

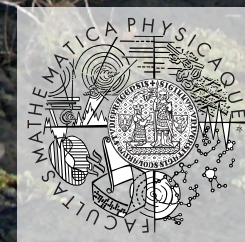
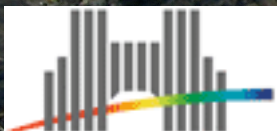
Modèle d'écoulement biphasé en sciences de la Terre : fusion partielle, compaction et différenciation

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20 décembre 2007

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Outline

- geophysical motivation
- theoretical and historical background
- two-phase model
- applications
 - coupling of deformation and melting
 - differentiation of a young planet

Shallow magmatism

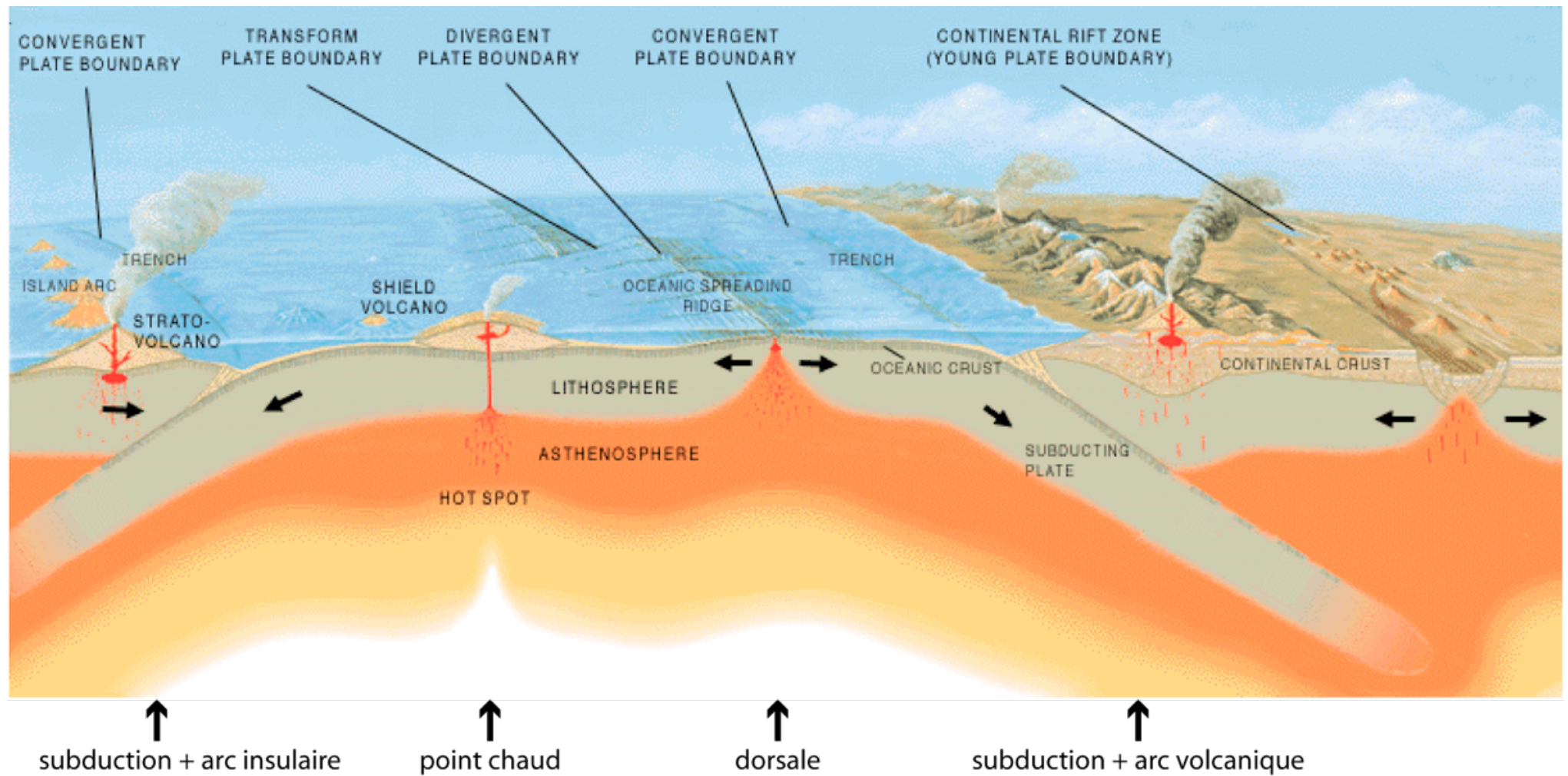
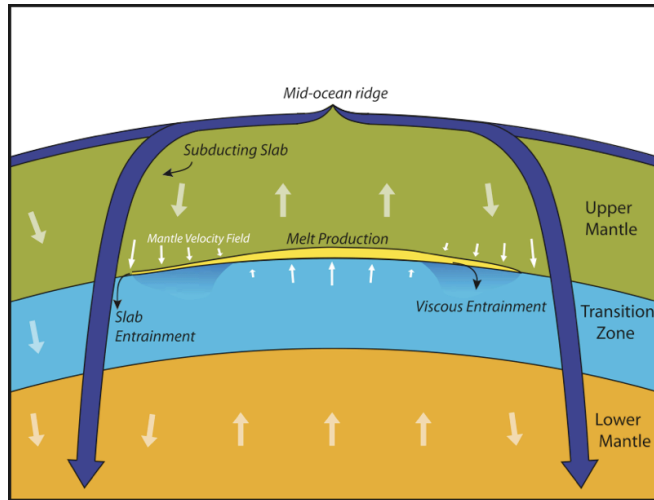


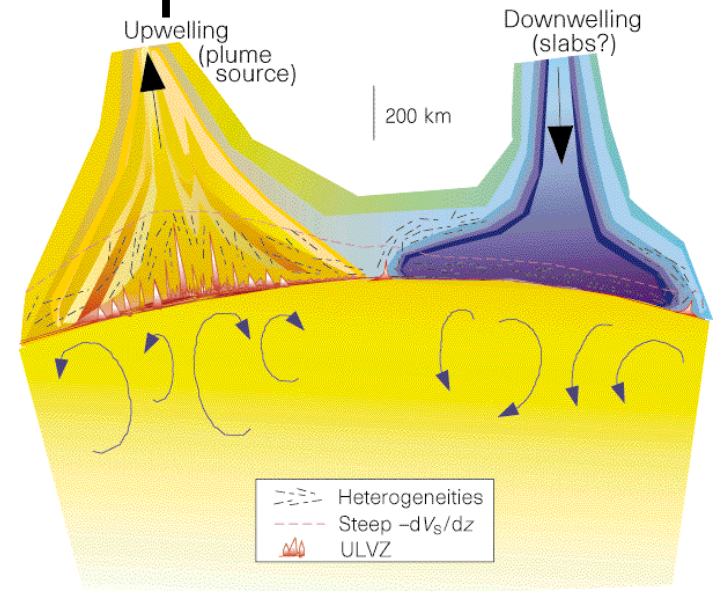
Figure modified after J. F. Vigil (USGS)

Partially molten regions in deeper Earth?



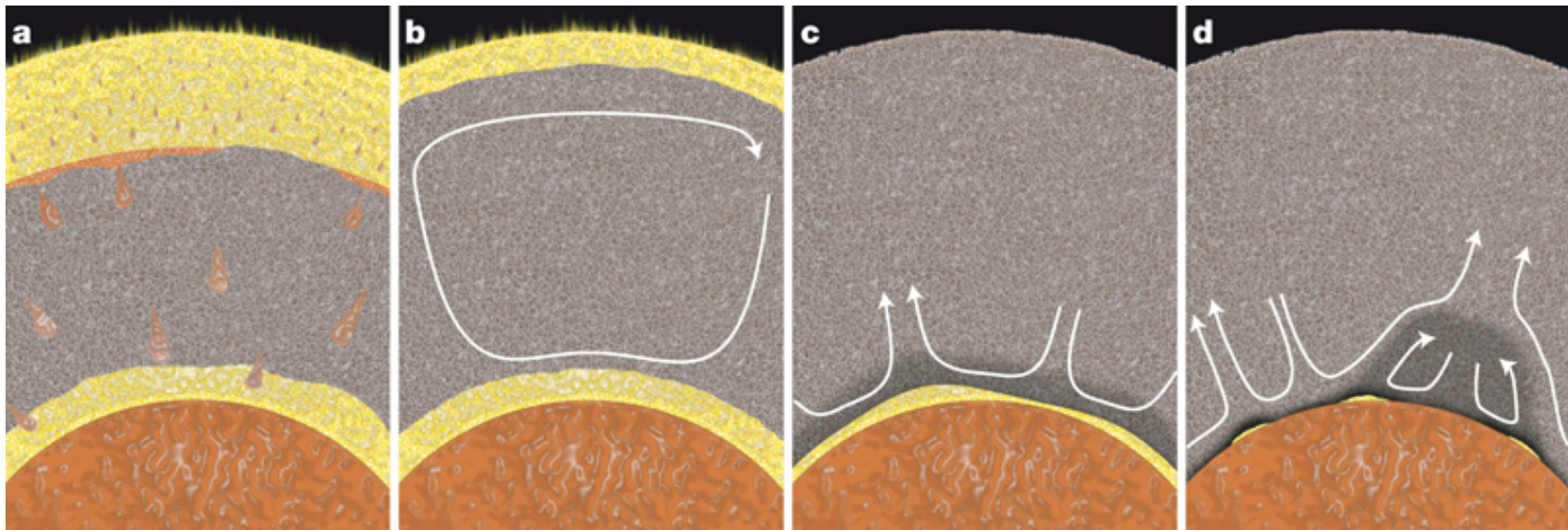
Bercovici & Karato (2003)

“transition zone water filter model”



Lay et al. (1998)

D'' region

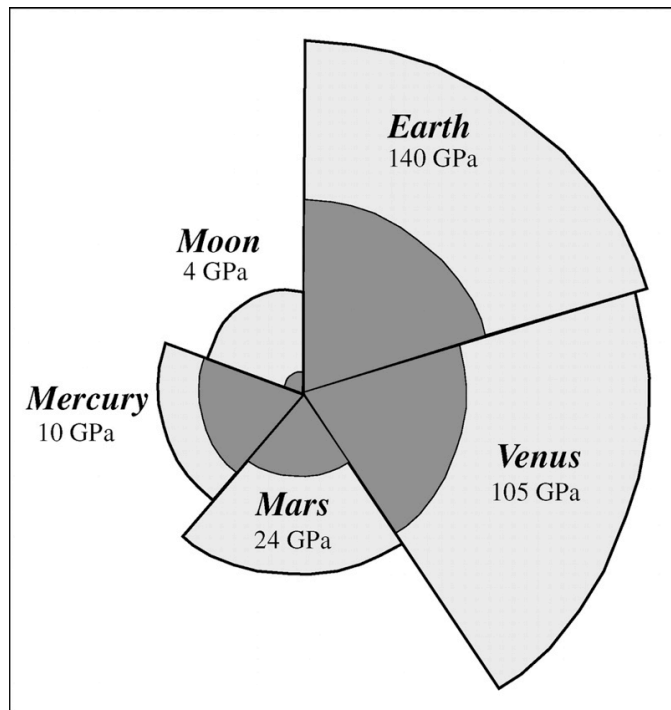


basal magma ocean

Labrosse et al. (2007)

Planetary differentiation

- terrestrial planets: metallic core & silicate mantle
- the core–mantle differentiation during/after the late stage of planetary growth

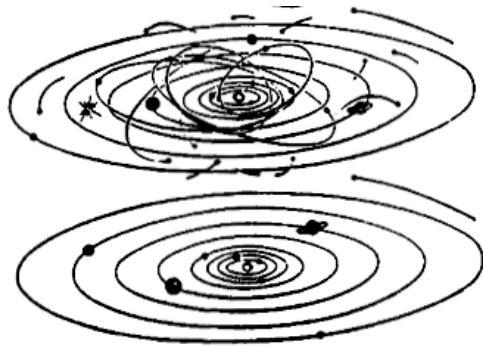


Richter (2003)



Raab (2005)

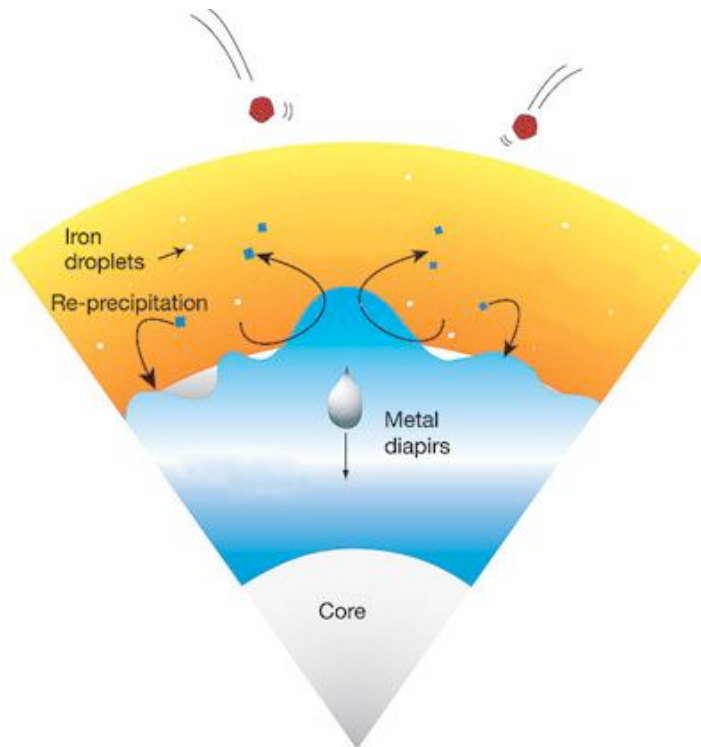
Planetary differentiation – giant impacts



after Ahrens (1990)

High T conditions

- radiogenic heat from short-lived isotopes
- accretion heat from impacts
- ΔT_{impact} may melt the metal, or even the silicates (magma ocean)
- heating by differentiation



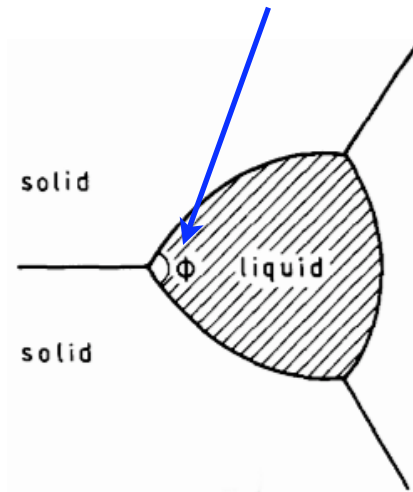
after Wood et al. (2007)

Physics of two-phase flow

I. Interconnectivity

dihedral angle ... crude assessment of interconnectivity

depends on the ratio of solid–solid
and solid–liquid interfacial energies

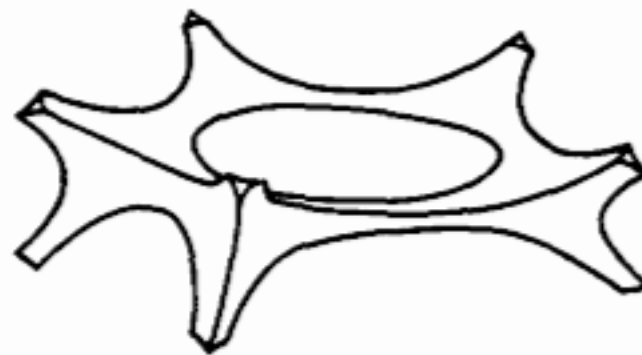
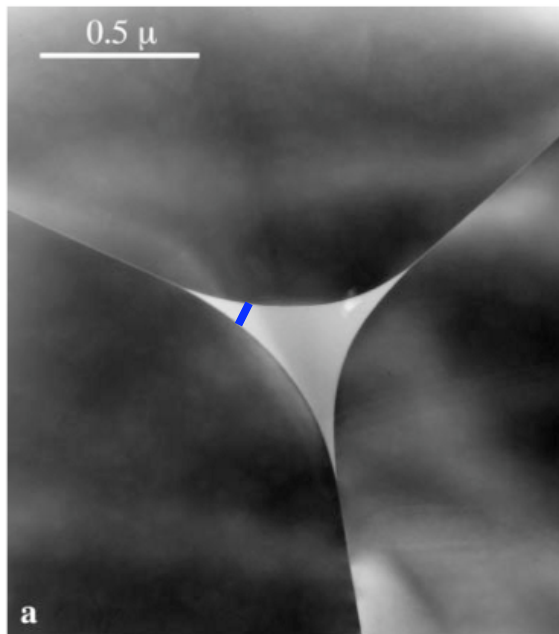


$< 60^\circ$

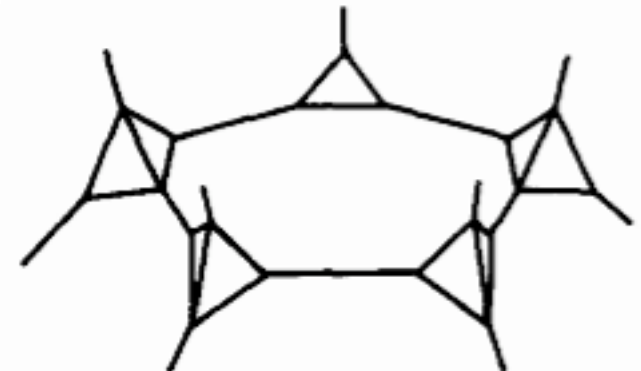
interconnected network

$> 60^\circ$

isolated pockets of liquid



$\phi = 45^\circ$



$\phi = 71^\circ$

after Schmelting (1985)

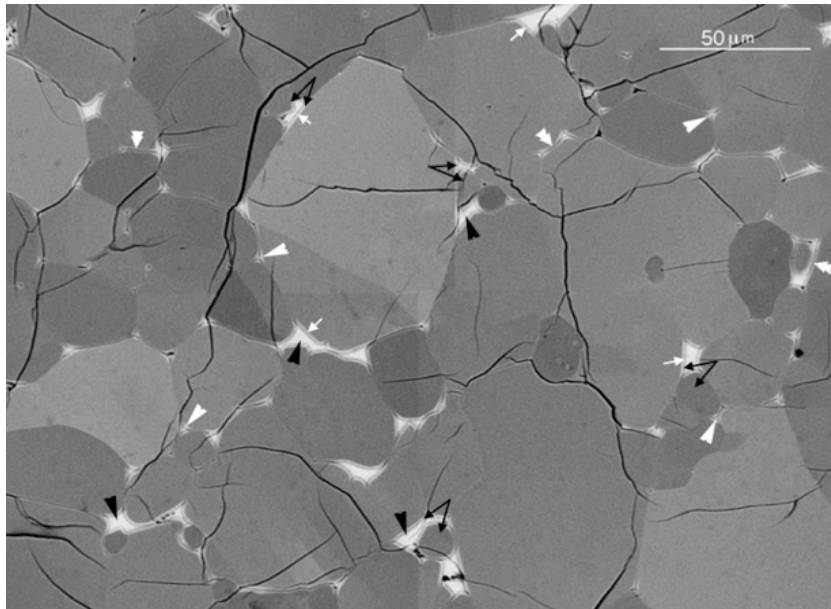
Cmíral et al. (1998)

Physics of two-phase flow

I. Interconnectivity

partial melting in the upper mantle
basaltic magma and olivine matrix

- dihedral angle 20° – 50°
- interconnected at low melt fraction

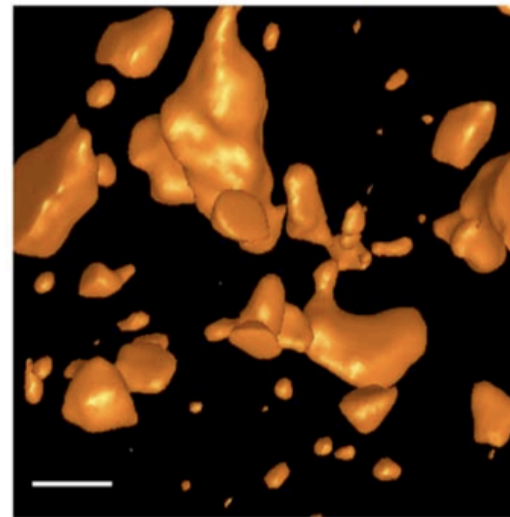


Cmíral et al. (1998)

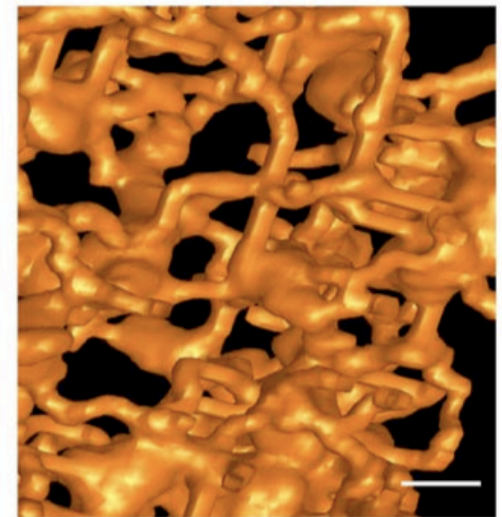
metal–silicate differentiation
molten iron alloy and silicate matrix

- dihedral angle probably $>60^{\circ}$
- interconnectivity threshold $\sim 5\%$ of metal

4% FeS melt



6% FeS melt

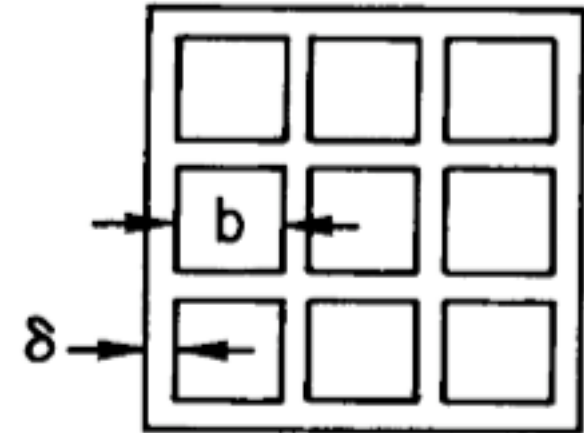


Roberts et al. (2007)

Physics of two-phase flow

2. Permeability and Darcy's law

$$\underset{\substack{\uparrow \\ \text{Darcy velocity}}}{\mathbf{v}_D} = - \frac{\overset{\substack{\nwarrow \\ \text{matrix permeability}}}{k}}{\underset{\substack{\nwarrow \\ \text{fluid viscosity}}}{\mu}} \nabla P$$

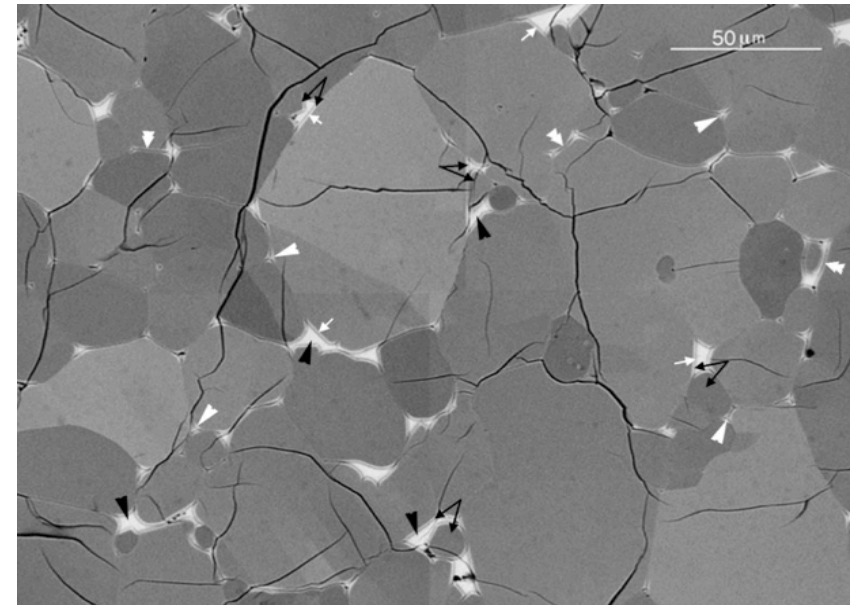


Ahern & Turcotte (1979)

- geometric representation of interconnected fluid network
- solve Poiseuille flow
- get permeability k

$$k(\phi) = \frac{b^2 \phi^n}{a} = k_0 \phi^n$$

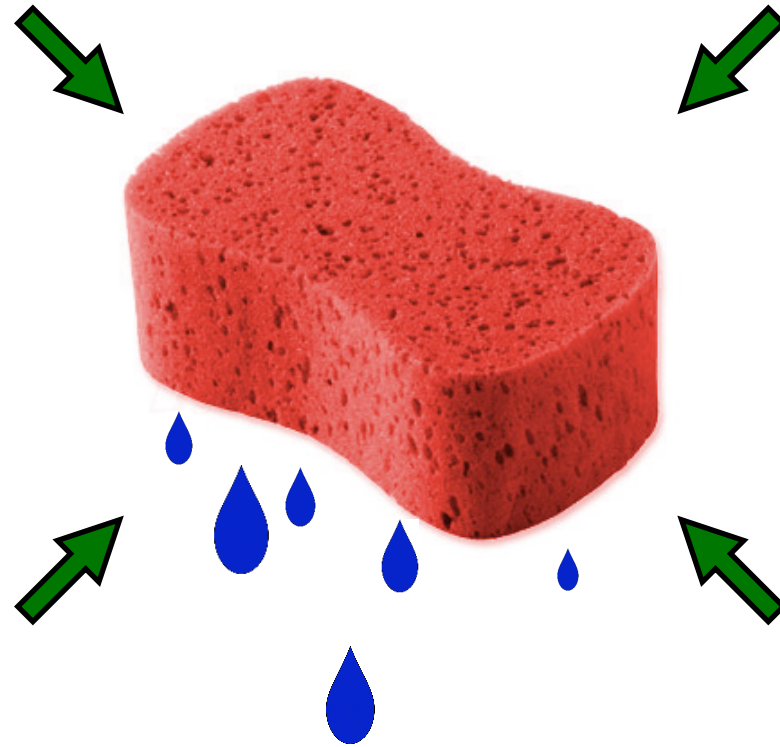
- ϕ porosity
- n typically 2 or 3
- $a \sim 100\text{--}1000$ “tortuosity”



Cmíral et al. (1998)

Physics of two-phase flow

3. Matrix deformation



Two-phase models is geophysics

- Sleep (1974), Turcotte & Ahern (1978), Ahern & Turcotte (1979)

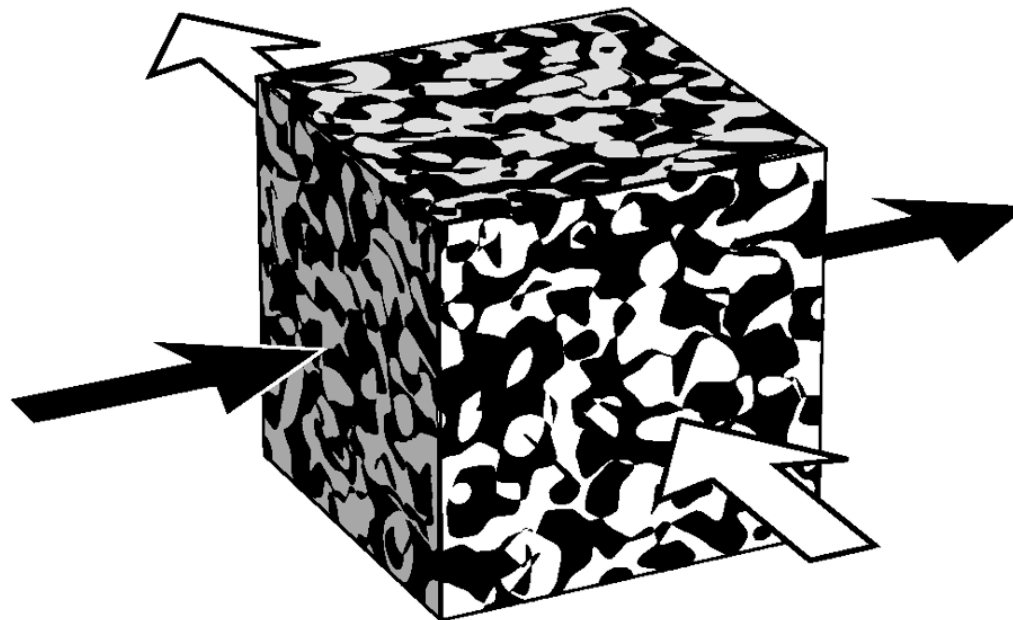
Mid 1980's:

- effort to understand divergent plate boundaries
- McKenzie (1984), Ribe (1985), Scott & Stevenson (1986)
 - general model of partially molten regions
 - porous flow of solid through viscous deformable matrix

Recent new two-phase model

- Bercovici, Ricard & Schubert (2001), Ricard & Bercovici (2003)
- accounts for the mechanical and thermodynamical effects of the interface (interfacial surface tension)
- requires a difference in pressures between the two phases
- does not include phase change

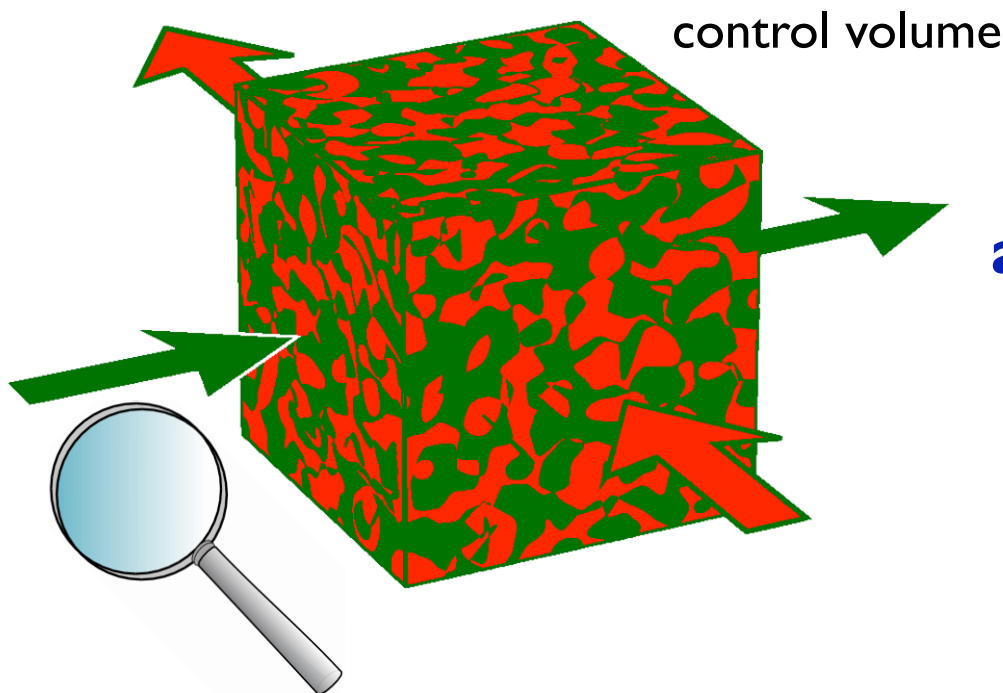
⇒ extend the model to include melting



Two-phase model

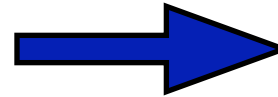
“fluid” phase ... index f

“matrix” phase ... index m

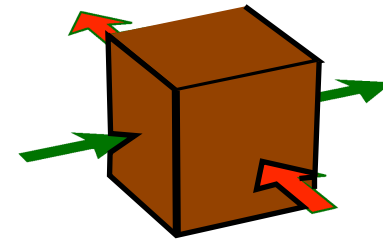


the 2 phases f and m occupy distinct regions at grain/pore scale

averaging



“point” in a two-phase continuum



the 2 phases are present at the same “point” with volume fractions ϕ (porosity) and $1-\phi$

other quantities (e.g., velocities v_f, v_m) are also averaged and assumed continuous in space

Two-phase equations

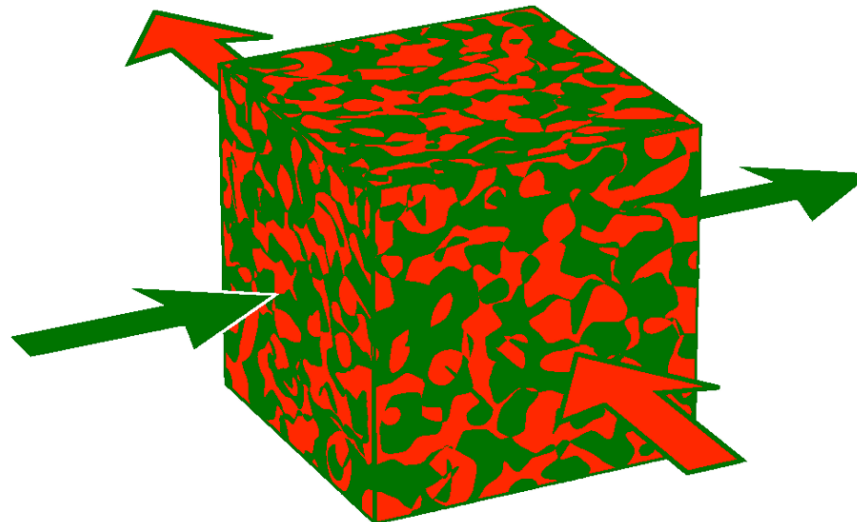
Mass conservation

- incompressible viscous fluids
- matrix skeleton/fluid network “compressible”

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \mathbf{v}_f) = \frac{\Delta \Gamma}{\rho_f}$$

$$-\frac{\partial \phi}{\partial t} + \nabla \cdot [(1 - \phi) \mathbf{v}_m] = -\frac{\Delta \Gamma}{\rho_m}$$

melting rate



Momentum equations

force balance for the individual phases in the limit $\mu_f \ll \mu_m$
inertial terms neglected

generalized Darcy's law

$$-\phi \left[\underbrace{\nabla P_f - \rho_f \mathbf{g}}_{\text{non-hydrostatic pressure gradient}} \right] + c \Delta \mathbf{v} = 0$$

non-hydrostatic pressure gradient

interaction coefficient (cf. Darcy's law)

$$c = \frac{\mu_f \phi^2}{k(\phi)} = \frac{\mu_f}{k_0}$$

permeability exponent = 2

matrix momentum equation

$$-(1 - \phi) [\nabla P_m - \rho_m \mathbf{g}] + \nabla \cdot [(1 - \phi) \underline{\tau}_m] - c \Delta \mathbf{v} + \Delta P \nabla \phi + \nabla (\sigma \alpha) = 0$$

deviatoric viscous stress in the matrix

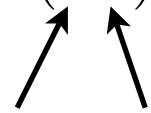
$$\tau_m = \mu_m \left(\nabla \mathbf{v}_m + [\nabla \mathbf{v}_m]^T - \frac{2}{3} \nabla \cdot \mathbf{v}_m \mathbf{I} \right)$$

interfacial surface tension



coefficient of
surface tension

interfacial
area density



Energy balance for the two-phase mixture

$$\begin{aligned}
 & \begin{array}{ccc} \text{phase } f & \text{phase } m & \text{interfacial terms} \\ \phi \rho_f C_f \frac{D_f T}{Dt} + (1 - \phi) \rho_m C_m \frac{D_m T}{Dt} - T \frac{D_m}{Dt} \left(\alpha \frac{d\sigma}{dT} \right) - T \alpha \frac{d\sigma}{dT} \nabla \cdot \mathbf{v}_m \end{array} \\
 &= \underbrace{-T \Delta s \Delta \Gamma}_{\substack{\downarrow \\ \text{latent} \\ \text{heat}}} + \underbrace{Q}_{\substack{\downarrow \\ \text{radioactive} \\ \text{heat}}} - \underbrace{\nabla \cdot \mathbf{q}}_{\substack{\downarrow \\ \text{heat} \\ \text{diffusion}}} + \underbrace{\Psi}_{\substack{\downarrow \\ \Psi = c \Delta v^2 + (1 - \phi) \nabla \mathbf{v}_m : \underline{\tau}_m \text{ dissipative heating}}} - \underbrace{\left(\Delta P + \sigma \frac{d\alpha}{d\phi} \right) \frac{D_m \phi}{Dt}}_{\substack{\text{heat source related to} \\ \text{change in porosity}}} + \underbrace{\left(\Delta \varepsilon - T \Delta s + \frac{P_m}{\rho_m} - \frac{P_f}{\rho_f} \right) \Delta \Gamma}_{\substack{\text{heat source related to} \\ \text{phase change}}}
 \end{aligned}$$

9 equations written
mass, momentum, energy
but 11 unknowns

2 velocities, 2 pressures, temperature, porosity, melting rate

non-equilibrium thermodynamics provides 2 additional relations

Non-equilibrium thermodynamics

$$\begin{array}{c} \text{flux} \\ \left(\begin{array}{c} \frac{D_m \phi}{Dt} \\ \Delta \Gamma \end{array} \right) \end{array} = \begin{array}{c} \text{force} \\ \left(\begin{array}{cc} \cdot & \cdot \\ \cdot & \cdot \end{array} \right) \end{array} \left(\begin{array}{c} -\Delta P - \sigma \frac{d\alpha}{d\phi} \\ \Delta \varepsilon - T \Delta s + \frac{P_m}{\rho_m} - \frac{P_f}{\rho_f} \end{array} \right)$$

2nd law of thermodynamics

Onsager's relations

micromechanical model

limiting cases

relation for pressure difference

$\frac{d\alpha}{d\phi}$ sum of principal
interface curvatures

$$\Delta P + \sigma \frac{d\alpha}{d\phi} = -\frac{4}{3} \frac{\mu_m}{\phi} \nabla \cdot \mathbf{v}_m$$

deviation of pressure difference
from the static Laplace's condition

$\propto$

compaction/dilation of
the matrix

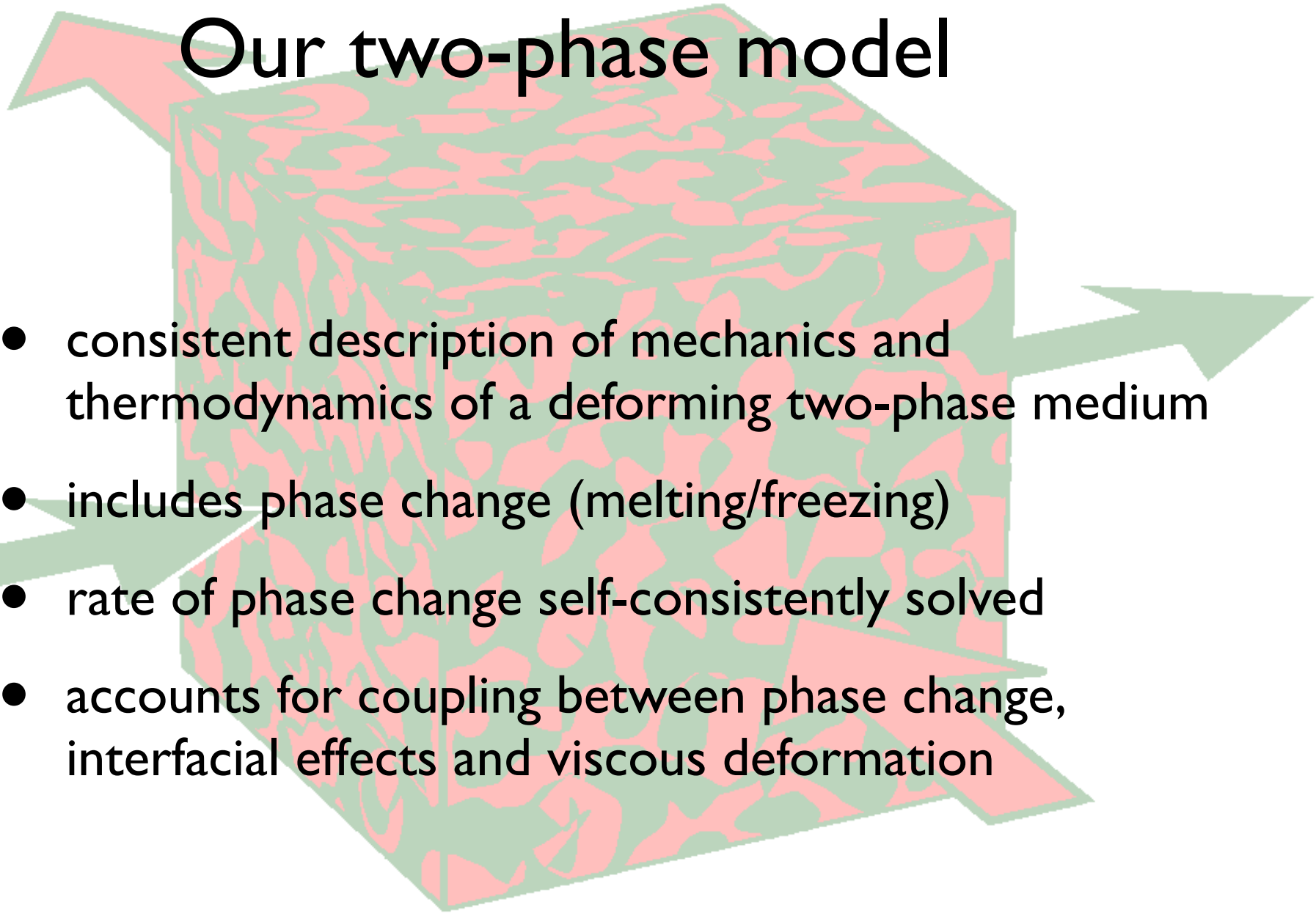
kinetic relation for melting rate

$$\Delta \Gamma = \chi \left(\Delta \varepsilon - T \Delta s - P_f \frac{\Delta \rho}{\rho_f \rho_m} - \frac{\sigma}{\rho_m} \frac{d\alpha}{d\phi} \right)$$

melting rate $\propto$

departure from equilibrium

Our two-phase model

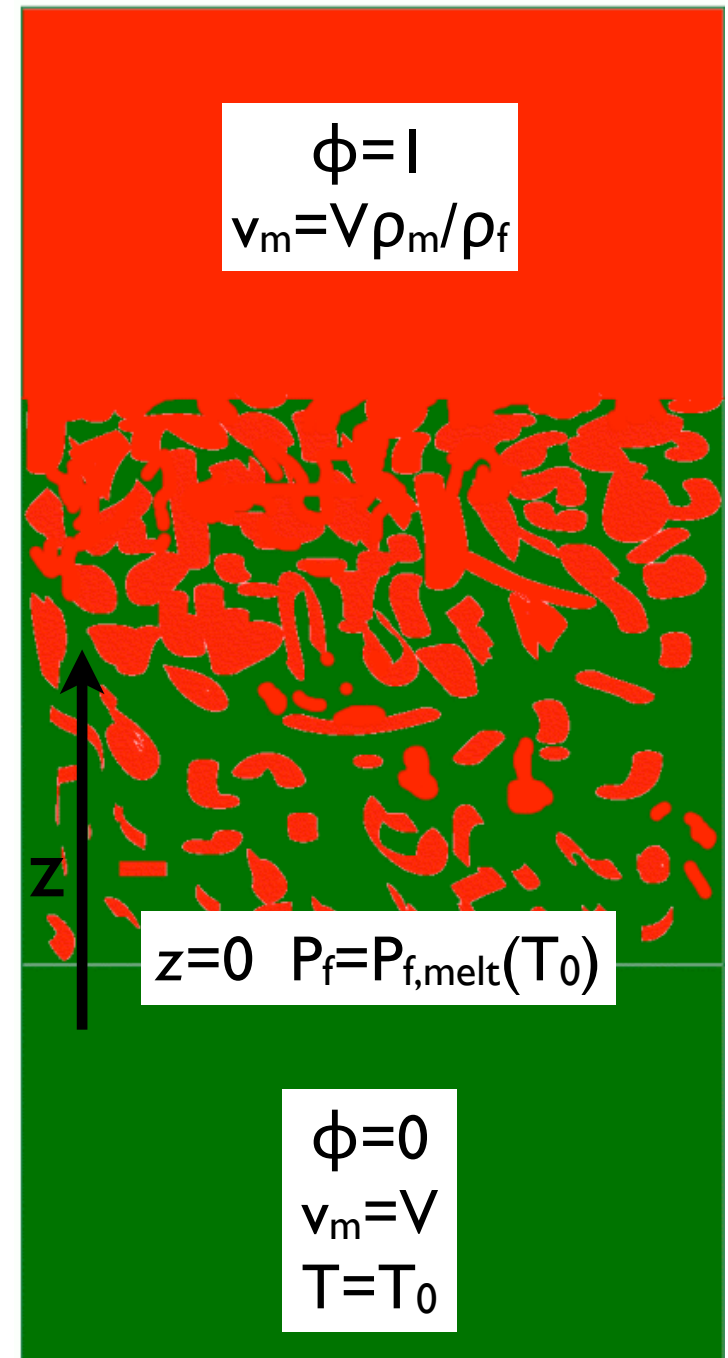


- consistent description of mechanics and thermodynamics of a deforming two-phase medium
- includes phase change (melting/freezing)
- rate of phase change self-consistently solved
- accounts for coupling between phase change, interfacial effects and viscous deformation

Coupling of compaction and melting

Šrámek et al. (GJI, 2007)

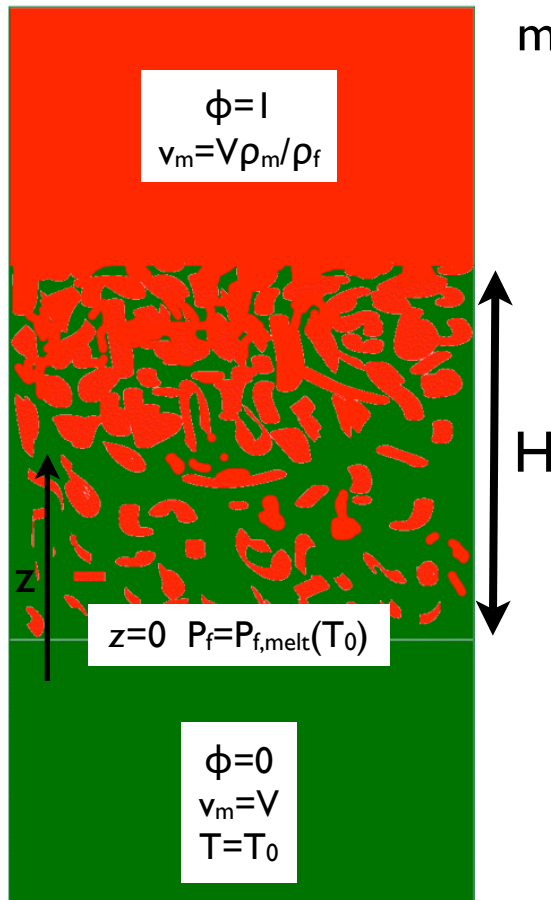
- equilibrium pressure release melting
- steady state, 1-D
- neglect surface tension
- univariant melting
- solid matrix upwelling at a prescribed velocity V
- two-phase region between pure solid ($\phi=0$) and pure melt ($\phi=1$)
- the extent of the melting zone not a priori known



Force balance in the two-phase zone

$$\delta^2 \phi \frac{d}{dz} \left(\frac{1 - \phi^2}{\phi} \frac{dv_m}{dz} \right) - \Delta v = V_B \phi (1 - \phi)$$

$\uparrow$ matrix viscous forces $\uparrow$ Darcy drag $\nwarrow$ buoyancy



$$\delta^2 = \frac{4\mu_m}{3cH^2} = \frac{4\mu_m k_0}{3\mu_f H^2}$$

dimensionless
compaction length δ

$$V_B = \frac{\Delta \rho g}{cV} = \frac{\Delta \rho g k_0}{\mu_f V}$$

dimensionless
buoyancy velocity V_B

spreading center:

$$\delta \sim 0.1$$

$$V_B \sim 100$$

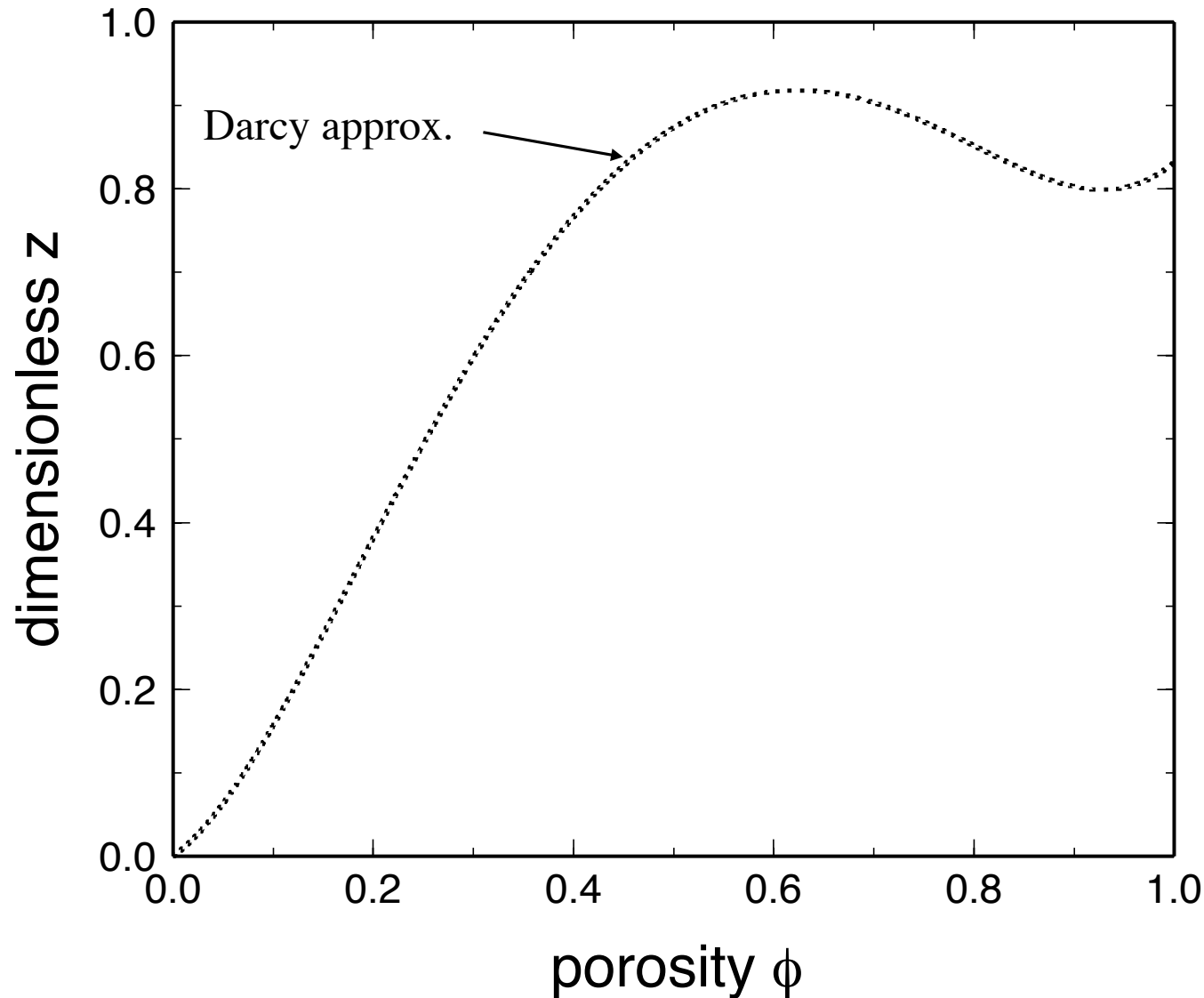
Darcy equilibrium ($\delta=0$) – porosity

~~$$\delta^2 \phi \frac{d}{dz} \left(\frac{1 - \phi^2}{\phi} \frac{dv_m}{dz} \right) - \Delta v = V_B \phi(1 - \phi)$$~~

matrix viscous forces

Darcy drag

buoyancy



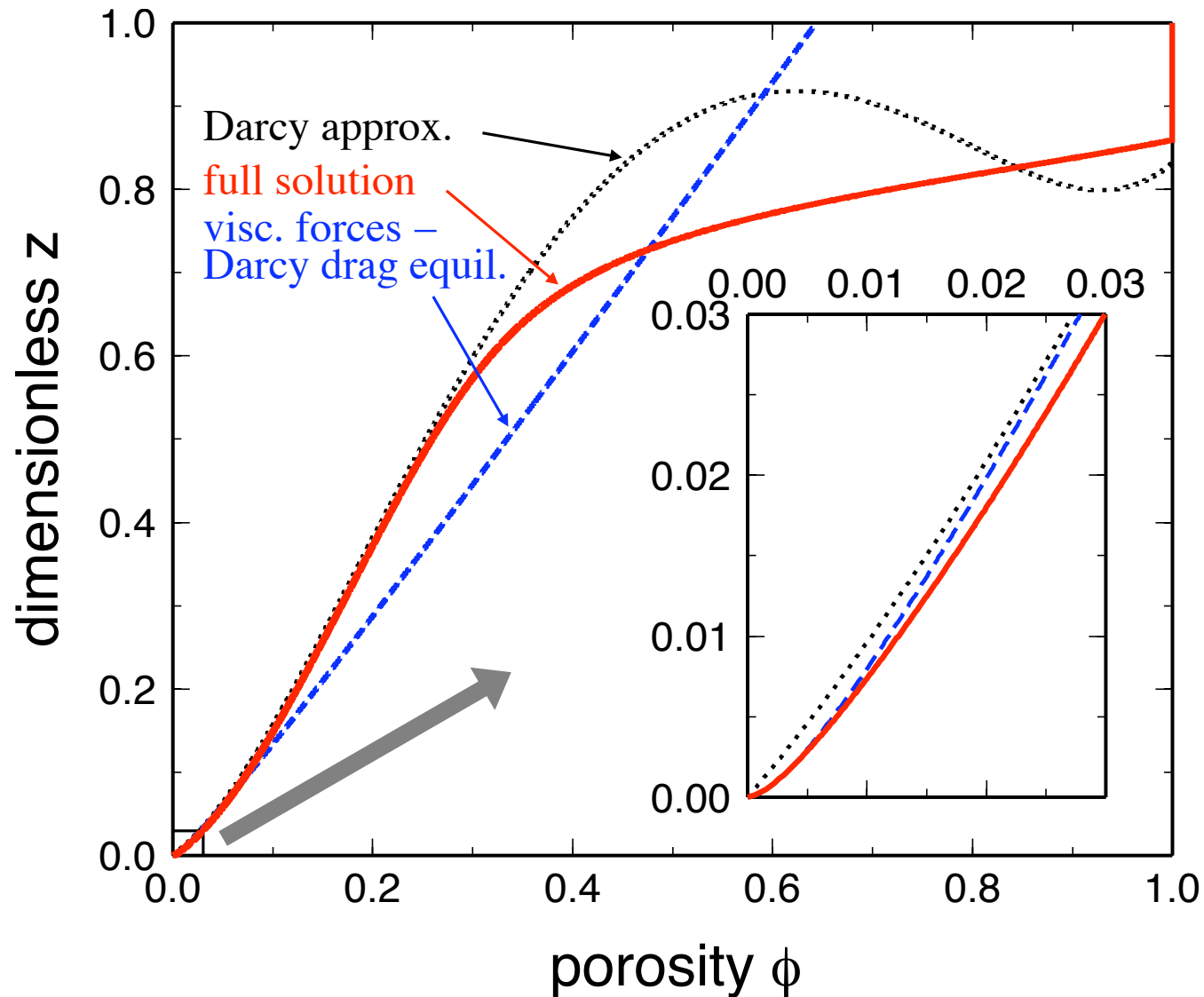
Full solution – porosity

$$\delta^2 \phi \frac{d}{dz} \left(\frac{1 - \phi^2}{\phi} \frac{dv_m}{dz} \right) - \Delta v = V_B \phi (1 - \phi)$$

matrix viscous forces

Darcy drag

buoyancy



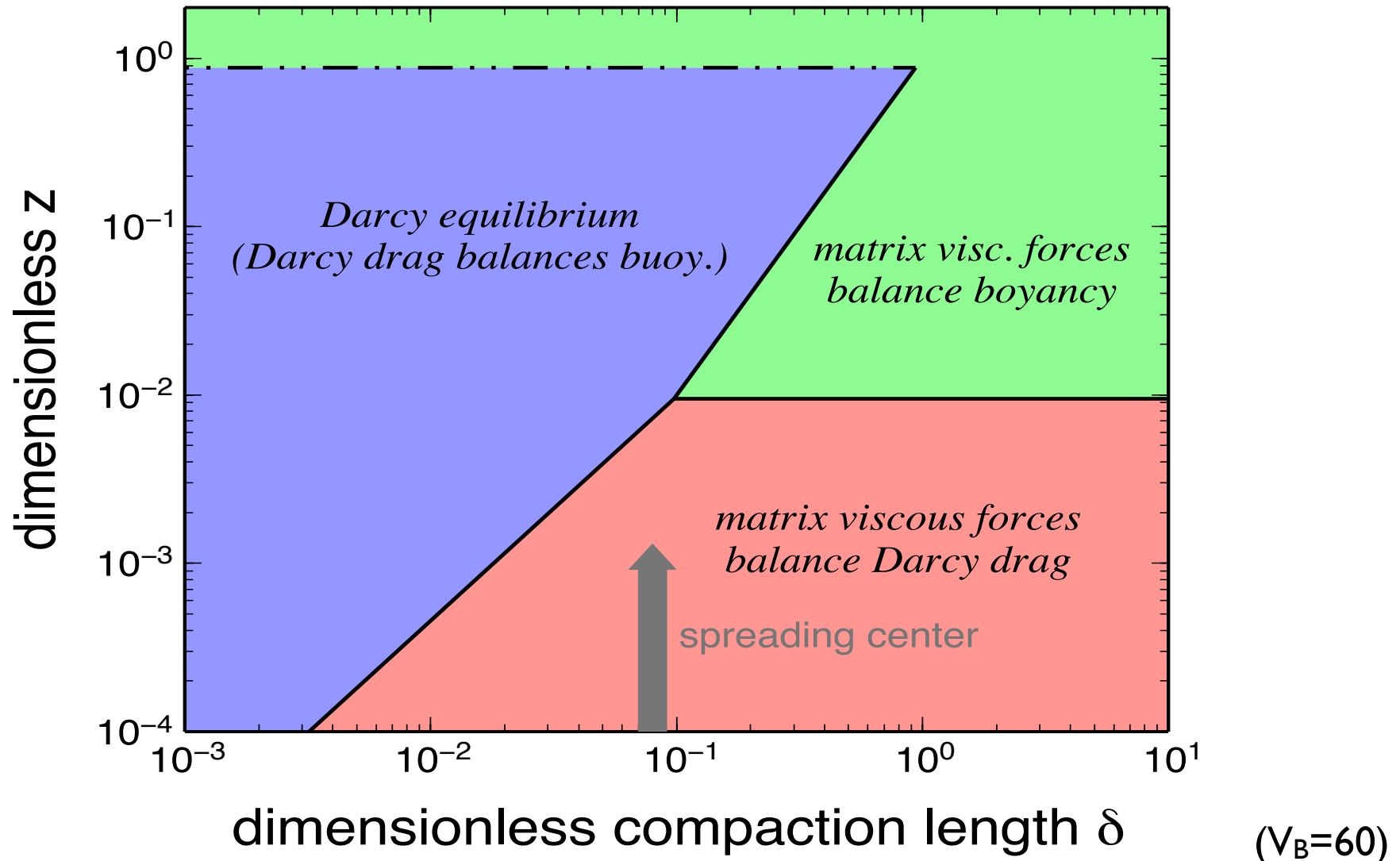
Force balance in the partially molten region

$$\delta^2 \phi \frac{d}{dz} \left(\frac{1 - \phi^2}{\phi} \frac{dv_m}{dz} \right) - \Delta v = V_B \phi (1 - \phi)$$

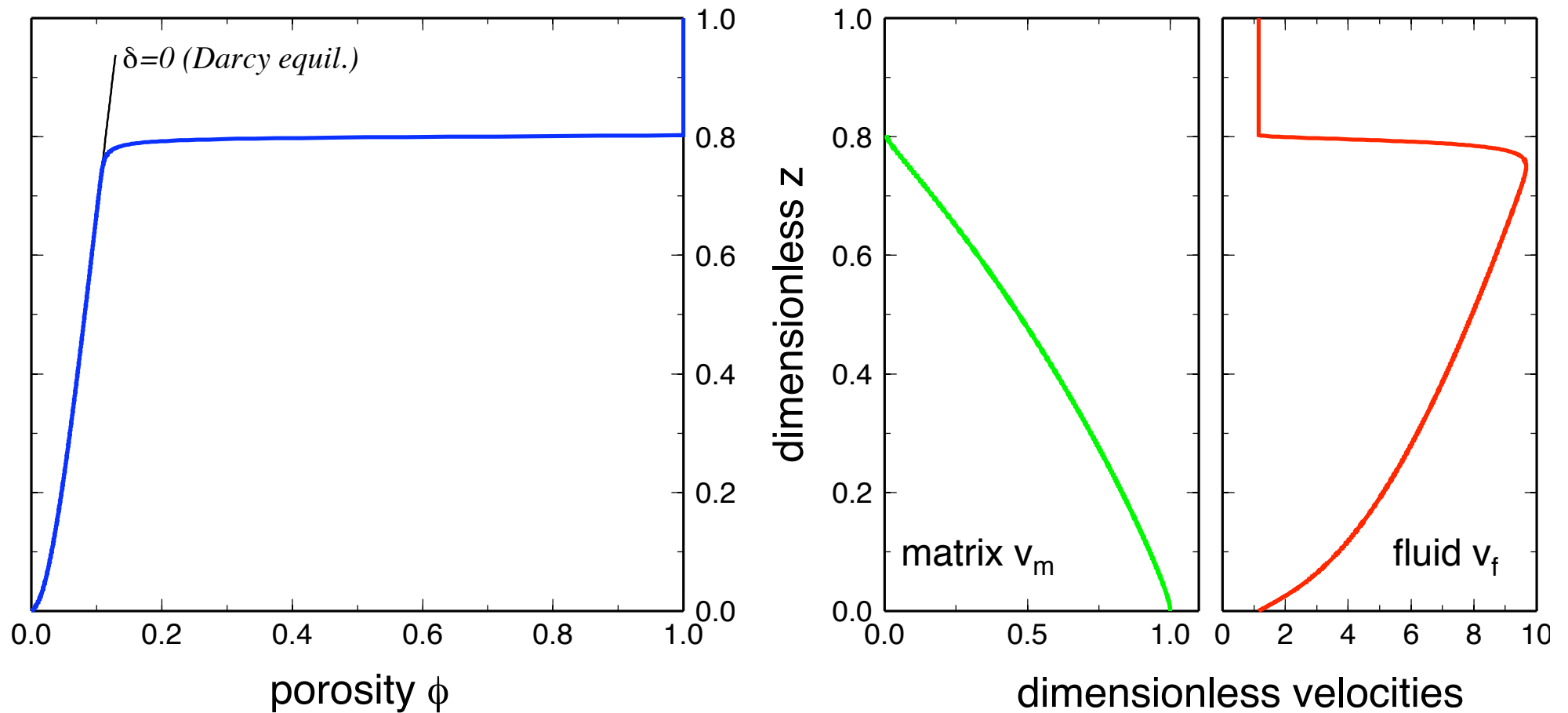
matrix viscous forces

Darcy drag

buoyancy

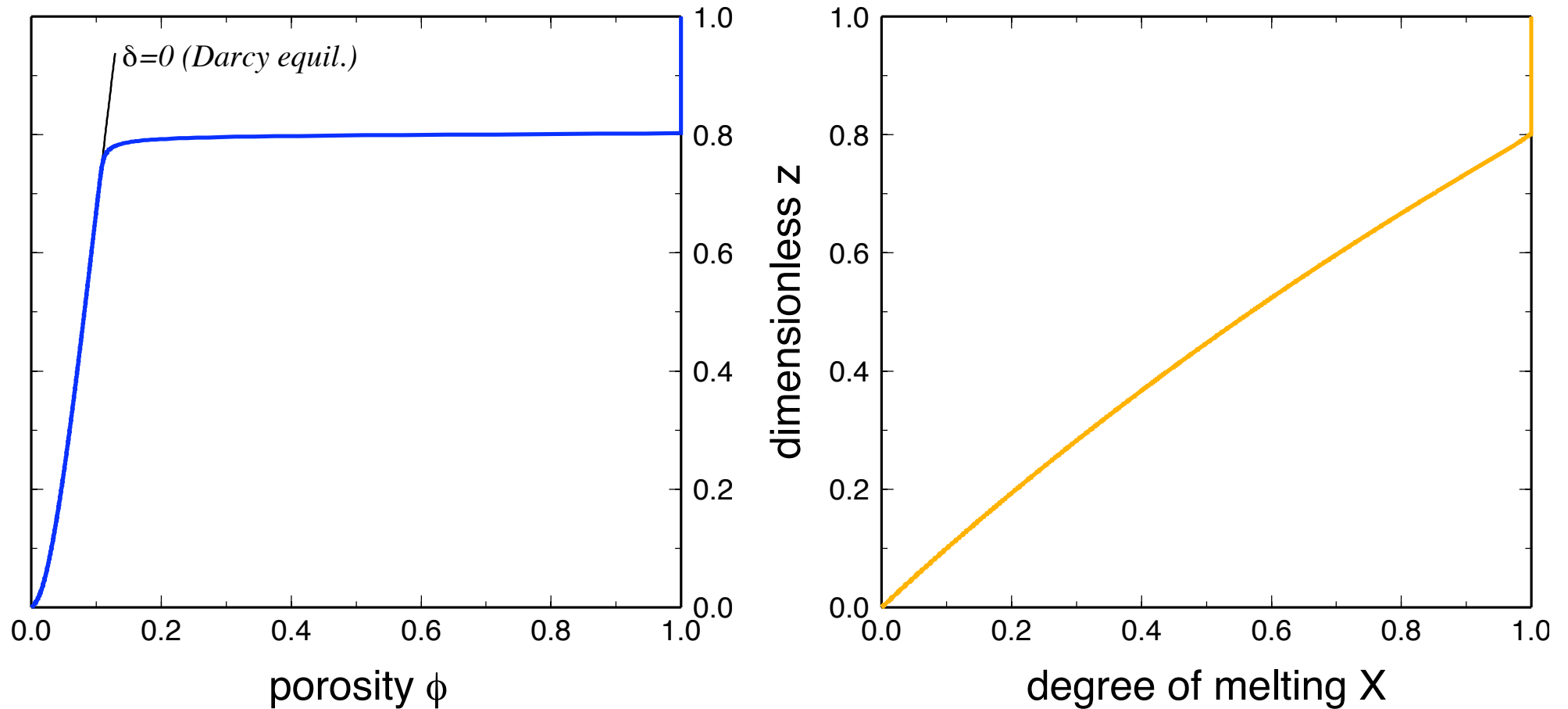


Porosity and velocities



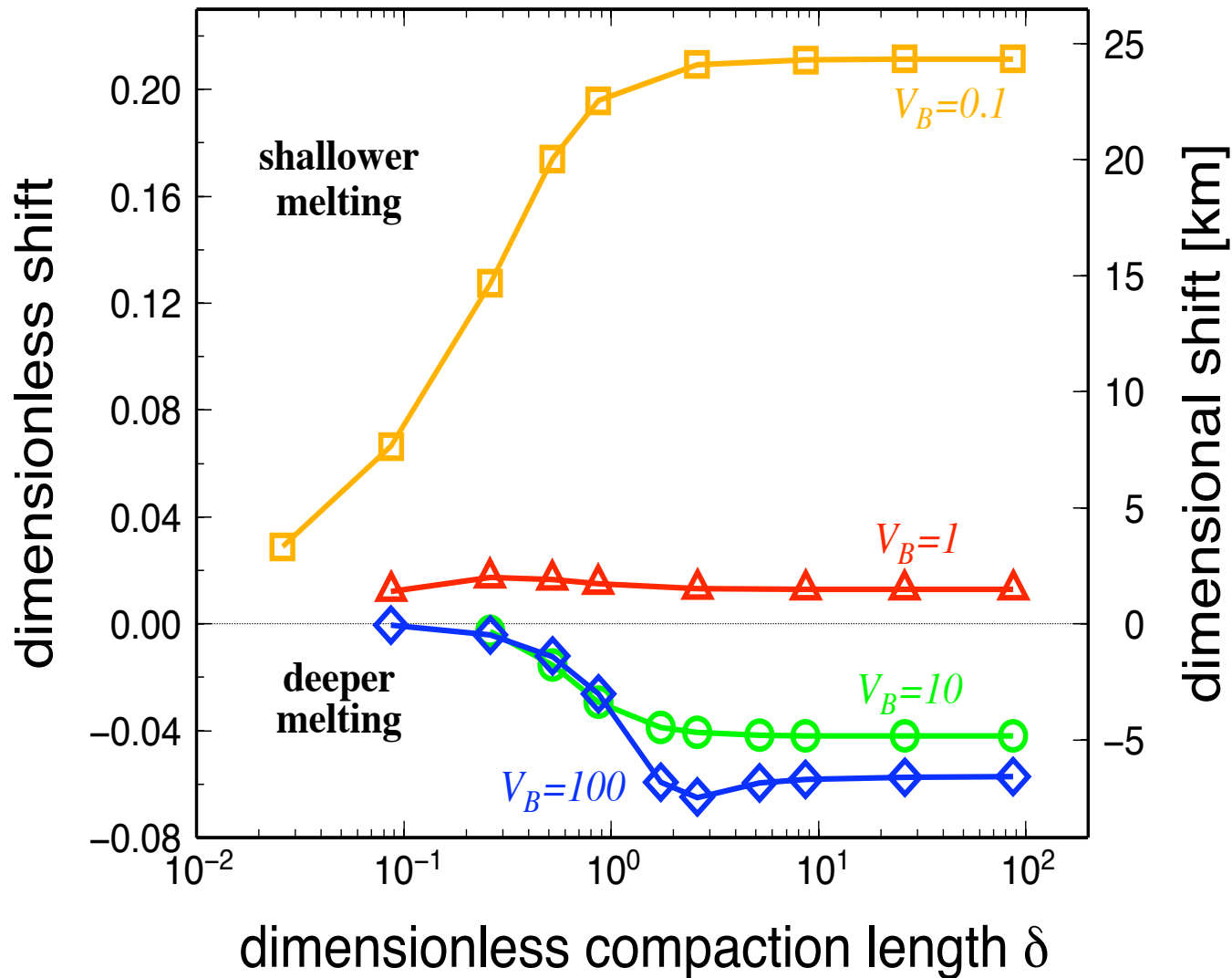
($\delta=0.26, V_B=100$)

Porosity and degree of melting



($\delta=0.26, V_B=100$)

Depth of incipient melting



$V_B < 1$
 inefficient melt extraction
 matrix dilates, $P_f > P_m$
 need lower average pressure,
 i.e., shallower depth to melt

$V_B > 1$
 melt readily extracted
 matrix compacts, $P_f < P_m$
 can melt at higher average
 pressure, i.e. deeper

Summary of the coupling compaction–melting

- Darcy equilibrium a reasonable approximation in the bulk of the partially molten zone
- but not valid at incipient melting
- melting begins at different depth than what predicts average pressure
- porosity much smaller ($\propto \sqrt{z}$) than degree of melting ($\propto z$) in most two-phase region

Metal–silicate differentiation

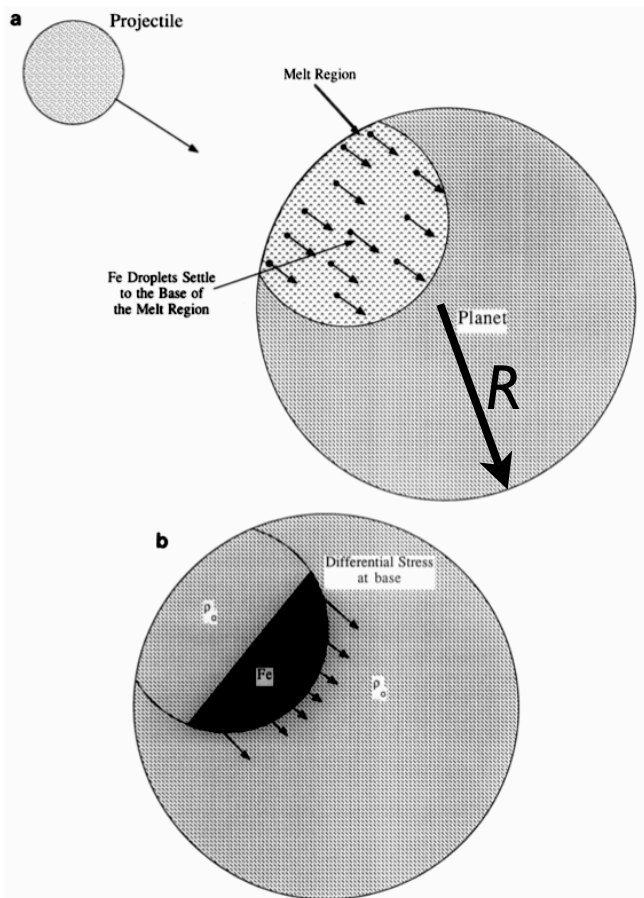
$$\Delta T_{\text{impact}} = \frac{4\pi}{3} \frac{f_1}{f_2} \frac{G\bar{\rho}R^2}{C}$$

Monteux et al. (2007)

we assume that a situation occurred where

- silicates are solid
- metal can be molten

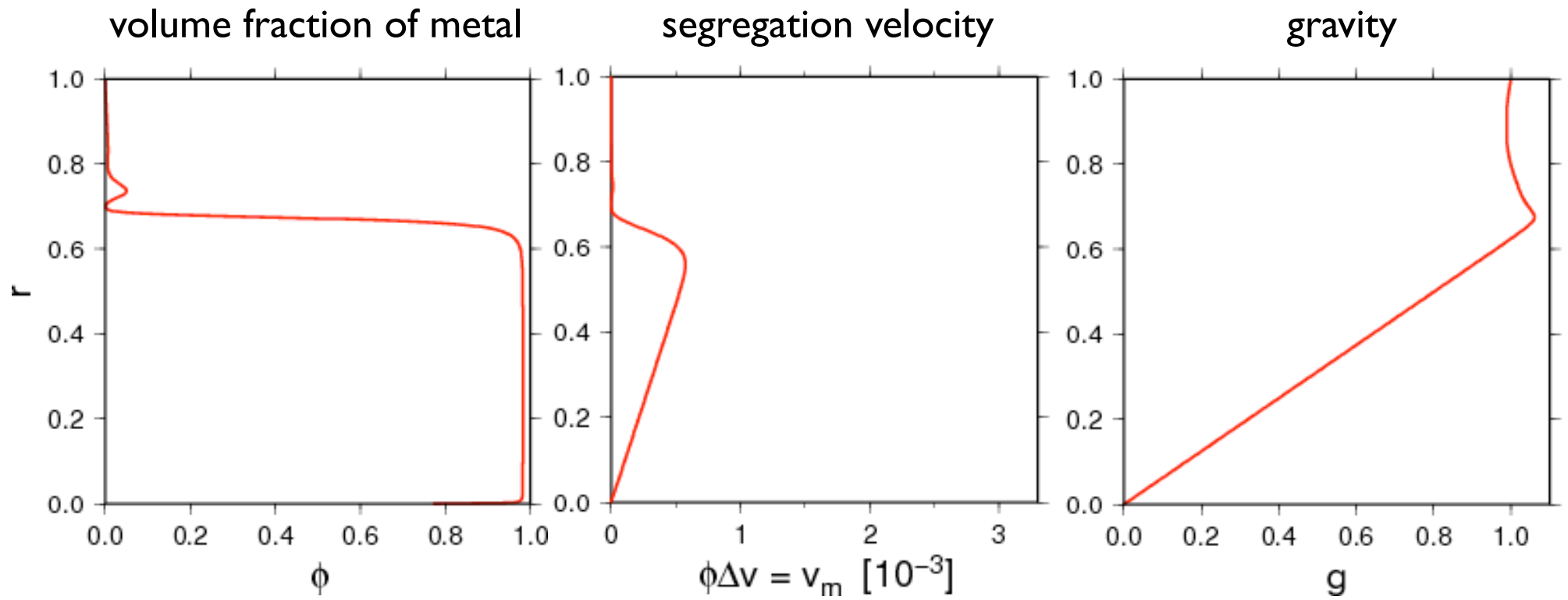
$$\Delta T_{\text{segreg}} \approx \Delta T_{\text{impact}}$$



Tonks & Melosh (1992)

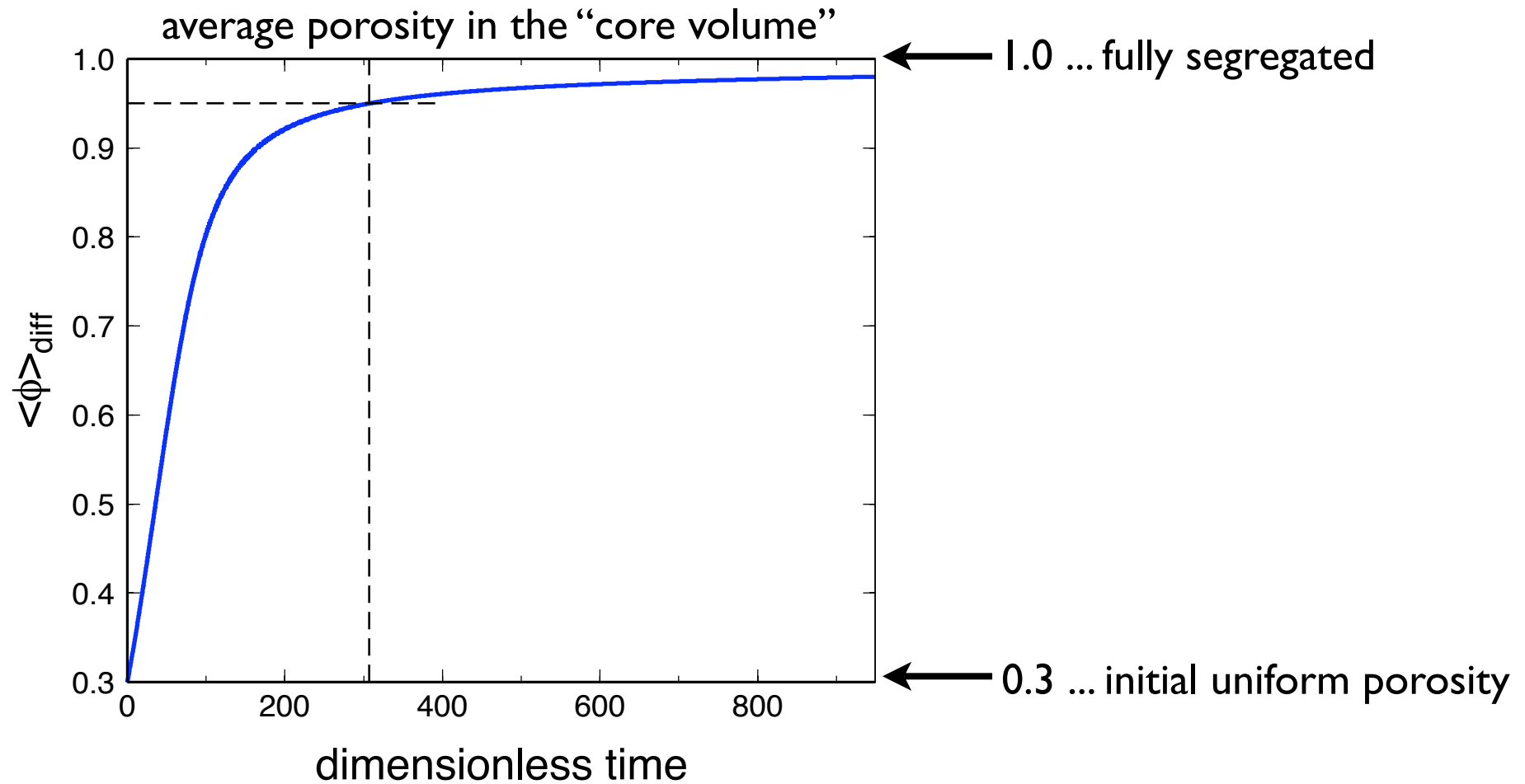
I-D spherically symmetric case

all the metal is liquid
only mechanical equations solved



dimensionless compaction length $\delta=0.2$

I-D spherically symmetric case



unit dimensionless time $\tau \sim 300$ yr ...

core segregation time ~ 100 kyr

$$\tau = \frac{\mu_m}{|\Delta\rho|g_0R}$$

Metal–silicate differentiation

2-D Cartesian model

accounts for conversion of potential energy into heat
upon differentiation

$$\phi(\mathbf{v}_f - \mathbf{v}_m) \begin{cases} =0 & \text{if metal solid ... single phase} \\ \neq 0 & \text{if metal molten ... phases separate} \end{cases}$$

Simplifications

- viscosities of the silicates and solid metal are equal
- the latent heat of fusion of the metallic phase is neglected

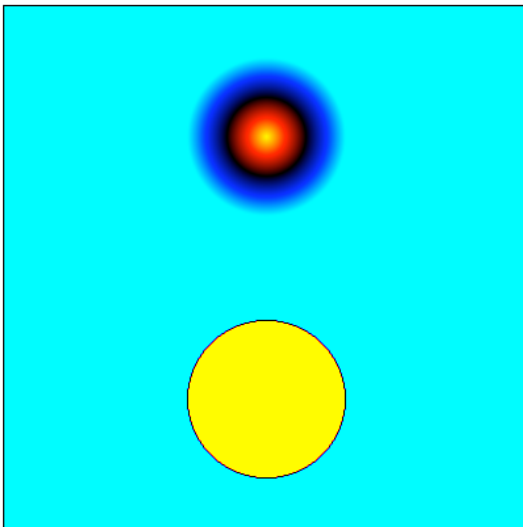
2-D Cartesian model – numerical method

average momentum equation:
direct solver (F. Dubuffet)

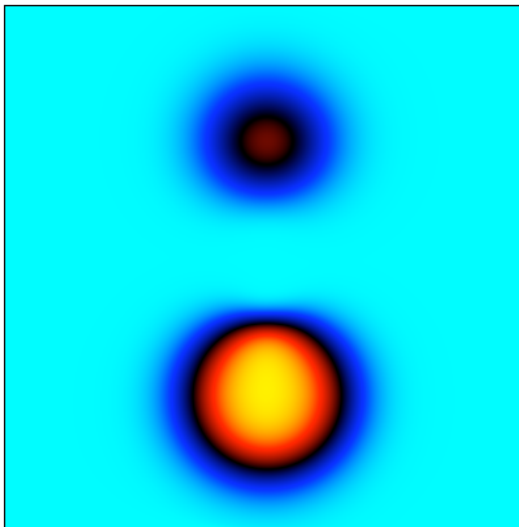
Darcy separation and compaction:
ADI iterative

porosity advection:
TVD conservative shock-preserving implicit “superbee”

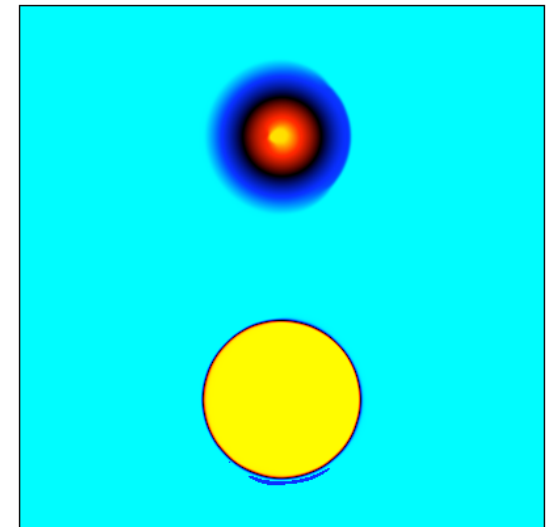
initial porosity



2π -rotation: upwind

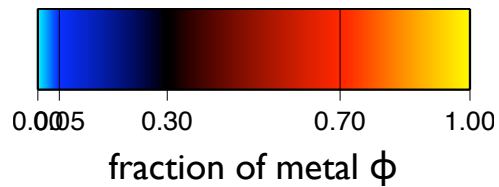
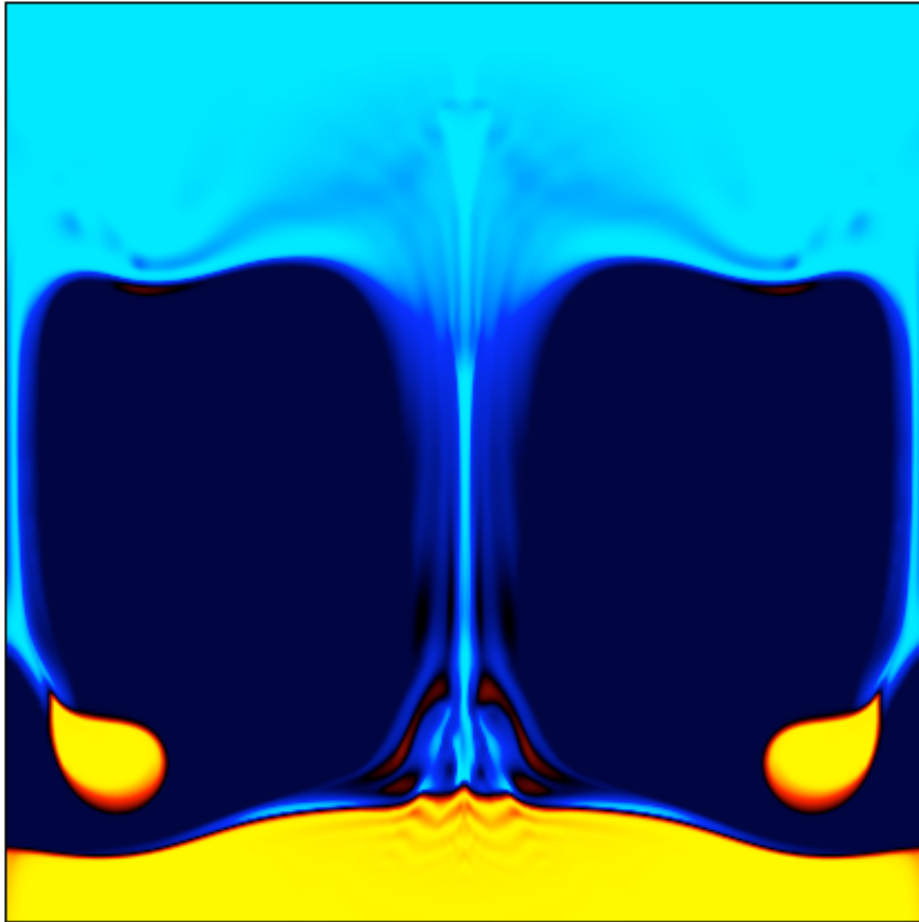


2π -rotation: our method



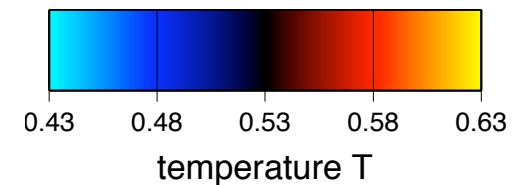
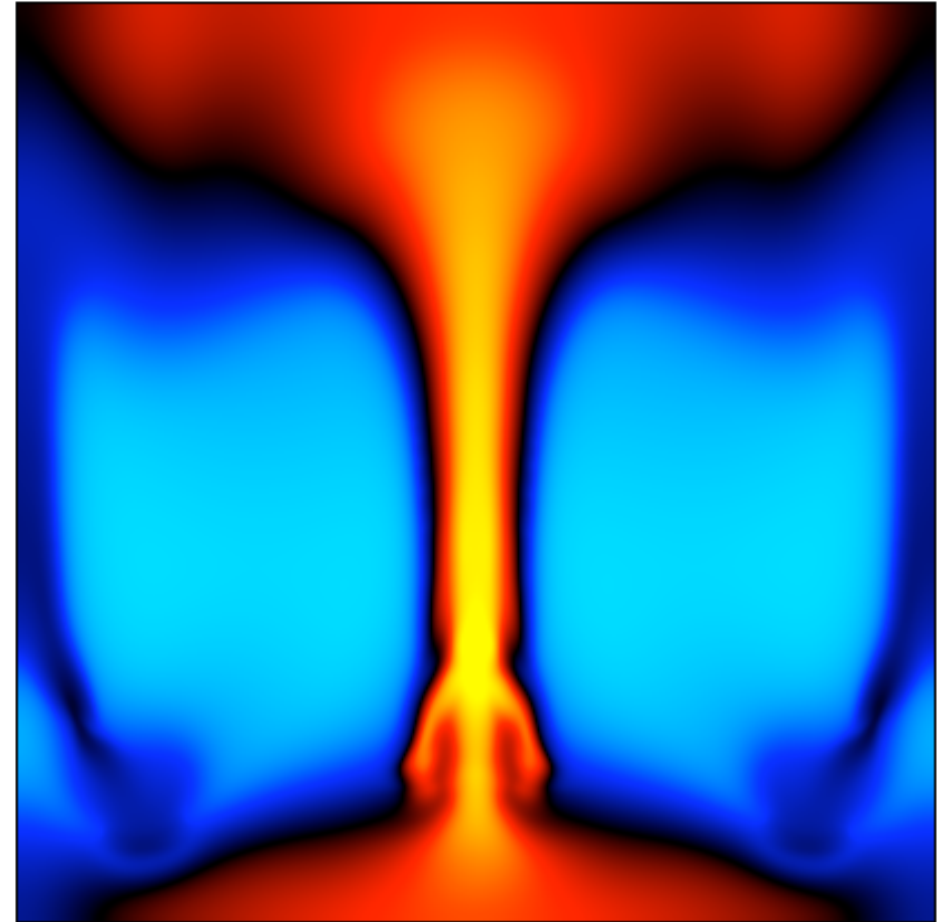
Simulation of an impact-induced differentiation

fraction of metal ϕ



initial fraction of metal
uniform, $\phi=0.25$

temperature T

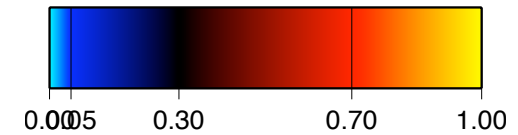


$T_{\text{melt}} = 0.48$ (1300 K)
 $T_{\text{in}} = 0.53$ (1430 K)
 $T_{\text{out}} = 0.43$ (1160 K)

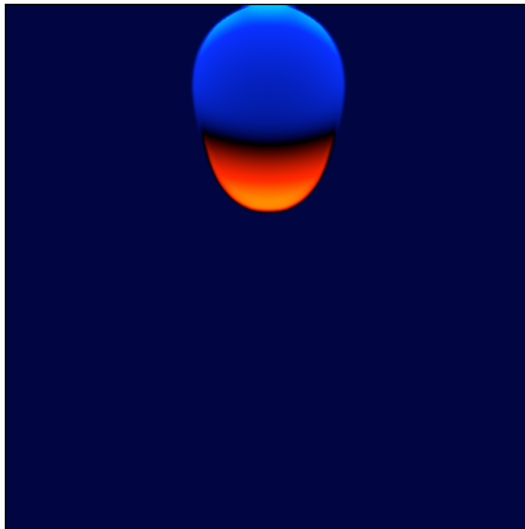
initial temperature above T_{melt}
of metal in a spherical region,
and below T_{melt} elsewhere

Simulation of an impact-induced differentiation

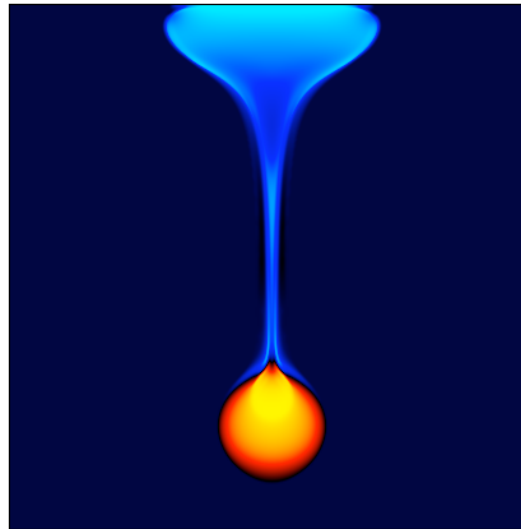
fraction of metal ϕ



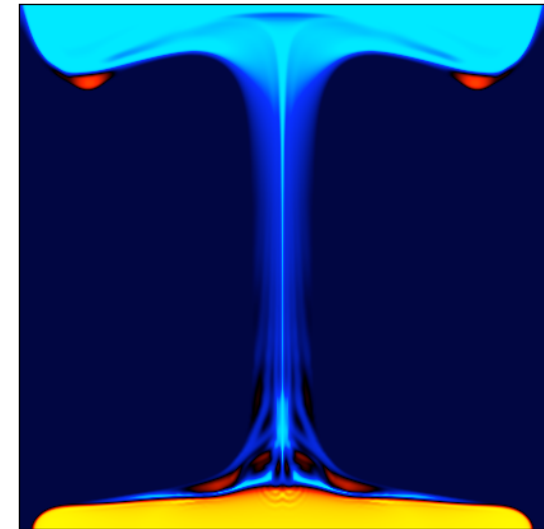
$t=150$ (40 kyr)



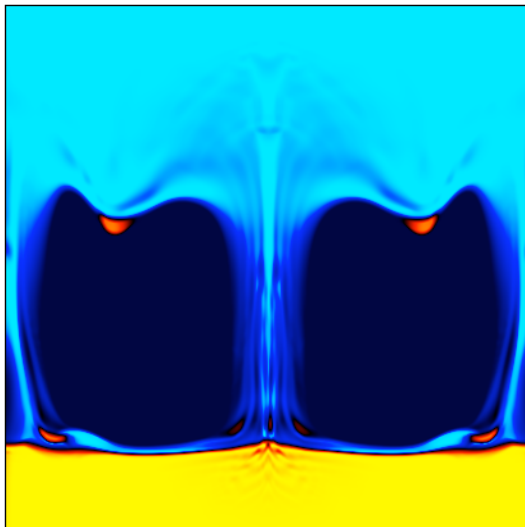
$t=400$ (108 kyr)



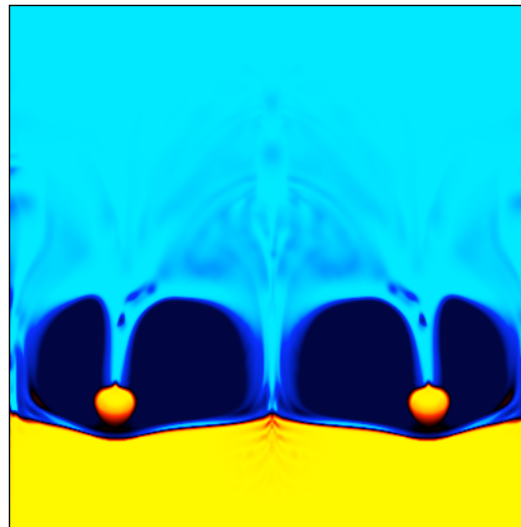
$t=1300$ (351 kyr)



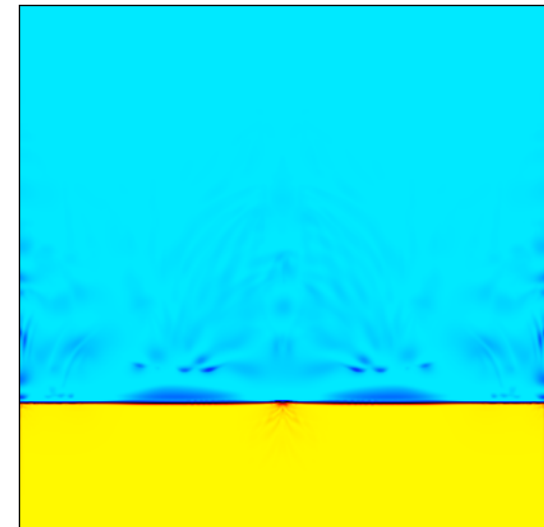
$t=3100$ (837 kyr)



$t=4000$ (1.1 Myr)



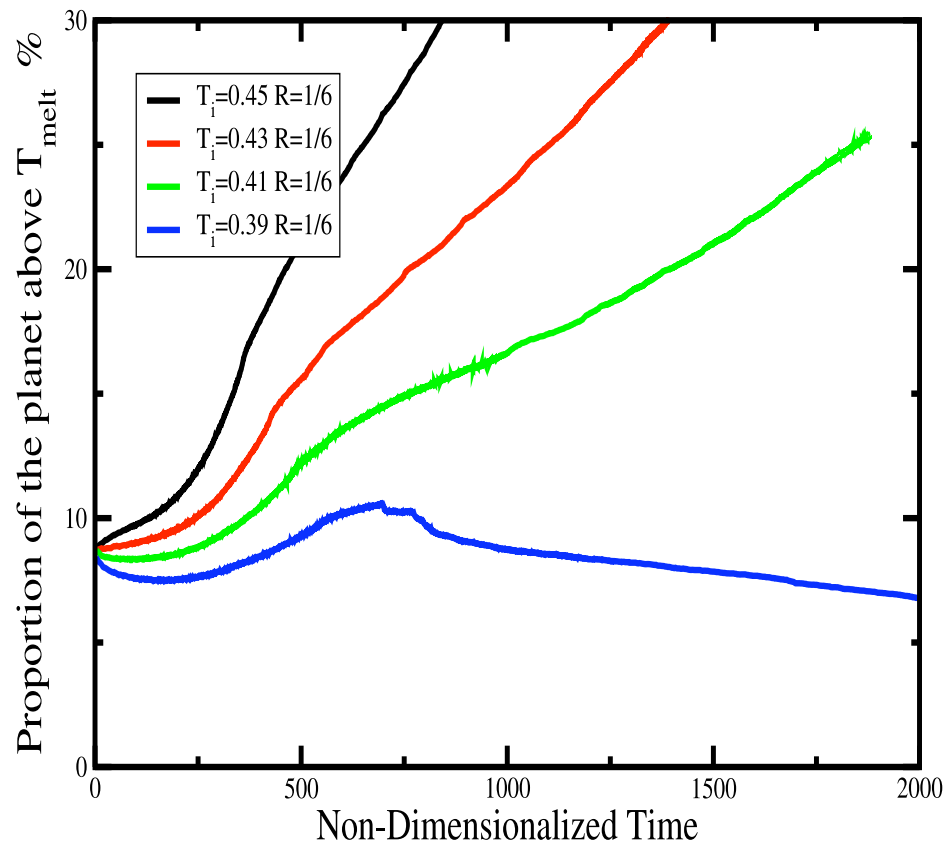
$t=5800$ (1.6 Myr)



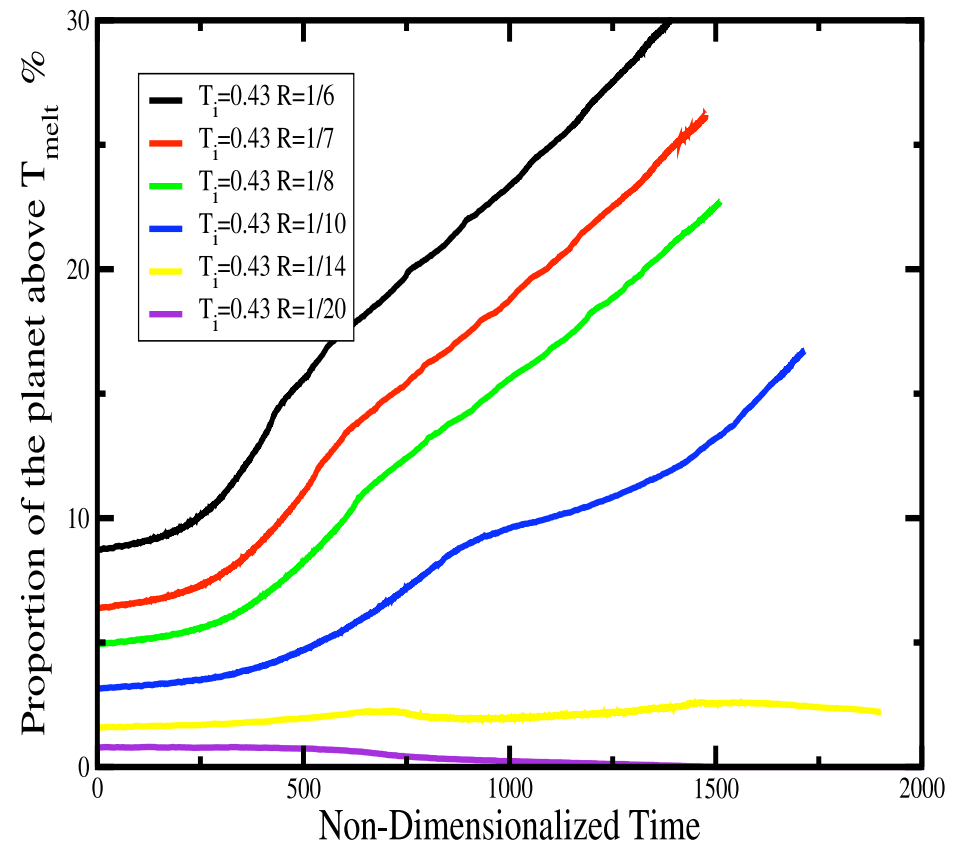
When does the instability develop?

the planet has to be hot enough
the impact has to be large enough

vary initial planetary temperature



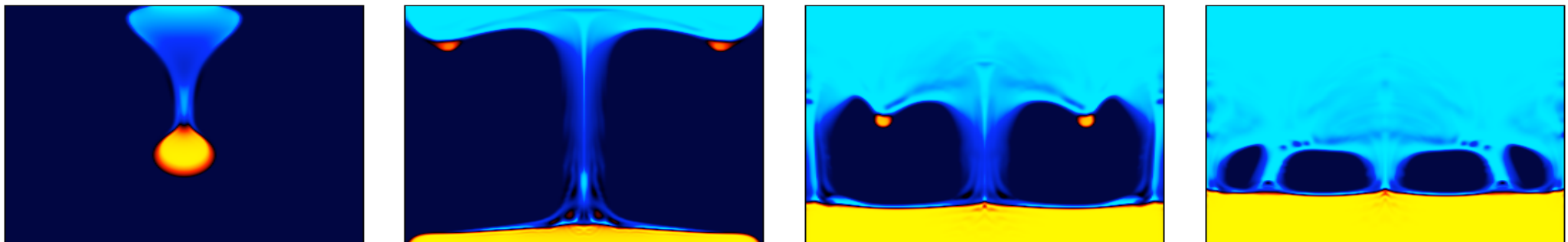
vary radius of the impacted zone



$T_{\text{melt}} = 0.48$ (1300 K), $\Delta T = 0.1$ (270 K)

Summary of metal–silicate differentiation modeling

- a single impact may trigger a large scale (whole planet) core segregation
- time scale of core formation ~ 1 Myr
- potential chemical equilibration at the base of the silicate mantle



Overall summary

- development of two-phase model with self-consistent inclusion of phase change
- study of coupling between compaction and melting
- modeling of metal–silicate differentiation

Perspectives

- calculations of partial melting in 2-D (spreading center)
- study of solitary waves with/without melting
- melting in a system with multiple components
- include radioactive heating in a model of early Earth evolution and differentiation
- account for latent heat of phase change of iron in the model of differentiation
- and many more ...

A photograph of a sandy beach with green seaweed and scattered white and grey stones. The text is overlaid in the center.

Merci, Yanick !

Et merci à tous !

