

Revisiting parametrized convection models: influence of heat from Earth's core

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Thermal history of the Earth: first estimates

(19th century) measurements of $\frac{\partial T}{\partial r} \approx 20\text{--}30 \text{ K/km}$

Lord Kelvin attributed the entire heat loss from the Earth's surface, Q_S , to Earth's secular cooling:

$$\Rightarrow Q_S = -C_E \frac{dT}{dt} \Rightarrow \text{Age of Earth} \approx 100 \text{ My}$$

Solution: radioactive sources H

$$C_E \frac{dT}{dt} = H - Q_S$$

Thermal history of the Earth: plate tectonics

Analog and numerical experiments of convecting systems

$$Q_S \sim S_m \frac{k(T_m - T_s)}{D} Ra^\beta \quad (\text{i.e. } Nu \sim Ra^\beta), \quad Ra = \frac{g\alpha(T_m - T_s)D^3}{\kappa\eta},$$

supported also by boundary layer theory (for $\beta=1/3$).

Plate-tectonics are very efficient way of losing heat, with 1st order estimate being $Q_S(t) \approx H(t)$

- this was attacked simultaneously in 1980 by Schubert et al. and by Davies

Thermal history of the Earth: Schubert et al. 1980

$$Q_S = A S_m \frac{k(T_m - T_s)}{D} Ra^\beta, \quad C_m \frac{dT_m}{dt} = H - Q_S$$

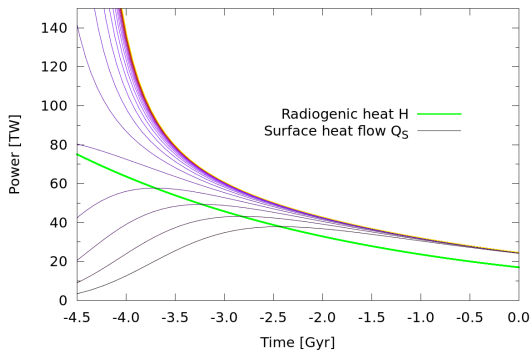


Figure : Radiogenic power $H(t)$ (green curve) follows $H = H_0 \exp(-t \log 2/\tau)$ with $H_0 = 10$ TW. The remaining curves show the surface heat flow Q_S , where the colour corresponds to the initial mantle temperature, evenly sampled from 1200 K (black) to 4200 K (yellow).

Thermal history of the Earth: Davies 1980

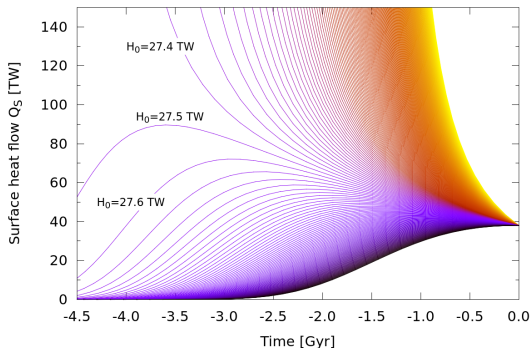


Figure : Backward-in-time integration. The various solutions correspond to different systematically varied values of the prescribed present-day radiogenic power H_0 . We show $Q_S(t)$, where colour distinguishes the different solutions: yellow stands for the “hot” end of the spectrum, with $Ur = 0$ ($H_0 = 0$ TW), while black represents the “cold” end with $Ur = 1$ ($H_0 = 38$ TW). For $H_0 \simeq 27.5$ TW, i.e. for Urey ratio H_0/Q_S^0 close to 0.724, the transition solution is reached.

Thermal catastrophe paradox

Parametrized convection models lead to extremely hot Earth in recent geological history when geochemical/cosmochemical estimates of H_0 are supplied to them.

Suggested solutions:

- $Nu \sim Ra^\beta$ scaling is invalid..
 - grain-size dependent viscosity, water, ...
 - resistance of plates toward bending
 - no plate-tectonics in the past
- The missing heat comes from the core

Separating the mantle and the core

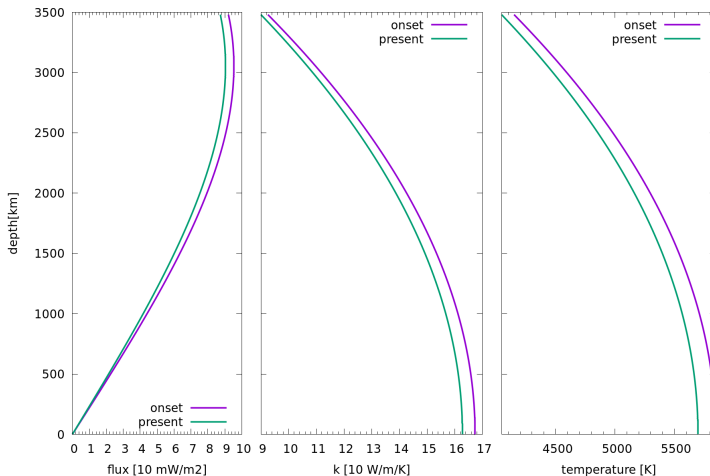
$$C_m \frac{dT_m}{dt} = H + Q_{\text{CMB}} - Q_S$$

Earth's core and mantle are likely to have different thermal histories, with secular cooling of the mantle being different from the secular cooling of the core.

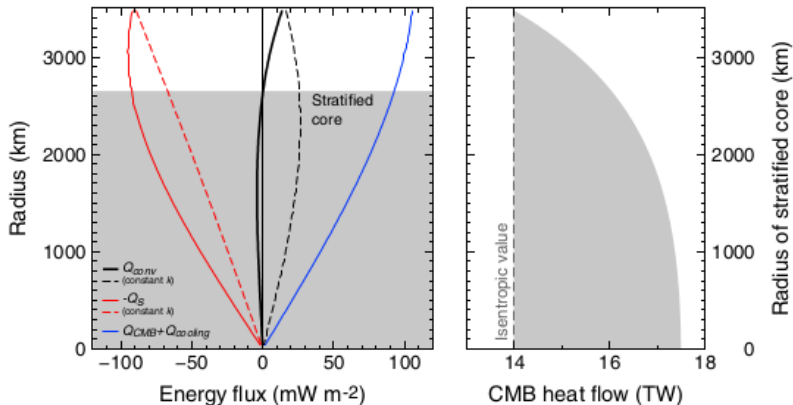
Operation of the geodynamo places a constraint on the minimum value of Q_{CMB} .

Temperature profile of the core

Outer core is a vigorously convecting liquid: $\Rightarrow T_a(r) = T_0 (\rho_a(r)/\rho_0)^\gamma$ isentropic
with $\rho_a(r)$ obtained from the Adams-Williamson equation of state

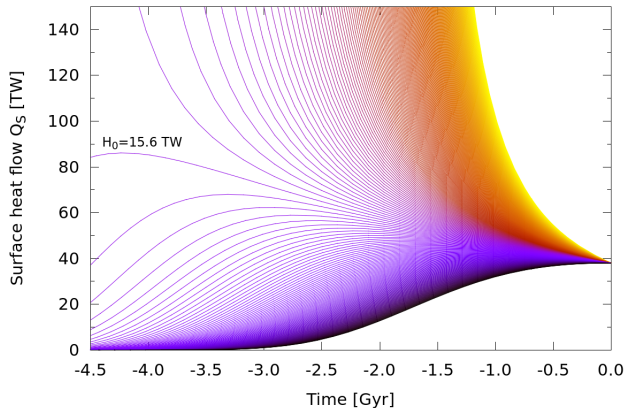


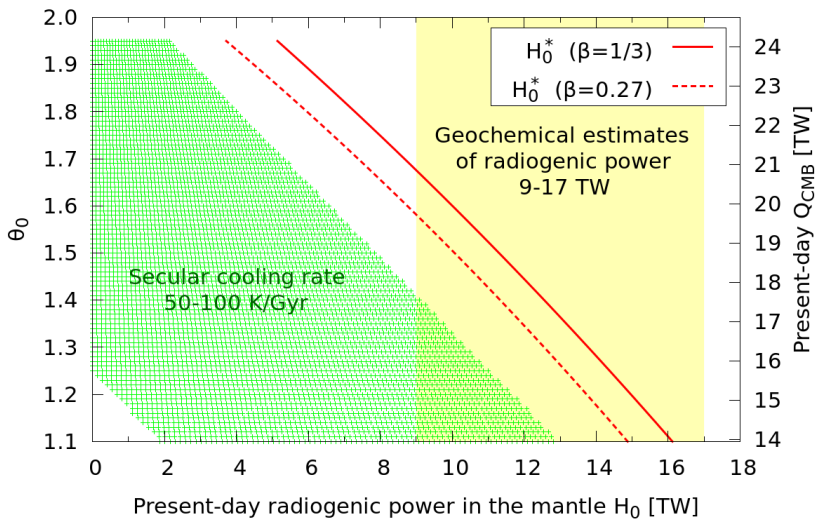
Labrosse, 2015, Convective heat flux in Earth's core with the recent estimates of k_{iron}



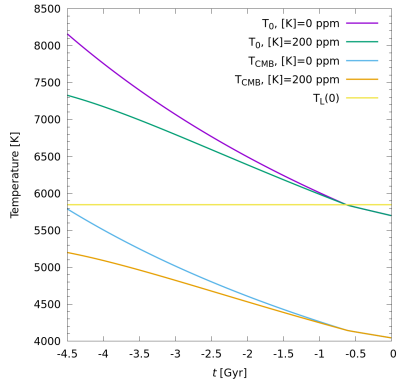
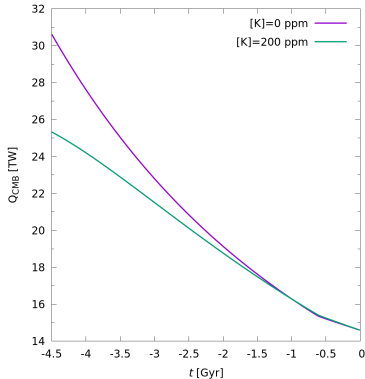
$$Q_{\text{CMB}} = \theta 4\pi r_C^2 k(r_C, T_{\text{CMB}}) \left(\frac{\partial T_a}{\partial r} \right)_{r=r_C}, \quad \theta = 1.15$$

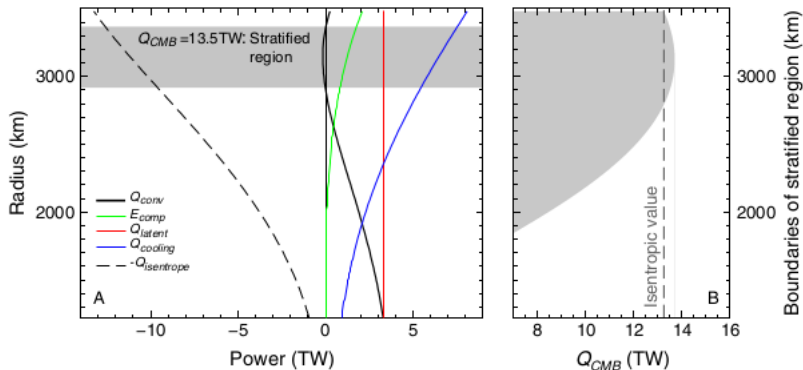
$$C_m \frac{dT_m}{dt} = H + Q_{\text{CMB}} - Q_S, \quad C_c \frac{dT_c}{dt} = H - Q_{\text{CMB}}$$





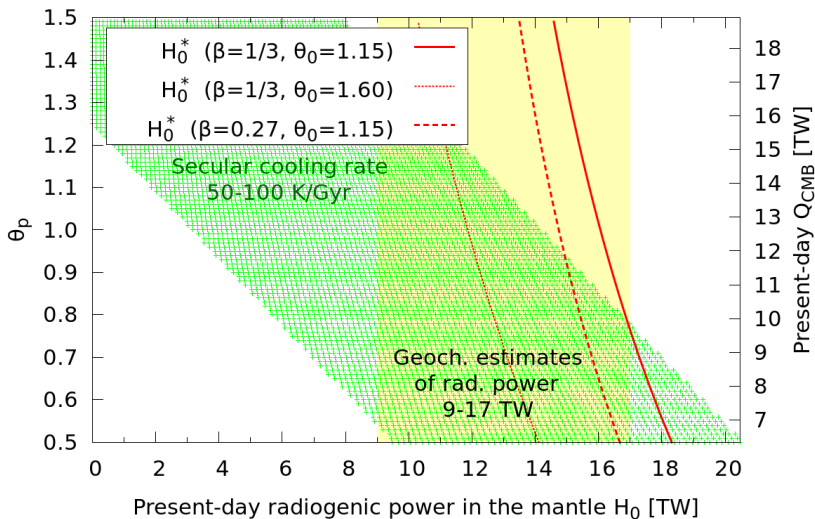
After the onset of IC, its convection becomes thermo-chemical and is driven by a combination of energy sources



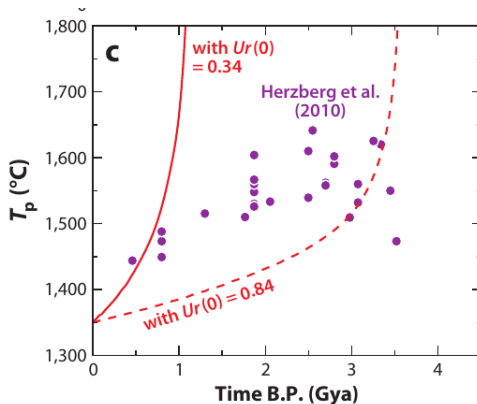


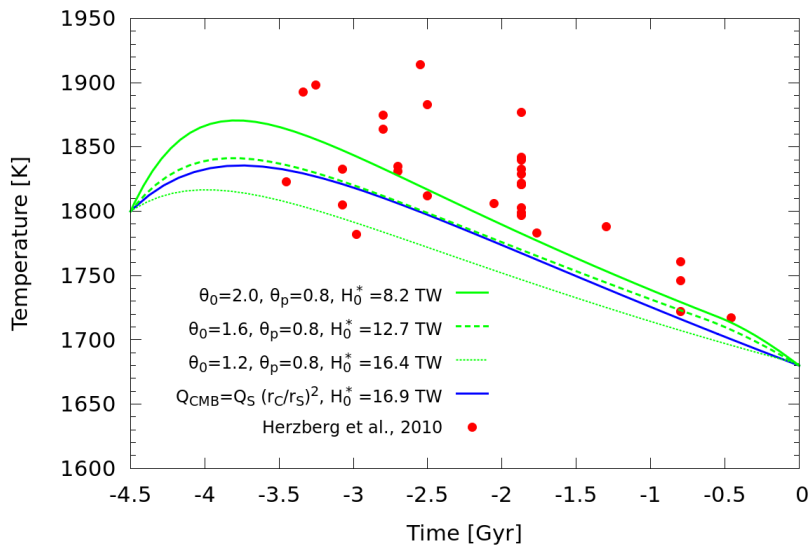
—from Labrosse, 2015

$$\theta(t) = \theta_p - (\theta_0 - \theta_p) \frac{t}{a_{IC}}, \quad t > -a_{IC}$$



Korenaga 2013: plate-tectonics review





Take home message

Is the scaling $Nu \sim Ra^\beta$ valid?

We don't know, but it cannot be ruled out on the basis of the thermal catastrophe argument.

