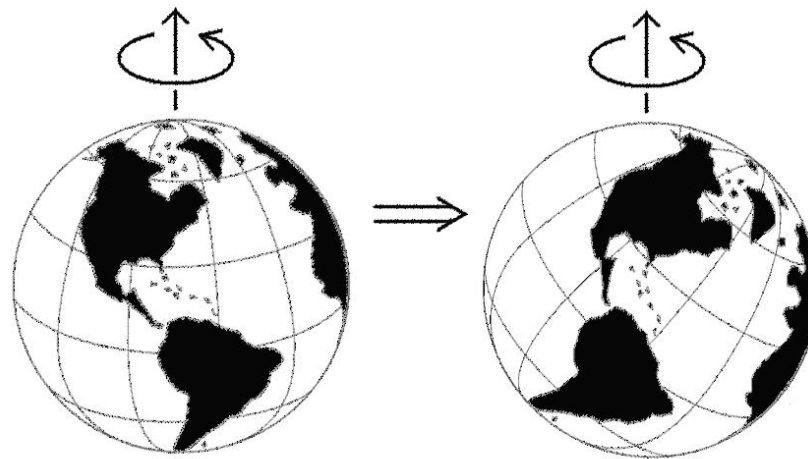


True Polar Wander on Dynamic Planets: Approximative Methods vs. Full Solution

Vojtěch Patočka

Faculty of Mathematics and Physics, Department of Geophysics,
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EUROPEAN UNION
European Structural and Investment Funds
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MINISTRY OF EDUCATION,
YOUTH AND SPORTS



Contents:

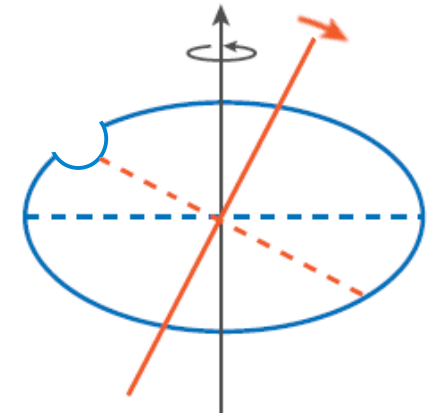
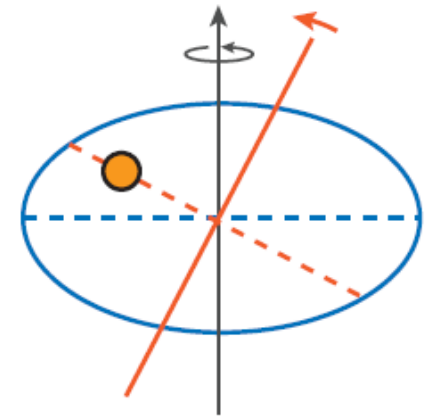
- Mechanism (basics, wobble, tidal axis)
- TPW on Earth and other bodies
- Methods: quasi-fluid approximation
- Future perspectives



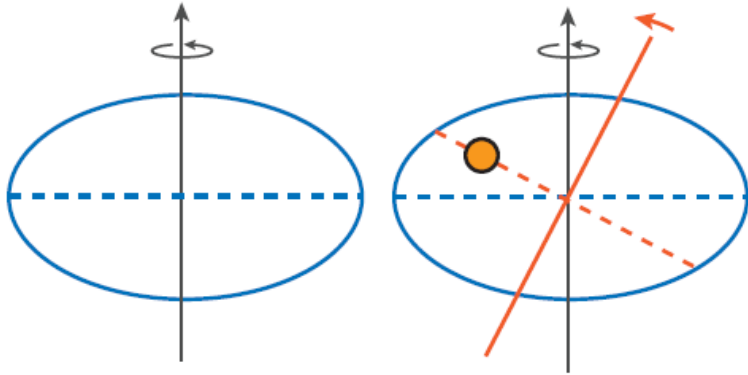
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Mechanism: basic concept

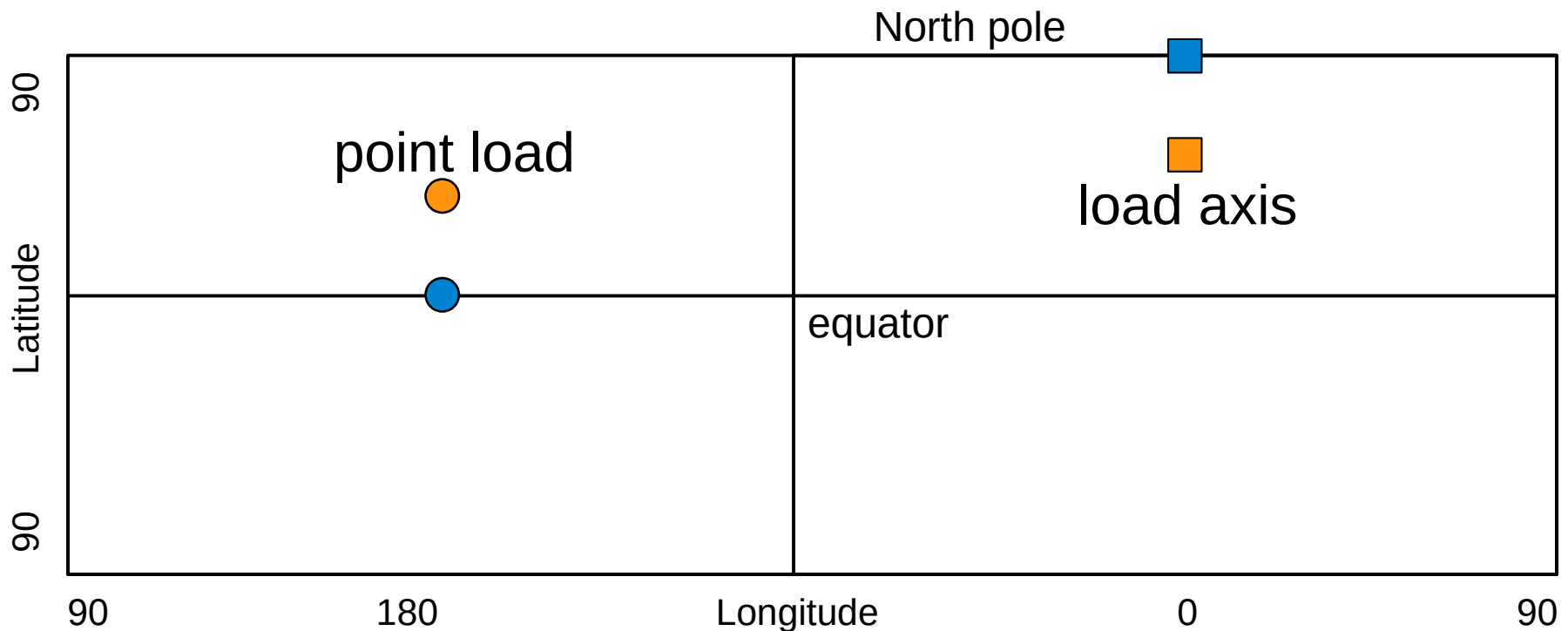


Mechanism: more detailed



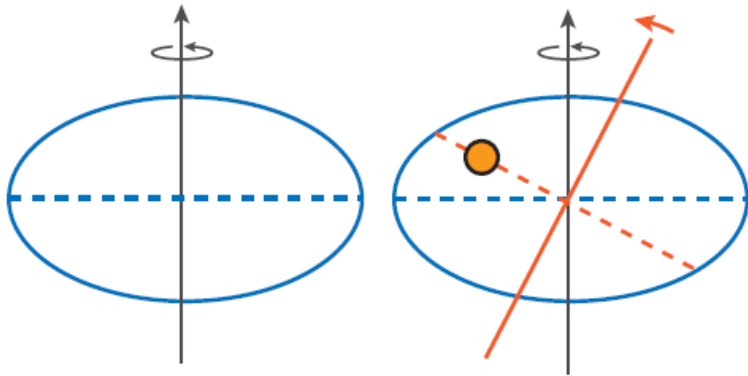
$$0 = \frac{d}{dt}(\mathbf{I} \cdot \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega})$$

Ann. Rev. Earth Planet. Sci.: Matsuyama I., Nimmo F., and Mitrovica J.X. (2014)



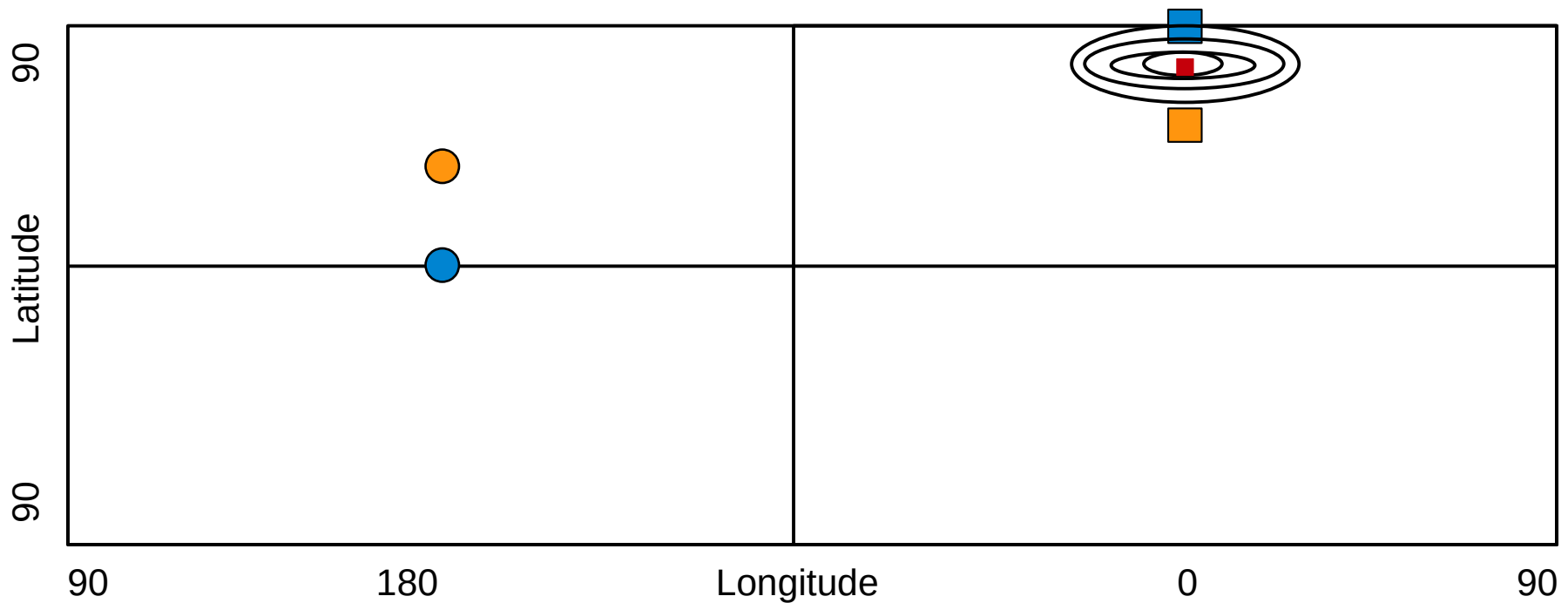
Mechanism: wobble

$\omega \parallel \text{MIA}$

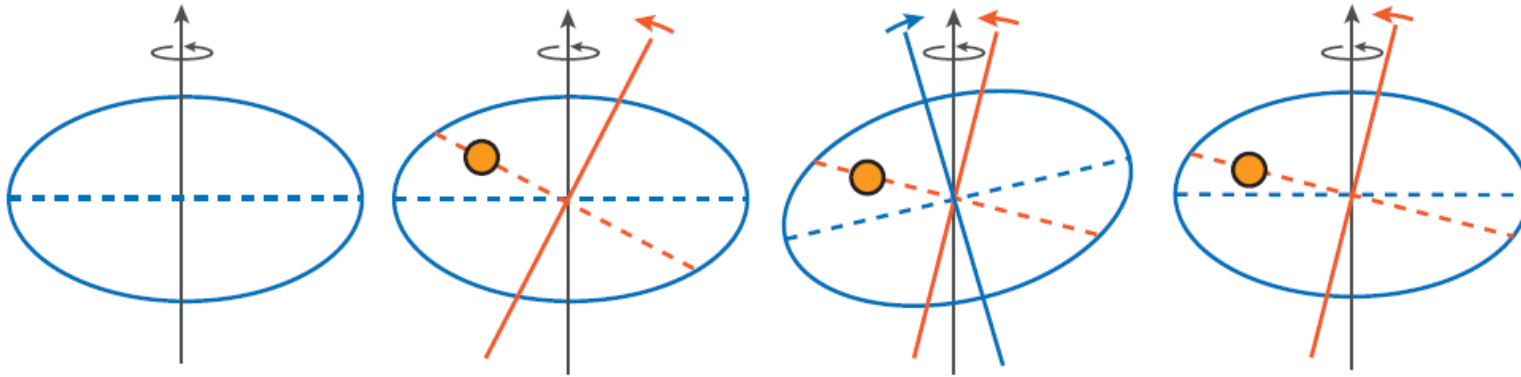


$$0 = \frac{d}{dt}(\mathbf{I} \cdot \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega})$$

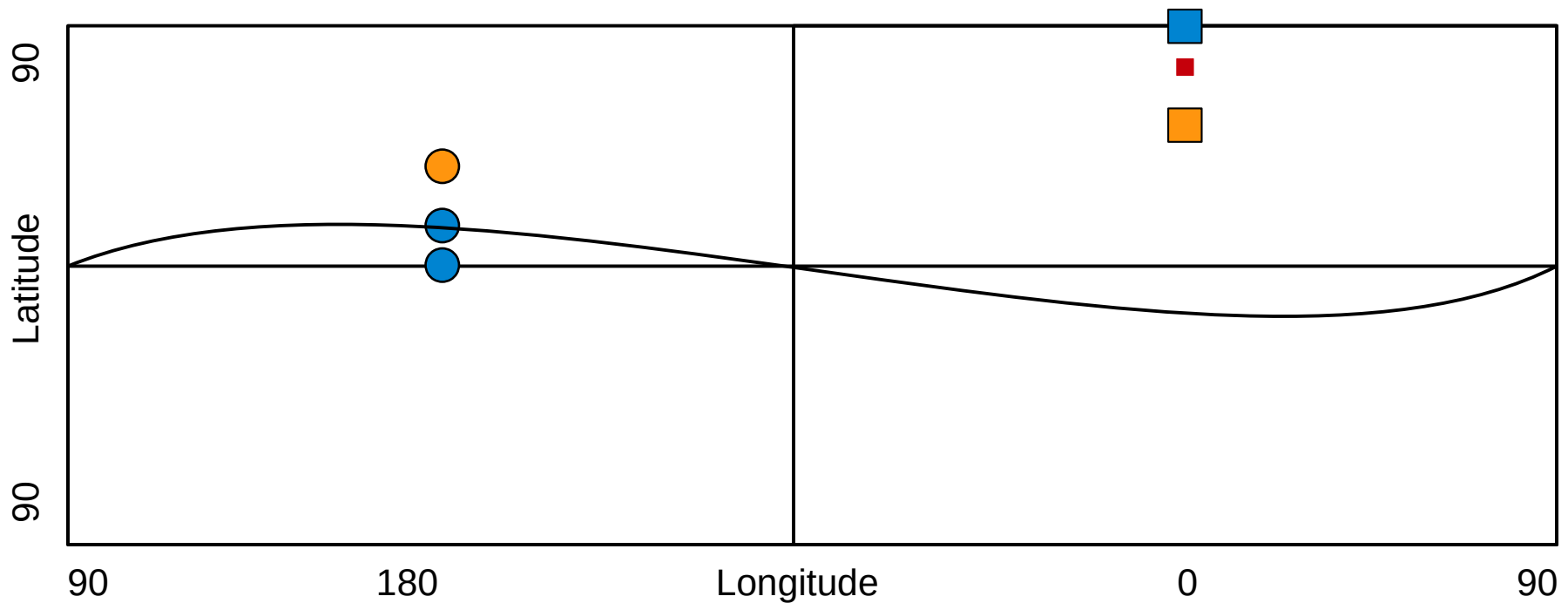
Ann. Rev. Earth Planet. Sci.: Matsuyama I., Nimmo F., and Mitrovica J.X. (2014)



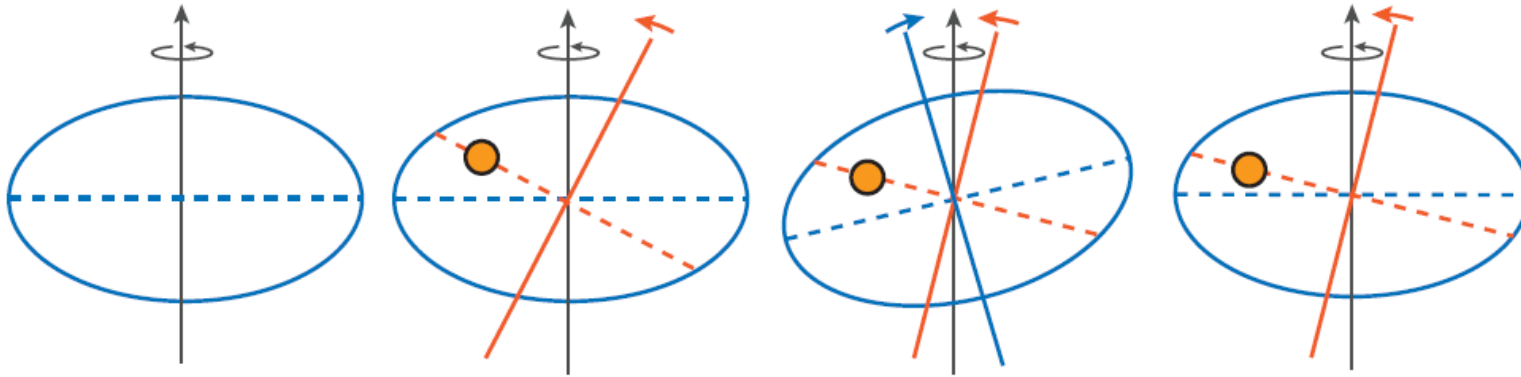
Mechanism: bulge readjustment



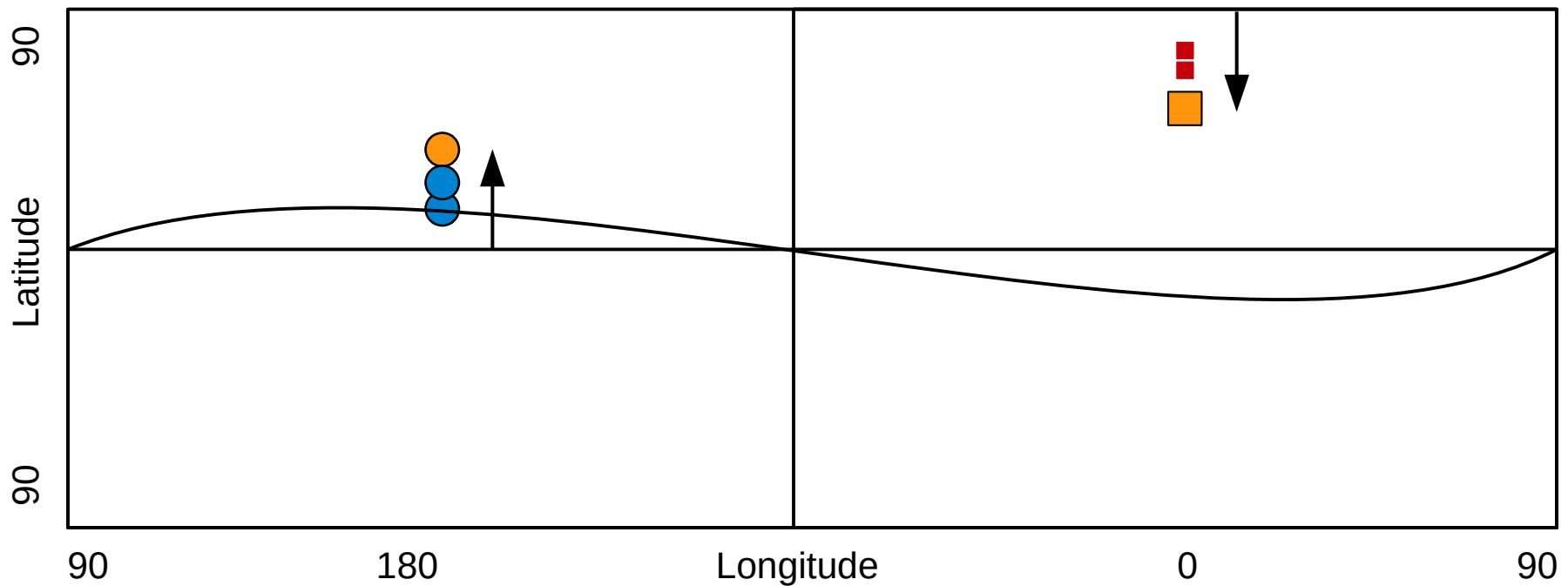
Ann. Rev. Earth Planet. Sci.: Matsuyama I., Nimmo F., and Mitrovica J.X. (2014)



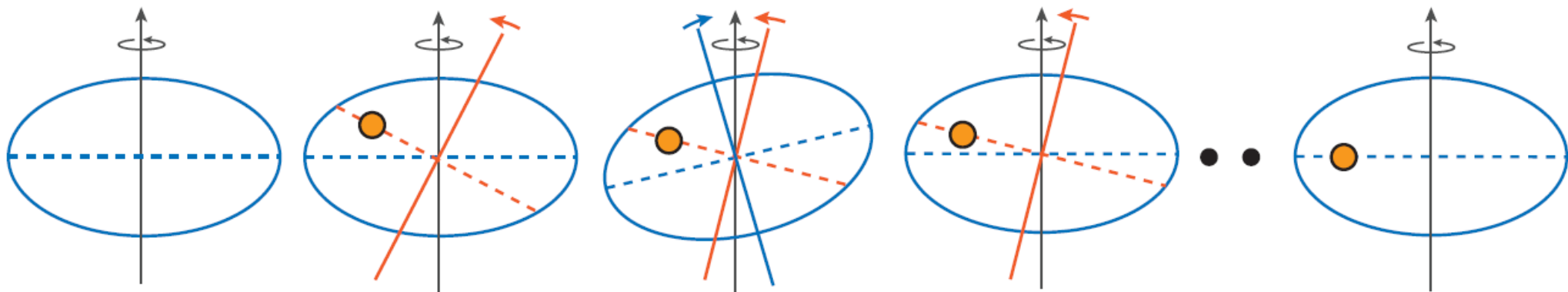
Mechanism: bulge readjustment



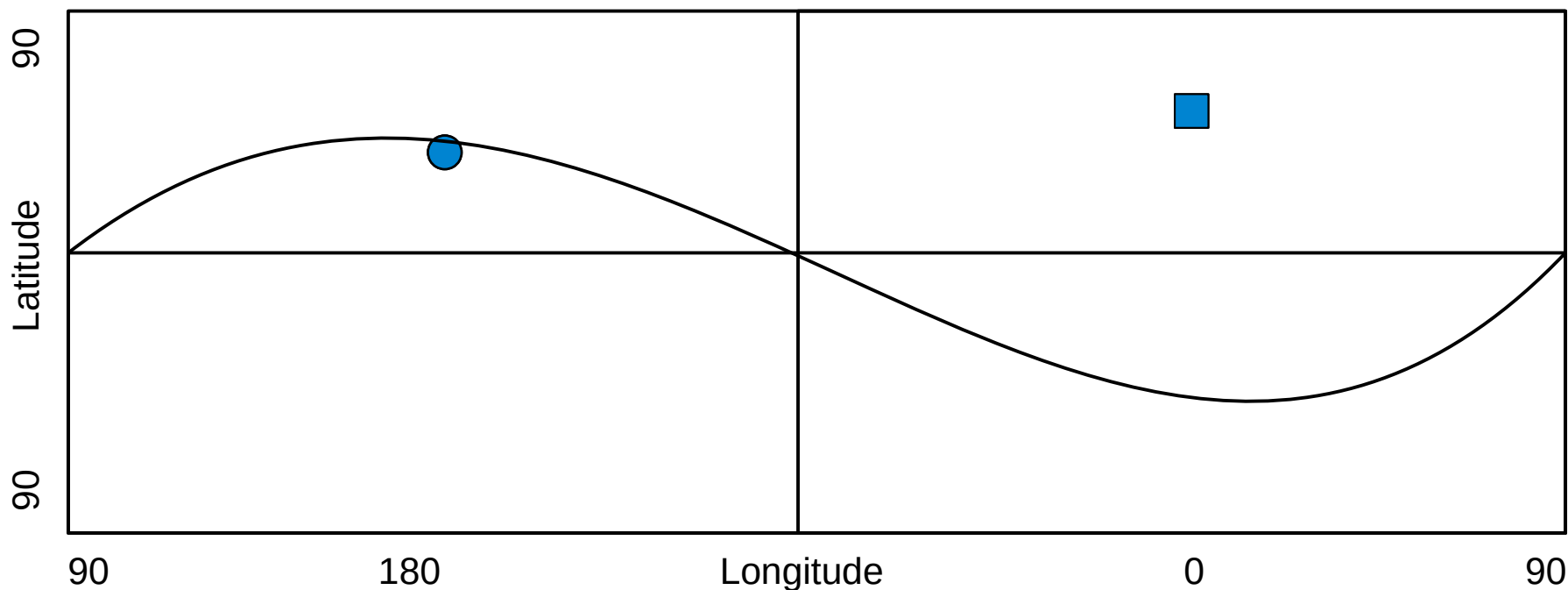
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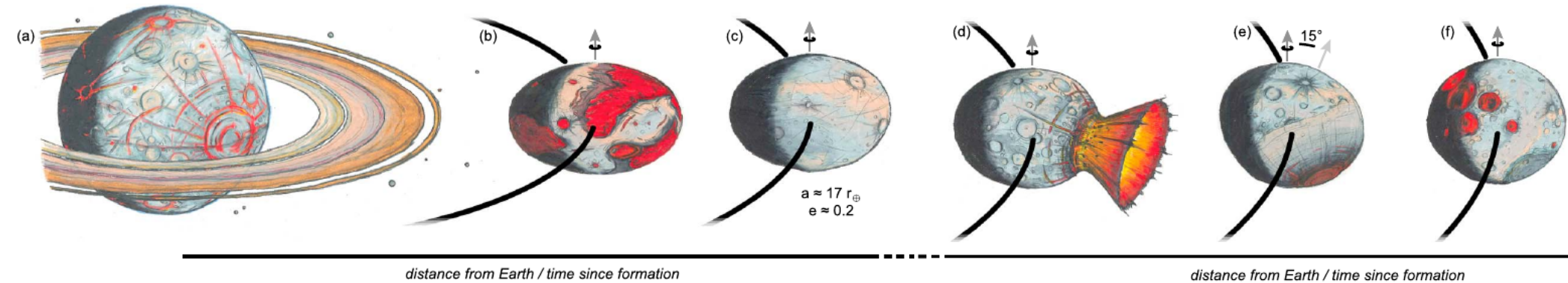
Mechanism: load $\rightarrow$ equator scenario



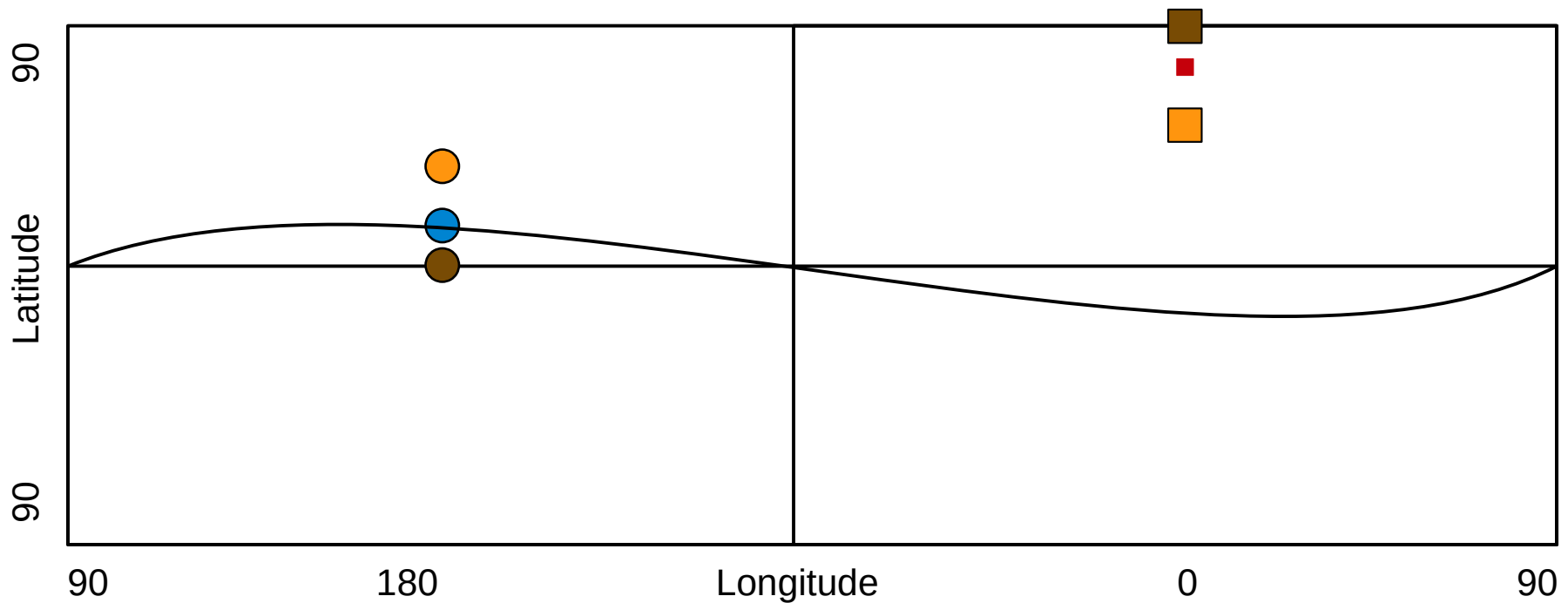
Ann. Rev. Earth Planet. Sci.: Matsuyama I., Nimmo F., and Mitrovica J.X. (2014)



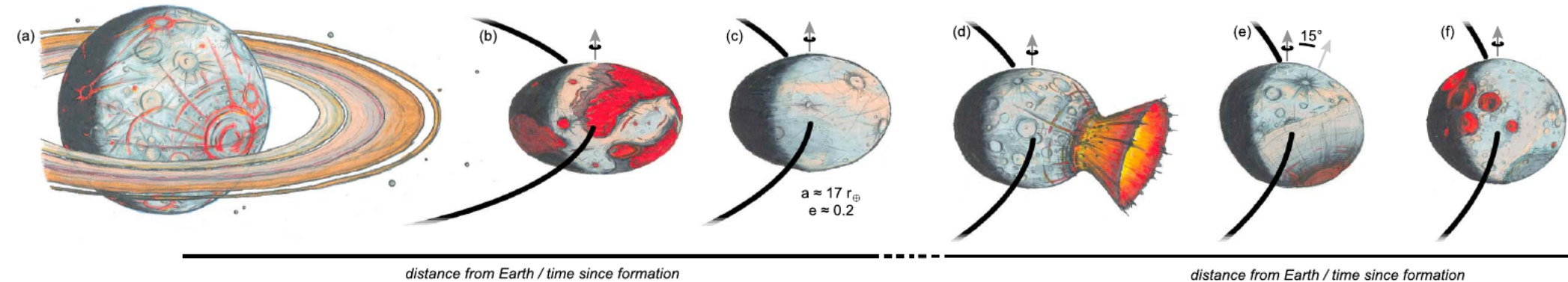
Mechanism: fossil (remnant) bulge



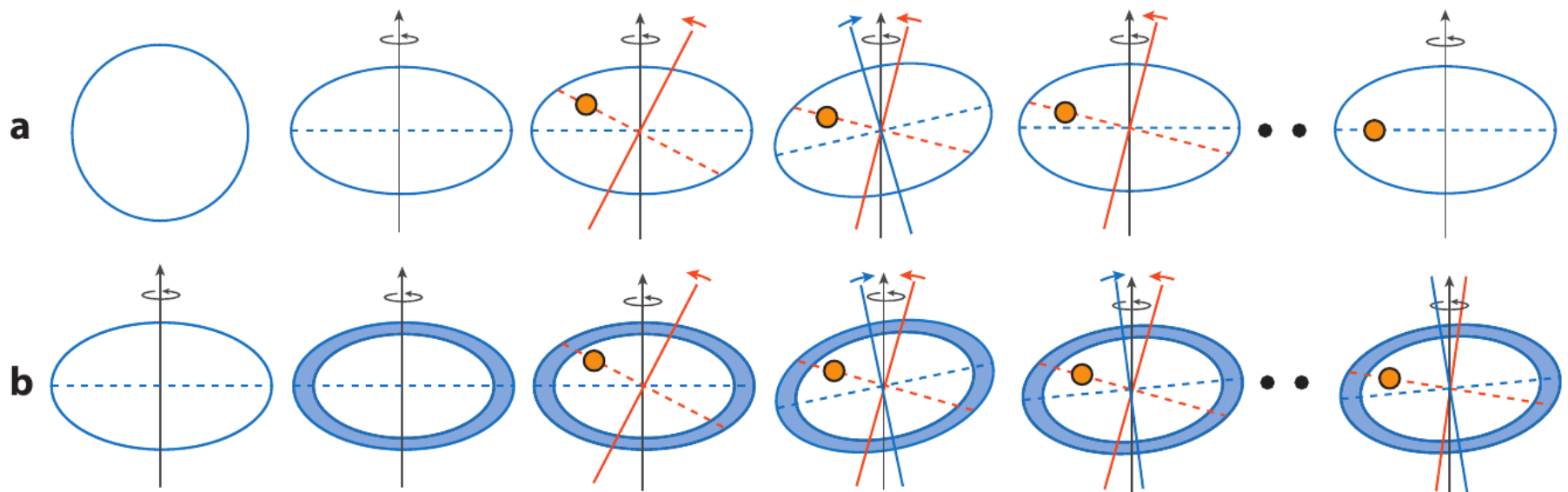
Keane and Matsuyama (2014): Evidence for lunar true polar wander and past low-eccentricity,...



Mechanism: fossil (remnant) bulge



Keane and Matsuyama (2014): Evidence for lunar true polar wander and past low-eccentricity,...



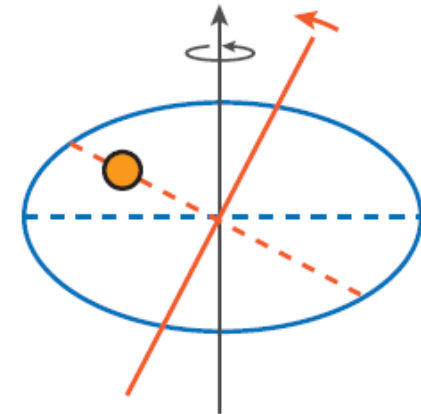
Ann. Rev. Earth Planet. Sci.: Matsuyama I., Nimmo F., and Mitrovica J.X. (2014)

Mechanism: rotation vs. tides

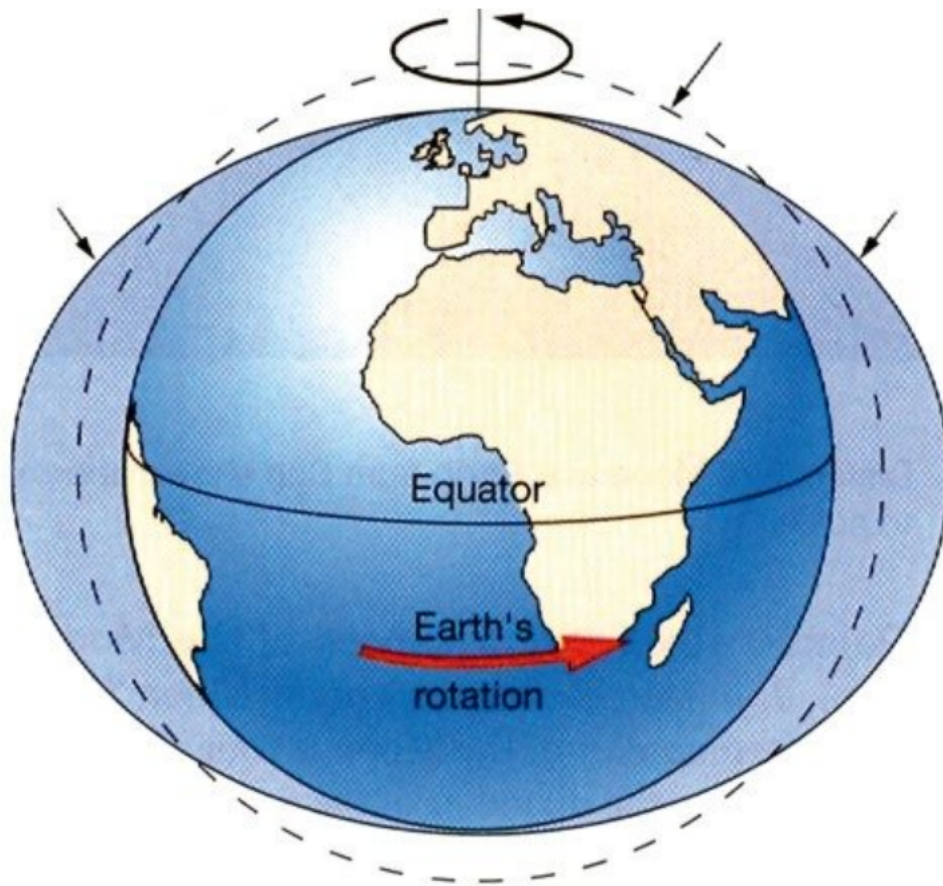
Rotation: bulge perpendicular to ω axis

Tides: bulge along the tidal axis

rotational axis

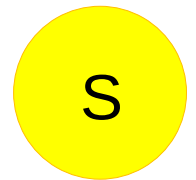


rotational axis

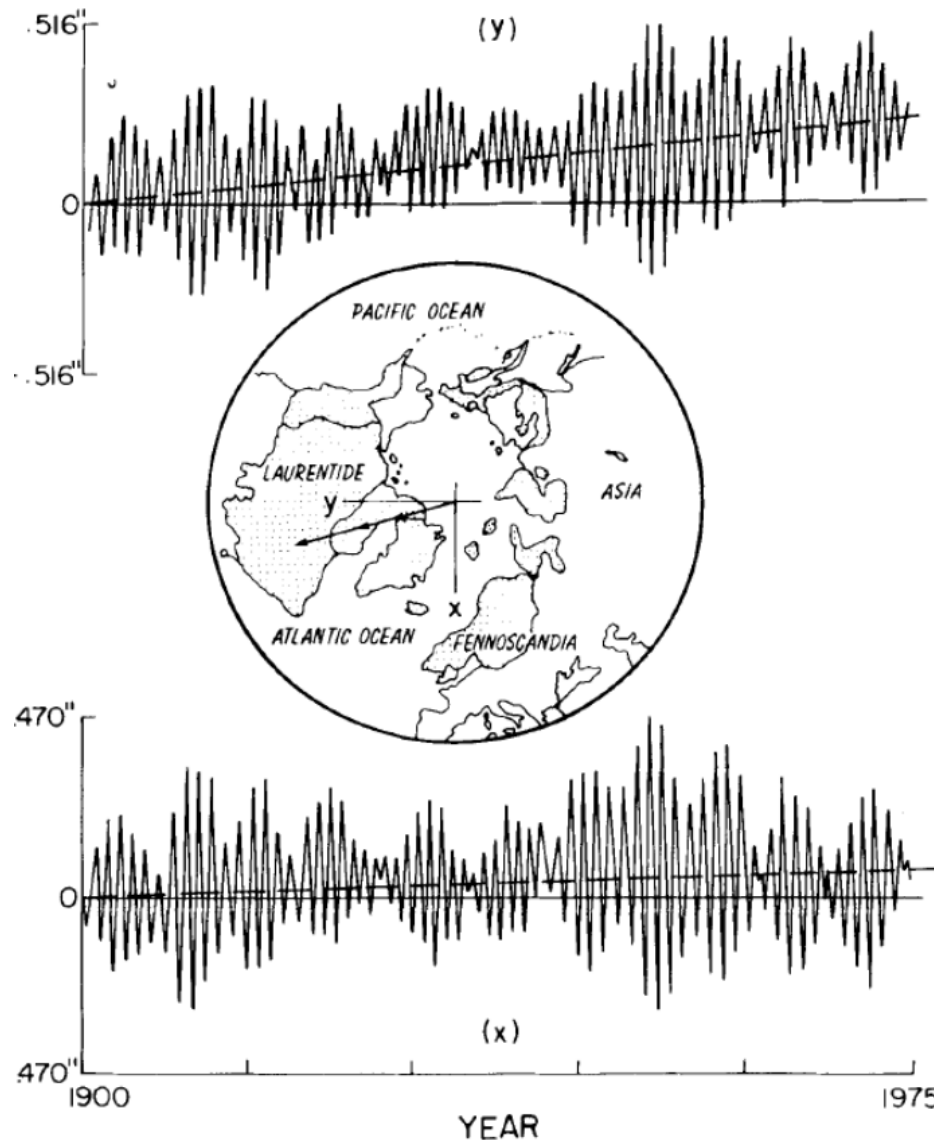


tidal axis

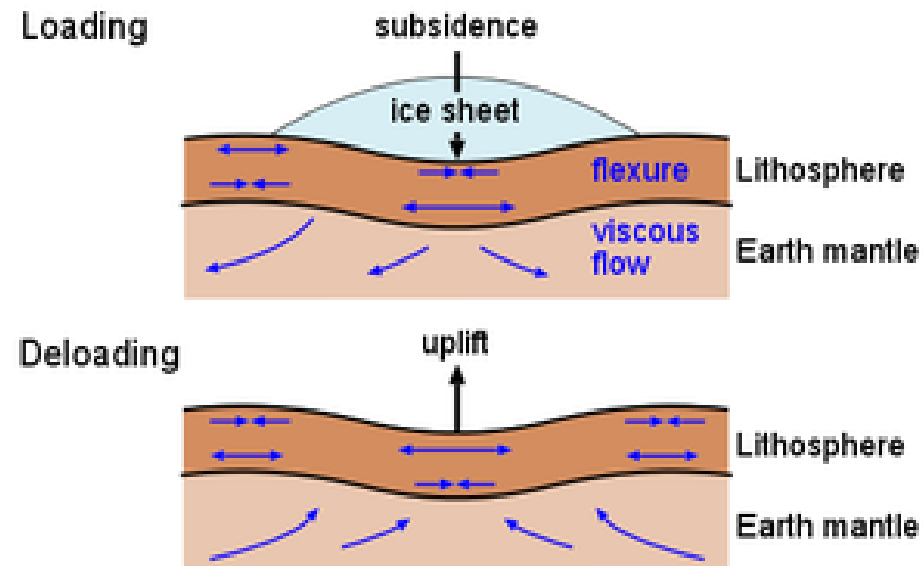
S



TPW: Earth vs. planets and moons



Wu and Peltier (1984):
Pleistocene deglaciation
and the Earth's rotation:
a new analysis

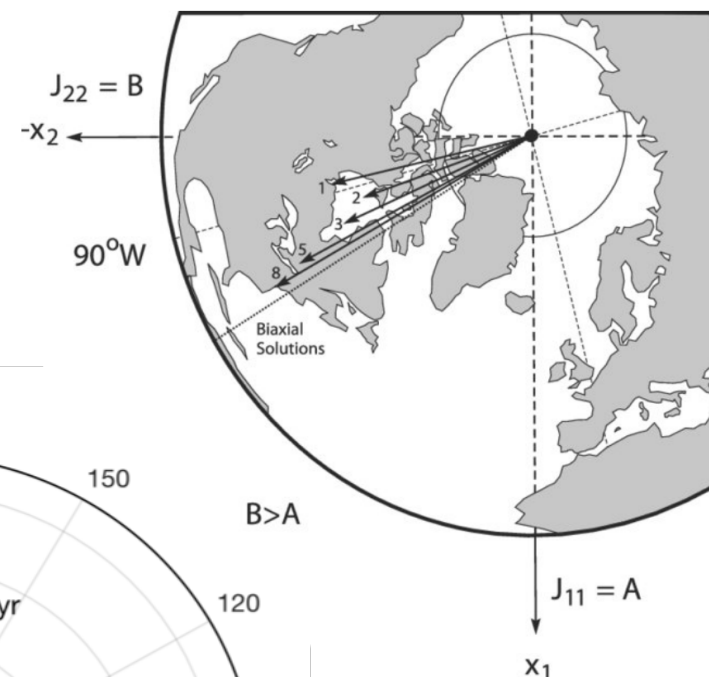
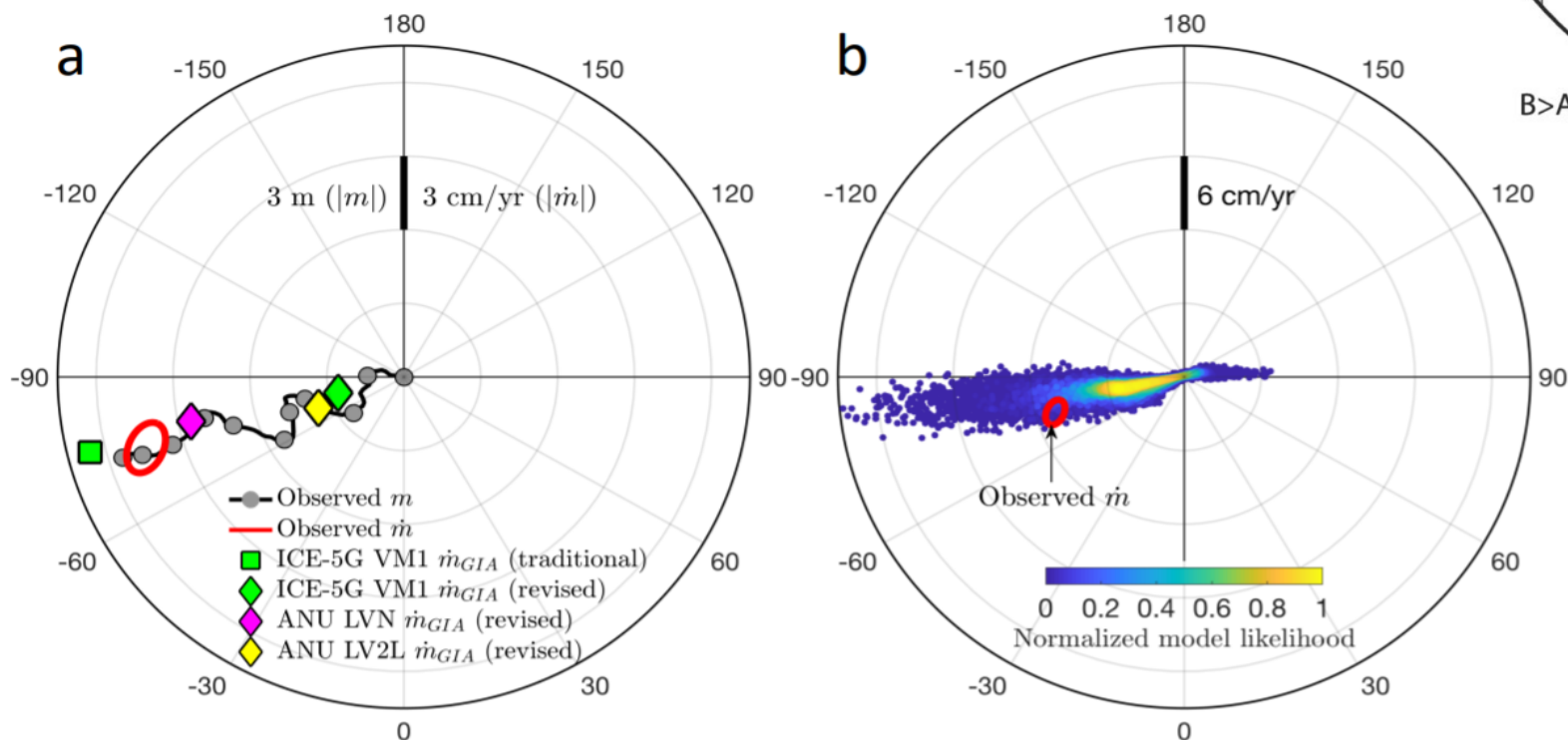


TPW: fine details on Earth

Recent advances:

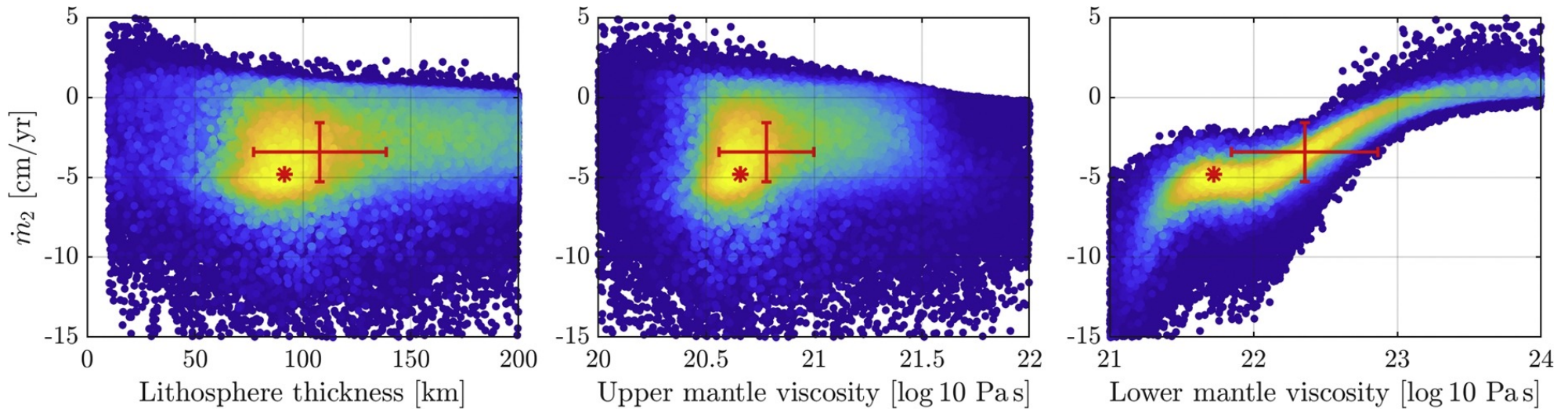
- Earth's (fossil) tri-axiality
- Land-ocean mass transfer
- Mantle convection as the missing source

Matsuyama et al. (2010)

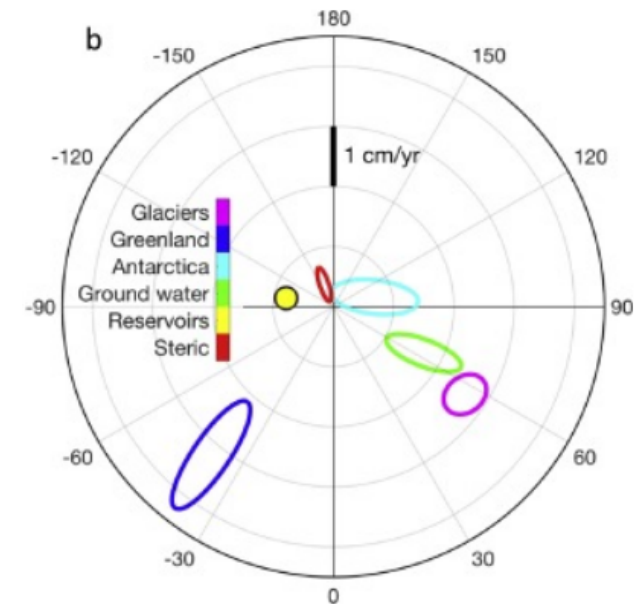


Adhikari et al. (2018, EPSL): What drives 20th century polar motion?

TPW: fine details on Earth

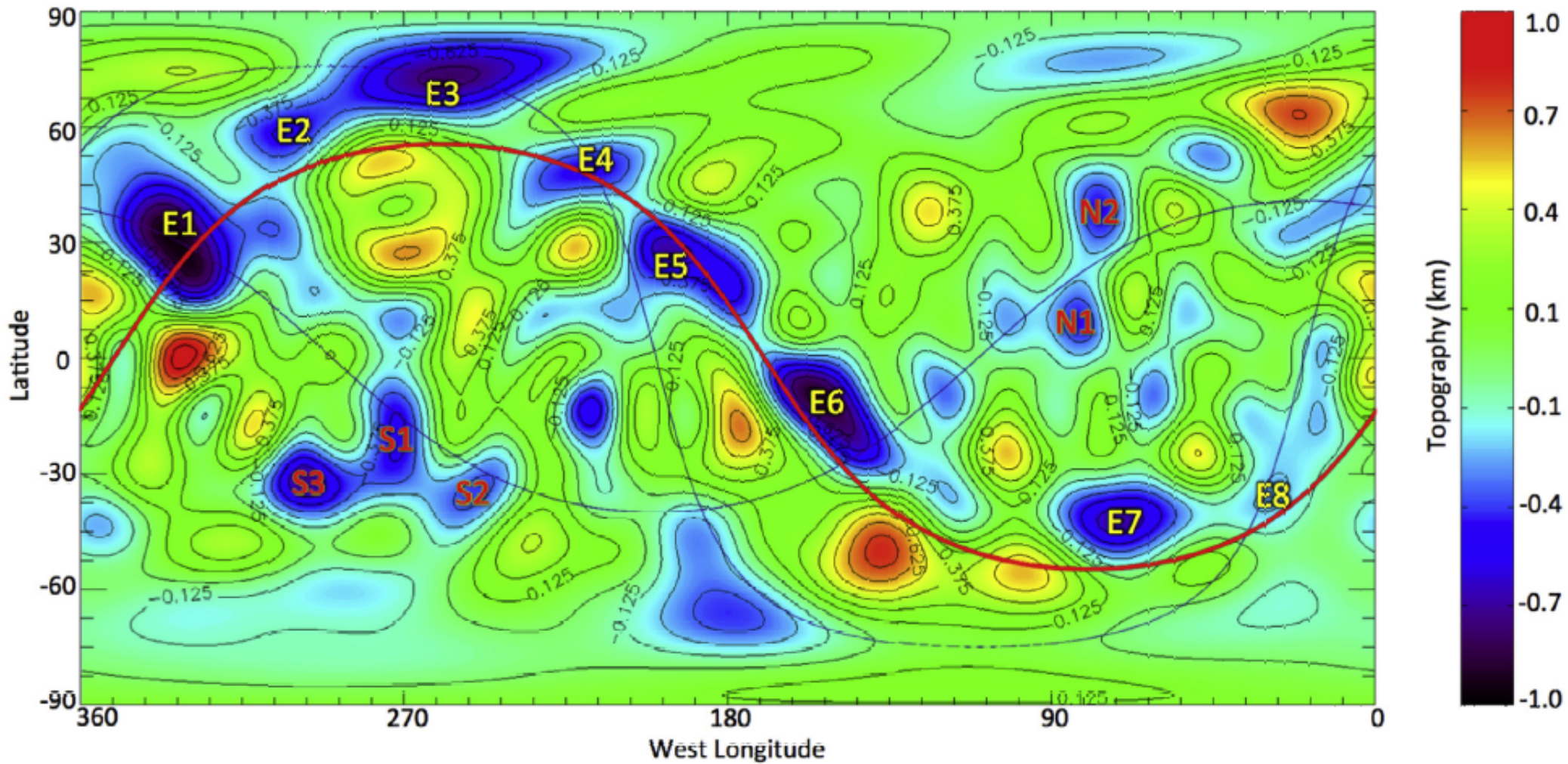


Source	GMSL rate (mm yr ⁻¹)	SPM rate $\dot{m}_1$ (cm yr ⁻¹)	SPM rate $\dot{m}_2$ (cm yr ⁻¹)	References
Groundwater	0.17 ± 0.04	0.77 ± 0.32	1.50 ± 0.62	Wada et al. (2012)
Dams/reservoirs	-0.29	-0.14	-0.78	Chao et al. (2008)



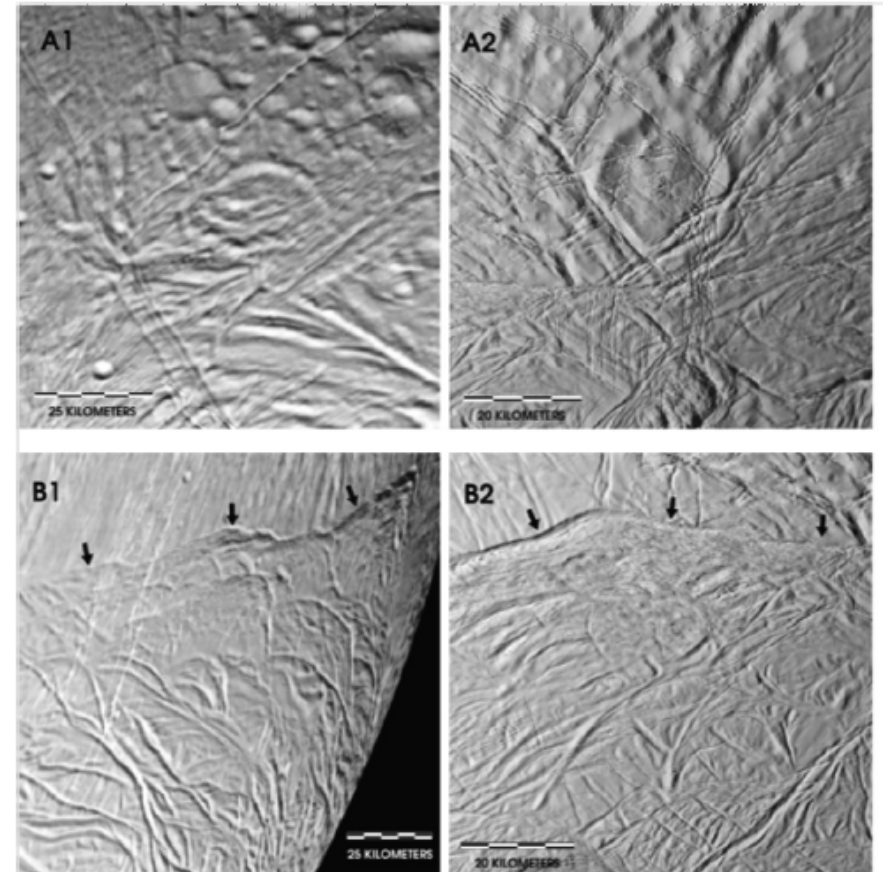
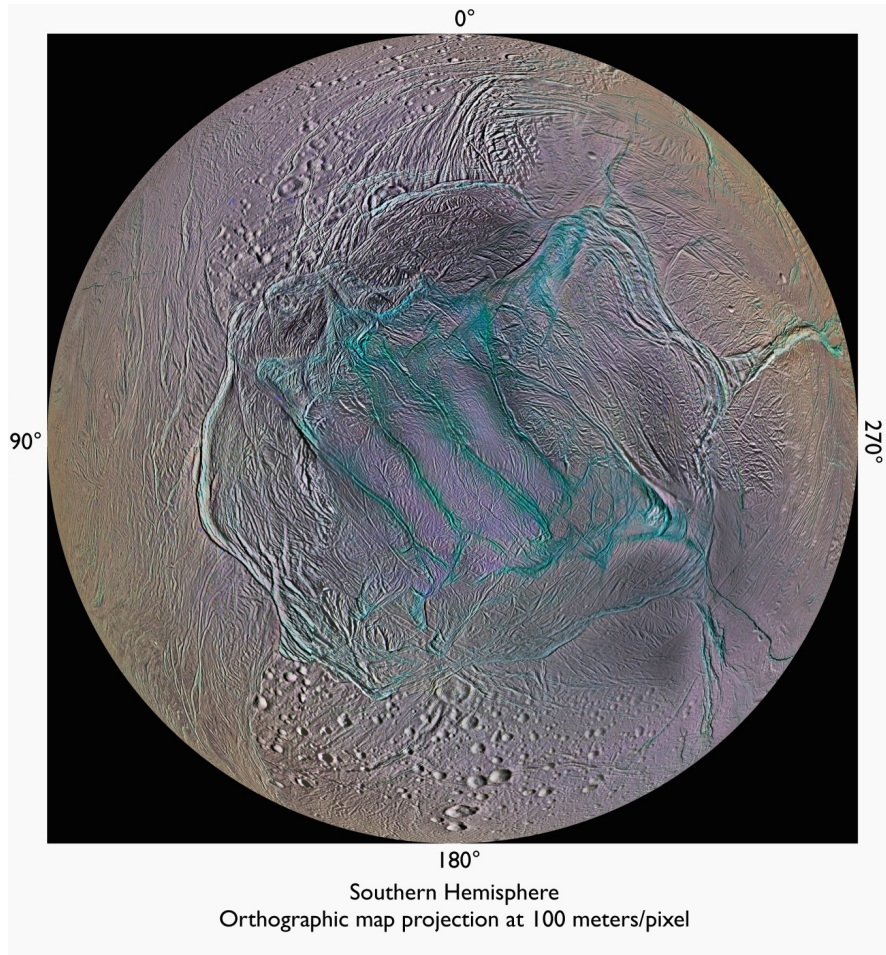
Adhikari et al. (2018, EPSL): What drives 20th century polar motion?

Enceladus



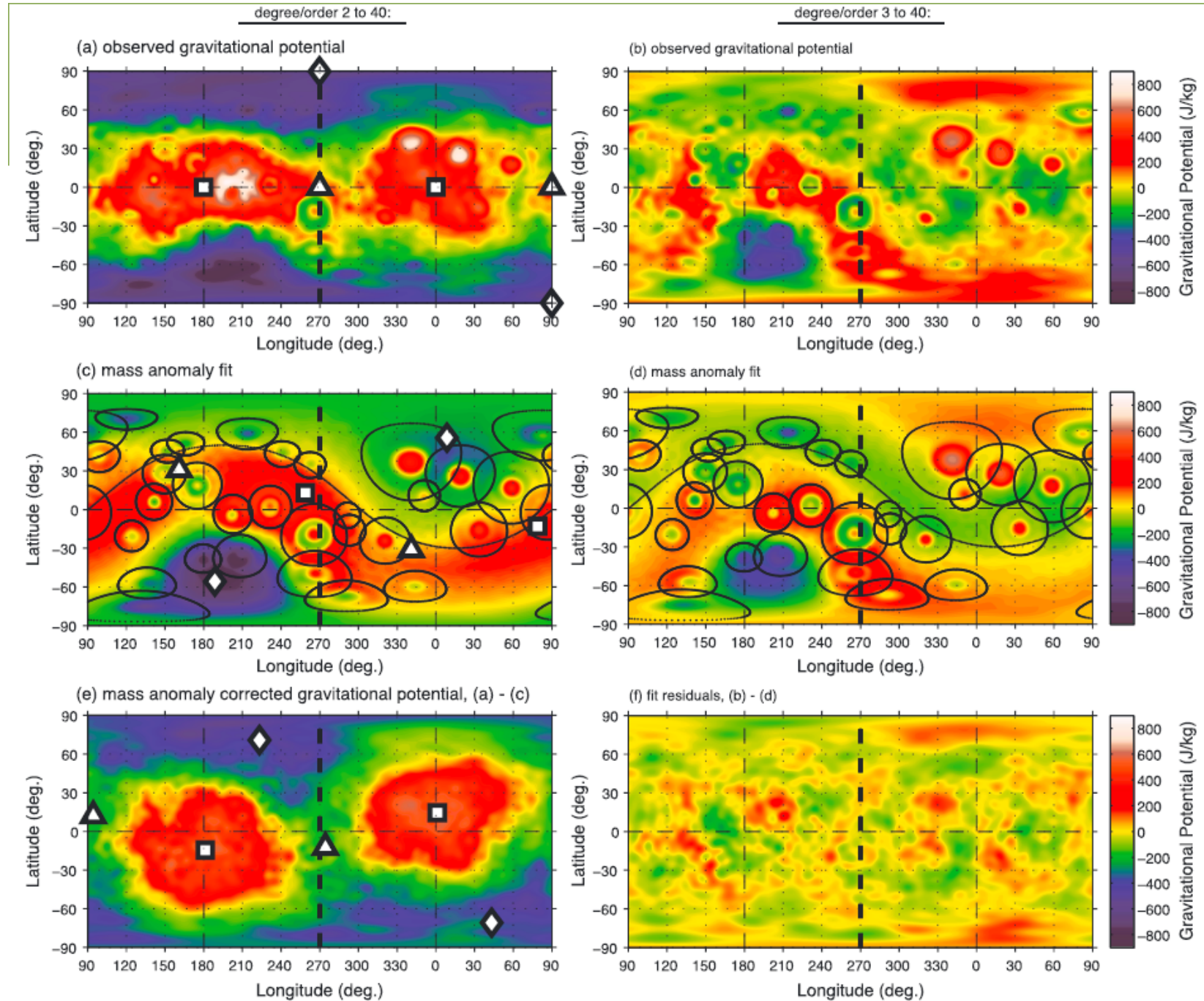
Tajeddine et al. (2017): True polar wander of Enceladus from topographic data

Enceladus

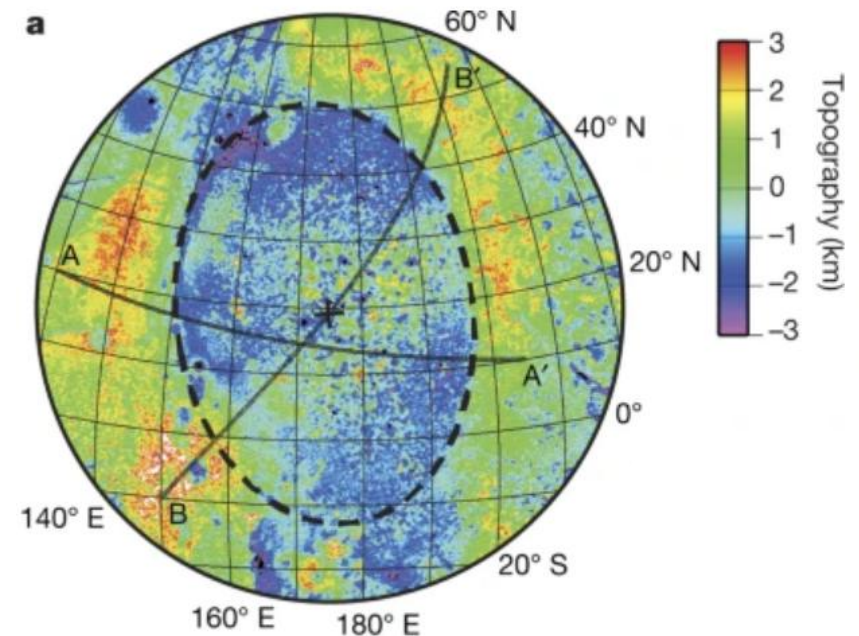
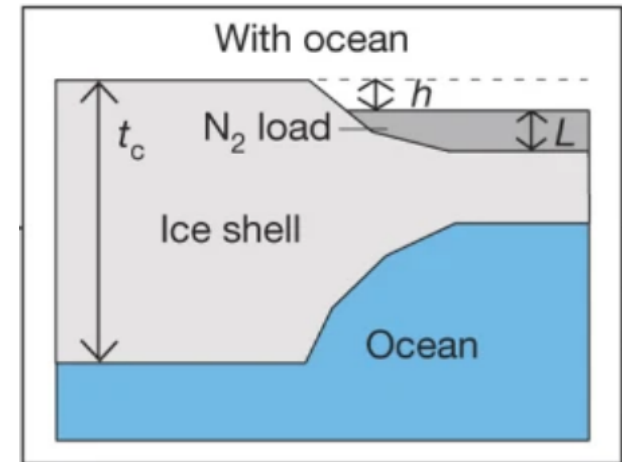
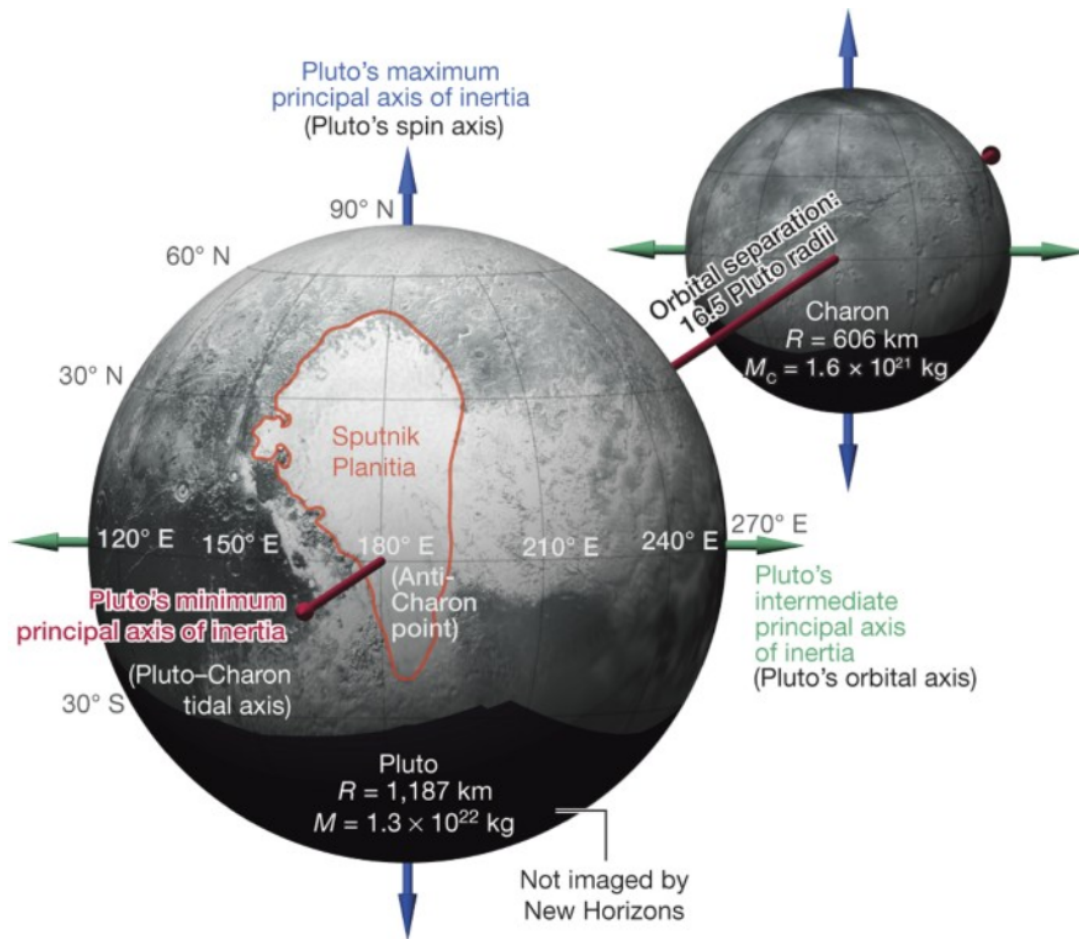


Tajeddine et al. (2017): True polar wander of Enceladus from topographic data

The Moon



Pluto

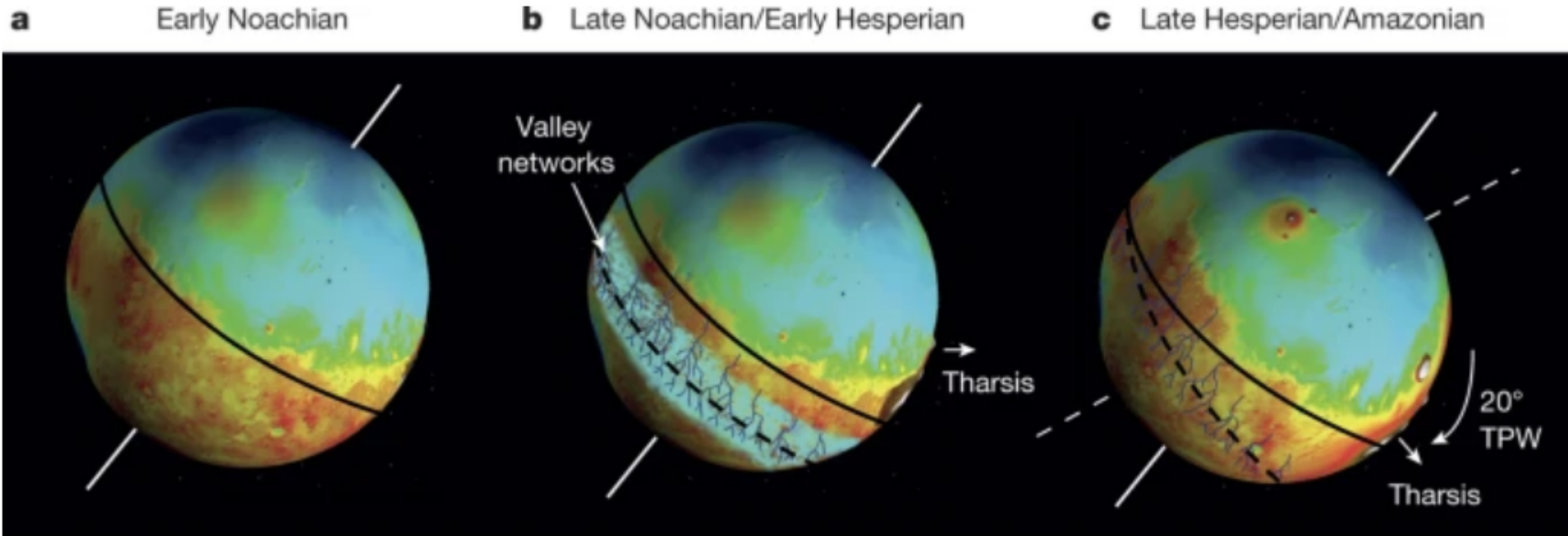
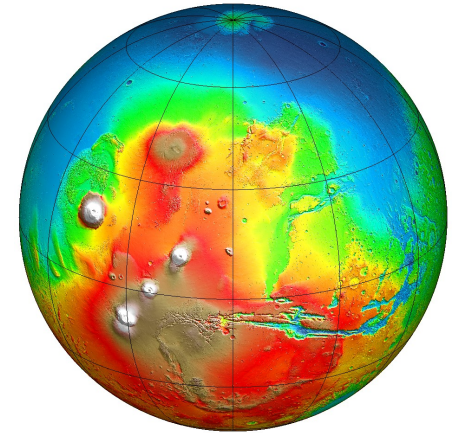


Keane et al., 2016: Reorientation and faulting of Pluto due to volatile loading within Sputnik Planitia (Nature)

Nimmo et al., 2016: Reorientation of Sputnik Planitia implies a subsurface ocean on Pluto (Nature)

Mars

Bouley et al. (2016, Nature): Late Tharsis formation and implications for early Mars



Formation of the dichotomy, heavy bombardment, dichotomy boundary in equatorial position

Tharsis dome formation, tropical precipitation with valley networks formation

Tharsis formation causes a TPW, valley networks on a small circle, Tharsis bulge on the equator

Large-wavelength convection (on exoplanets)

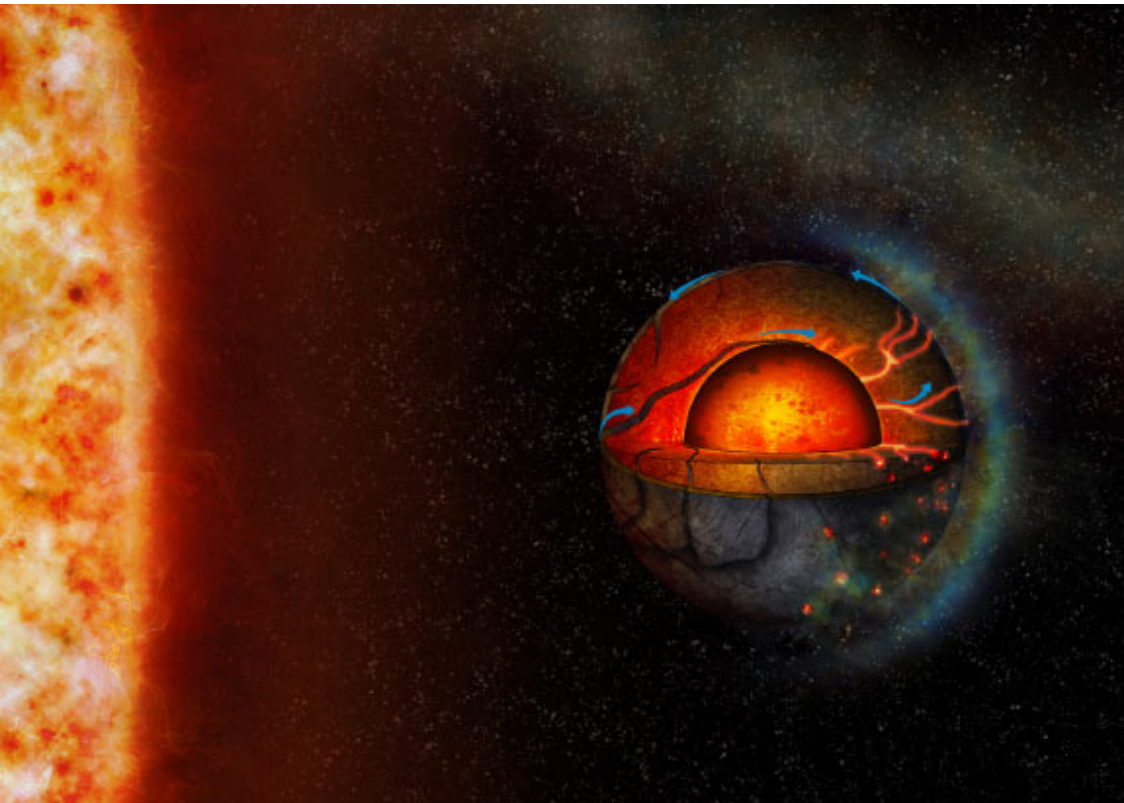
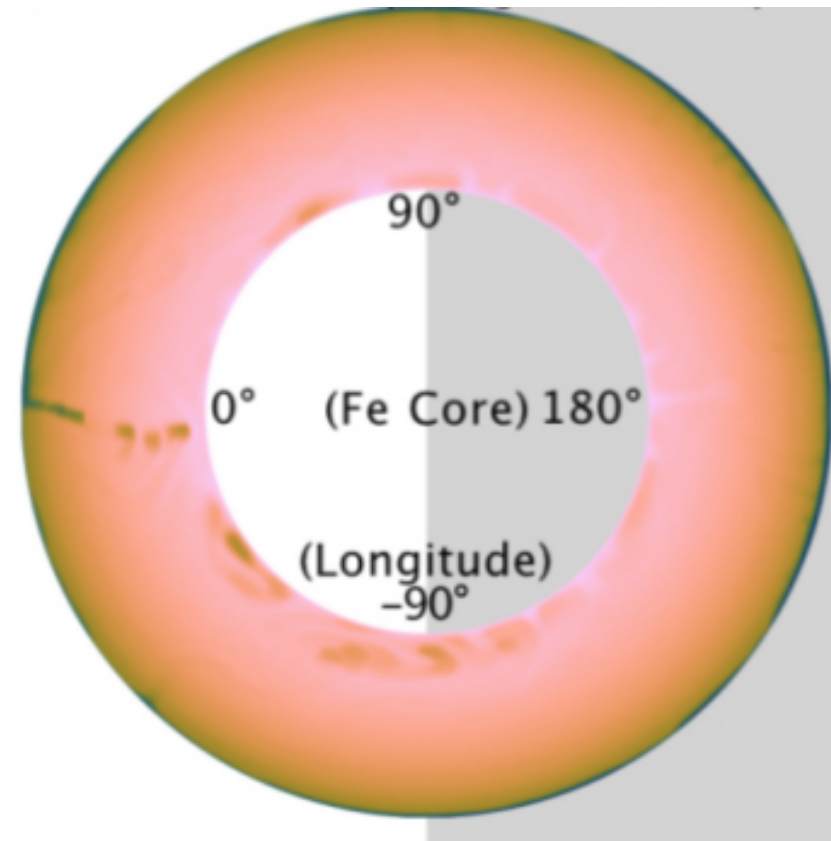


Image: Thibault Roger



Meier et al. (2021): Hemispheric tectonics

TPW: methods overview

Peltier, W. R. (1974). The impulse response of a Maxwell Earth.

Willemann, R. (1984). Reorientation of Planets with Elastic Lithospheres.

Ricard, Y., Spada, G., & Sabadini, R. (1993, MAY). Polar wandering of a dynamic

Lefftz, M., Legros, H., & Hinderer, J. (1991, MAR). Non-linear equations for the rotation of a viscoelastic planet taking into account the influence of a liquid

Vermeersen, L., & Sabadini, R. (1996, NOV). Significance of the fundamental mantle rotational relaxation mode in polar wander simulations.

Recently:

Hu, H., van der Wal, W., & Vermeersen, L. L. A. (2017a, JAN). A numerical method for reorientation of rotating tidally deformed viscoelastic bodies.

Hu, H., van der Wal, W., & Vermeersen, L. L. A. (2017b, DEC). A Full-Maxwell Approach for Large-Angle Polar Wander of Viscoelastic Bodies.

Hu, H., van der Wal, W., & Vermeersen, L. L. A. (2019, MAR 15). Rotational dynamics of tidally deformed planetary bodies and validity of fluid limit and quasi-fluid approximation.

TPW: methods overview

Linearized Liouville Equation (LLE)

$$m_1(t) = \frac{\Delta I_{13}(t)}{C - A} + \frac{C \Delta \dot{I}_{23}(t)}{\Omega(C - A)(C - B)}$$

1)

$$m_2(t) = \frac{\Delta I_{23}(t)}{C - B} - \frac{C \Delta \dot{I}_{13}(t)}{\Omega(C - A)(C - B)}$$

$$m_3(t) = -\frac{\Delta I_{33}}{C}$$

Liouville Equation (LE)

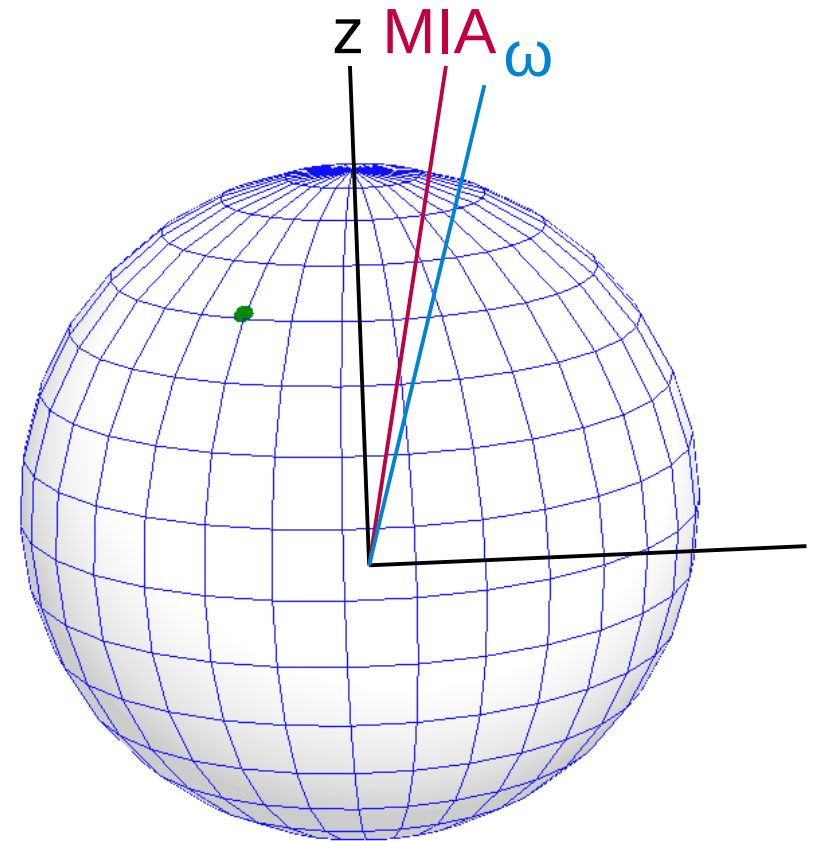
$$2) \quad 0 = \frac{d}{dt}(\mathbf{I} \cdot \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega})$$

Lefftz, M., Legros, H., & Hinderer, J.

$$3) \quad \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) = 0$$

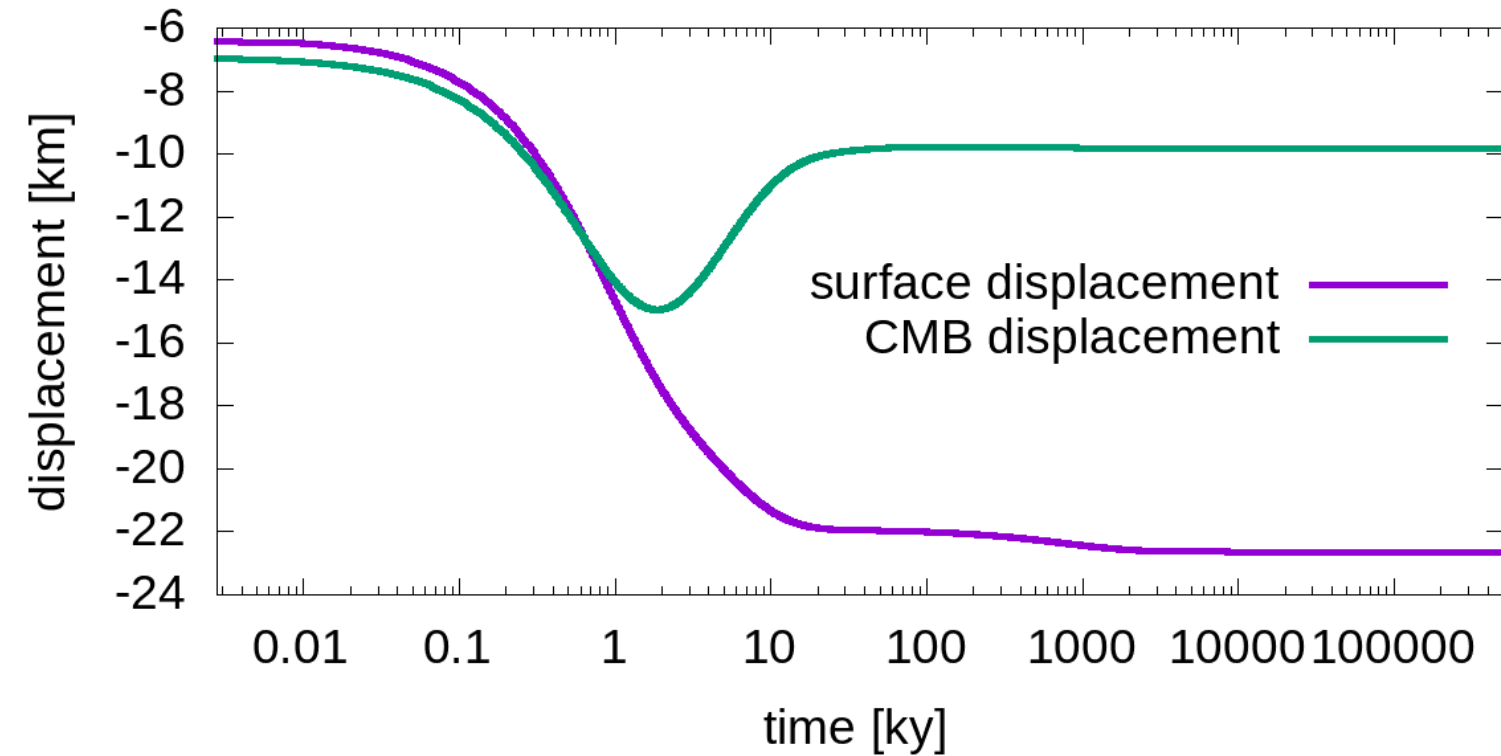
Hu, H., van der Wal, W., & Vermeersen, L. L. A.

4) LLE + iterative transformation of coordinated



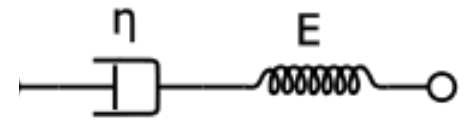
here:
2) - 4)

TPW: bulge formation / readjustment

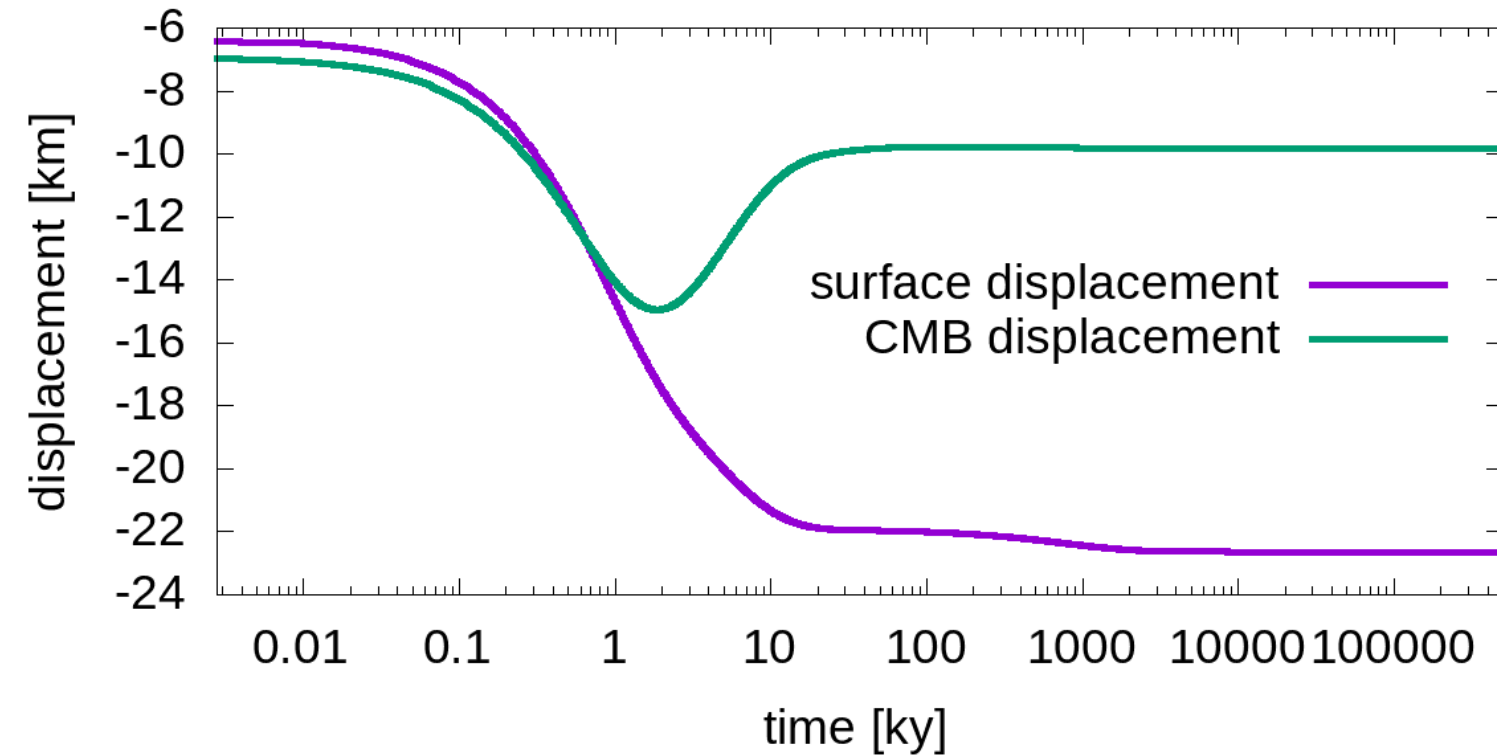


$$\nabla \cdot \boldsymbol{\tau} + \boldsymbol{f} = 0$$

$$\nabla \cdot \boldsymbol{u} = 0$$

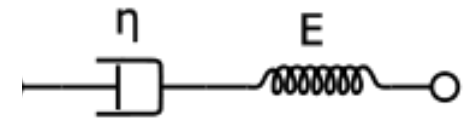


TPW: methods overview



$$\nabla \cdot \boldsymbol{\tau} + \boldsymbol{f} = 0$$

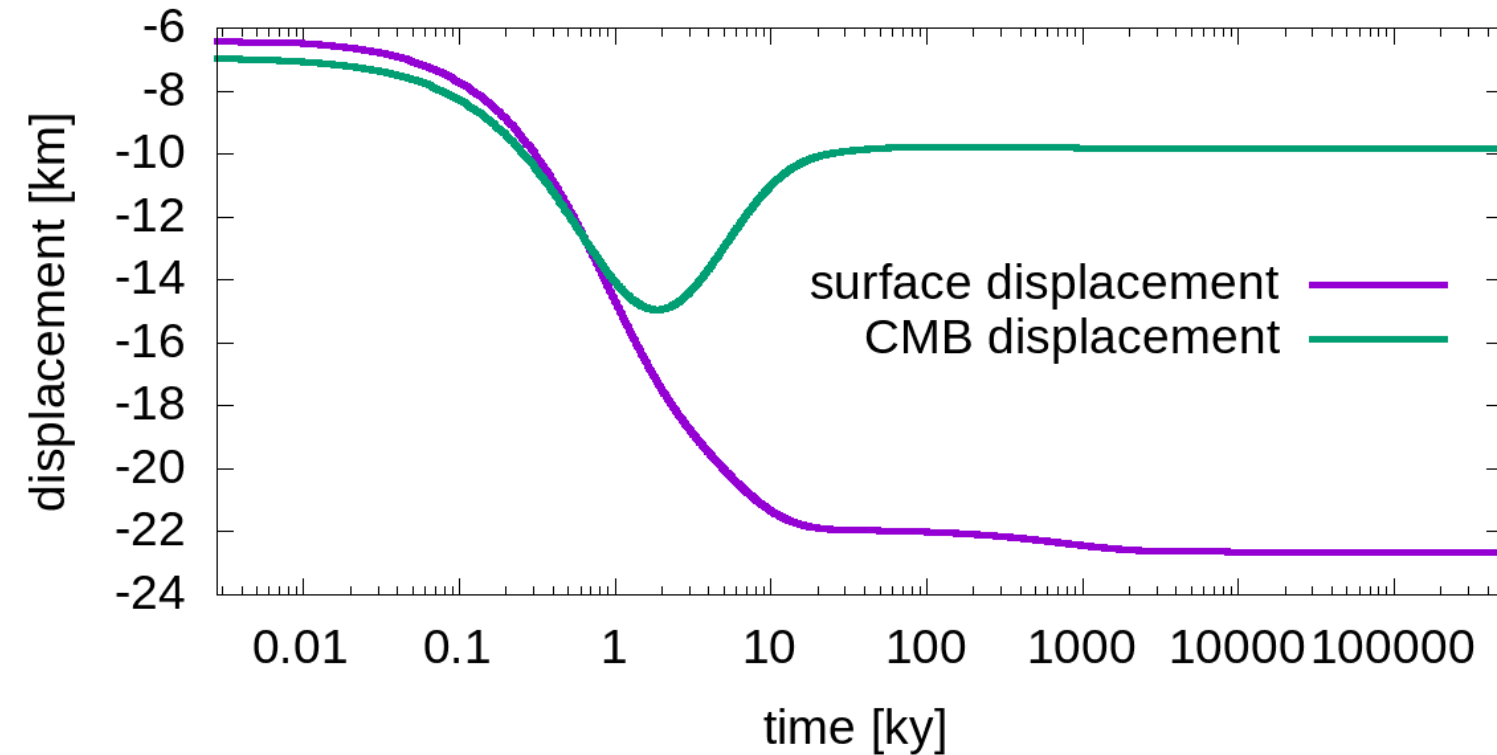
$$\nabla \cdot \boldsymbol{u} = 0$$



$$\boldsymbol{k}^T(s) = \boldsymbol{k}_e^T + \sum_{i=1}^M \frac{\boldsymbol{k}_i^T}{s - s_i}$$

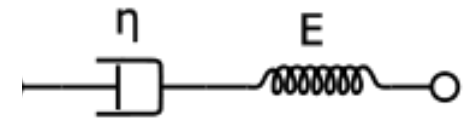
$$0 = \frac{d}{dt}(\boldsymbol{I} \cdot \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\boldsymbol{I} \cdot \boldsymbol{\omega})$$

TPW: methods overview



$$\nabla \cdot \boldsymbol{\tau} + \boldsymbol{f} = 0$$

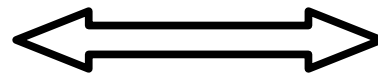
$$\nabla \cdot \boldsymbol{u} = 0$$



$$k^T(s) = k_e^T + \sum_{i=1}^M \frac{k_i^T}{s - s_i}$$

$$0 = \frac{d}{dt}(\mathbf{I} \cdot \boldsymbol{\omega}) + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega})$$

Lefftz et al. (1991)



$$\approx k_e^T - \sum_{i=1}^M \frac{k_i^T}{s_i} = k_f^T (1 - T_1 s)$$

$$\boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) = 0$$

Hu et al. (2017a, 2017b, 2019)

Hu, H., van der Wal, W., & Vermeersen, L. L. A. (2017b, DEC). A Full-Maxwell Approach for Large-Angle Polar Wander of Viscoelastic Bodies.

Figure 5. A point mass of -5×10^{18} kg is attached at the surface at 45° colatitude. (top row) The displacement of the rotational axis in the along-track and normal direction. (bottom row) The movement of the mass anomaly in the bulge-fixed frame where the rotational axis is always pointing upward at the center. Blue lines are obtained by applying the quasi-fluid approximation, and red lines are from the full-Maxwell method. The black dots are the original locations of the mass anomalies in the bulge-fixed frame.

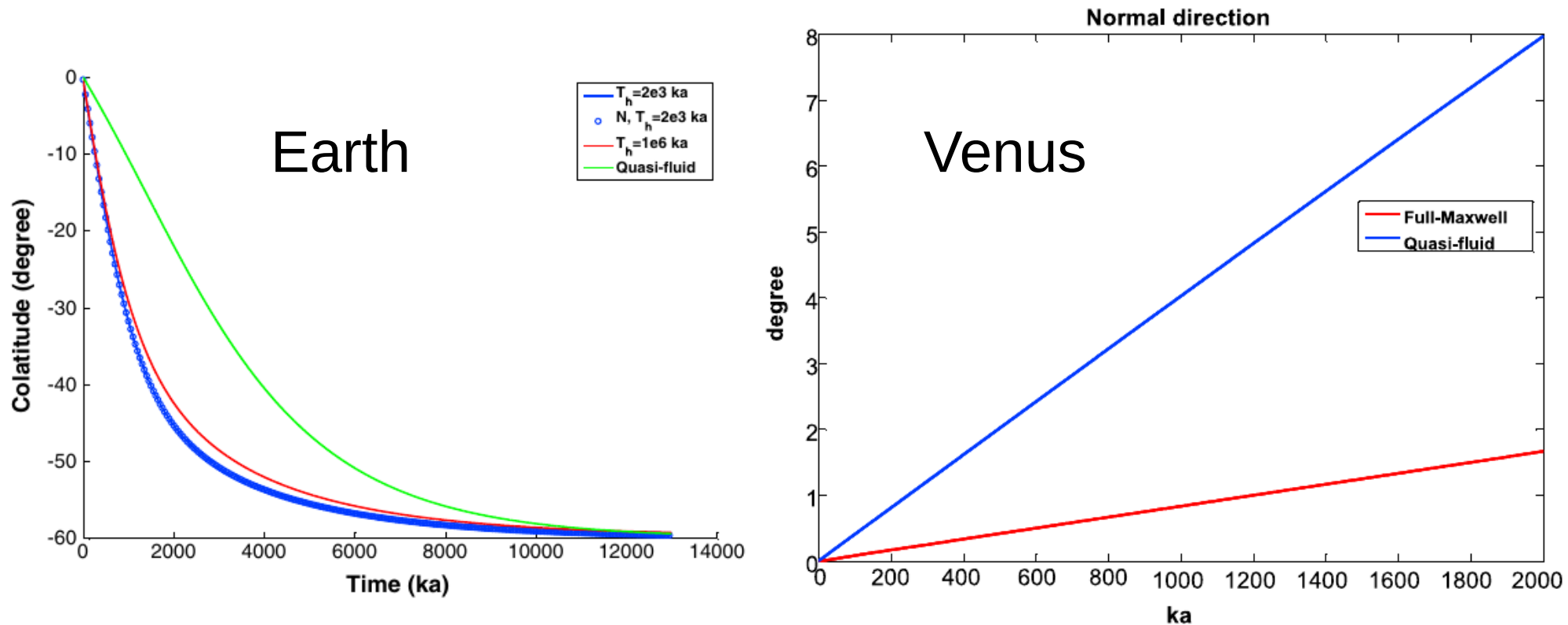
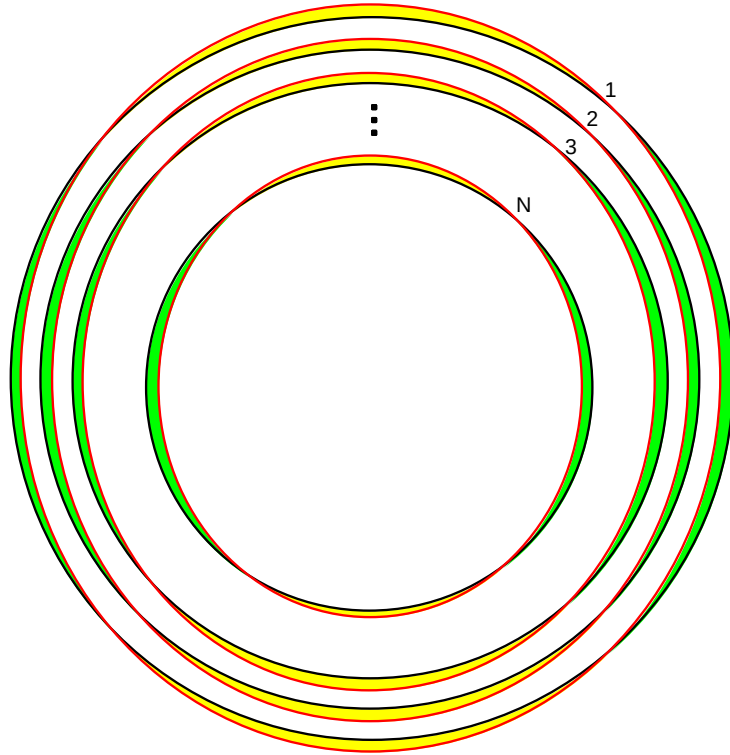


Figure 4. Polar wander in the X-Z plane for the SG6 Earth model triggered by a point mass of 2×10^{19} kg attached at the surface at 30° colatitude.

LIOSHHELL

open access GitHub repository

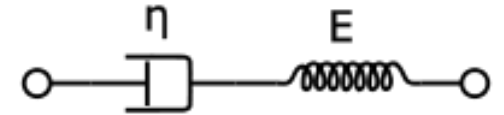


- $\delta\rho = [\rho_0]_i$
- $\delta\rho = -[\rho_0]_i$
- $\delta\rho = 0$
- deformed body
- reference sphere

Smooth-profiled or layered bodies

Maxwell viscoelastic rheology

Spherical harmonic decomposition



$$\nabla \cdot \boldsymbol{\tau} + \boldsymbol{f} = 0$$

$$\nabla \cdot \boldsymbol{u} = 0$$

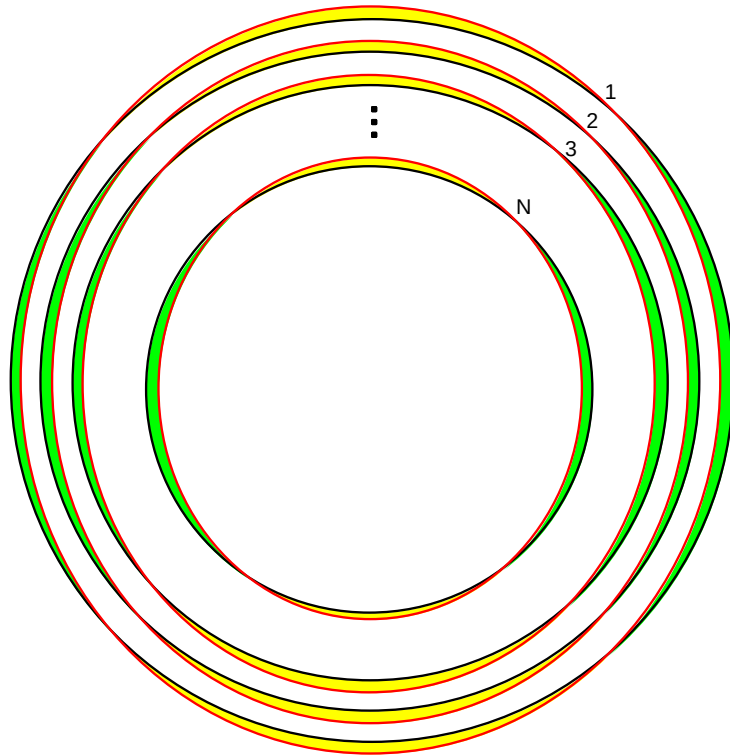
$$\boldsymbol{\tau}^d - \mu(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T) = -\frac{\mu}{\eta} \int_0^t \boldsymbol{\tau}^d dt'$$

True Polar Wander on Dynamic Planets: Approximative Methods vs. Full Solution

Vojtěch Patočka

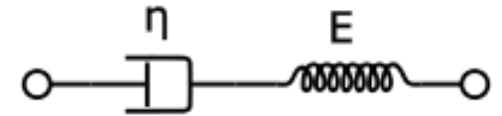
e2021JE006948 | First Published: 01 November 2021

JGR Planets



- $\delta\rho = [\rho_0]_i$
- $\delta\rho = -[\rho_0]_i$
- $\delta\rho = 0$
- deformed body
- reference sphere

Smooth-profiled or layered bodies
Maxwell viscoelastic rheology
Spherical harmonic decomposition



$$\nabla \cdot \boldsymbol{\tau} + \mathbf{f} = 0$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\boldsymbol{\tau}^d - \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) = -\frac{\mu}{\eta} \int_0^t \boldsymbol{\tau}^d dt'$$

Liouville equation: conservation of angular momentum in a rotating frame

$$\frac{d\boldsymbol{\omega}}{dt} = -\mathbf{I}^{-1} \cdot \left(\frac{d\mathbf{I}}{dt} \cdot \boldsymbol{\omega} + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) \right) \quad \text{OR} \quad \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) = 0$$

Slow rotating bodies: Venus

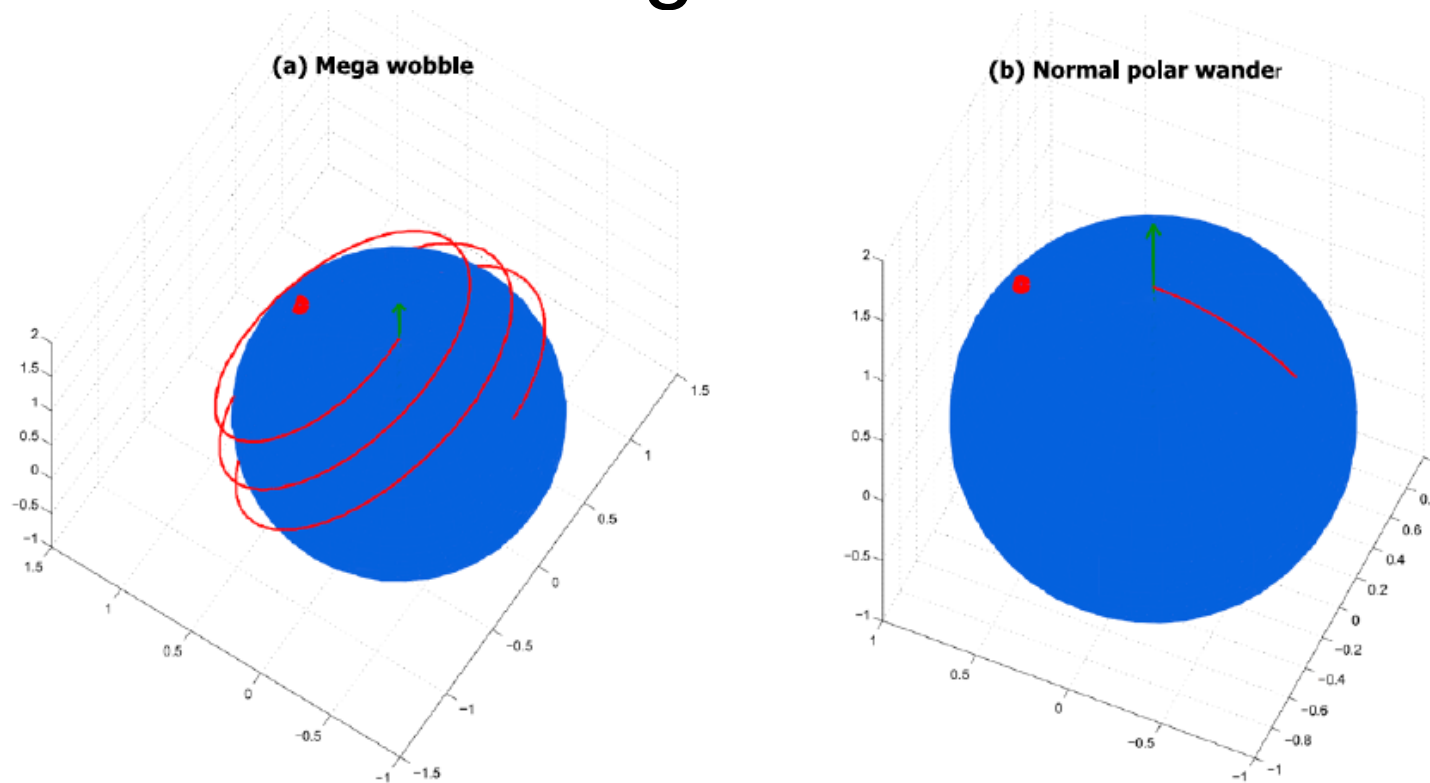


Figure 5. Two types of polar wander path. The green arrow is the initial position of the rotational axis and the red dot is the point mass load.

Slow rotating bodies: Venus

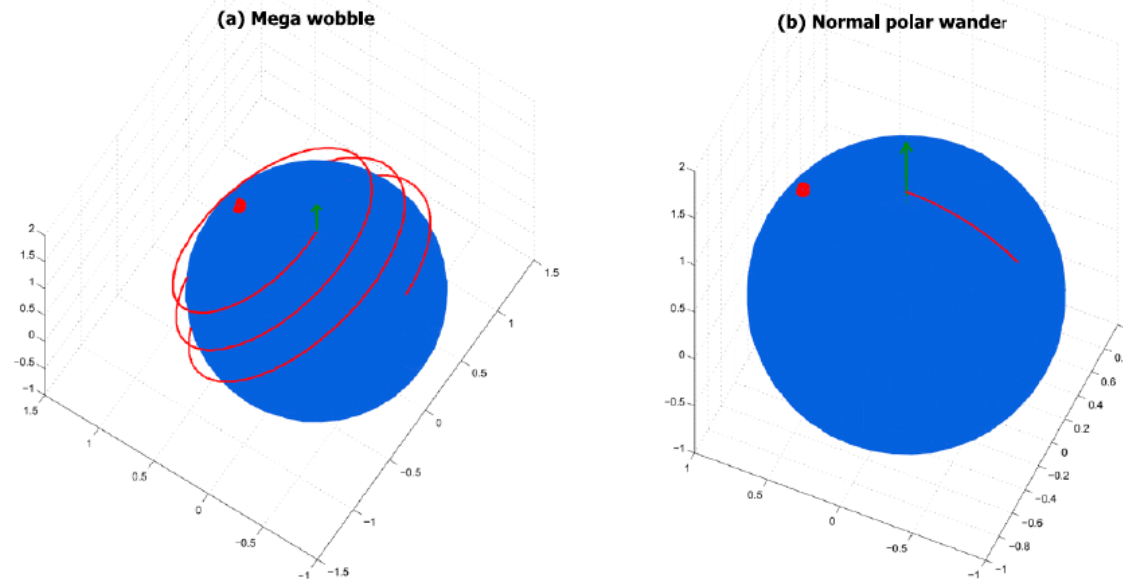
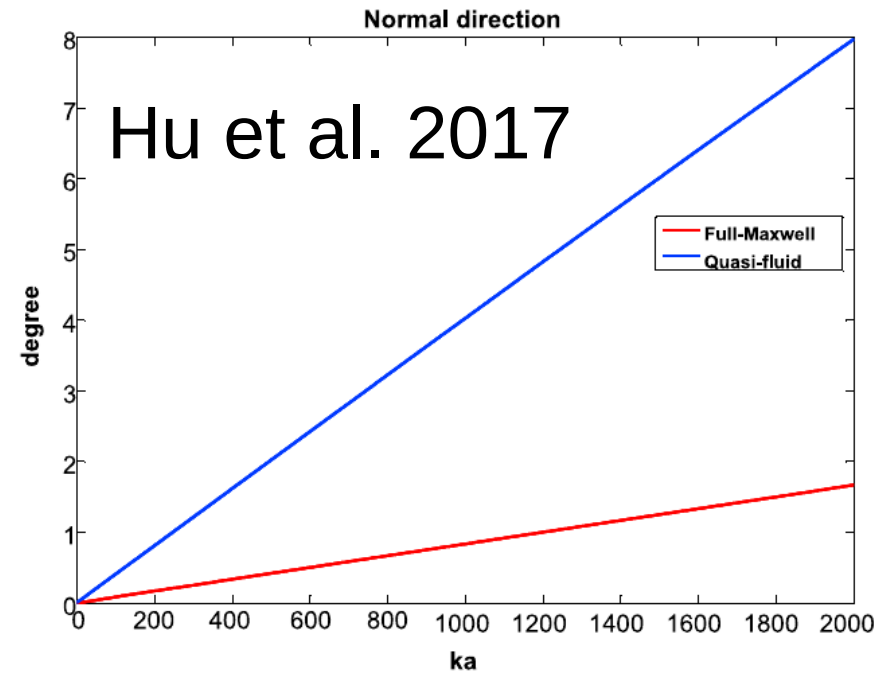
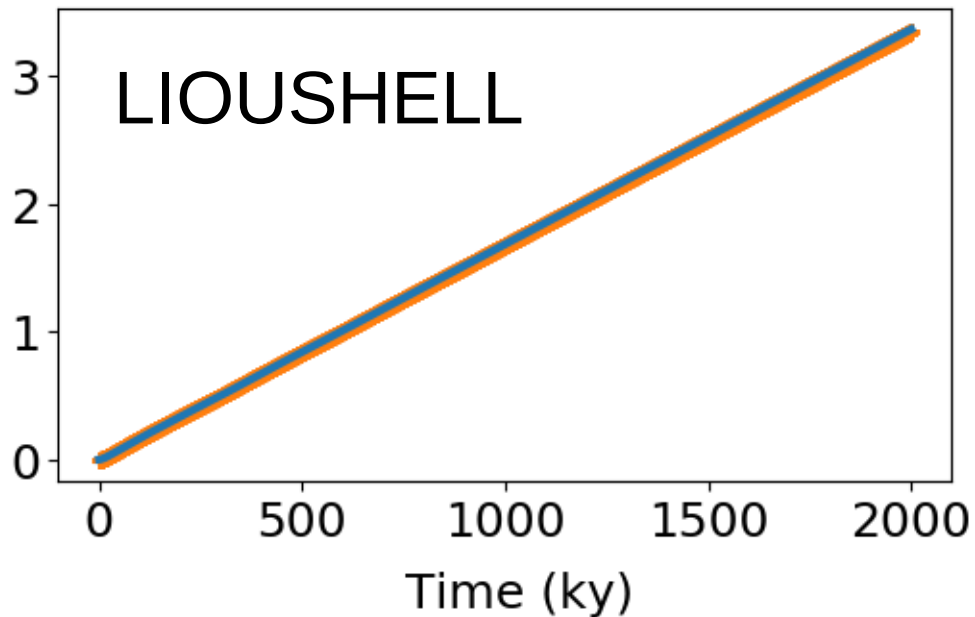
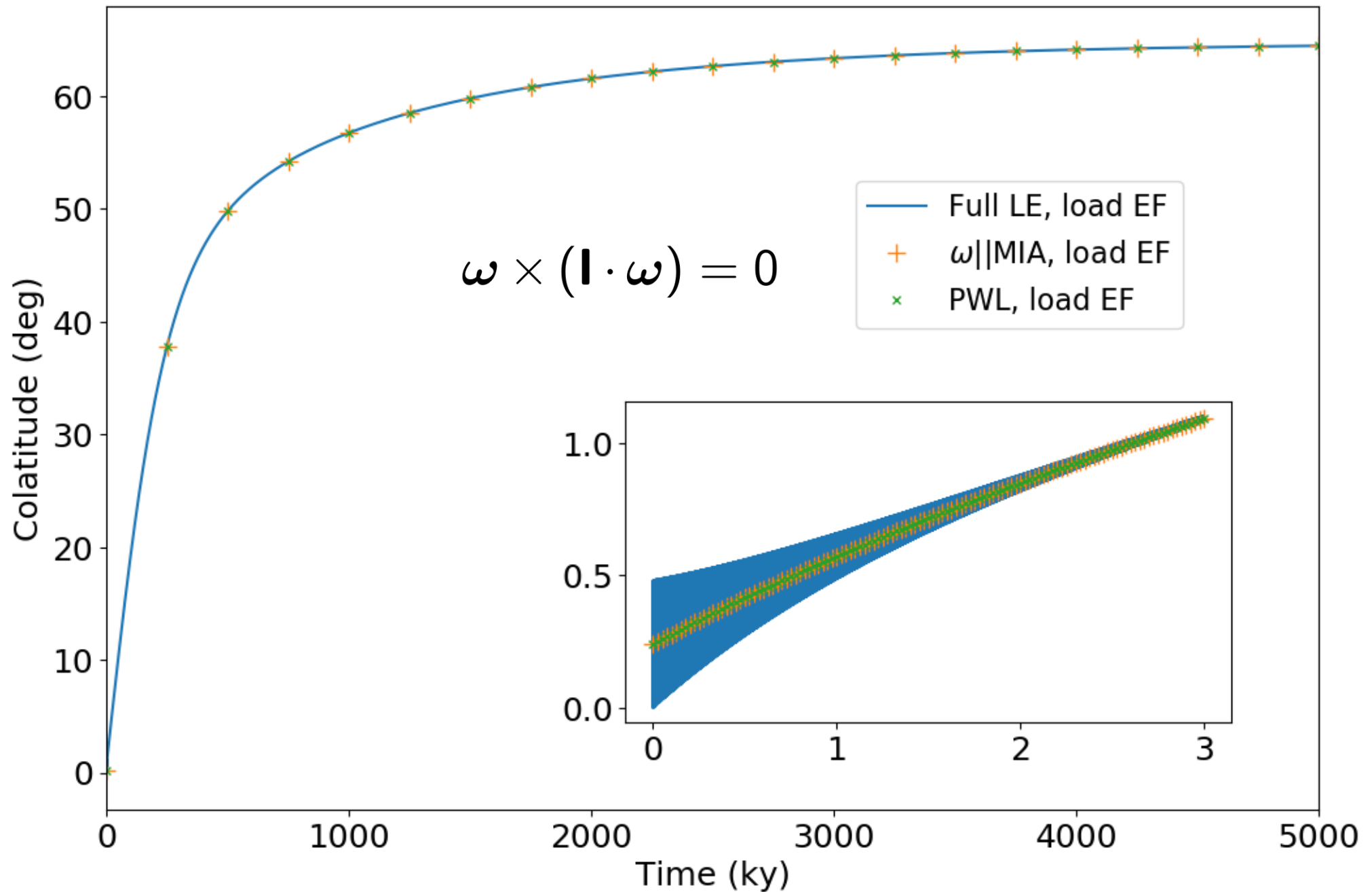


Figure 5. Two types of polar wander path. The green arrow is the initial position of the rotational axis and the red dot is the point mass load.



Fast rotating bodies: Earth



PWL approach of Hu et al.

Linearized Liouville equation:

$$m_1(t) = \frac{\Delta I_{13}(t)}{C - A} + \frac{C \Delta \dot{I}_{23}(t)}{\Omega(C - A)(C - B)}$$

$$m_2(t) = \frac{\Delta I_{23}(t)}{C - B} - \frac{C \Delta \dot{I}_{13}(t)}{\Omega(C - A)(C - B)}$$

$$m_3(t) = -\frac{\Delta I_{33}}{C}$$

Quasi-fluid approach:

$$\boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) = 0$$

+ Iterative transformation of coordinates:

$$\mathbf{Q} = \begin{pmatrix} \omega_3 + \frac{\omega_2^2}{1+\omega_3} & -\frac{\omega_1\omega_2}{1+\omega_3} & \omega_1 \\ -\frac{\omega_1\omega_2}{1+\omega_3} & 1 - \frac{\omega_2^2}{1+\omega_3} & \omega_2 \\ -\omega_1 & -\omega_2 & \omega_3 \end{pmatrix}$$

PWL approach of Hu et al.

Linearized Liouville equation:

$$m_1(t) = \frac{\Delta l_{13}(t)}{C - A} + \frac{C \Delta \dot{l}_{23}(t)}{\Omega(C - A)(C - B)}$$

$$m_2(t) = \frac{\Delta l_{23}(t)}{C - B} - \frac{C \Delta \dot{l}_{13}(t)}{\Omega(C - A)(C - B)}$$

$$m_3(t) = -\frac{\Delta l_{33}}{C}$$

+ Iterative transformation of coordinates:

$$\mathbf{Q} = \begin{pmatrix} \omega_3 + \frac{\omega_2^2}{1+\omega_3} & -\frac{\omega_1\omega_2}{1+\omega_3} & \omega_1 \\ -\frac{\omega_1\omega_2}{1+\omega_3} & 1 - \frac{\omega_2^2}{1+\omega_3} & \omega_2 \\ -\omega_1 & -\omega_2 & \omega_3 \end{pmatrix}$$

$$l_{ij}(t) = l\delta_{ij} + \frac{k^T(t)a^5}{3G} * \left(\Omega^2 \left[\omega_i(t)\omega_j(t) - \frac{1}{3}\delta_{ij} \right] \right) + [\delta(t) + k^L(t)] * C_{ij}(t)$$

Quasi-fluid approach:

$$\boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) = 0$$



Lefftz et al. (1991)

$$\approx k_e^T - \sum_{i=1}^M \frac{k_i^T}{s_i} = k_f^T (1 - T_1 s)$$

Quasi-fluid approximation

Traditional derivation of the full Liouville equation (LE):

$$\frac{d\mathbf{H}}{dt} + \boldsymbol{\omega} \times \mathbf{H} = \mathbf{L}$$

$$\frac{d}{dt}(\mathbf{I} \cdot \boldsymbol{\omega} + h) + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega} + h) = \mathbf{L}$$

$$\frac{d\boldsymbol{\omega}}{dt} = -\mathbf{I}^{-1} \cdot \left(\frac{d\mathbf{I}}{dt} \cdot \boldsymbol{\omega} + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) \right) \quad \blacksquare$$

$$\frac{d\mathbf{h}}{dt} = \mathbf{L} + \int_{v(t)} \mathbf{r} \times \mathbf{f}_{\text{int}} \rho dv + \int_{v(t)} \mathbf{r} \times \mathbf{f}_{\text{fic}} \rho dv$$

$$\mathbf{f}_{\text{fic}} = -\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - 2\boldsymbol{\omega} \times \mathbf{v} - \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r} - \frac{d\mathbf{v}_0}{dt} - \boldsymbol{\omega} \times \mathbf{v}_0$$

Centrifugal force:

$$\begin{aligned} - \int_{v(t)} \mathbf{r} \times (\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})) \rho dv &= - \int_{v(t)} \mathbf{r} \times (\boldsymbol{\omega}(\boldsymbol{\omega} \cdot \mathbf{r}) - \mathbf{r}(\boldsymbol{\omega} \cdot \boldsymbol{\omega})) \rho dv = \\ &- \int_{v(t)} \mathbf{r} \times \boldsymbol{\omega}(\boldsymbol{\omega} \cdot \mathbf{r}) \rho dv = \boldsymbol{\omega} \times \int_{v(t)} \mathbf{r} (\boldsymbol{\omega} \cdot \mathbf{r}) \rho dv = -\boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) \end{aligned}$$

Coriolis force:

$$\begin{aligned} - \int_{v(t)} \mathbf{r} \times (2\boldsymbol{\omega} \times \mathbf{v}) \rho dv &= \int_{v(t)} (\mathbf{u}(\mathbf{r} \cdot 2\boldsymbol{\omega}) - 2\boldsymbol{\omega}(\mathbf{r} \cdot \mathbf{u})) \rho dv = \\ - \int_{v(t)} (\mathbf{r}(\boldsymbol{\omega} \cdot \mathbf{u}) - \mathbf{u}(\mathbf{r} \cdot 2\boldsymbol{\omega})) \rho dv &- \int_{v(t)} 2\boldsymbol{\omega}(\mathbf{r} \cdot \mathbf{u}) - \mathbf{r}(\mathbf{u} \cdot \boldsymbol{\omega}) - (\mathbf{u}(\mathbf{r} \cdot \boldsymbol{\omega})) \rho dv = \\ - \int_{v(t)} \boldsymbol{\omega} \times (\mathbf{r} \times \mathbf{u}) \rho dv &- \left(\int_{v(t)} 2(\mathbf{r} \cdot \mathbf{u}) \mathbb{1} - \mathbf{r} \otimes \mathbf{u} - \mathbf{u} \otimes \mathbf{r} \right) \rho dv \Big) \cdot \boldsymbol{\omega} = \\ &- \boldsymbol{\omega} \times \mathbf{h} - \frac{d\mathbf{I}}{dt} \cdot \boldsymbol{\omega} \end{aligned}$$

Euler force:

$$- \int_{v(t)} \mathbf{r} \times \left(\frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r} \right) \rho dv = - \int_{v(t)} \frac{d\boldsymbol{\omega}}{dt} r^2 - \mathbf{r} \left(\frac{d\boldsymbol{\omega}}{dt} \cdot \mathbf{r} \right) \rho dv = -\mathbf{I} \cdot \frac{d\boldsymbol{\omega}}{dt}$$

$$\frac{d\mathbf{h}}{dt} = \mathbf{L} + \int_{v(t)} \mathbf{r} \times \mathbf{f}_{\text{int}} \rho dv + \int_{v(t)} \mathbf{r} \times \mathbf{f}_{\text{fic}} \rho dv$$

Quasi-fluid approx:
WHY DOES IT
WORK?

$$\mathbf{f}_{\text{fic}} = -\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - 2\boldsymbol{\omega} \times \mathbf{v} - \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r} - \frac{d\mathbf{v}_0}{dt} - \boldsymbol{\omega} \times \mathbf{v}_0$$

Centrifugal force:

$$\begin{aligned} - \int_{v(t)} \mathbf{r} \times (\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})) \rho dv &= - \int_{v(t)} \mathbf{r} \times (\boldsymbol{\omega}(\boldsymbol{\omega} \cdot \mathbf{r}) - \mathbf{r}(\boldsymbol{\omega} \cdot \boldsymbol{\omega})) \rho dv = \\ &- \int_{v(t)} \mathbf{r} \times \boldsymbol{\omega}(\boldsymbol{\omega} \cdot \mathbf{r}) \rho dv = \boldsymbol{\omega} \times \int_{v(t)} \mathbf{r} (\boldsymbol{\omega} \cdot \mathbf{r}) \rho dv = \boxed{-\boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega})} \end{aligned}$$

Coriolis force:

$$\begin{aligned} - \int_{v(t)} \mathbf{r} \times (2\boldsymbol{\omega} \times \mathbf{v}) \rho dv &= \int_{v(t)} (\mathbf{u}(\mathbf{r} \cdot 2\boldsymbol{\omega}) - 2\boldsymbol{\omega}(\mathbf{r} \cdot \mathbf{u})) \rho dv = \\ - \int_{v(t)} (\mathbf{r}(\boldsymbol{\omega} \cdot \mathbf{u}) - \mathbf{u}(\mathbf{r} \cdot 2\boldsymbol{\omega})) \rho dv &- \int_{v(t)} 2\boldsymbol{\omega}(\mathbf{r} \cdot \mathbf{u}) - \mathbf{r}(\mathbf{u} \cdot \boldsymbol{\omega}) - (\mathbf{u}(\mathbf{r} \cdot \boldsymbol{\omega})) \rho dv = \\ - \int_{v(t)} \boldsymbol{\omega} \times (\mathbf{r} \times \mathbf{u}) \rho dv &- \left(\int_{v(t)} 2(\mathbf{r} \cdot \mathbf{u}) \mathbb{1} - \mathbf{r} \otimes \mathbf{u} - \mathbf{u} \otimes \mathbf{r} \right) \rho dv \cdot \boldsymbol{\omega} = \end{aligned}$$

$$\boxed{\frac{d}{dt}(\mathbf{I} \cdot \boldsymbol{\omega} + \mathbf{h}) + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega} + \mathbf{h}) = \mathbf{L}}$$

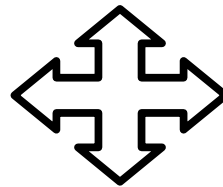
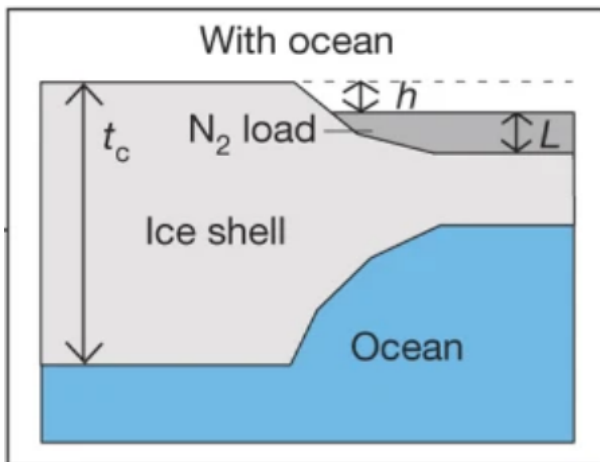
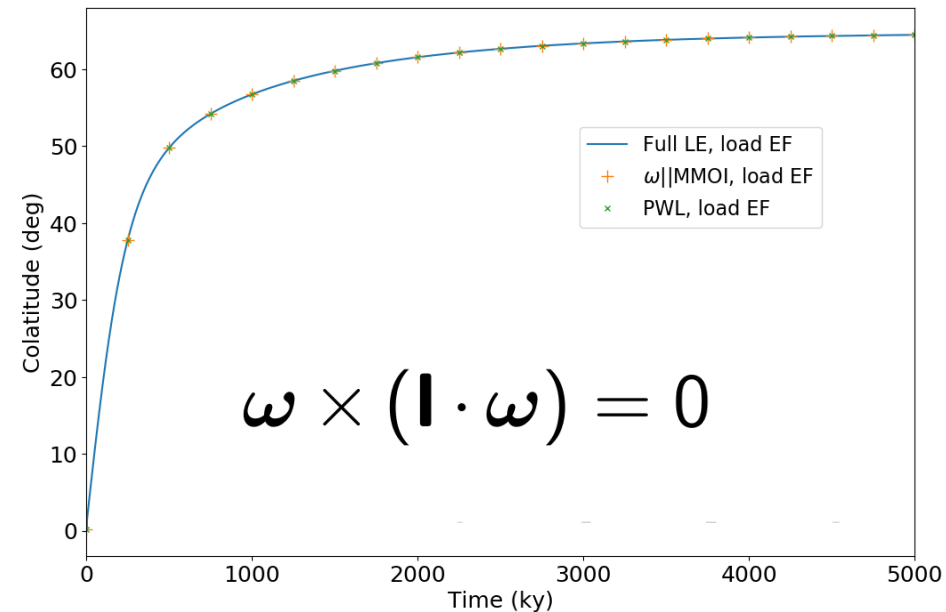
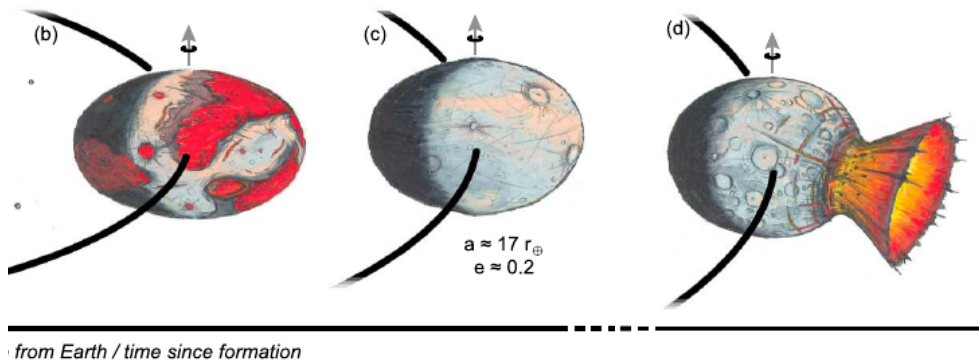
$$\boxed{-\boldsymbol{\omega} \times \mathbf{h} - \frac{d\mathbf{I}}{dt} \cdot \boldsymbol{\omega}}$$

Euler force:

$$- \int_{v(t)} \mathbf{r} \times \left(\frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r} \right) \rho dv = - \int_{v(t)} \frac{d\boldsymbol{\omega}}{dt} r^2 - \mathbf{r} \left(\frac{d\boldsymbol{\omega}}{dt} \cdot \mathbf{r} \right) \rho dv = \boxed{-\mathbf{I} \cdot \frac{d\boldsymbol{\omega}}{dt}}$$

Future perspectives

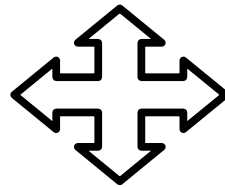
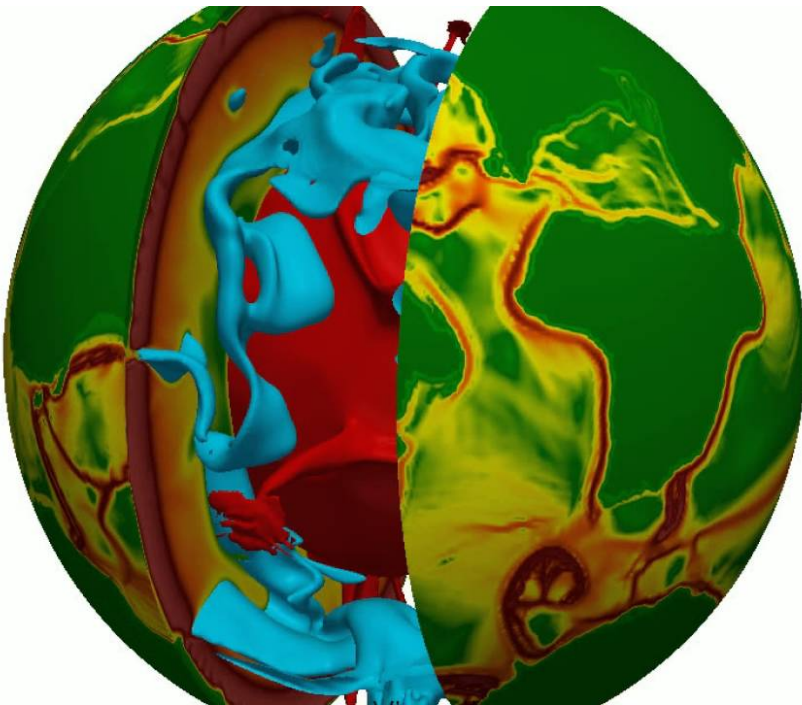
- TPW rates can vary by orders of magnitude **and should be checked**



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Summary

- ❑ LIOUSHELL, free license, user-interface, fast
- ❑ TPW rates can vary by orders of magnitude and should be checked
- ❑ Quasi-fluid response and quasi-fluid LE are not the same thing



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