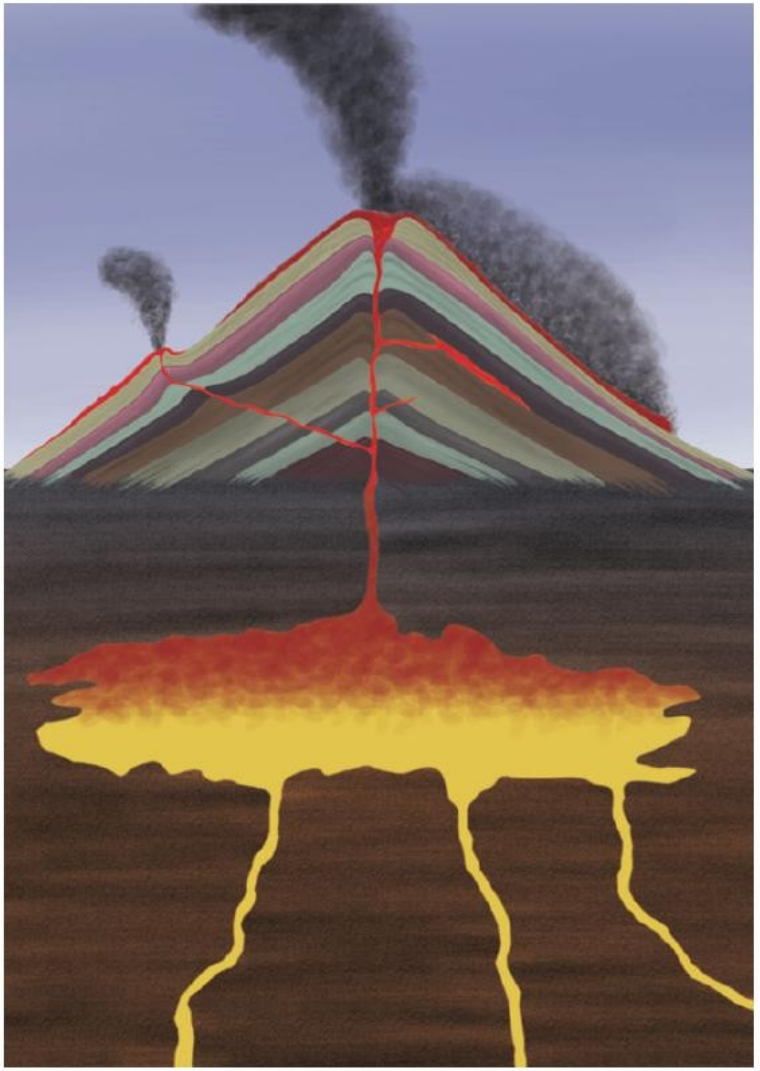




Content

- Magma chamber paradigm
- Settling of inertial particles in turbulent Rayleigh-Bénard convection
- Unified model of residence time
- Future perspectives





Sparks et al. (2018):
Formation and dynamics of magma reservoirs

VS

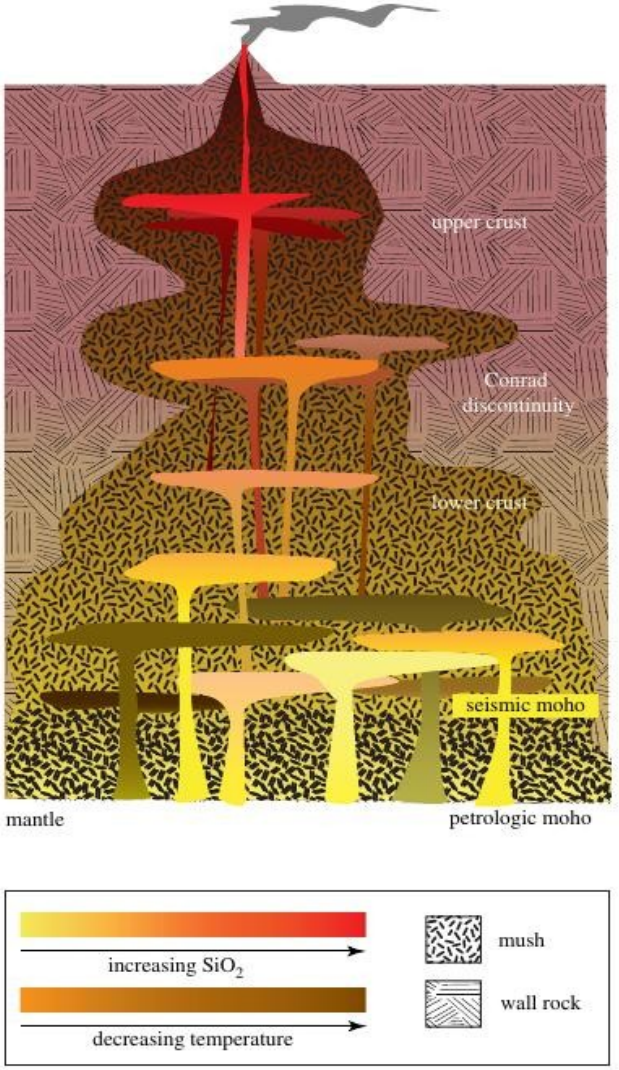
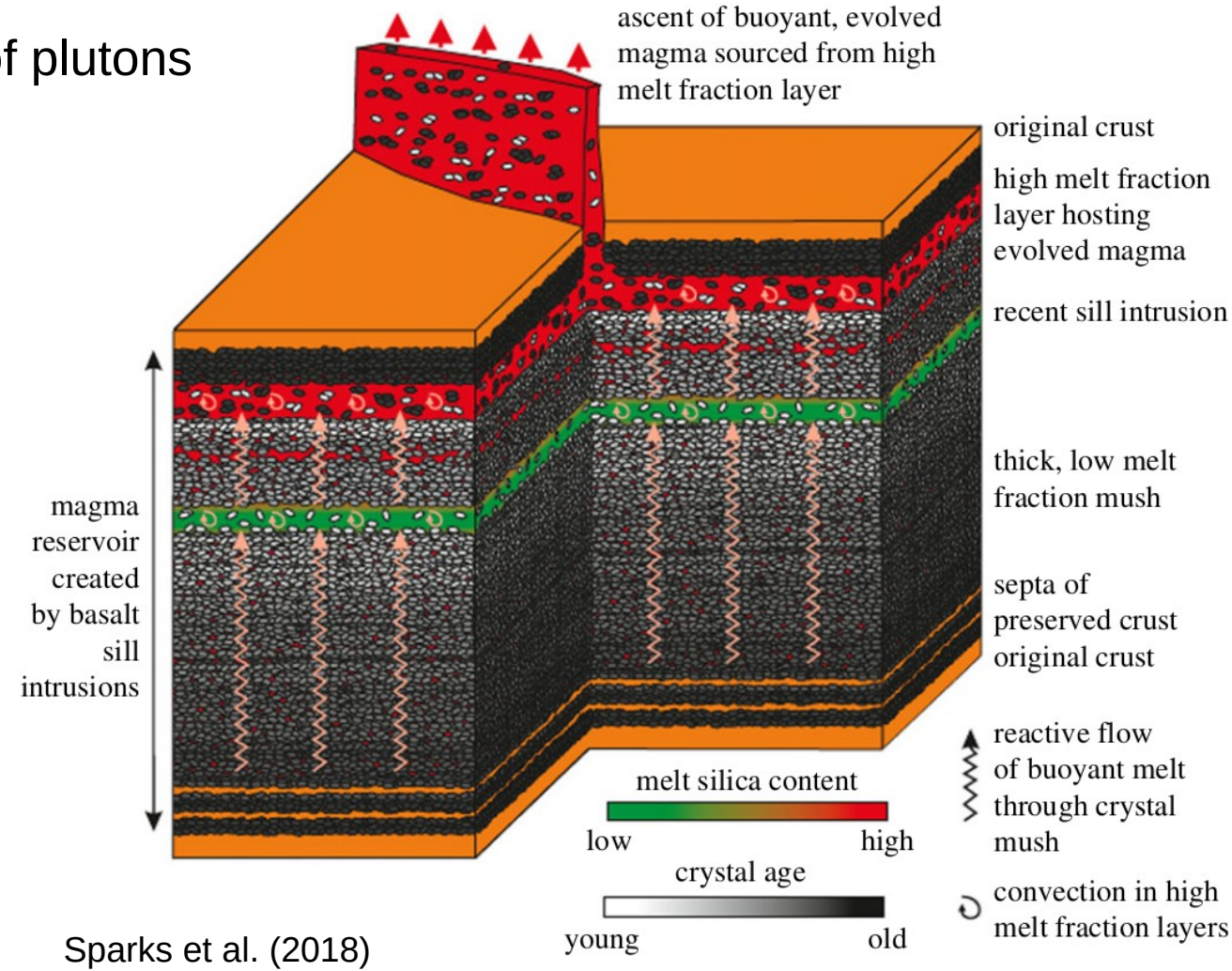


Figure 1. Depiction of a classic magma chamber beneath a volcano in which the chamber is envisaged as a melt-dominated

- igneous differentiation
- refractory characteristics of plutons
- crystal-melt separation

Chamber vs Mush

- tomography/EM sounding
- dating of zoned crystals
- long-lasting mush, ephemeral chamber



Sparks et al. (2018)

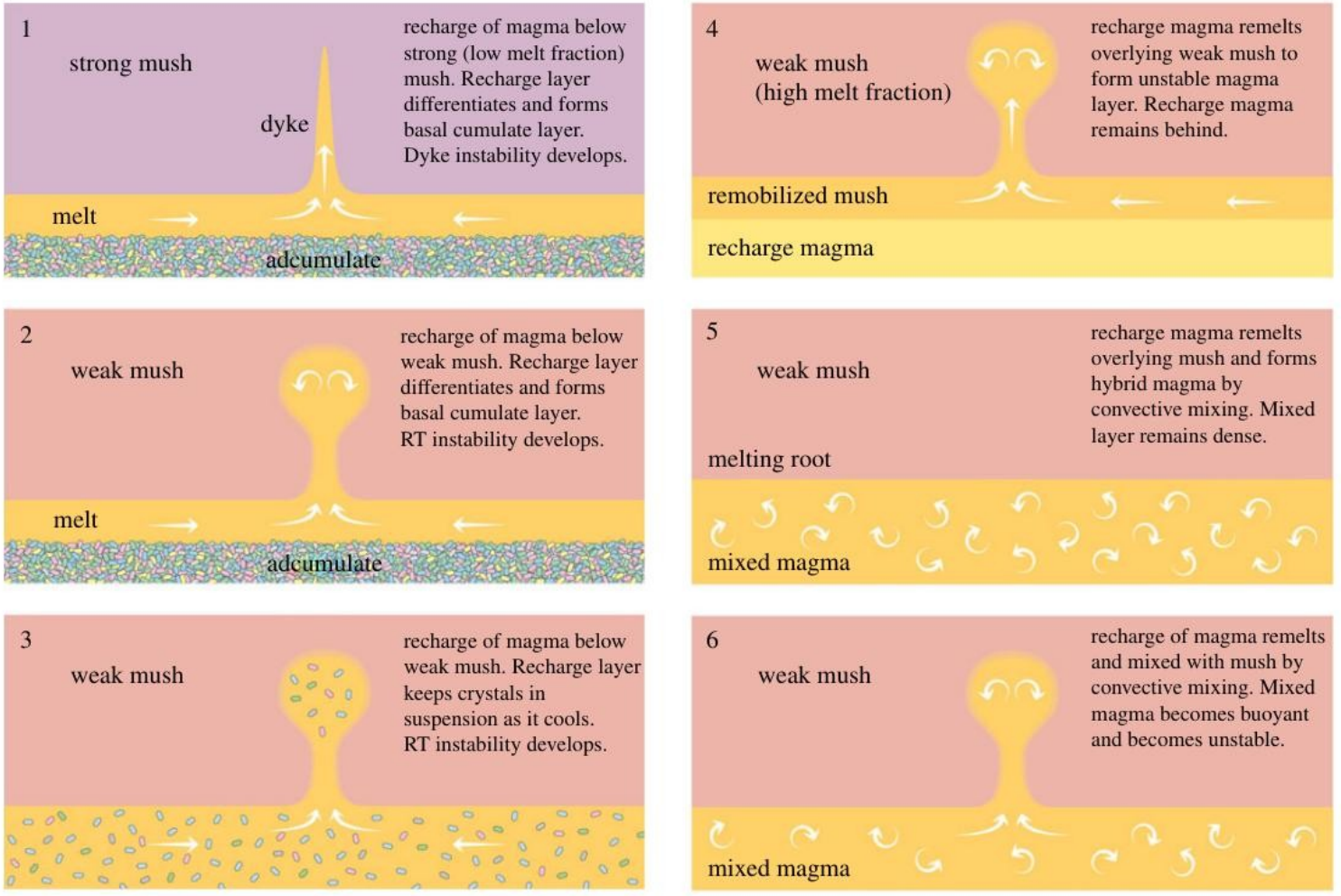
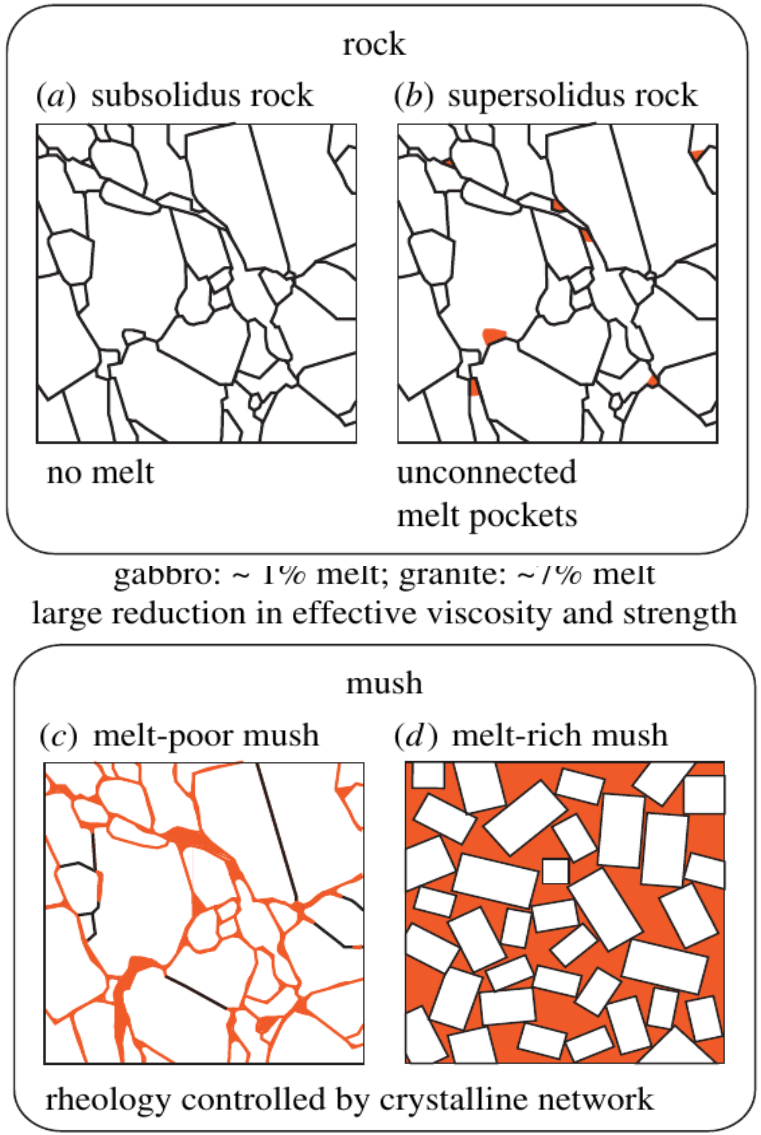
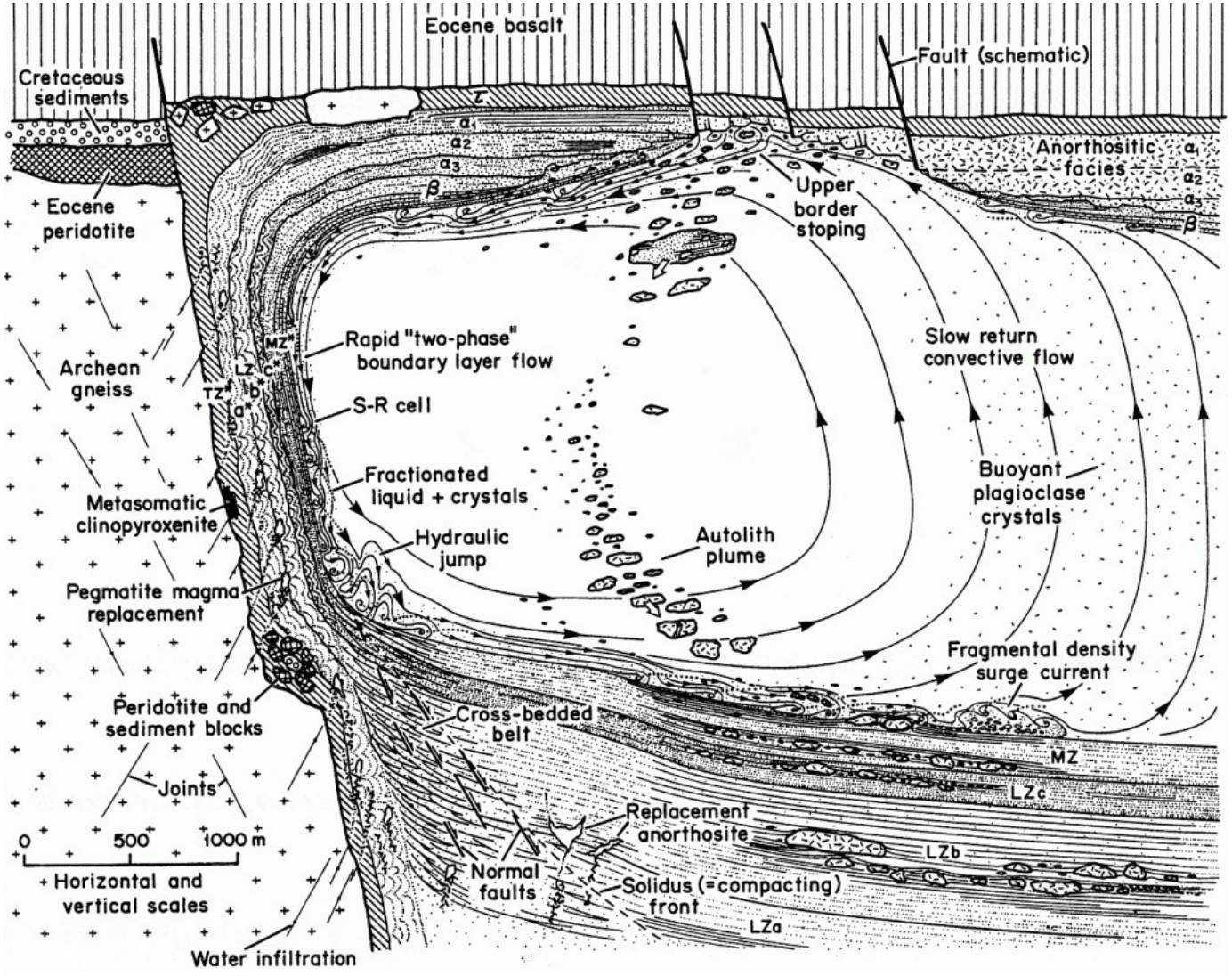


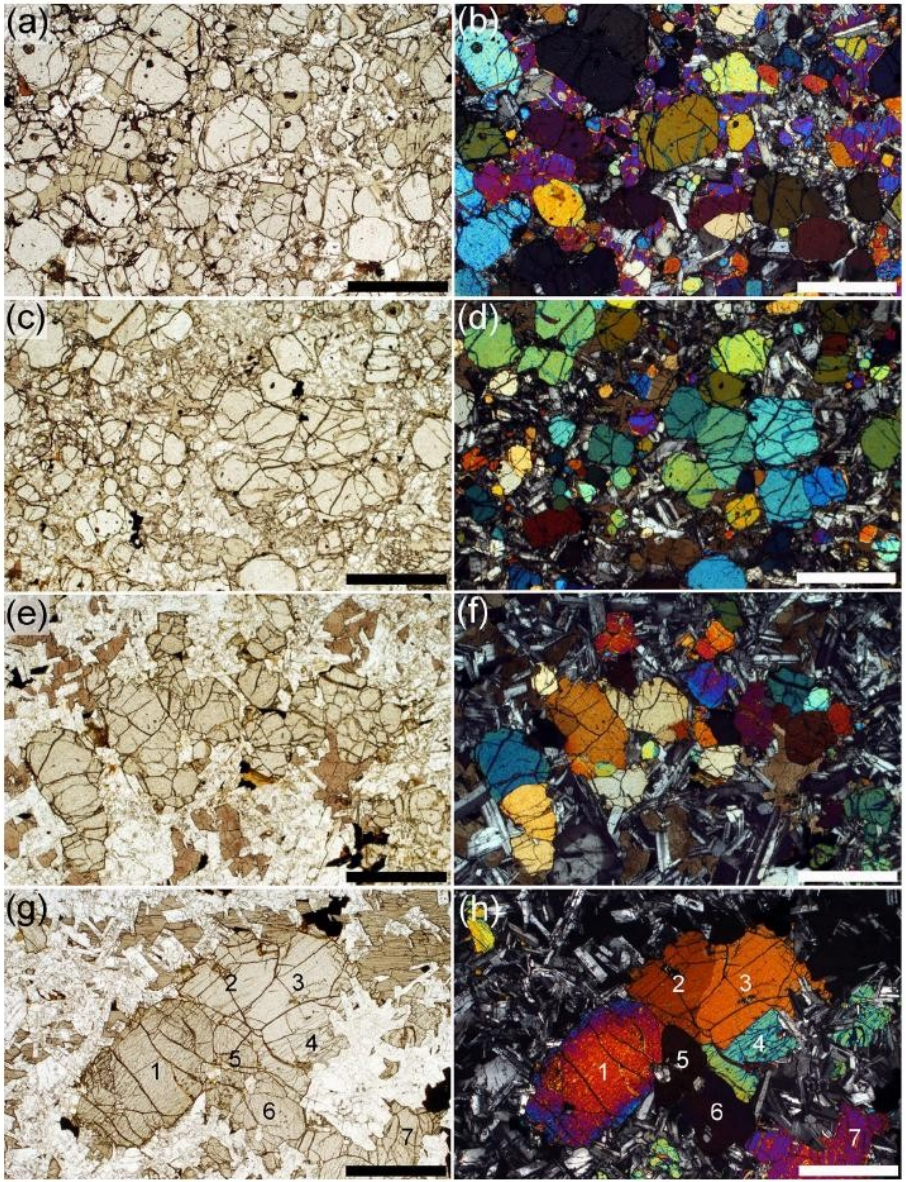
Figure 7. Cartoons of mush replenishment showing some variants of possible scenarios (see text and labels).

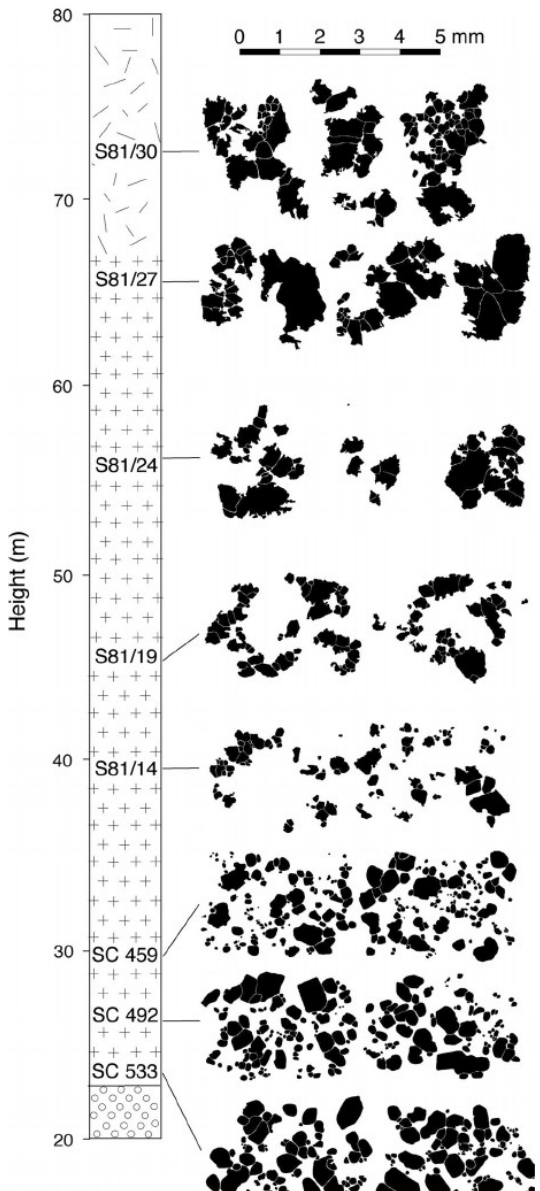
Sparks et al. (2018)

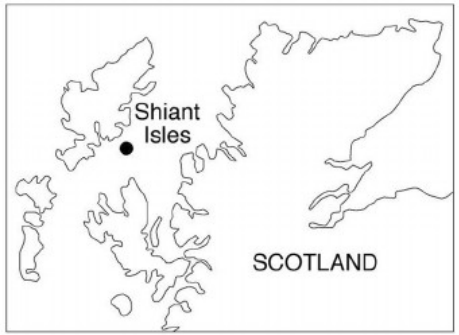


Sparks et al. (2018)


Eocene basalt
Fault (schematic)
Cretaceous sediments
τ
α1
α2
α3
β
Anorthositic facies
Upper border stoping
Eocene peridotite
Archean gneiss
MZ*
LZ c*
TZ b*
a*
Rapid "two-phase" boundary layer flow
S-R cell
Slow return convective flow
Fractionated liquid + crystals
Buoyant plagioclase crystals
Metasomatic clinopyroxenite
Hydraulic jump
Autolith plume
Pegmatite magma replacement
Fragmental density surge current
Peridotite and sediment blocks
Cross-bedded belt
MZ
LZc
Joints
Replacement anorthosite
LZb
0
500
1000 m
Normal faults
Solidus (=compacting) front
Horizontal and vertical scales
LZa
Water infiltration









Holness et al. (2017):
Crystal settling and convection in the Shiant Isles Main Sill

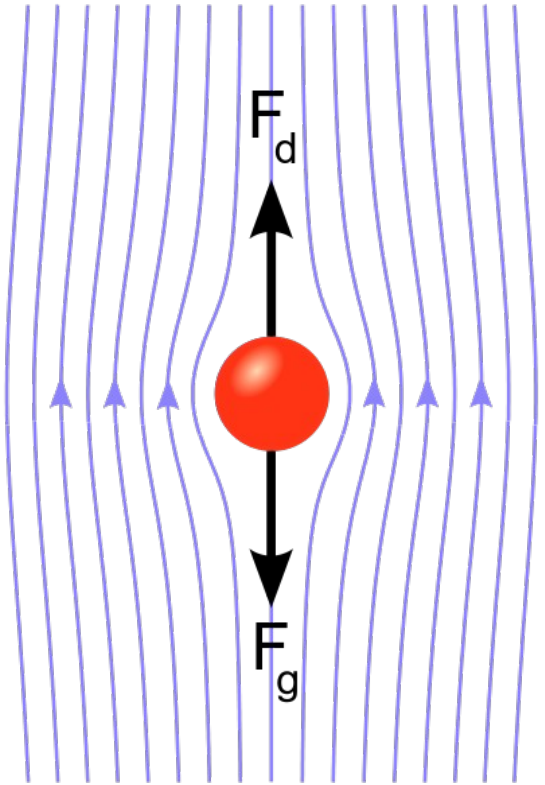


Magma oceans / chambers

Table 1
Model parameters used for evaluating St, Λ, and β.

Parameter	Symbol	Value	Units
Mantle depth	H	2890	km
Grav. acceleration	g	9.8	m/s^2
Thermal expansivity[a]	α	$5{\cdot}10^{-5}$	K^{-1}
Thermal diffusivity[b]	κ	$5{\cdot}10^{-7}$	m^2/s
Kinematic viscosity[c]	ν	$[1, 10, 100] \cdot \kappa$	m^2/s
Temperature contrast[d]	ΔT	1	K
Crystal size[a]	r_c	$0.5 - 10$	mm
Density ratio	ρ_p/ρ_f	$0 - 2$	–

Flow past particles

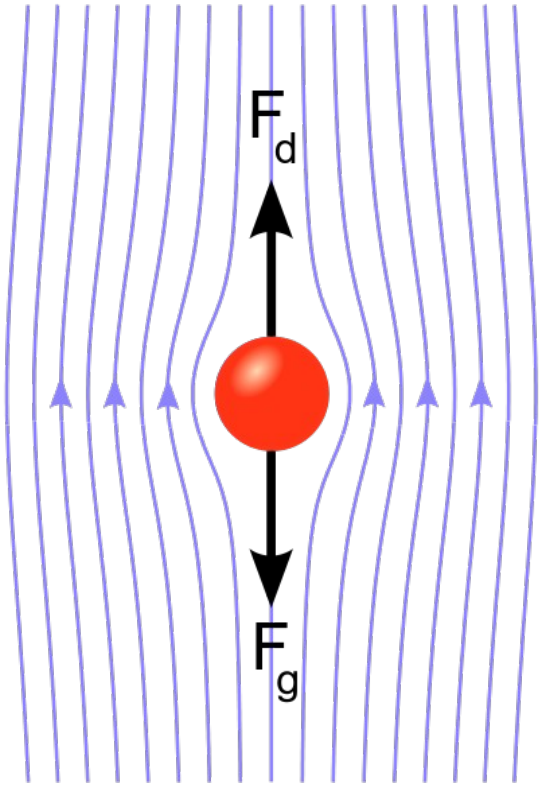


Stokes law:

$$F_d = 6\pi\,\mu\,R\,v$$

$$F_g = (\rho_p - \rho_f)\; g\,\frac{4}{3}\pi\,R^3$$

Flow past particles (focus of this study)



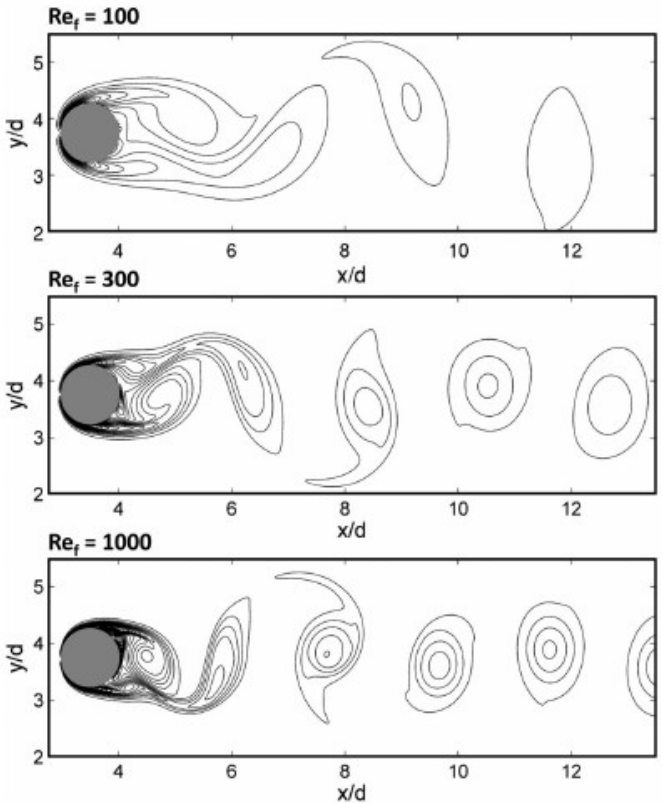
$$Re_p := v_t r_p / \nu$$

← **laminar**, turbulent →

Stokes law:

$$F_d = 6\pi\,\mu\,R\,v$$

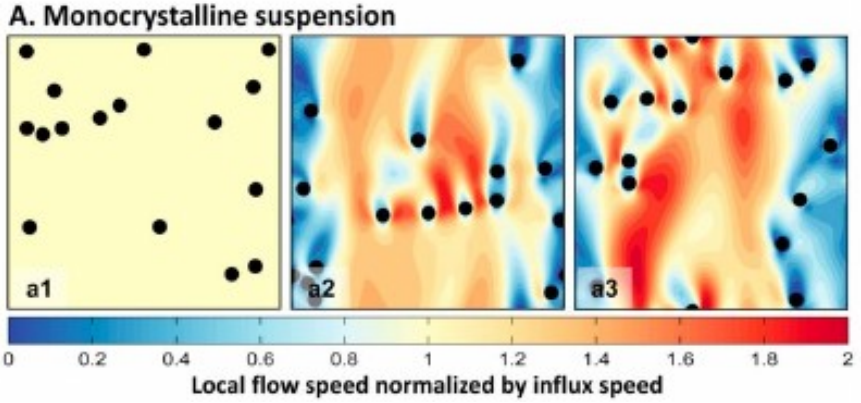
$$F_g = (\rho_p - \rho_f)\; g\,\frac{4}{3}\pi\,R^3$$

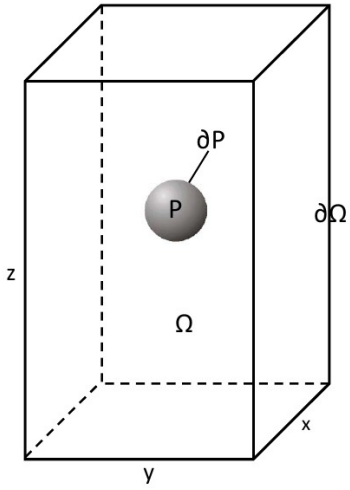


Suckale et al., 2012

Existing numerical methods

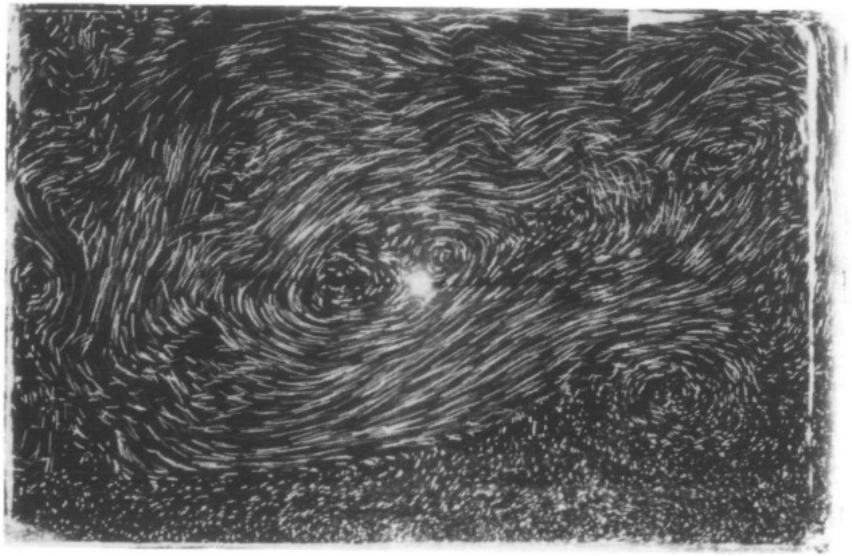
- Macroscopic methods that incorporate particles via Lagrangian tracers
- **Semi-analytical and numerical methods based on Basset-Boussinesq-Oseen eq.**
- Direct numerical simulations of fluid-solid systems
  (Verhoeven & Hansen, 2009, Suckale et al., 2012, Maas & Hansen, 2015, 2019)





Small particle dilemma

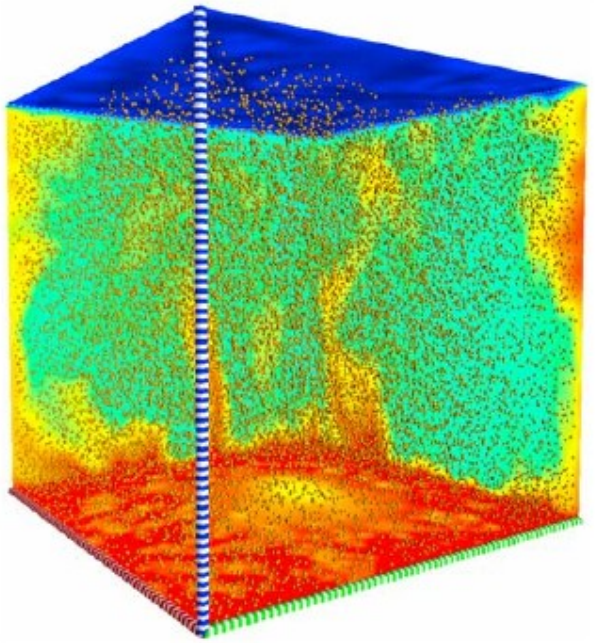
Martin & Nokes, 1989



$$N(t) = N_0 \exp(-v_t t/H)$$

?

Verhoeven et al., 2009
Maas & Hansen, 2015, 2019



$$N(t) = N_0$$

Numerical model

NVS fluid:

$$\begin{aligned} \partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} &= -\nabla p + \sqrt{\frac{Pr}{Ra}}\nabla^2\mathbf{u} + \theta\,\hat{\mathbf{z}} \\ \nabla\cdot\mathbf{u} &= 0 \\ \partial_\tau \theta + (\mathbf{u}\cdot\nabla)\theta &= \frac{1}{\sqrt{PrRa}}\nabla^2\theta, \end{aligned}$$

Particle model: Simplified Maxey-Riley equation (Basset-Boussinesq-Oseen model)

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \beta\frac{D\mathbf{u}}{Dt} + \frac{1}{St}(\mathbf{u}-\mathbf{v}) + \Lambda\,\hat{\mathbf{z}},$$

$$\beta = \frac{3\rho_\mathrm{f}}{\rho_\mathrm{f}+2\rho_\mathrm{p}}; \quad St = \frac{r_\mathrm{c}^2\sqrt{\alpha g\Delta T}}{3\nu\beta\sqrt{H}}; \quad \Lambda = \frac{\beta-1}{\alpha\Delta T}.$$

PHYSICAL REVIEW FLUIDS **5**, 114304 (2020)

Editors' Suggestion

Settling of inertial particles in turbulent Rayleigh-Bénard convection

Vojtěch Patočka [1,*] Enrico Calzavarini [2] and Nicola Tosi [1]

[1] *Institute of Planetary Research, German Aerospace Center (DLR), Berlin, Germany*

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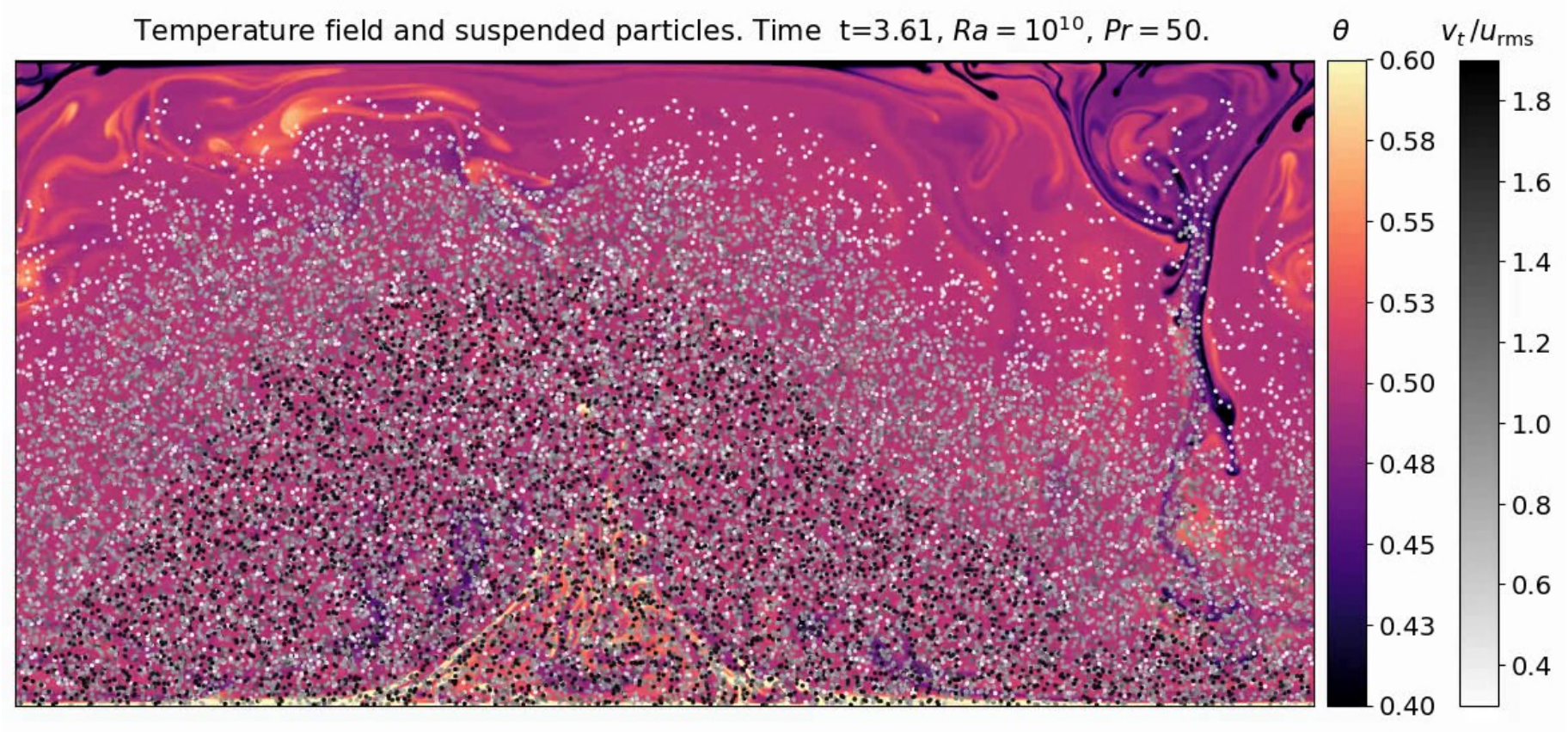
Earth and Planetary Science Letters



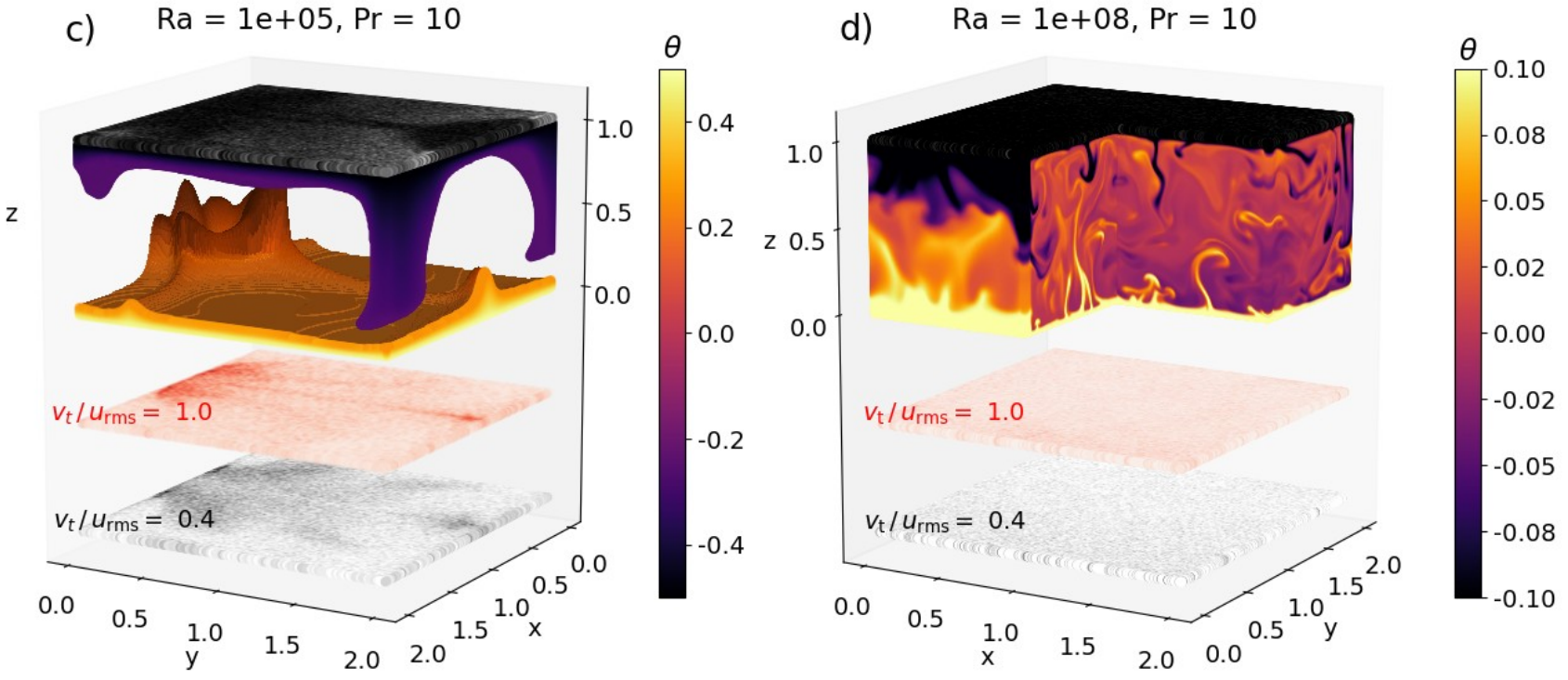
Residence time of inertial particles in 3D thermal convection: Implications for magma reservoirs

Vojtěch Patočka [a,*], Nicola Tosi [b], Enrico Calzavarini [c]

Example simulation (2D)



Results: settling curves

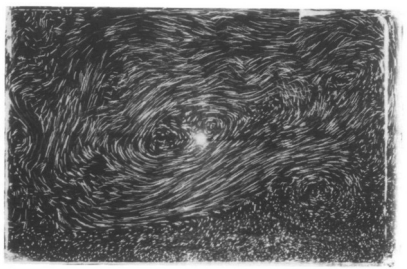


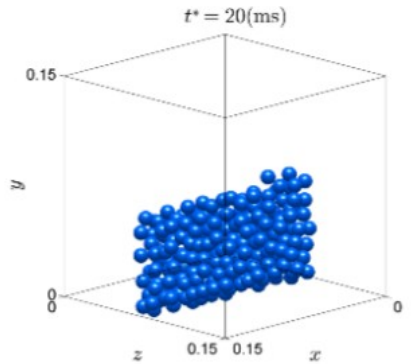
End-member scenarios

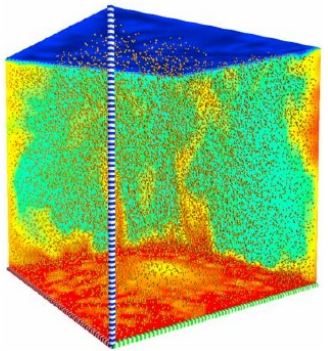
1. M&N (1989): $\frac{dN}{dt} = -A v_t c_{bnd}, \quad c_{bnd} = \frac{N}{A H} \quad \Rightarrow \quad N(t) = N_0 \exp \frac{-v_t t}{H}$

2. Stoke's settling: $\frac{dN}{dt} = -A v_t c_{bnd}, \quad c_{bnd} = \frac{N}{A(H - v_t t)} \quad \Rightarrow \quad N(t) = N_0 \left(1 - \frac{v_t t}{H}\right)$

3. Convective suspension: $\frac{dN_{eq}}{dt} = 0$



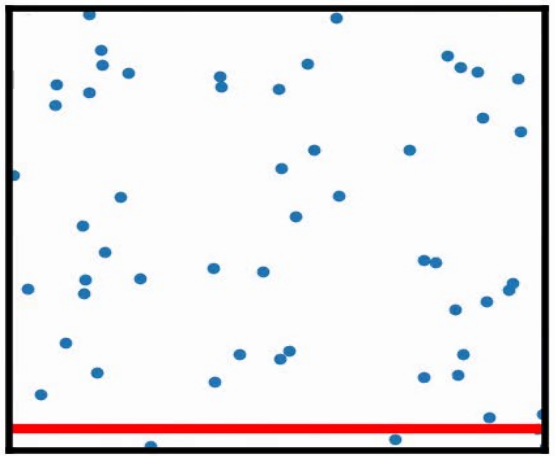


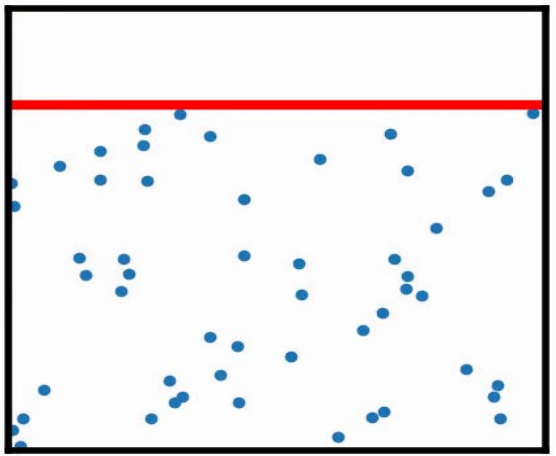


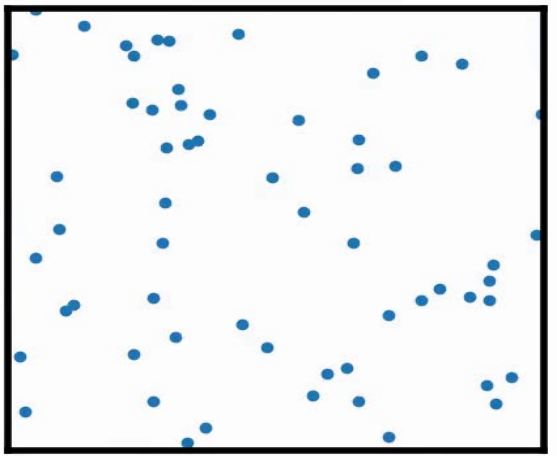
$$N(t)=N_0\exp\frac{-\nu_t t}{H}$$

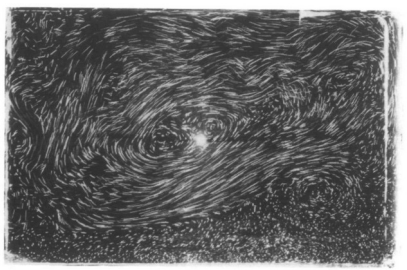
$$N(t)=N_0\left(1-\frac{\nu_t t}{H}\right)$$

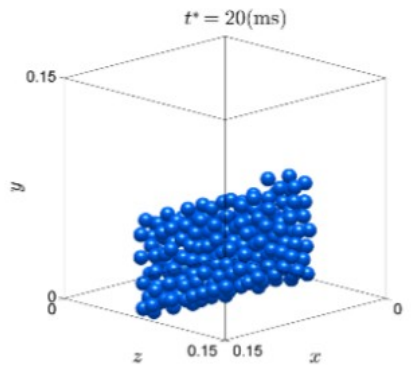
$$\frac{dN_{eq}}{dt}=0$$

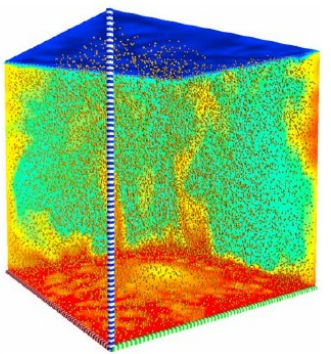




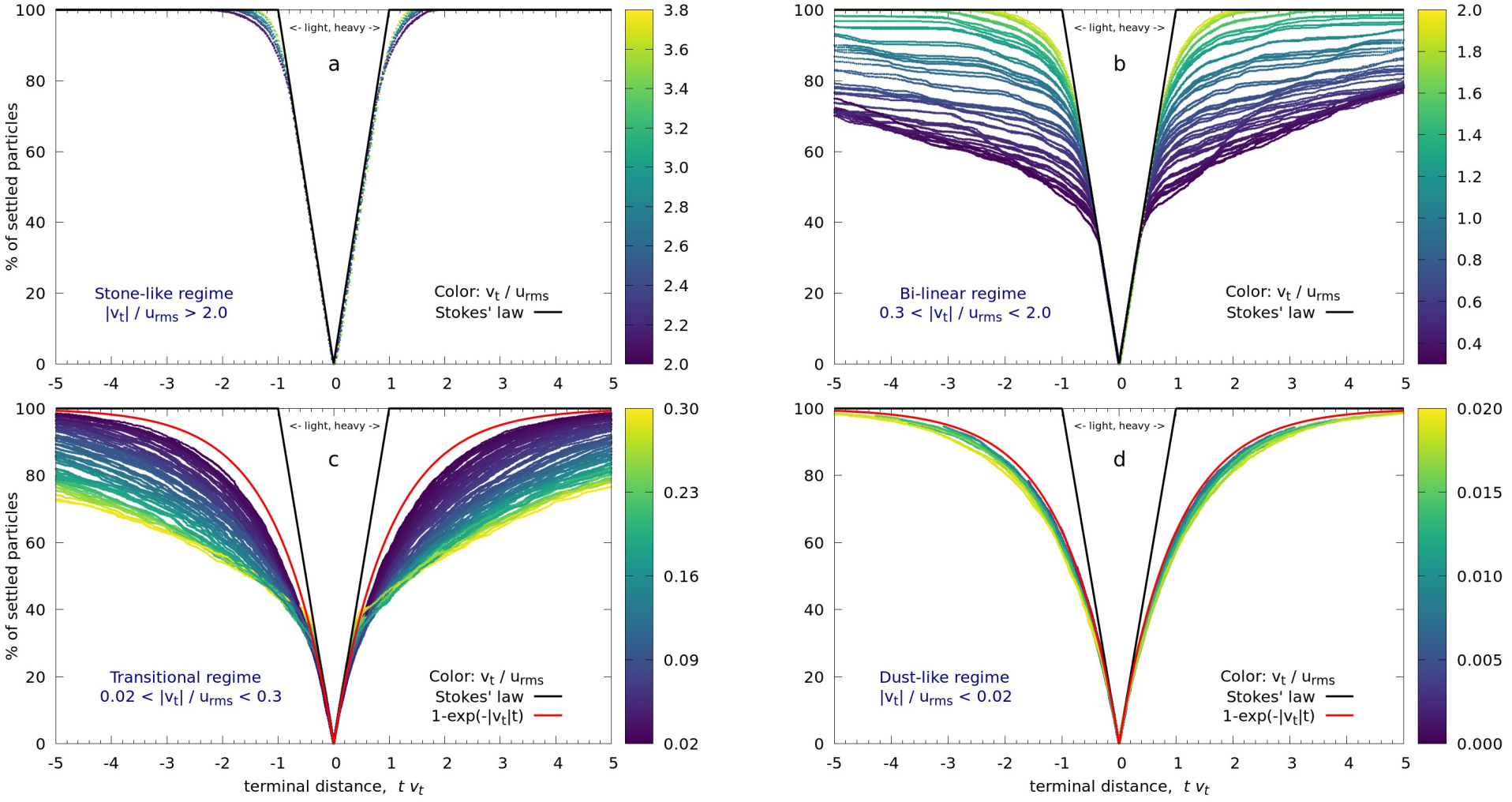




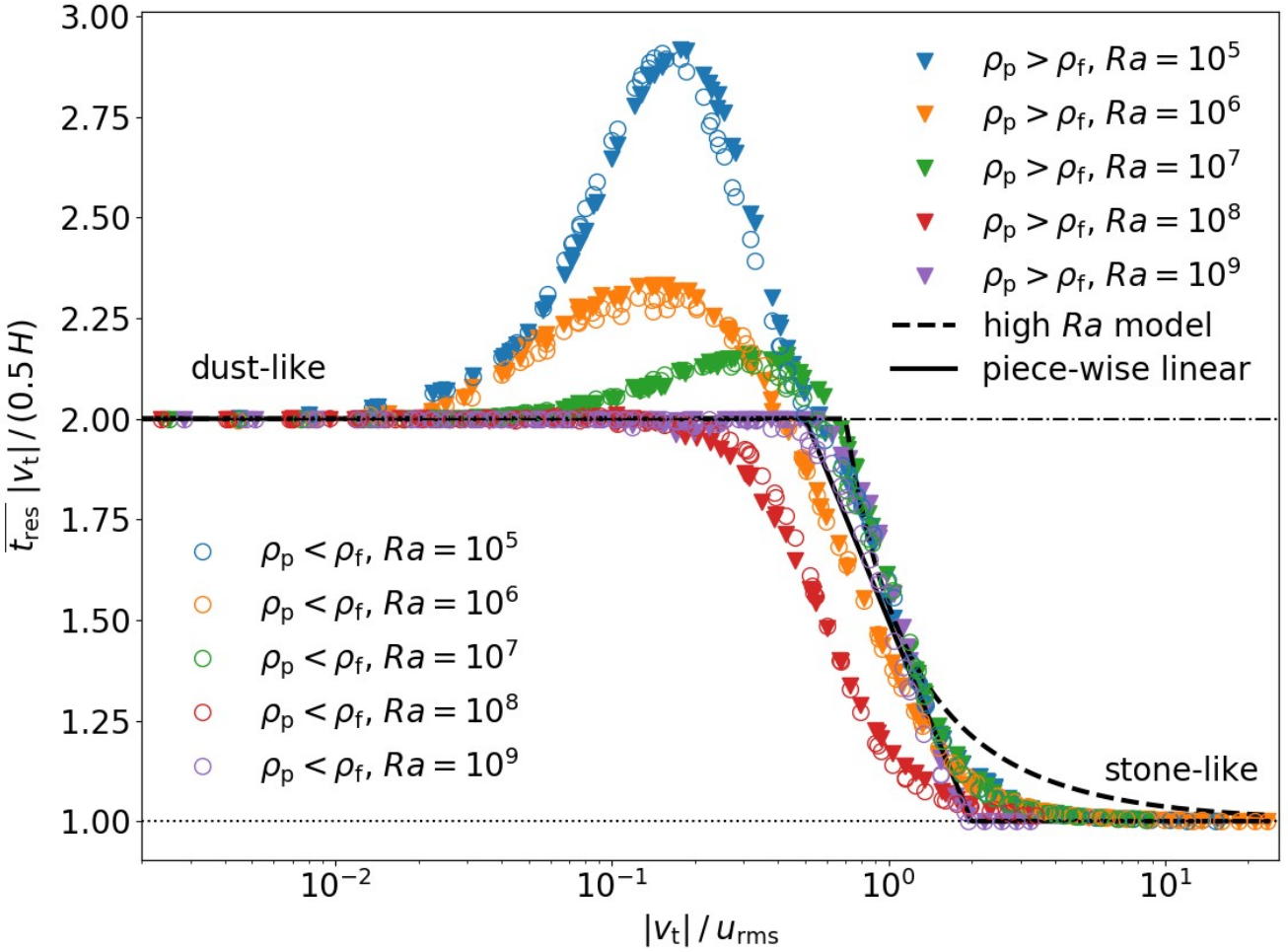




Results: settling curves (*Ra* 10^{10}, *Pr* 50)



Results: high-*Ra* limit



residence time formula

Below we construct an idealized model that roughly captures the observed residence times. The particle mass conservation imposes that the rate of particles leaving the suspension is

$$\frac{\mathrm{d}}{\mathrm{d}t}N(t) = -|v_\mathrm{t}|\,A\,c_\mathrm{bnd}, \tag{8}$$

where $c_\mathrm{bnd} = \langle c(z\to 0, t)\rangle$ is the mean particle concentration at the bottom boundary (or $\langle c(z\to 1, t)\rangle$ in case of buoyant particles), for which we use the following estimation:

$$c_\mathrm{bnd} = \frac{N(t)}{A\,(H - t\,v_\mathrm{sh})}. \tag{9}$$

The shrinking velocity v_sh describes how the particle cloud, i.e. the region that contains the $N(t)$ particles, changes in volume with time. Its value is different for each particle type, approaching 0 in the dust-like limit (particles always occupy the entire volume AH), and $|v_\mathrm{t}|$ in the stone-like limit (the volume $Av_\mathrm{t}t$ at the top of the tank is particle-free). With the help of Eq. (9), the solution to Eq. (8) can be obtained:

$$N(t) = N_0\left(1 - \frac{t\,v_\mathrm{sh}}{H}\right)^{\frac{|v_\mathrm{t}|}{v_\mathrm{sh}}}, \tag{10}$$

which gives the desired $N_0\exp(-v_\mathrm{t}t/H)$ and $N_0(1 - v_\mathrm{t}\,t/H)$ respectively in the limits of $v_\mathrm{sh}\to 0$ and $v_\mathrm{sh}\to|v_\mathrm{t}|$.

$$\frac{|v_\mathrm{t}|}{0.5\,H}\,\overline{t_\mathrm{res}} = \frac{2|v_\mathrm{t}|}{v_\mathrm{sh} + |v_\mathrm{t}|}. \tag{11}$$

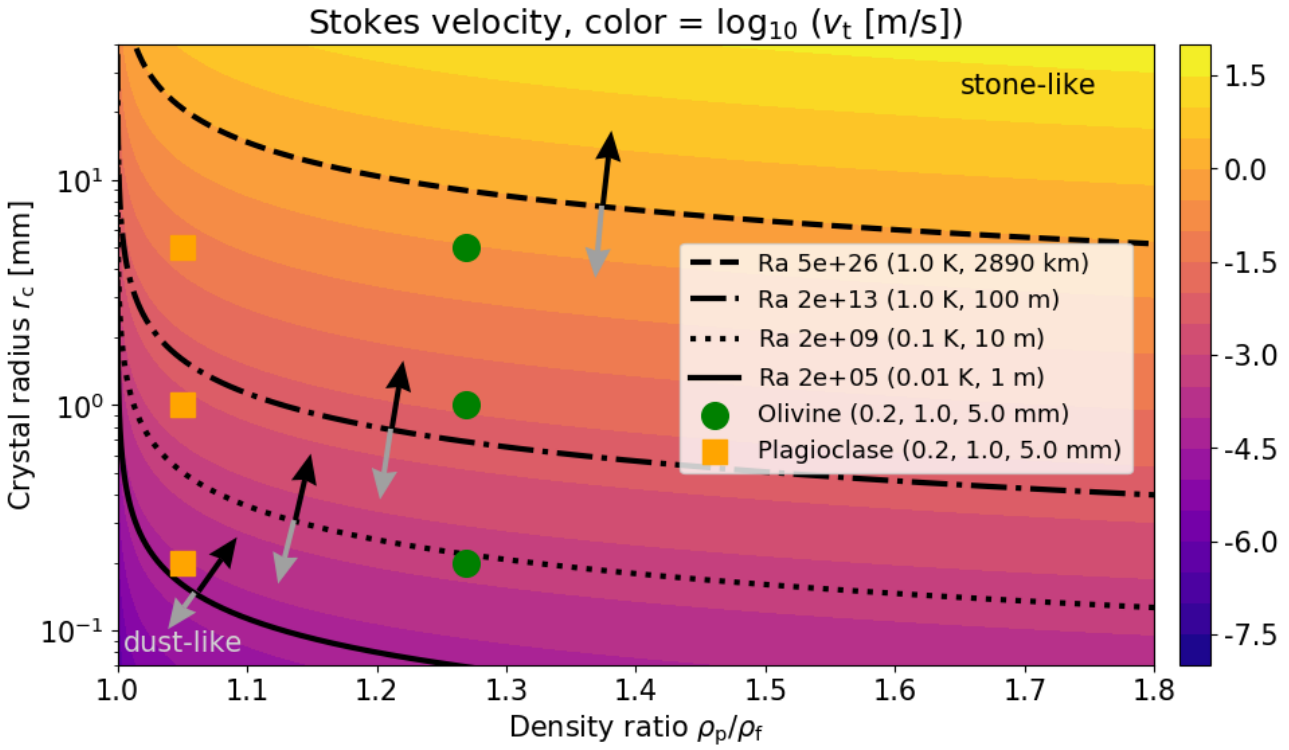
Finally, we assume that the shrinking velocity can be approximated as

$$v_\mathrm{sh} = \begin{cases} |v_\mathrm{t}| - \gamma\,u_\mathrm{rms}, & \text{if } |v_\mathrm{t}| > \gamma\,u_\mathrm{rms} \\ 0, & \text{if } |v_\mathrm{t}| \le \gamma\,u_\mathrm{rms} \end{cases} \tag{12}$$

where γ is a constant representing the mixing of particles - it mimics the fact that some particles are transported above the settling front due to the presence of fluid flow. The second branch in Eq. (12) expresses the constraint $v_\mathrm{sh} > 0$, because the model domain does not expand in response to mixing. The normalized residence time is then

$$\frac{|v_\mathrm{t}|}{0.5\,H}\,\overline{t_\mathrm{res}} = \begin{cases} \frac{2}{2 - \gamma\,u_\mathrm{rms}/|v_\mathrm{t}|}, & \text{if } |v_\mathrm{t}|/u_\mathrm{rms} > \gamma \\ 2, & \text{if } |v_\mathrm{t}|/u_\mathrm{rms} \le \gamma \end{cases} \tag{13}$$

Application to magma reservoirs



Application to magma reservoirs

```
# THE MAIN PROCEDURE - COMPUTING CRYSTAL SIZE DISTRIBUTIONS IN SEDIMENT AND BULK
# Below the crystallization is modelled via Eqs 8,9, and 12
# This direct approach allows for straightforward addition of new complexities, e.g. of grain growth
# A quicker option to reproduce Fig. 5 would be to simply use Eq. 10 - see the end of the script
volc_bulk[0] = np.sum( dbulk[:]*(4./3.)*np.pi*np.power(rbin[:], 3.0) )
volfac = (phibot + phitop)/2.0 * Htarget / H0 / volc_bulk[0]
volc_bulk[0] *= volfac
maxheight = volc_bulk[0]*H0/phised
print("#Particles in the bulk %.1f (N0 %.2e), Maximum sediment height %.2f" % (np.sum(dbulk), N0, maxheight))
print()
for i in range(0,maxstep):
    for isel in isel_list:
        nbulk_sel[i,isel] = dbulk[isel]
    time[i] = i*dt
    Hnow = H0
    Hchamber[i] = Hnow
    if(shrink_chamber and i>0):
        Hnow = H0 * (1.0 - volc_sed[i-1])
    Heff[:] = Hnow - Vshrink[:]*time[i]
    # Sedimentation (masks: effective volume cannot be negative, and only what is in the bulk can fall down)
    for j in range(nbin):
        if Heff[j]>0.0:
            sedstep[j] = stv[j]*dbulk[j]/Heff[j]*dt
        if sedstep[j]>dbulk[j]:
            sedstep[j] = dbulk[j]
    dsed[:] += sedstep[:]
    dbulk[:] -= sedstep[:]
    nbulk[i] = np.sum(dbulk)
    nsed[i] = np.sum(dsed)
    rpmean_bulk[i] = np.sum(dbulk*rbin) / nbulk[i]
    rpmean_sed[i] = np.sum(dsed*rbin) / nsed[i]
    rpanal = rpref / np.power(1.+intt*time[i],0.5)
    volc_bulk[i] = volfac * np.sum( dbulk[:]*(4./3.)*np.pi*np.power(rbin[:], 3.0) )
    volc_sed[i] = volfac * np.sum( dsed[:]*(4./3.)*np.pi*np.power(rbin[:], 3.0) ) / phised
    flux[i] = volfac * np.sum( sedstep[:]*(4./3.)*np.pi*np.power(rbin[:], 3.0) ) / dt
    # WRITING h5 FILES FOR LATER POSTPROCESSING
```

$$\frac{\mathrm{d}}{\mathrm{d}t}N(t) = -|v_\mathrm{t}|\,A\,c_\mathrm{bnd},$$

$$c_\mathrm{bnd} = \frac{N(t)}{A\,(H - t\,v_\mathrm{sh})}.$$

$$\frac{|v_\mathrm{t}|}{0.5\,H}\,\overline{t_\mathrm{res}} = \begin{cases} \frac{2}{2-\gamma\,u_\mathrm{rms}/|v_\mathrm{t}|}, & \text{if } |v_\mathrm{t}|/u_\mathrm{rms} > \gamma \\ 2, & \text{if } |v_\mathrm{t}|/u_\mathrm{rms} \le \gamma \end{cases}$$

Application to magma reservoirs

V. Patočka, N. Tosi and E. Calzavarini

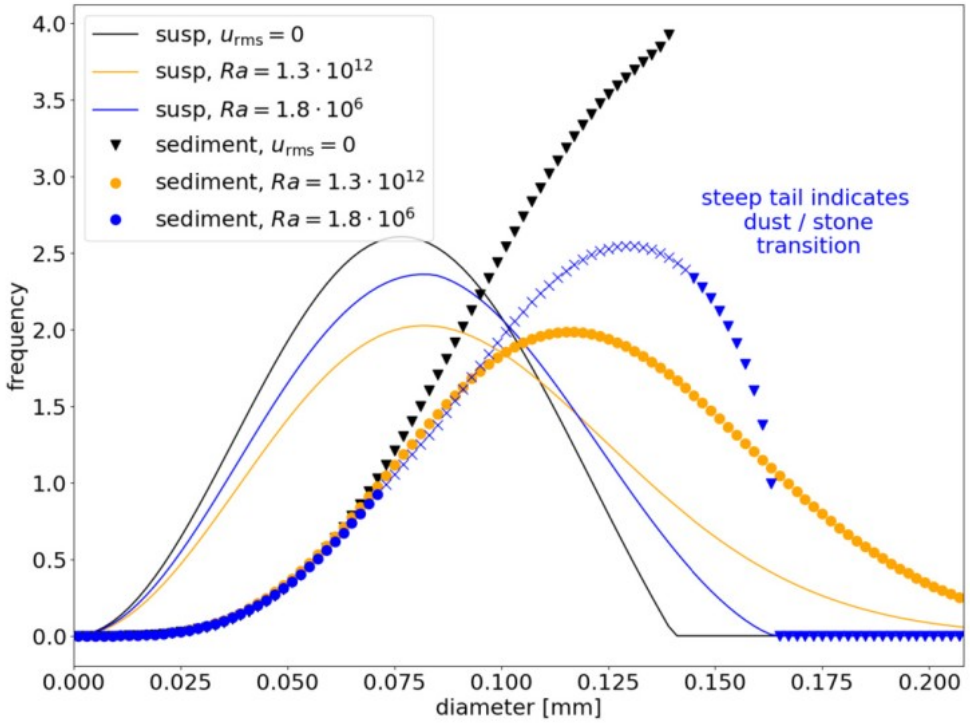
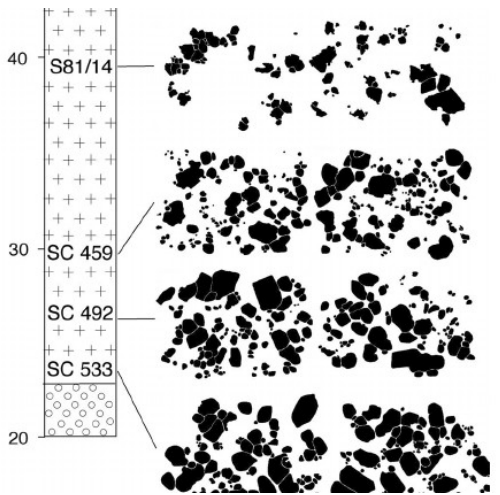


Fig. 5. Crystal size distribution in suspension and sediment for a simple application that follows the Discussion in Holness et al. (2017). We test three values of the fluid mean velocity u_{rms}, corresponding to a static magma (black), vigorously convecting magma (orange), and to an intermediate case (blue). Solid lines show the size distribution of particles remaining in suspension at the time $t = 40$ weeks, at which more than 60% of all the particles have settled in all three cases. Symbols display the size distribution in the currently forming layer of sediment. Triangles represent particles with $v_t > 2.0\,u_{rms}$ (stone-like), circles show particles with $v_t < 0.5\,u_{rms}$ (dust-like), and crosses mark the transition.

Holness et al. (2017)



Side outcome: drift equation

$$\frac{d\boldsymbol{v}}{dt} = \beta \frac{D\boldsymbol{u}}{Dt} + \frac{1}{\tau_D}(\boldsymbol{u} - \boldsymbol{v}) + (1 - \beta)\boldsymbol{g}, \tag{16}$$

where $\tau_D = r_c^2/(3\nu\beta)$ is the particle response time and r_c is the particle radius.

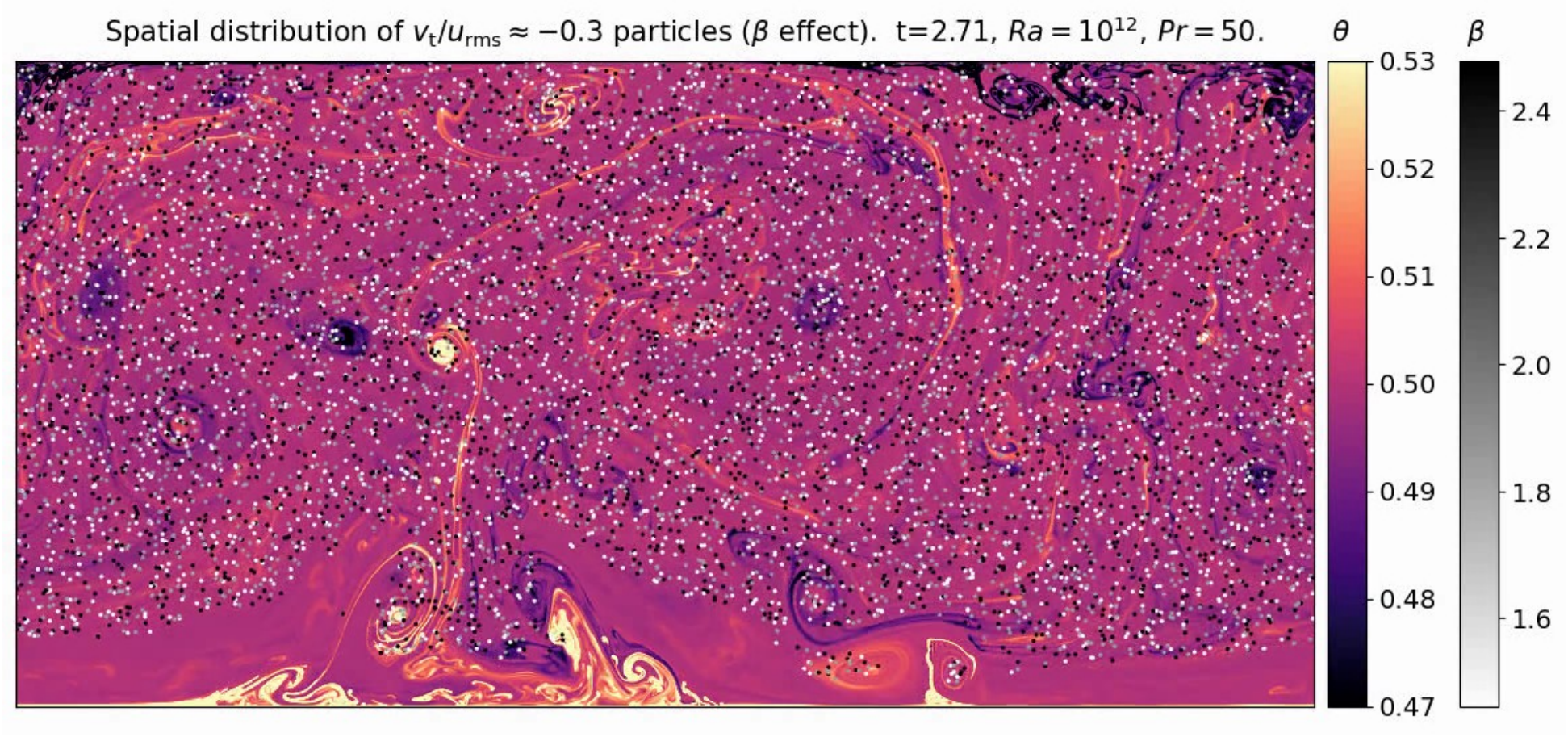
$$\boldsymbol{v} = \boldsymbol{u} + \tau_D \boldsymbol{v}_1 + \mathcal{O}(\tau_D^2).$$

$$\boldsymbol{v} = \boldsymbol{u} + \tau_D(1 - \beta)\left(\boldsymbol{g} - \frac{D\boldsymbol{u}}{Dt}\right) + \mathcal{O}(\tau_D^2).$$

$$\boldsymbol{v} = \boldsymbol{u} - St\,\Lambda\hat{\boldsymbol{z}} + St\,(1 - \beta)\frac{D\boldsymbol{u}}{Dt},$$



Vortex sink


Spatial distribution of $v_t/u_{rms} \approx -0.3$ particles (β effect). t=2.71, $Ra = 10^{12}$, $Pr = 50$.
θ
0.53
0.52
0.51
0.50
0.49
0.48
0.47
β
2.4
2.2
2.0
1.8
1.6

Side outcome: drift equation

$$\frac{d\boldsymbol{v}}{dt} = \beta \frac{D\boldsymbol{u}}{Dt} + \frac{1}{\tau_D}(\boldsymbol{u} - \boldsymbol{v}) + (1-\beta)\boldsymbol{g}, \qquad (16)$$

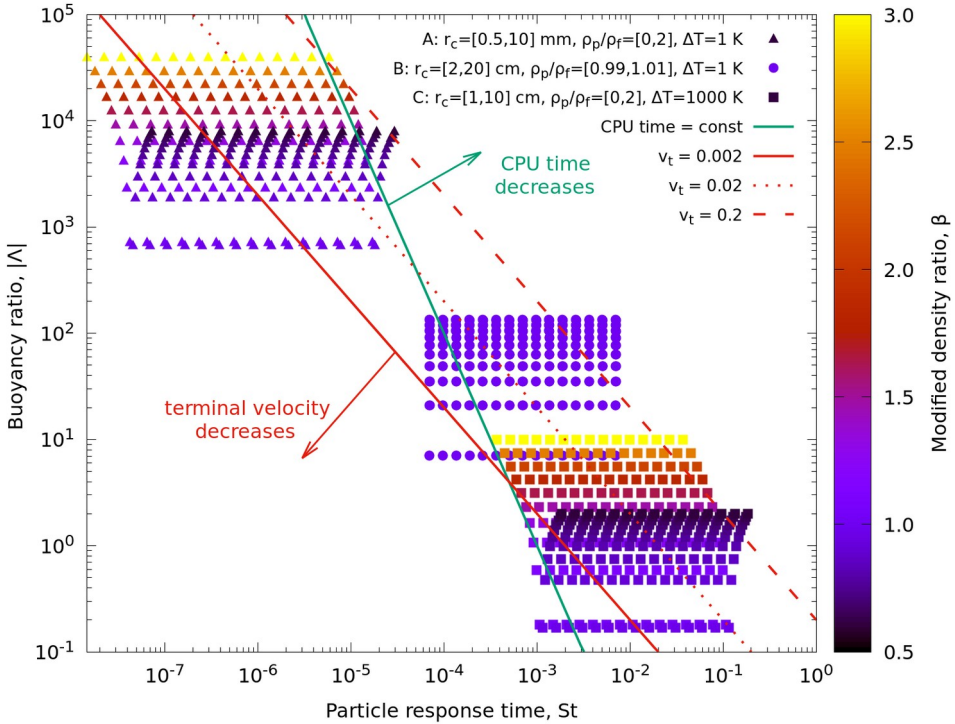
where $\tau_D = r_c^2/(3\nu\beta)$ is the particle response time and r_c is the particle radius.

$$\boldsymbol{v} = \boldsymbol{u} + \tau_D \boldsymbol{v}_1 + \mathcal{O}(\tau_D^2).$$

$$\boldsymbol{v} = \boldsymbol{u} + \tau_D(1-\beta)\left(\boldsymbol{g} - \frac{D\boldsymbol{u}}{Dt}\right) + \mathcal{O}(\tau_D^2).$$

Back to non-dimensional:

$$\boldsymbol{v} = \boldsymbol{u} - St\,\Lambda\hat{\boldsymbol{z}} + St\,(1-\beta)\frac{D\boldsymbol{u}}{Dt},$$



Summary

- Characteristic time scale of settling ~ terminal time
- Formula for the residence time of particles in the high-*Ra* limit

$$\frac{|v_\mathrm{t}|}{0.5\,H}\,\overline{t_\mathrm{res}} = \begin{cases} \dfrac{2}{2-\gamma\,u_\mathrm{rms}/|v_\mathrm{t}|}, & \text{if } |v_\mathrm{t}|/u_\mathrm{rms} > \gamma \\ 2, & \text{if } |v_\mathrm{t}|/u_\mathrm{rms} \le \gamma \end{cases}$$

- Microstructure of intrusions can indicate flow vigor of cooling magma
- Construction of a better parametrized model is possible

What next? Dark (blind?) alleys...

- Nucleation and growth of crystals
- Rotational flows
- Crystal feedback on the flow
- Re-entrainment

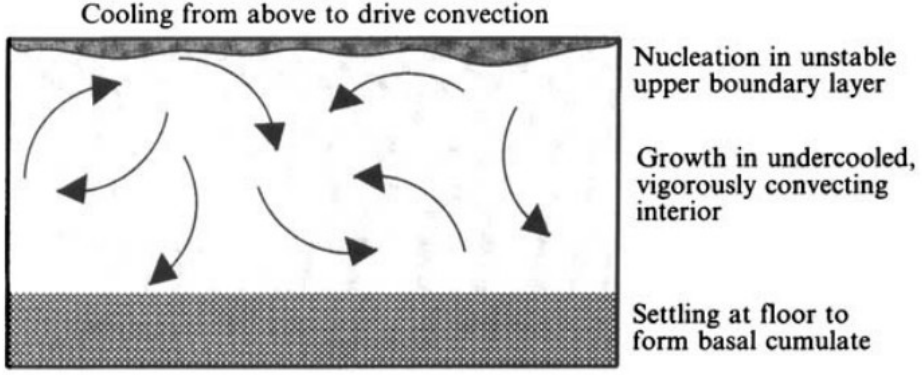
J. Fluid Mech. (1994), *vol.* 273, *pp.* 83–107
Copyright © 1994 Cambridge University Press

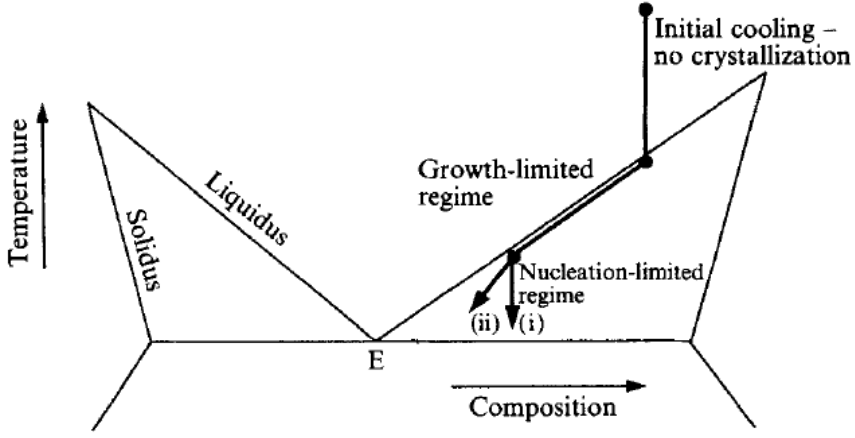
The nucleation, growth and settling of crystals from a turbulently convecting fluid

By RICHARD A. JARVIS[1]† AND ANDREW W. WOODS[2]

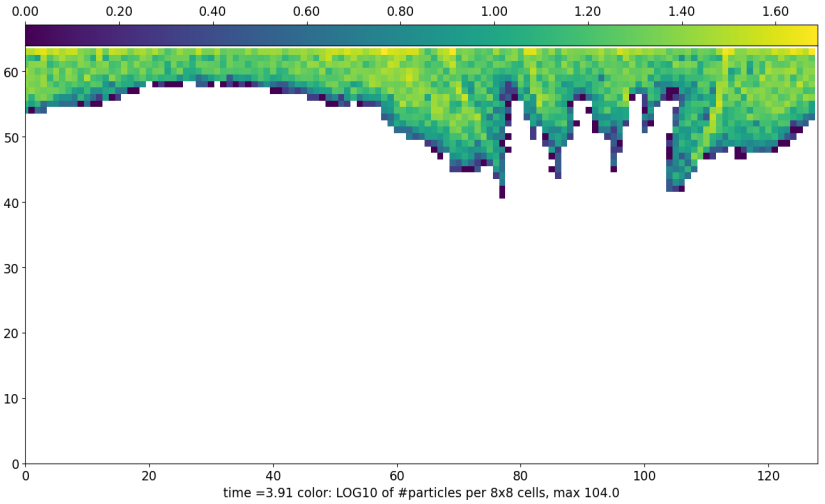
[1] Department of Geology and Geophysics, Yale University, New Haven, CT 06511, USA

[2] Institute of Theoretical Geophysics, Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, UK

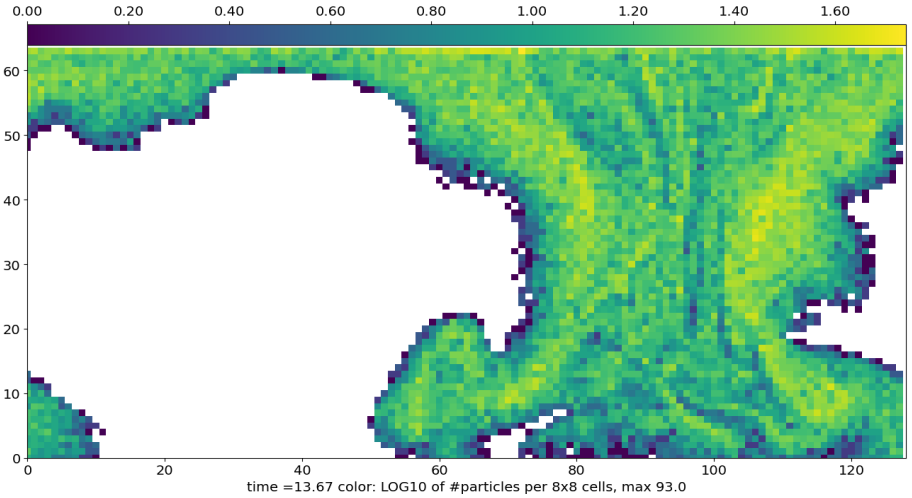




Nucleation and growth



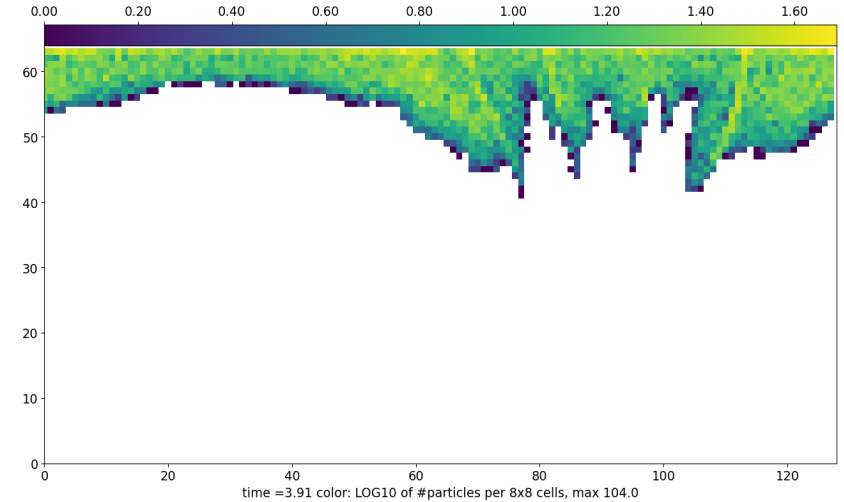
UC:
T_S-T



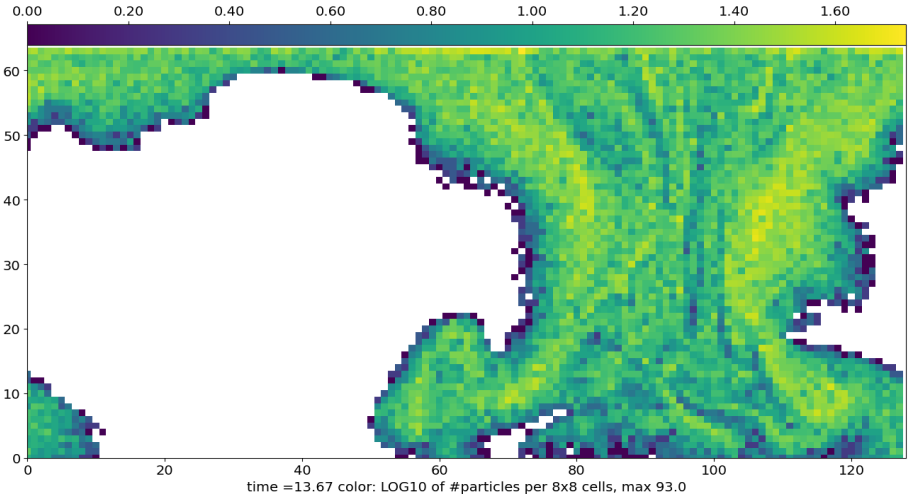
time

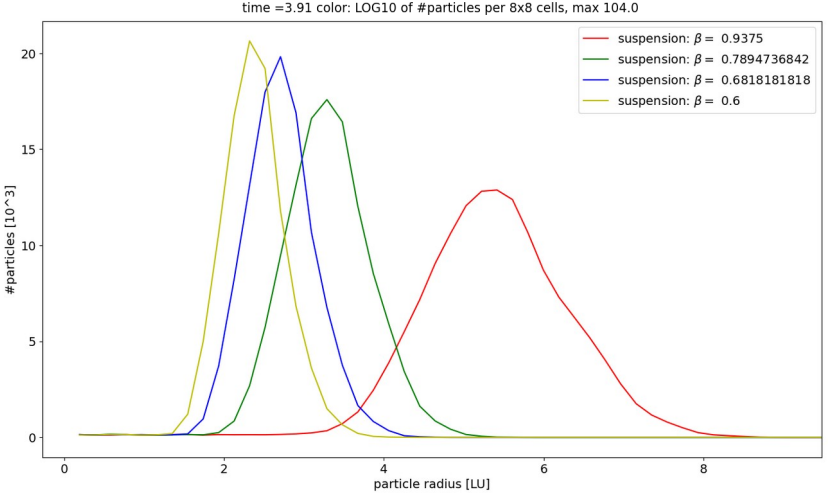


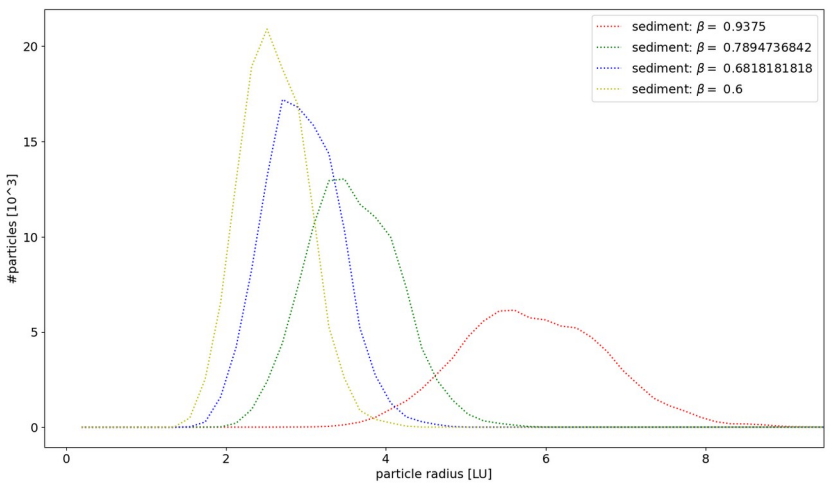
Nucleation and growth

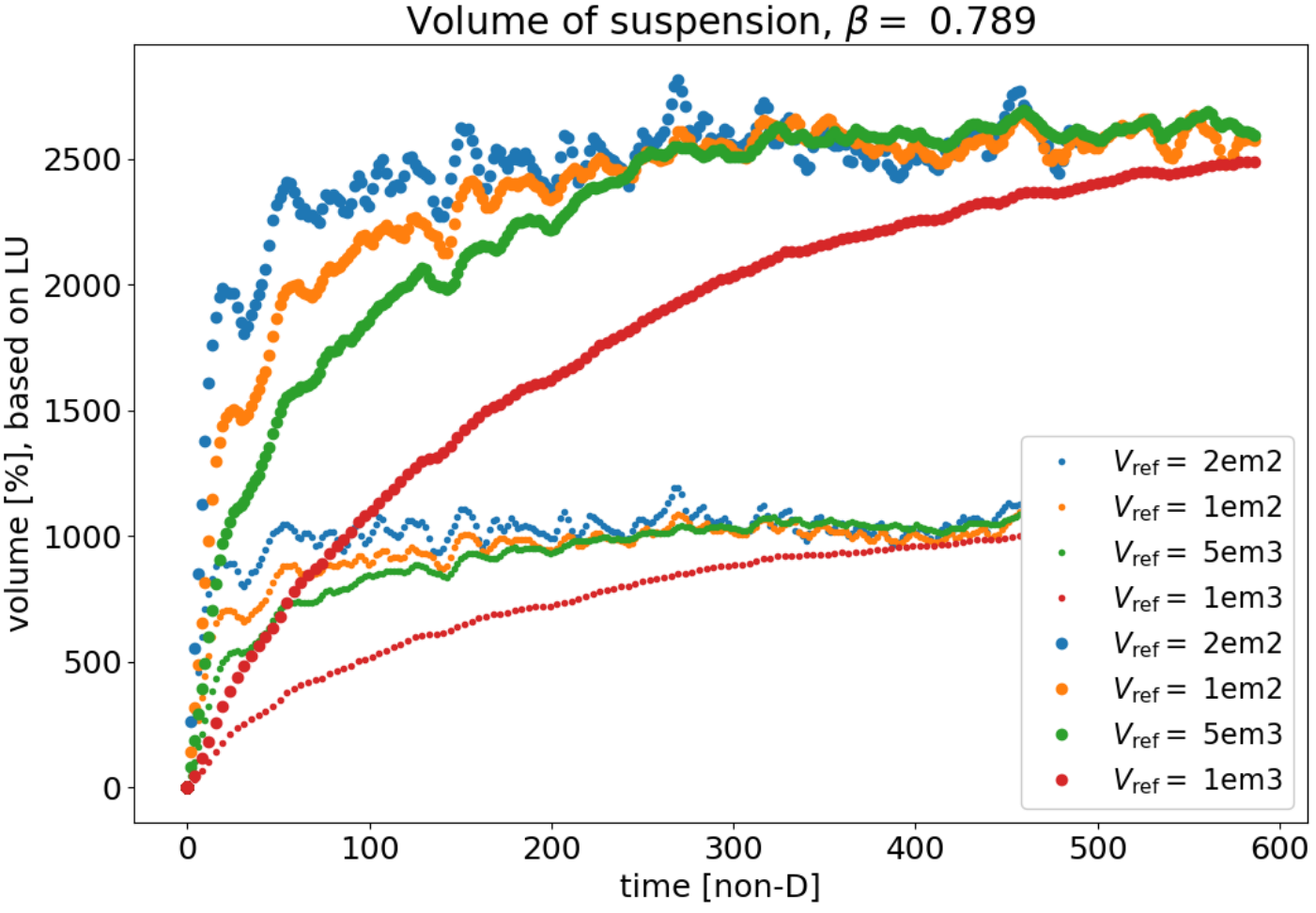


UC:
T_S-T

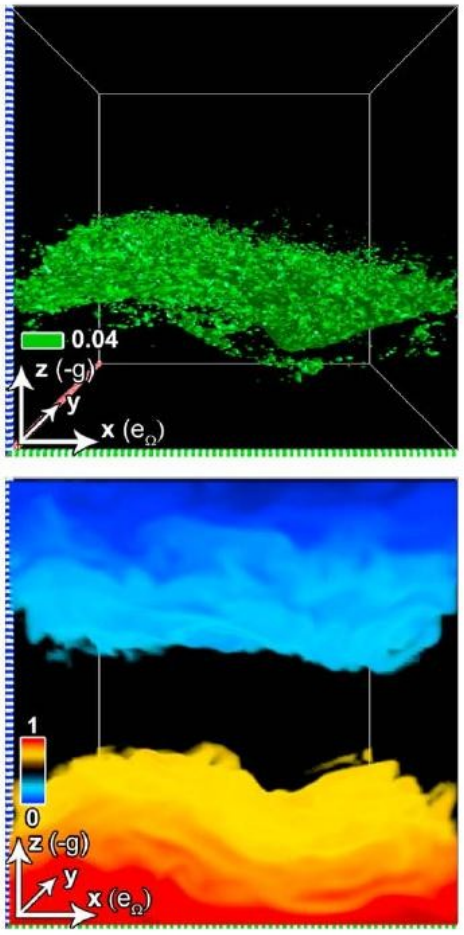







Volume of suspension, $\beta =$ 0.789
volume [%], based on LU
time [non-D]
0
500
1000
1500
2000
2500
0
100
200
300
400
500
600
$V_{ref} =$ 2em2
$V_{ref} =$ 1em2
$V_{ref} =$ 5em3
$V_{ref} =$ 1em3
$V_{ref} =$ 2em2
$V_{ref} =$ 1em2
$V_{ref} =$ 5em3
$V_{ref} =$ 1em3

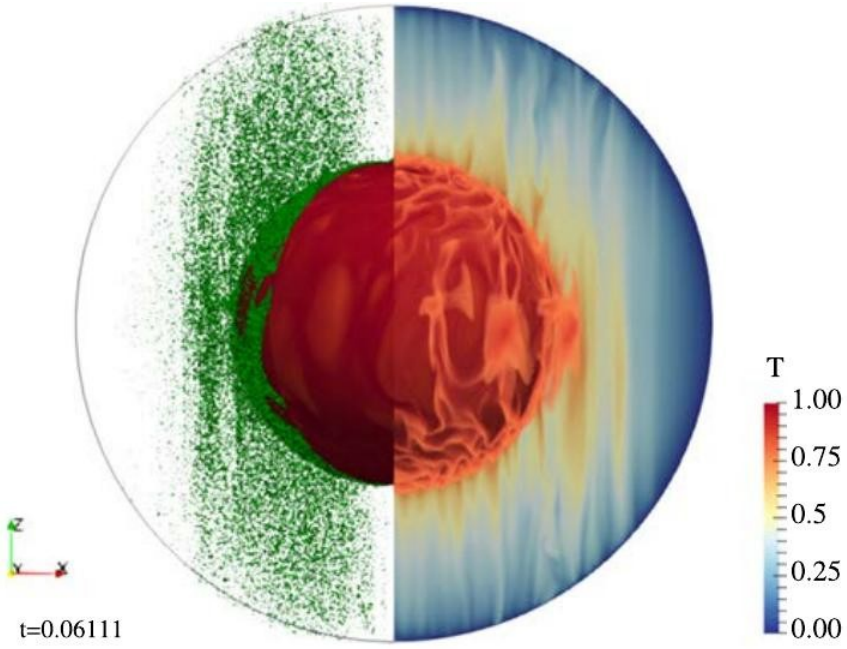
Rotational flows



(f) Ro = 0.1.

Floating “ribbon”:
- balance between Coriolis force and gravity

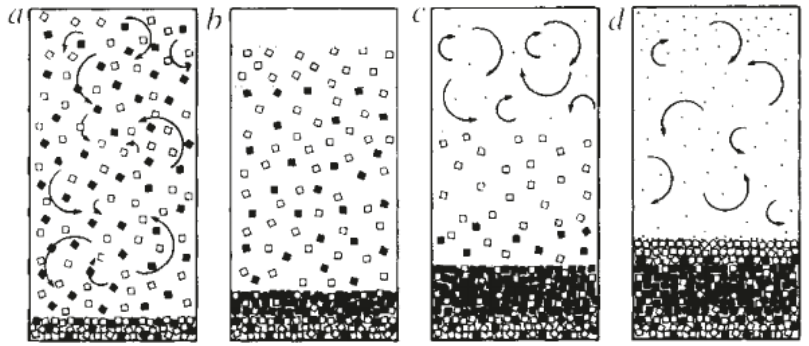
Boundary conditions:
- free-slip, no-slip ?



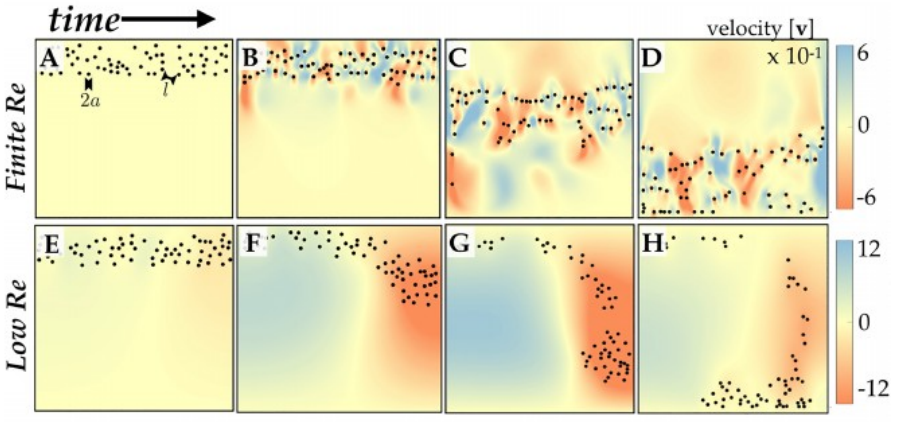
(c) particle and temperature field snapshot.

Crystal feedback on the flow

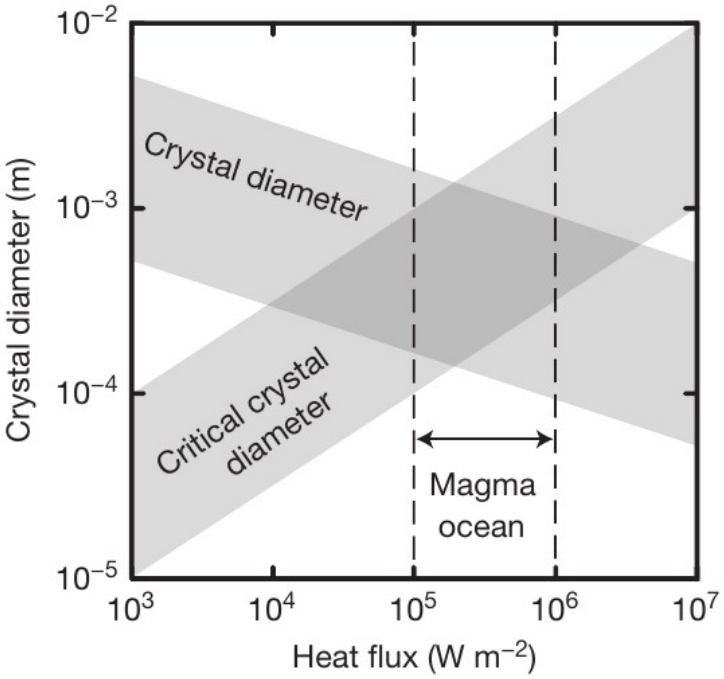
- Sedimentation events
  (Koyaguchi et al, Sparks et al., Nature, 1990)



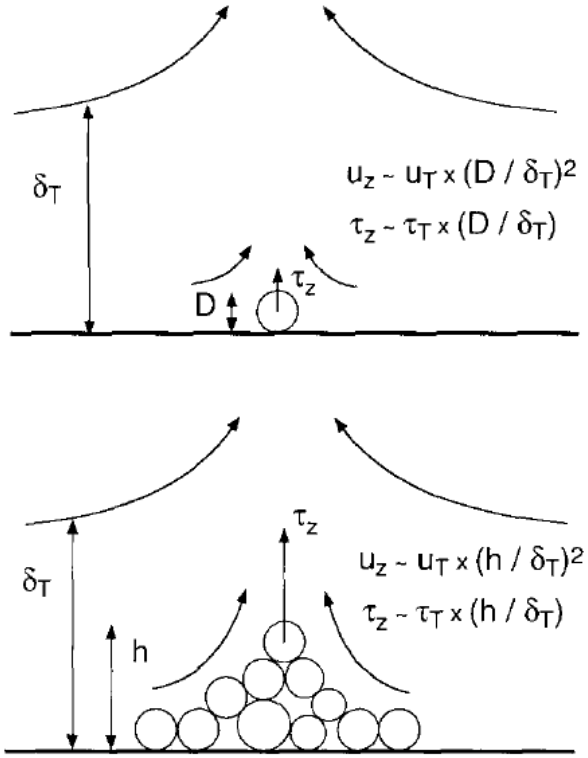
- Crystal-driven convection
  (Culha et al., JGR, 2020)

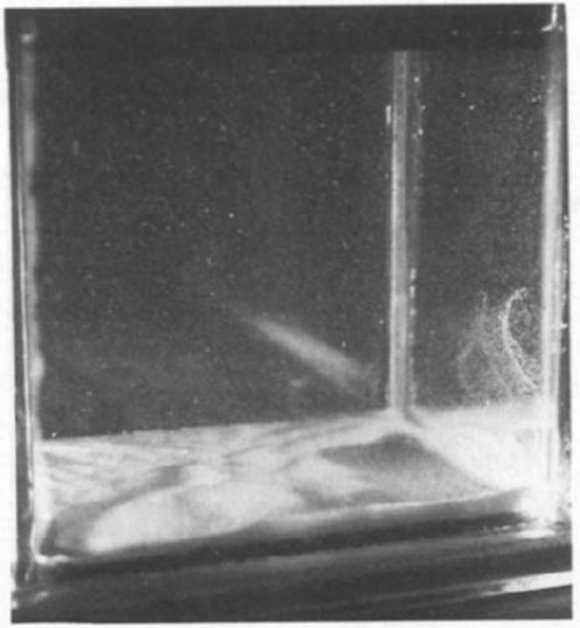


Re-entrainment



Solomatov, ToG 2015





Series of papers by Solomatov, Stevenson, and Olson in 1993