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Multiresolution tectonic features over the Earth inferred from a wavelet transformed geoid

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Abstract Geoid signals give information about the underlying density structure and can be used to locate the source depth of the mass anomalies. Wavelet analysis allows a multiresolution analysis of the signal and permits one to zoom into a specific area bounded by a particular length scale. The ability of wavelets to resolve the geoid signal into individual wavelength components without losing the spatial information makes this method superior to the more common spherical harmonic method. The wavelet analysis allows one to zoom into a specific area and look at the regional geology. We have used a wavelet transform of the geoid to study the regional geology of Japan and the Philippine Plate, South America, Europe, North America, East Africa and the Middle East, India and the Himalayas, China and Southeast Asia, and Australia. By filtering the Earth's geoid anomalies with 2-D Gaussian wavelets at various horizontal length scales, one can detect the subduction zones along South America, the Aleutians, and the western Pacific; the Himalayas; the Zagros Mountains; the Mid-Atlantic ridge; and the island chains of the mid-Pacific. We have processed geoid data with a horizontal resolution down to approximately 200 km. Using an adjustable wavelet, one can detect

structures that can only be picked up visually with much higher resolution spherical harmonic gravity data. We have also looked at the wavelength at which the maximum signal occurs over a range of scales. This method, known as E-max and k-max, is especially effective for detecting plate tectonic boundaries and ancient suture zones along with areas of strong non-isostatic gravitational potential due to high differential stress. These areas are likely to be at high risk of earthquakes. These methods will be especially useful to future studies of the geoid potentials of other planets, such as Mars and Venus, since they will allow careful studies of the regional geology variations with geoid data of the resolution available from satellites.

Keywords Geoid · Wavelets · Tectonics · Plate boundaries

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Introduction

Geoid anomalies are undulations of the Earth's equipotential surface with respect to a reference ellipsoid and have been used to constrain the viscosity structure of the entire mantle in the past two decades (Hager 1984; Ricard et al. 1984; Forte and Mitrovica 1996; King 1995; Čadek et al. 1995; Kido and Čadek 1997; Čadek and Fleitout 1999). There are various mechanisms for producing geoid anomalies, depending on the horizontal wavelength, such as the rotation of the Earth, which can cause sea-level fluctuations and operates over long horizontal wavelengths (Sabadini et al. 1990). Longer wavelength geoid anomalies have sources in the lower mantle (Chase 1979) and shorter wavelength anomalies are caused by heterogeneities in the lithosphere (Hager 1983; Le Stunff and Ricard 1995). One can use the geoid locally to study a particular area (Calmant and Cazenave 1987) or globally to probe the deep mantle (Cazenave et al. 1989). The gravity dataset continues to grow with the current GRACE and CHAMP missions, which

should allow measurements of the gravity field to spherical harmonic degree and order 160 from satellite data alone with surface data being needed only for higher degrees and orders (Gruber et al. 2000). New techniques are needed to datamine and analyze the large amount of data being produced by these missions.

Wavelet analysis is a relatively new mathematical technique in image processing (Antoine et al. 1993). It is especially powerful for simultaneous detection of both the position and shape of a signal (Antoine et al. 1993). It is basically a local spectral analysis, controlled by a specified position and a scale parameter, which acts to contract or magnify the image, like a microscope. A wavelet basis exists on many scales of resolution (Tymczak et al. 2002), which means that the same wavelet basis can be used to represent features that range from small-scale microscopic phenomenon to large-scale flow in the Earth's mantle. Lifting techniques have been developed that allow wavelet bases to be built quickly in any dimension for any lattice with any number of vanishing moments (Kovačević and Sweldens 2000). The wavelet analysis is carried out everywhere, and this aspect endows it with a global character. Thus, the principal advantage of wavelets is the ability to view the whole scenario at different scales.

Wavelets are more satisfactory than the classical windowed Fourier transform because they are determined self-consistently within a sound mathematical framework, and the window size is adjusted automatically according to the length scale under consideration. These characteristics make wavelets an excellent tool for addressing problems in many geophysical fields, including seismic tomography (Bergeron et al. 1999, 2000a, 2000b; Piromallo et al. 2001), convection studies (Hier Majumder et al. 2002; Yuen et al. 2002; Vecsey and Matyska 2001), crystal zoning (Wallace and Bergantz 2002), and the geoid (Yuen et al. 2002; Vecsey et al. 2001; Vecsey 2002). Wavelets have also been used to invert for crustal gravity and magnetic data in exploration geophysics (Boschetti et al. 2001; Hornby et al. 1999).

The wavelet transform provides a multiresolution representation of the function (Strang and Nguyen 1996), which allows one to examine the behavior of the function at different spatial resolutions (Kumar and Foufoula-Georgiou 1994). Wavelet multiresolution edge detection has been used in the medical sciences to separate the edges of real structures from noise in magnetic resonance images (Laine 2000) and in exploration geophysics to outline structures on gravity and magnetic maps (Hornby et al. 1999). We have developed a technique similar to edge detection for locating both modern and fossil plate tectonic boundaries in the geoid field.

In this paper, we will report the geological results obtained by applying a 2-D continuous wavelet filter at discrete scales to discrete data from a 90 spherical harmonic degree non-hydrostatic geoid model, which is obtained by recomputing and truncating the full model of 360 degrees (Rapp and Paulis 1990). We will

demonstrate that, although this geoid model may not have the highest resolution, the wavelet-filtered version will show some impressive fine features. First, we will discuss the mathematical aspects of the wavelet filter. Then we display the filtered geoid for various length scales. We then zoom into the signal at short length scales to study the regional geology of an area. Finally, we discuss the results and the prospects of this blossoming field.

The continuous Gaussian wavelet transform

A continuous and isotropic wavelet transform, Ψ , can be viewed as a convolution of a spatial function, f , with a given wavelet function, ψ , which is shifted in the spatial domain by $\mathbf{b}$ and equally dilated in each direction by the scale, a . The continuous wavelet filter is applied to the discrete data of the geoid model at discrete scales. In two dimension this takes the form:

$$\Psi_f(a, \mathbf{b}) = \frac{1}{a} \int_0^{L_x} \int_0^{L_y} f(\mathbf{x}) \psi^* \left(\frac{\mathbf{x} - \mathbf{b}}{a} \right) d^2 \mathbf{x} \quad (1)$$

where L_x and L_y are the lengths of the Cartesian domain, and $\mathbf{x}$ is the position space vector. The above integral is a true convolution only if:

$$\psi^*(\mathbf{x}) = \psi(-\mathbf{x}) \quad (2)$$

which is the case in our study. Since the wavelet transformation (Daubechies 1992) corresponds to the convolution of the signal, it is most efficiently computed in Fourier space using the Fast Fourier Transform (Bergeron et al. 1999).

The convolution kernel chosen here for the filtering is the second derivative of the Gaussian function, called the Mexican hat (Daubechies 1992). Since it is real and relatively narrow in space, it has good spatial resolution with an excellent ability to isolate peaks and discontinuities in space (Torrence and Compo 1998; Antoine et al. 1993). The expression of the Mexican hat function is given in Fourier space:

$$\psi(a\mathbf{k}_F) = (2\pi)^{\frac{1}{2}} |a\mathbf{k}_F|^2 e^{-\frac{1}{2}|a\mathbf{k}_F|^2} \quad (3)$$

where $\mathbf{k}_F$ is the global Fourier wavenumber. Higher order Gaussian wavelets, including the fourth and eighth derivative of the Gaussian, are also available in our code (Piromallo et al. 2001).

The wavelet scale, a is not necessarily equal to the horizontal Fourier wavelength. In this case, a is defined as:

$$a = \frac{1}{\exp(0.22k)} \quad (4)$$

where k is the wavenumber or wavelet mode. The wavelength, λ , of a given scale, a , is defined as twice the diameter of the circular structure:

$$C = \exp \left(i \frac{2\pi}{\lambda} (x + y) \right) \quad (5)$$

with the diameter that maximizes the value of the wavelet transform (Eq. 1) for that scale. The diameter of this circular structure gives an approximation of the size of features that give the highest wavelet values at a given scale, a . We have defined the diameter of this circle as the resolution of the wavelet at a given scale. For our case, with the isotropic Mexican hat wavelet, the resolution is approximately:

$$d = a\pi R \sqrt{\frac{2}{3}} \quad (6)$$

where R is the radius of the Earth, 6,378 km. Details concerning the numerical implementation can be found in Bergeron et al. (2000a) and Yuen et al. (2002).

Since the Fourier transform is needed for the entire surface of the Earth, we must apply a window to the dataset. For this purpose we have selected the Parzen window and the entire Mercator projection. The window deletes the signal at the poles, but this is not a problem because the Mercator projected geoid is not useable for latitudes over about 60–65°.

Computing the wavelet transform with the fast Fourier transform can lead to edge effects because the signal must be extended periodically. Large-scale wavelets may pick up on the extended parts of the signal. Small-scale wavelets are also susceptible to edge effects. They may pick up spurious features, especially in the corners of the box, where the signal drops to zero. Since we were using the Mercator projection in this study, the geoid signal was already significantly distorted at the top and bottom of the box. Therefore, we simply did not study any signals in detail along the edge of the box. To study the geoid in areas near the edge of the box, one can simply rotate the data so that the region of interest is in the center of the Mercator projection. Spherical wavelet

filters are currently being developed for cases such as the geoid (Kido et al. 2002).

The wavelet filtered geoid

The non-hydrostatic geoid used in our study covers up to spherical harmonic degree 90, which has a horizontal spatial resolution up to about 200 km. For the spherical harmonic transform, we have defined the resolution as:

$$d = \pi R / l \quad (7)$$

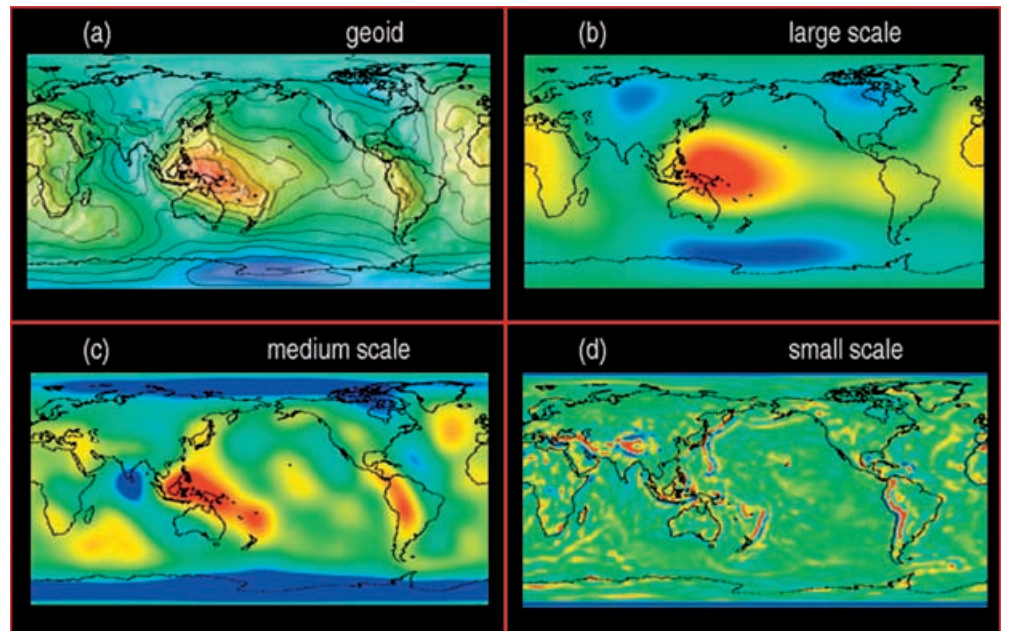
where R is the radius of the Earth, 6,378 km, and l is the spherical harmonic degree. We have normalized the geoid to 1 m²/s².

Since the kernel for the geoid is the Green's function, there is a relationship between the horizontal wavelength of the geoid anomaly and the depth of the source of the geoid anomaly (Richards and Hager 1984; Kido and Yuen 2000): the longer the wavelength, the deeper the source of the signal, and the shorter the wavelength, the shallower the source of the signal.

The geoid is displayed in Fig. 1a in the Mercator projection. Only large-scale features, such as a strong maximum over Indonesia and a local minimum over India, are clearly observed (Fig. 1a; Chase 1979; Hager 1983; Richards and Hager 1984; Le Stunff and Ricard 1995).

The discrete data from the spherical harmonic model is filtered with a continuous wavelet transform. The wavelet transform is taken at discrete scales, a (Eq. 4), where k is assigned the integer values from 1 to 20. The wavelet filtered gravity potential (Fig. 1b) at a wavelet resolution of 5,400 km, as defined by Eq. (6), shows only the largest structures shown in Fig. 1a. More details are displayed at a resolution of 1,500 km (Fig. 1c).

Fig. 1 **a** Geoid of Rapp and Paulis (1990) truncated at spherical harmonic degree 90 in the Mercator projection. **b** Large-scale (5400 km) wavelet transform. **c** Medium scale (1500 km) wavelet transform. **d** Small-scale (200 km) wavelet transform. Color map ranges from red for large positive signals to green/yellow for areas with very small signals to blue for large negative signals



Large depressions in the geoid appear over the southern tip of India and the West Atlantic, and pronounced maxima appear over the western and central Pacific and the Andes. The maximum over Iceland may be significantly distorted because of the strong deformation of the Mercator projection at latitudes over 60–65°. Nonetheless, there is still a significant signature over Iceland, which cannot be ignored. These features are well known from previous studies (Yuen et al. 2002; Bergeron et al. 1999). The wavelet analysis allows us to selectively dial into different scales to see at which scales various features appear. This is an important advantage over both conventional truncated window analysis in spherical harmonics and the full spherical harmonic expansion.

At a horizontal resolution of about 200 km, the visible anomalies only sample the Earth down to about 100 km depth. Therefore, anomalies seen at this scale are caused by lithospheric structures. The wavelets allow us to outline topographic features from our dataset of spherical harmonic degree 90 and compare them with the Earth's relief (see Figs. 1d, 2). The subduction zones along the west coast of South America (number 1 in Fig. 2), the Aleutian Arc (2), and the East Pacific trenches down to the latitude of Tonga (3–9) are clearly visible. Mountain belts formed by convergent plate processes, such as the Himalayas (12) and the Andes (11), are apparent. The African rift system, the Mid-Atlantic Ridge (16), Hawaii (18), the Atlas Mountains (15) of northwest Africa, and some details of the plates of the Indian Ocean at the latitude of Madagascar (17) are also visible.

The appearance of the Australian coastline in the scalogram is due to the absence of a continental margin and can be picked out only on very high-resolution

spherical harmonic gravity maps (Sandwell and Smith 1997). Areas with more complicated tectonic structures, such as Europe, can be studied by zooming into the region (see below).

Low-dimensional data compression of the wavelet spectra

The complete information of the local wavelet spectra analysis is difficult to visualize because at each location an entire spectrum must be rendered. This requires operation over a grid size of $N \times N \times M$ where N is the number of grid points, and M is the number of scales computed. This demands an extremely large memory capacity, and it is prohibitively expensive to manipulate a high-resolution dataset, such as the gravity dataset of Sandwell and Smith (1997).

We have employed a data compression procedure called low-dimensional parameterization for culling the wavelet spectra into two distinct quantities, E-max, the maximum of the local wavelet energy, and k-max, the associated local wavenumber, to synthesize the data for visualization as a 2-D map and data compression (Bergeron et al. 1999; Yuen et al. 2002). The wavelet energy, which is a proxy for the local gravitational energy, is represented as the L_2 norm:

$$E = |\Psi(a, \mathbf{b})|^2 \quad (8)$$

The energy values calculated at each scale are then normalized by the average value for that scale. The largest value of all the scales is then defined as the E-max at each spatial point. After the maxima have been chosen, they are multiplied by the sign of the signal at that spatial point so that negative and positive signals can be distinguished on the E-max map.

The k-max map represents the distribution of scales for which the absolute value of the gravity potential is maximal locally. For a given range, if k-max were the smallest wavenumber, we would have a flat plateau on

Fig 2 Topography map of the world in Mercator projection (Smith and Sandwell 1997). *Numbered areas* are discussed in text. *Red (medium gray)* indicates high topography and *blue (dark gray)* indicates low topography

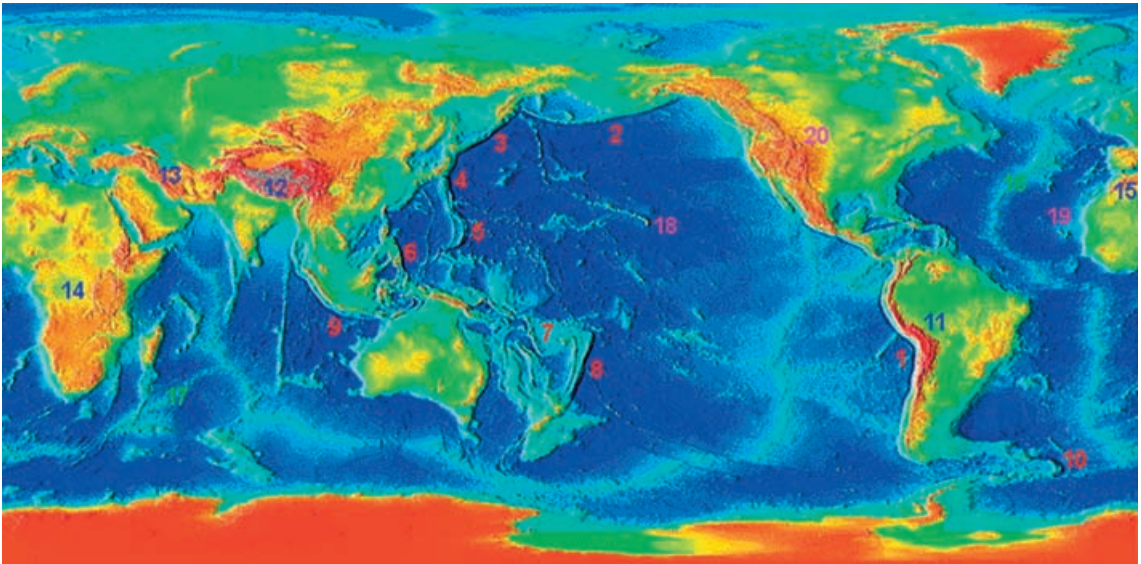
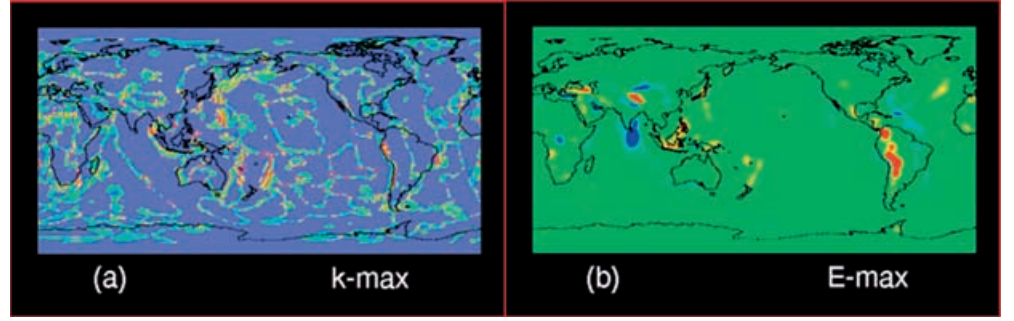


Fig. 3a, b The 2-D map describing k-max and E-max values taken for resolutions ranging from 600 to 200 km. *a* Red indicates high k values (resolution 200 km) and *purple* indicates low k values (resolution 600 km). *b* E-max. The color map is the same as in Fig. 1



the k-max map. The k-max increases sharply along boundaries where there are sharp gradients in E-max. Therefore, the k-max helps to facilitate the detection of the boundaries of structures. Similar techniques have been used to find the edges of source structures in exploration magnetic and gravity data (Hornby et al. 1999) and in biomedical images (Laine 2000).

The spatial distributions of k-max and E-max are plotted in Fig. 3a, b. We examined only the smaller scale features with resolutions (Eq. 6) of about 600–200 km. We computed the wavelet transform at 20 scales, a (Eq. 4), with modes or wavenumbers, k , varying from 15 to 20. The wavenumber was stepped according to:

$$k = \frac{5(n-1)}{19} + 15 \quad (9)$$

where n was assigned the integer values from 1 to 20.

In the E-max map we can immediately discern strong positive signals in the Andes (see 1) in Fig. 2), the Tibetan Plateau (12), the Alpine-Tethys ranges (13), and a negative signal in southern India. We can also discern Hawaii (18) and the Azores. The k-max map provides an outline of the plates of the Earth, highlighting the transform, ridge, and subduction boundaries along the major plate edges (Fig. 3a; Gordon 1998). Fracture zones in the three major oceans can also be determined from the k-max map.

Many convergent zones, such as those of South America, Indonesia, Japan, and Tonga, can be seen as having delineated features. In fact, E-max gradients are related to small values of k-max. Boundaries of the zones associated with high E-max values, such as ridges, are correlated with strong gradients in gravity potential. This implies very small spatial scales due to the sharp front of the adjacent high and low gravity potential values along ridges. The scale of these gradients could be used as a criterion for limiting the horizontal extent of the gravity influence coming from the oceanic ridges.

Wavelet transform analysis compared with spherical harmonics truncation methods

Very often we have datasets presented in the spherical harmonic representation, since this has been a tradi-

tional approach going back to Gauss. Truncating the spherical harmonics at a given number of terms offers an easy but rather deceptive means of depicting features of limited scales in the dataset. In order to display the hazard associated with such truncation methods, we compared the spherical harmonic geoid cut at degree 20 or 40 with the wavelet spectrum of mode 20 (or wavenumber, k), which has a horizontal resolution (Eq. 6) of 200 km (Fig. 4). In Fig. 4a we show the spherical harmonic transform where only the coefficients for $l=20-90$, with resolutions (Eq. 7) of about 1,000–200 km, have been plotted. Figure 4b shows the spherical harmonic transform with only the coefficients for $l=40-90$ (resolutions from 500–200 km) plotted. We see the same structure in all places where the signal is strong; but due to the global character of the spherical harmonic basis, the truncation method fails in places with a weak signal. A strong signal in one place can excite the formation of false signals in other places. These false signals can mask any real but weaker signals in their area. This problem can only be fixed by adding harmonics of higher orders. This is known as the Gibbs phenomenon. Thus after the truncation, we obtain false spotted signals in places of low magnitude signals, which can be confusing.

A more precise comparison of the geoid that is cut at degree 20 or 40 with wavelet spectrum of mode 20 (or wavenumber, k) reveals more of the ability of the wavelet filter. As we have already noted, the Fourier spectrum of the Mexican hat wavelet is the 2-D Gaussian function. On the scalogram it picks up features not only at mode 20, but also slightly bigger or smaller modes. Therefore, we can localize not only the small-scale subduction zones but also the medium-scale Mid-Atlantic Ridge. In the truncated spherical harmonic geoid, some areas are of the same clarity as in the wavelet spectrum, for example, subduction zones, the Himalaya, and the Atlas Mountains. Others can be found in the truncated spherical harmonic geoid only when we know where to look for them, such as the Mid-Atlantic Ridge and the Australian coastline; and some of them, like the circular structure around Hawaii, can only be found in the wavelet transform (Fig. 4c). Thus, we again emphasize the wavelet transform as a powerful tool for local analysis, which picks up more features than the spherical harmonic transform.

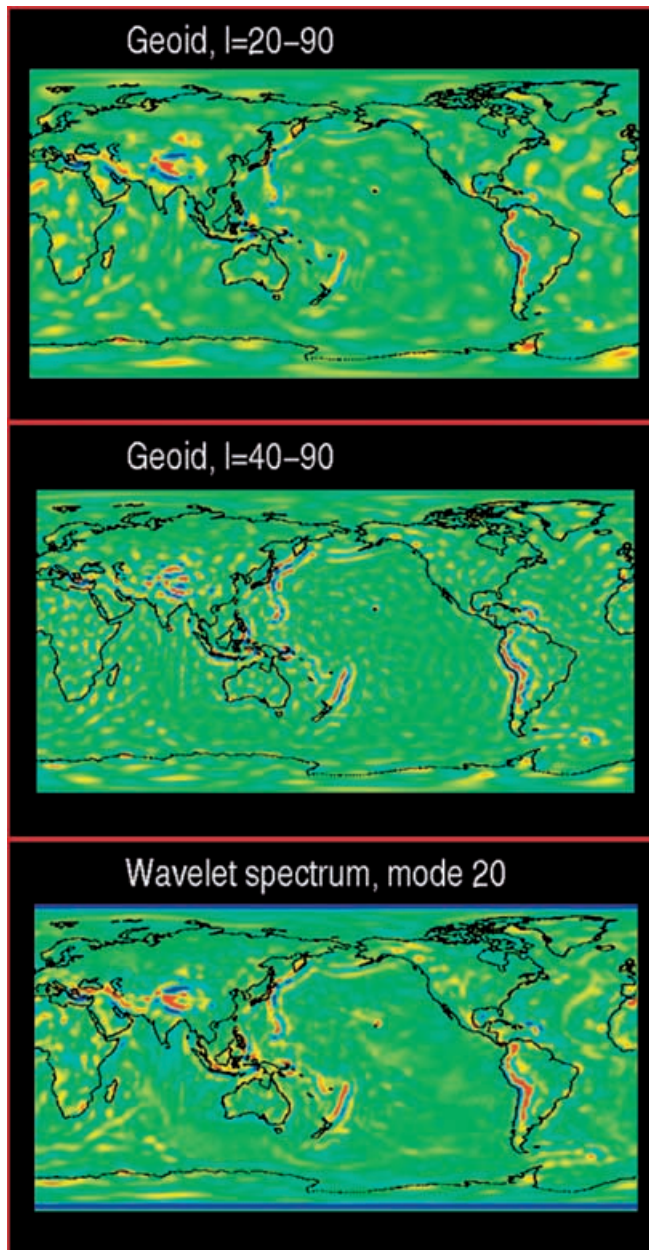


Fig. 4 **a** Spherical harmonics for $l=20-90$. **b** Spherical harmonics for $l=40-90$. **c** Wavelet transform at a resolution of 200 km. The color map is the same as in Fig. 1

The most powerful advantage of wavelets, however, is the ability to quickly zoom into the wavelet dataset to study a particular region without having to recalculate the entire wavelet transform. Spherical harmonics can be forced to better imitate the localized features of wavelets. The resulting image will still be plagued by Gibbs oscillations, however, for an insufficiently long window (Yuen et al. 2002). This technique requires a lot of trial and error efforts to find the appropriate weighting function, and the computations tend to be at least 200 times slower than with wavelet computation for l of around 100 (Yuen et al. 2002).

Regional geological features

In this section we will employ wavelets to investigate different portions of the Earth's surface with the aim of gleaned some interesting geological information on a local regional basis. Wavelets allow us to zoom into a region and adjust the color map to the areal average to study geologic features, such as the location of terrane boundaries. We have selected eight regions to zoom into: the area of Japan and the Philippine Plate, South America, Europe, North America, East Africa and the Middle East, India and the Himalayan region, China and Southeast Asia, and Australia. Each region is shown at the scale associated with 200 km resolution (Eq. 6) and regions with larger-scale features are also shown at 600 km resolution. The E-max and k-max of each region are taken for 20 scales ranging from about 600 to 200 km resolution, using the same method as for the global geoid.

Japan and the Philippine Plate

Japan and the Philippine Plate area are shown in Fig. 5 and Table 1. The E-max map (Fig. 5c) shows two strong maxima over Japan (see number 1 in Fig. 6) and the Philippines (2). These are areas associated with very prominent compressional features (Seno and Maruyama 1984; Seno et al. 1993). Areas of strong gravitational potential around 200 km resolution, such as seen here, indicate a high earthquake risk (Tanimoto and Okamoto 2000). Therefore, the wavelet transform can be used as a voltmeter to detect high earthquake risk areas on a planet. This could be especially useful for finding regions to put seismometers on other planets. The E-max map (Fig. 5c) examines resolutions of 600 to 200 km, which are several times the plate thickness. Normally, at this scale, the isostatic geoid becomes small due to compensation. The large, non-isostatic geoid signatures in this area indicate that high differential tectonic stresses must exist to support the non-isostatic load. The Japan maximum occurs over an area of a major triple junction of the Eurasian (3), Philippine (4), and Pacific (5) Plates. Another strong maximum appears over the Philippines (2), which are being compressed by trenches on both sides. Small-scale minima occur in the vicinity of the Japan (6) and Mariana (7) Trenches and over the deepest ocean depth on Earth, the Challenger Deep (8). A large-scale minimum on the E-max map covers the Banda Sea (9), which looks like a forgotten promontory of the Eurasian Plate between New Guinea and the Philippines. The outline of the Pacific and Philippine Plate trench systems is clearly visible in the smaller scales (Fig. 5). The trench systems can be also discerned in the k-max map along with the region of back-arc spreading that formed the Philippine Plate (Seno and Maruyama 1984).

South America

The South American region is shown in Fig. 7 and Table 2. A very strong maximum appears over the Altiplano Plateau (1 in Fig. 8) in the E-max map. The source has a dipole nature, with a large positive geoid anomaly of 26 m compensated by a large negative Bouguer anomaly from a deep-seated crustal root (Froidevaux and Ricard 1987). The Altiplano Plateau is

Table 1 Wavelet coefficients at a given resolution and E-max values for Japan and the Philippine Plate

	Resolution		E-max
	600 km	200 km	
Japan (1 in Fig. 6)	0.27	0.071	0.35
Philippines (2 in Fig. 6)	0.32	0.061	0.60
Challenger Deep (8 in Fig. 6)	-0.09	—	—
Banda Sea (9 in Fig. 6)	-0.19	-0.082	-0.36
Taiwan	—	—	0.03

associated with an E-max value of 3.7, the largest of any of the mountain ranges we examined (Table 2). This indicates an area of very high tectonic stresses in this subduction zone. The plateau is associated with a 1,500-km segment of shallow subduction, which transmits seismic energy 250–800 km into the upper lithospheric plate, where it is released at rates three to five times greater than in areas of steeper subduction (Gutscher et al. 2000). The area of shallow subduction is held up by the buoyancy of the hot Nazca Ridge and the Inca Plateau, which was formed by the same hotspot as the Marquesas Plateau (Gutscher et al. 1999). The absolute westward motion of the South American plate at about 3 cm/year also contributes to the flat subduction due to continuous lithospheric displacement relative to the mantle (van Hunen et al. 2002). The Peru-Chile trench (2) systems and the Andes Arc (1) are visible on the smaller-scale geoid and the k-max maps (Fig. 7a, b).

Fig. 5 Region around Japan and the Philippine Plate. **a** Wavelet transform at a resolution of 200 km. **b** k-max map. **c** E-max map. The color maps are the same as in Figs. 1 and 3

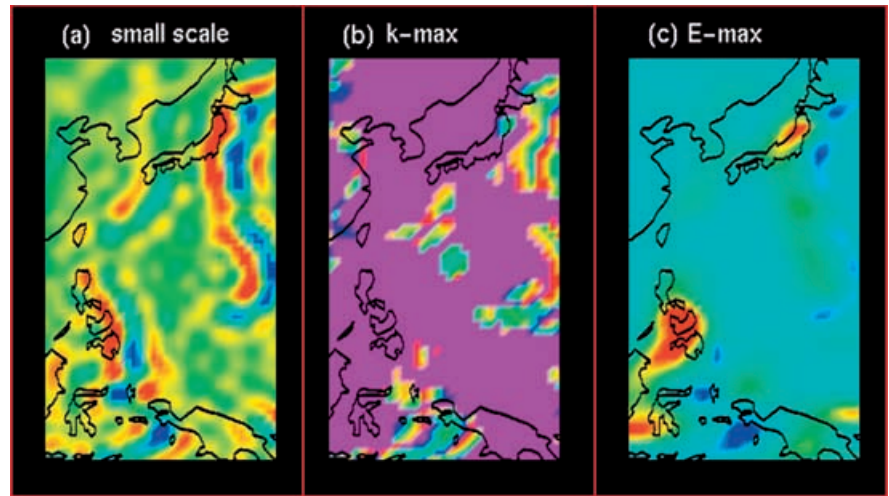
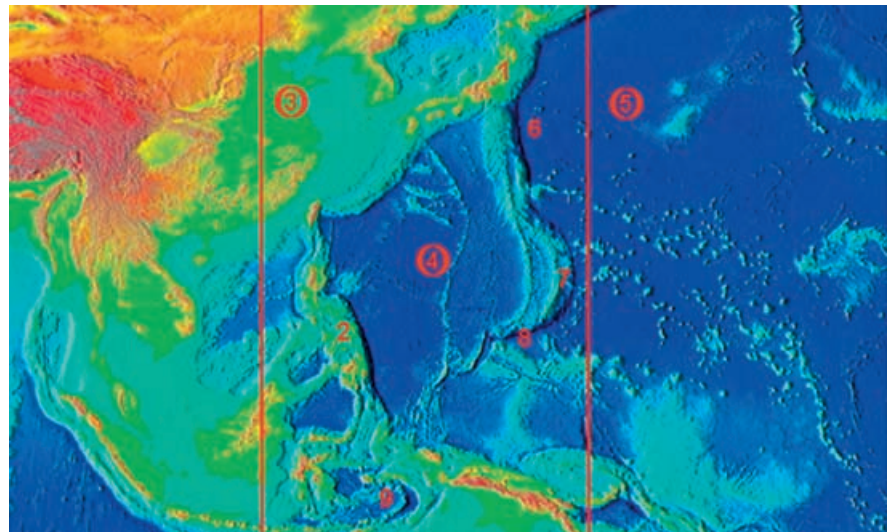


Fig. 6 Topography of region around Japan and the Philippine Plate (Smith and Sandwell 1997). *Numbered areas* are discussed in text. The color map is the same as in Fig. 2



Europe

The European area is shown in Fig. 9 and Table 3. In the medium-scale and E-max maps (Fig. 9a, c), a strong maximum over the middle part of the Alpine-Tethys range (with the mountainous areas of Turkey in the west, the Caucasus in the north, and the Zagros Mountains in the east, 1–3 in Fig. 10) and a maximum with similar magnitude over the Atlas Mountains (4) of northwest Africa are visible. A minimum is apparent in the vicinity of the Hellenic Trench (7), especially towards its eastern part. The European E-max map (Fig. 9c) shows clearly the ability of the E-max to pick out areas of active subduction, where high tectonic stresses support the non-isostatic load. For example, the Caucasus

and the Atlas Mountains appear as highs, whereas mountain belts in the stable continental interior, such as the Urals, have lower values because they are compensated by thick crustal roots.

In the small scale (Fig. 9b), we see a very strong maximum over the Aegean Sea (8). This is an area of high heat flow associated with back-arc spreading (Fytikas and Kolios 1979). The Hellenic Trench appears as a strong minimum. The Atlas Mountains also appear as a strong maximum along with a weaker maximum associated with the Sierra Morena (9) on the Spanish side of the collision zone.

A very weak maximum (see Table 3) is visible over the Swiss Alps (5) and a barely detectable high is seen over the Austrian Alps (6). These values are lower than even what is seen for other ranges in stable continental interiors with isostatic compensation, such as the Colorado Rockies. The disconnected nature of the roots of the eastern and western Alps is confirmed by seismic studies (Babuška et al. 1990). The low highs seen over the Alps appear to indicate a deficit of mass in this area. A similar deficit of mass that is detected in the Tien Shan region of the Himalayas has been proposed to be due to a cold, dense, thickened lithospheric root inducing convective downwelling in the mantle (Burov et al.

Table 2 Wavelet coefficients at a given resolution and E-max values for South America

	Resolution		E-max
	600 km	200 km	
Altiplano (1 in Fig. 8)	0.54	0.100	3.70
Peru–Chile Trench (2 in Fig. 8)	–	–	–0.18

Fig. 7a–c Region around South America. **a** Wavelet transform at a resolution of 200 km. **b** k-max map. **c** E-max map. The color maps are the same as in Figs. 1 and 3

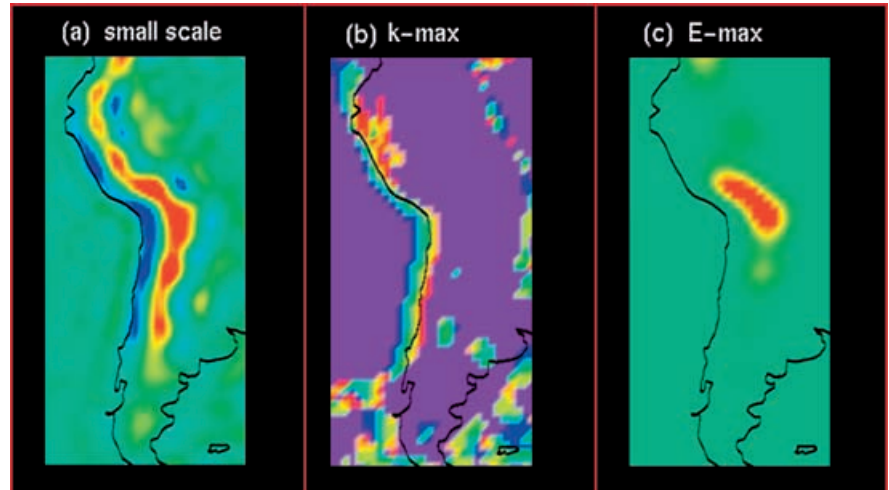
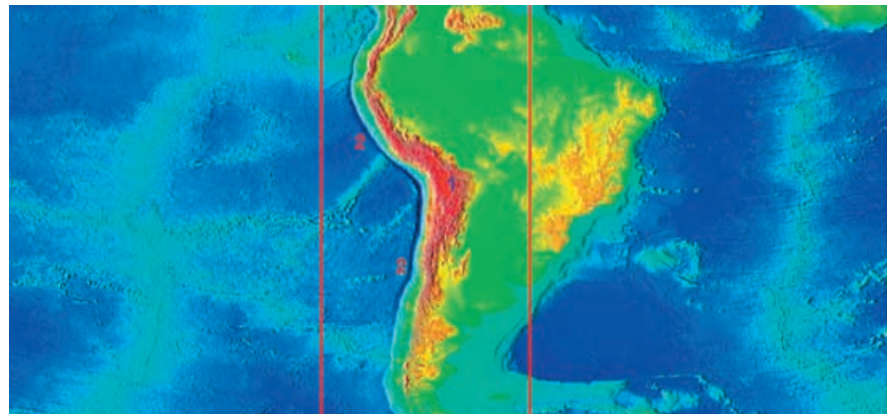


Fig. 8 Topography of region around South America (Smith and Sandwell 1997). *Numbered areas* are discussed in text. The color map is the same as in Fig. 2



1990). The presence of a high-density body below the Alpine orogeny is known from seismic studies (Kissling 1993; Yegorova et al. 1998). Tomographic studies also indicate a region of fast seismic velocity at ≤ 100 km below the Alps, perhaps associated with a cold subducted slab beneath the region (Spakman 1991; Spakman et al. 1993; Morelli and Piromallo 2000). This

signature further supports the idea of a cold downwelling beneath the Alps.

North America

The United States and surrounding regions are shown in Fig. 11 and Table 4. On the large-scale and E-max maps we can see maxima in the western United States and the Sierra Madre Oriental of Mexico (2 in Fig. 12). Although the Sierra Madre Oriental (2) mountains are not as wide and high as the Sierra Madre Occidental (1) mountains, the maximum over the Sierra Madre Oriental is higher, and it is probably influenced by a volcanic region to the south of the mountains (outside the map), where the volcanoes Popocatépetl, Pico de Orizaba and Iztaccihuatl are situated (Condie 1997). As with Europe, we see a much higher signature in regions of active subduction with high tectonic stresses with compressional origins, like Mexico; whereas less active

Table 3 Wavelet coefficients at a given resolution and E-max values for Europe

	Resolution		
	600 km	200 km	E-max
Caucasus Mountains (2 in Fig. 1010)	0.31	0.033	.80
Atlas Mountains (4 in Fig. 1010)	0.26	0.044	0.42
Alps (5 in Fig. 10)	–	–	0.03
Hellenic Trench (7 in Fig. 10)	–0.37	–0.076	–1.71
Aegean Sea (8 in Fig. 10)	–	0.078	0.26
SierraMorena (9 in Fig. 10)	–	–	0.03

Fig. 9a–d Region around Europe. **a** Wavelet transform at a resolution of 600 km. **b** Wavelet transform at a resolution of 200 km. **c** k-max map. **d** E-max map. The color maps are the same as in Figs. 1 and 3

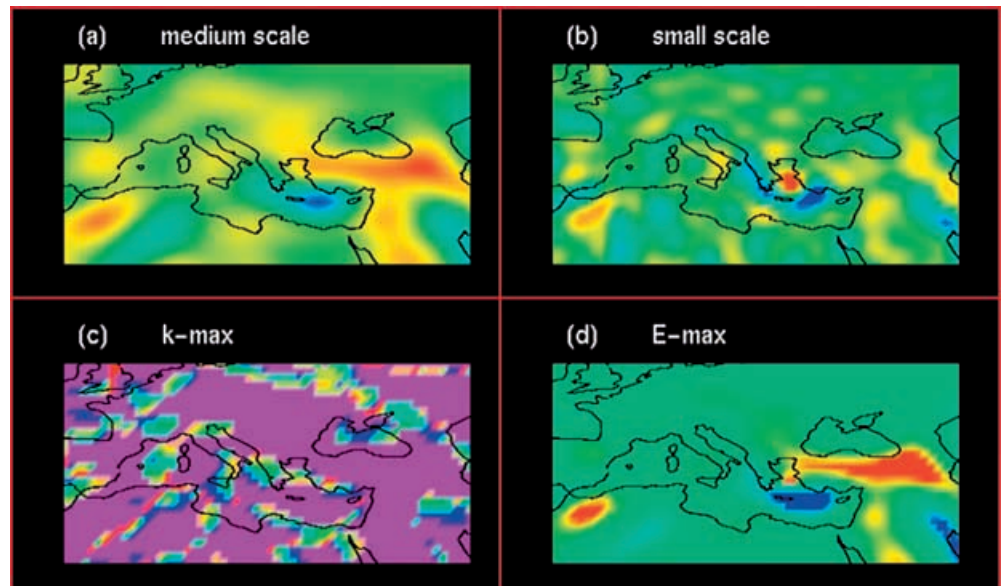
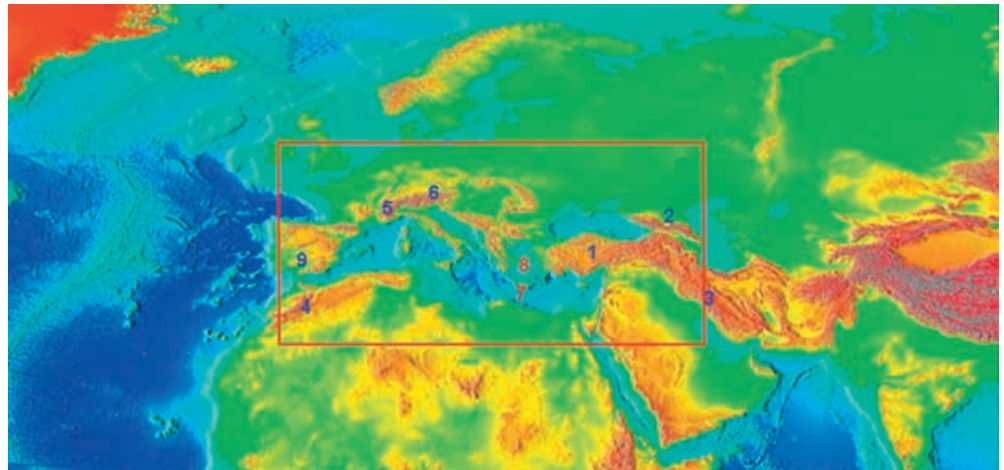


Fig. 10 Topography of region around Europe (Smith and Sandwell 1997). *Numbered areas* are discussed in text. The color map is the same as in Fig. 2



mountains within the stable continental interior, like the Colorado Rockies, have isostatic compensation. Minima in the geoid can be seen over the West Atlantic abyssal plains surrounding the Bermuda Rise (3), which appears as having an average gravity signature for the region. A minimum is also visible over the Gulf of Mexico (4) in the Mexico Basin.

Table 4 Wavelet coefficients at a given resolution and E-max values for North America

	Resolution		E-max
	600 km	200 km	
S. Sierra Madre Oriental (2 in Fig. 12)	0.25	0.037	0.43
Hatteras Plain near Bermuda (3 in Fig. 12)	-0.24	-0.025	-0.36
Mexico Basin (4 in Fig. 12)	-0.18	-0.039	-0.22
Colorado Rockies (5 in Fig. 12)	—	0.025	—
Yellowstone (6 in Fig. 12)	0.19	0.020	0.14
Midcontinent Rift (11 in Fig. 12)	—	-0.015	—
Puerto Rico Trench	-0.33	-0.055	-1.22

In the smaller-scale map (Fig. 11b), we can see the detailed structure of the western United States. We detect two relatively strong maxima, one associated with the Colorado Rockies (5) and one over the Yellowstone hotspot (6) (Condie 1997). Other weak maxima can be found in the small-scale map (Fig. 11b), e.g., along the Sierra Nevada (7) and the Cascades (8). We can see the linear oscillations of gravity highs and lows corresponding to buckling and folding of the Moho below the Basin and Range (Froidevaux and Ricard 1987).

The Mexico Basin in the Gulf of Mexico (4) appears as a strong minimum. Detailed structures of the West Atlantic begin to appear at this scale (cf. Fig. 12). The abyssal plains appear as minima. The Blake Plateau (see the Bahamas, 9) off the east coast of Florida is visible along with the location of fracture zones cutting the weak maximum of the Bermuda Rise (3). The relatively flat area in the middle of the continent exhibits two interesting minima related to the 1-billion-year-old Midcontinent Rift, which starts in Lake Superior in Minnesota (10) and continues to the southwest into Oklahoma (11) (Weiblen

Fig. 11a–d North America. **a** Wavelet transform at a resolution of 600 km. **b** Wavelet transform at a resolution of 200 km. **c** k-max map. **d** E-max map. The color maps are the same as in Figs. 1 and 3

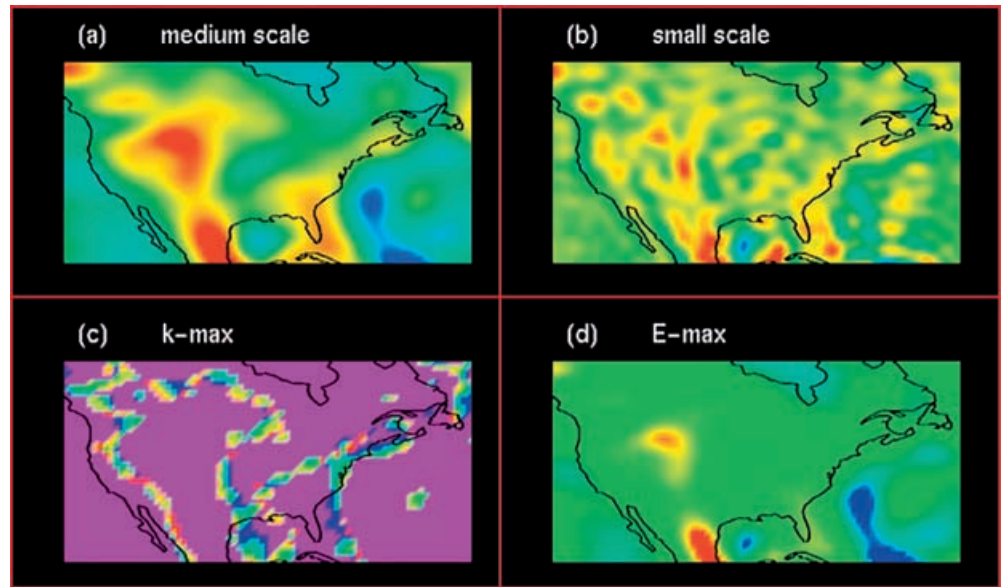
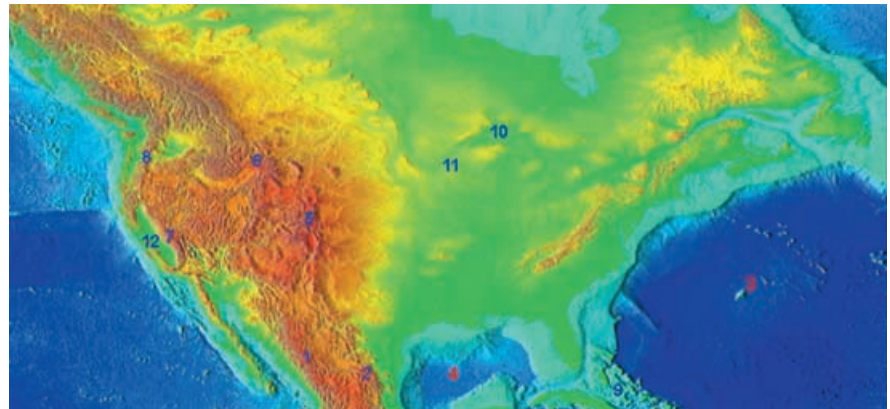


Fig. 12 Topography of North America (Smith and Sandwell 1997). Numbered areas are discussed in text. The color map is the same as in Fig. 2



1982). The rift is now buried by nearly 1 km of younger sedimentary rocks (Cannon 1992).

On the k-max map, one of the most astounding features is the delineation of Precambrian rift zones, including the Midcontinent Rift (10–11) and the Reelfoot Rift along the Mississippi Valley. The Reelfoot Rift is a 600-Ma failed rift that was reactivated by the passage of the Bermuda hotspot in the Cretaceous (Pollitz et al. 2001). It was the site in 1811–1812 of the largest historical earthquakes in the conterminous United States (Hough et al. 2000). These large events triggered earthquakes in the stable continental crust as far away as Cincinnati, Ohio (Hough 2001). There is a good chance for reoccurrence of large earthquakes in this area that could be especially destructive due to the lack of earthquake preparation in this region (Hough 2001). A large mafic body about 10^4 km^3 exists in the lower crust below the area of active earthquakes (Ginzburg et al. 1983; Mooney et al. 1983). It has been proposed that the earthquakes are due to the sinking of this mafic body after the lower crust was weakened in the last glaciation (Pollitz et al. 2001).

Many other features are displayed on the k-max map. The boundary between the continental shelf and the abyssal plain is delineated off the eastern coast. The Bermuda Rise (3) appears as a green point. In the western United States and northern Mexico, the boundary of the Sierra Madre Occidental (1), the Sierra Nevada (7), the Cascade Range (8), and the Great Valley (12) are delineated. The k-max distributions in Canada cannot be interpreted easily because of the distortion due to the Mercator projection above 60–65° latitude.

East Africa and the Middle East

East Africa and the Middle East are shown in Fig. 13 and Table 5. In the medium scale (Fig. 13a), we can

Table 5 Wavelet coefficients at a given resolution and E-max values for East Africa and the Middle East

	Resolution		
	600 km	200 km	E-max
Zagros Mountains (1 in Fig. 14)	0.21	0.035	0.08
Mesopotamia (2 in Fig. 14)	–	–	–0.05
Red Sea, East African Rift, Gulf of Aden triple junction (6 in Fig. 14)	–	0.035	–
Somali Basin (7 in Fig. 14)	–	–	–0.04
Mecca (10 in Fig. 14)	0.22	0.032	0.11
Dead Sea (12 in Fig. 14)	0.21	0.030	0.16
Central Makran Range (17 in Fig. 14)	–	0.026	–
Caspian Sea	–	–	–0.03
Persian Gulf	–0.36	–	–0.29

observe the broad structure of mountains, stretching from Turkey through the Zagros Mountains (Fig. 14, 1) to the Himalayas, associated with the closing of the Tethyan Sea. The existence of strong tectonic stresses in this region supporting the non-isostatic load is clearly evident. The thickened crust in the Zagros region of eastern Turkey supplies the gravitational energy to drive Turkey westward towards Eurasia, making this continental region one of the most seismically active and rapidly deforming on Earth (Taymaz et al. 1991). Turkey is driven westward along the North Anatolian Fault where the gravitational potential energy is converted episodically into large earthquakes (Taymaz et al. 1991; Nalbant 2002; Papazachos 2002).

We can also see the low spot in the Persian Gulf/Mesopotamia region (2). The triple junction structure is clearly seen with the Red Sea, East African Rift, and Gulf Aden (6) appearing as highs separated by the low of the Somali Basin (7). Tomography shows a plume of hot mantle rising from the core-mantle boundary in this region (Zhao 2001). The broad high of the Carlsberg Ridge (8) is also visible.

Fig. 13 East Africa, the Middle East, and the Indian subcontinent. **a** Wavelet transform at a resolution of 600 km. **b** Wavelet transform at a resolution of 200 km. **c** k-max map. **d** E-max map. The color maps are the same as in Figs. 1 and 3

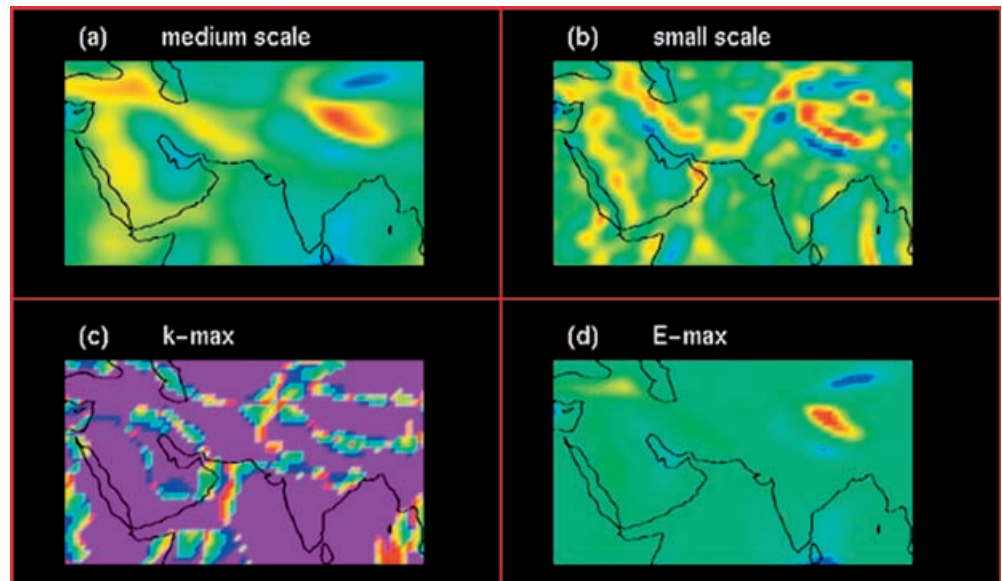
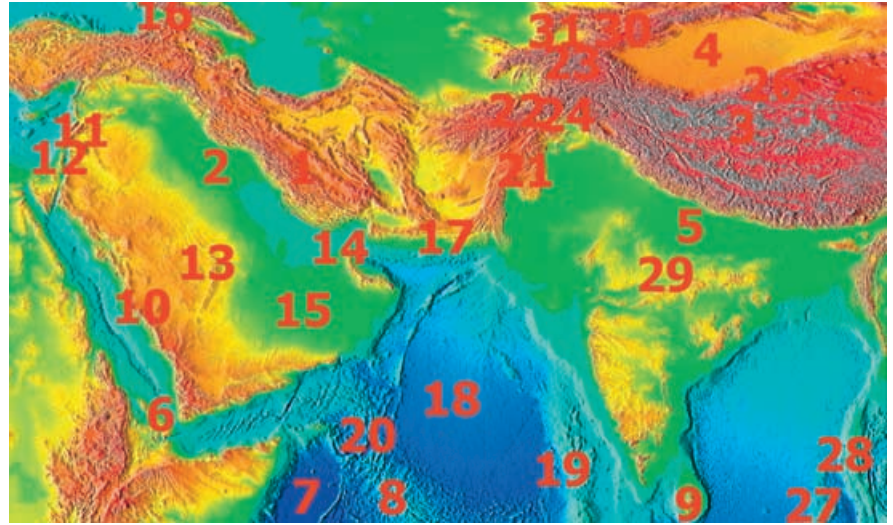


Fig. 14 Topography of East Africa, the Middle East, and the Indian subcontinent (Smith and Sandwell 1997). *Numbered areas* are discussed in text. The color map is the same as in Fig. 2



The structures of the individual mountain belts become much clearer at the 200-km resolution (Fig. 13b). We can see highs associated with the Mecca (10) and Damascus (11) regions along with a low in the Dead Sea region (12). The Tuwayq Mountains (13) of Saudi Arabia and the Oman Promontory (14) are also visible. Mesopotamia, the Persian Gulf, and the Rub al Khali (15) of Saudi Arabia appear as lows. The individual ranges of the Caucasus Mountains (16), the Zagros Mountains (1), and the Central Makran Range (17) are all visible. In the Arabian Sea, the Somali Basin (7) and the Arabian Basin (18) are lows. The Carlsberg Ridge (8) and the Chagos-Laccadive Plateau (19) are highs. The break in the Carlsberg Ridge along the Owens Fracture Zone (20) is also visible. The k-max map (Fig. 13c) clearly outlines both the northern and southern borders of the Tethyan Suture Zone through Turkey and Iran (Condie 1997).

India and the Himalayan Region

In the medium-scale wavelet transform (Fig. 13a, Table 6), the high of the Tibetan Plateau (Fig. 14, 3) is surrounded by lows of the Tarim Basin (4) on the north and the Ganges Plain (5) on the south. The stresses supporting the strong non-isostatic signatures in the Himalayan region are created by the underthrusting of the Indian crust beneath Asia, replacing the Asian lithosphere, which is sinking into the mantle (Chemenda et al. 2000). This structure also shows up clearly in the E-max map (Fig. 13d).

One surprising feature that cannot be well correlated with lithospheric tectonics is the prominent low over Sri Lanka (9), which is also seen in the E-max map (Fig. 13d). The Sri Lankan low has a wavelet coefficient of -1.07 at the 600-km resolution (Table 6). This is the largest low found on Earth at this resolution. Since a low in this region is also found at the 5,400-km resolution (Fig. 1b), we would expect this signal to have a lower mantle source (Richards and Hager 1984; Cazenave et al.

Table 6 Wavelet coefficients at a given resolution and E-max values for India and the Himalayan region

	Resolution		
	600 km	200 km	E-max
Himalayas/Tibetan Plateau (3 in Fig. 14)	0.40	–	1.22
Tarim Basin (4 in Fig. 14)	–0.93	–0.046	–0.37
Ganges Plain (5 in Fig. 14)	–0.59	–0.066	–0.06
Sri Lanka (9 in Fig. 14)	–1.07	0.010	–0.38
Pamir Knot (23 in Fig. 14)	–	0.048	–
Hindu Kush (22 in Fig. 14)	–	0.038	–
Nanga Parbat (24 in Fig. 14)	–	0.063	–
Qaidam Basin (25 in Fig. 14)	–	–0.034	–
Chagos-Laccadive Plateau (27 in Fig. 14)	–	0.024	–

1989). The large magnitude of the signal at the 600-km resolution also indicates that it is probably associated with a longer wavelength lower mantle source (Kaula 1966). It also appears that this non-isostatic load must be supported by lower mantle flow because there are no known tectonic stresses that could be supporting it. The lack of correlation of lower mantle geoid signals like the Sri Lankan low with surface tectonics supports the idea of layered mantle convection regionally (Chase 1979; Kido and Yuen 2000).

The finer structure of the Himalayan region is seen in the small-scale wavelet transform (Fig. 13b). The Sulaiman Range (21) of Pakistan merges into the sharp high of the Hindu Kush (22) of Afghanistan. Larger highs are seen for the Pamir Knot (23) of Tajikistan. The Nanga Parbat (24) of Pakistan, which is uplifting at the fastest rate on Earth (1 cm/year) (Windley 1982), has the largest small-scale wavelet coefficient in the region, 0.063 (Table 6). This area consists of the Kohistan arc complex, which was thrust onto the Indian passive margin, exposing its lower crust and upper mantle (Burg et al. 1996, 1998). The Ganges Plain forms a prominent low, which is bordered on the north by the prominent high of

the Himalayas. The Tarim (4) and Qaidam (25) Basins appear as prominent lows to the north of the highs of the Kunlun Range (26), which forms the northern boundary of the Tibetan Plateau. The southern boundary of the Plateau, the Yarlung Zangbo Suture Zone, where the crustal thickness and the Moho depth decreases (Burg and Chen 1984), can be clearly discerned in the sharp low to high transition in the geoid.

The high of the Tien Shan (30) is located north of the Tarim Basin. The high signal is much stronger in the western than the eastern part of the range. This low high over an active mountainous region is similar to the situation in the Alps. It has been proposed that convective downwelling instigated by a cold, descending lithospheric root may increase the negative Bouguer anomaly, thus decreasing the geoid high (Burov et al. 1990). A sharp low beneath the Ferghana Valley (31) surrounded by smaller highs has been interpreted as being caused by folding of the Moho, which has deepened the Moho beneath the Ferghana Valley (Burov and Molnar 1998). Seismic studies show evidence for a sharp transition at the boundary of the Kunlun Range and Tibetan Plateau, where the crust suddenly thins under the Tarim and Qaidam Basins (Zhu and Helmberger 1998). This crustal thinning is manifested as a sharp transition to geoid lows on our small-scale wavelet map (Fig. 13b).

At smaller scales, a high appears over Sri Lanka (9) (Fig. 13b); rather than the low seen at larger scales (Fig. 13a). This confirms that the Sri Lankan low has a lower mantle origin. The high of the Chagos-Laccadive Plateau (27) also appears prominently in the Indian Ocean next to the low of the Java Trench (28).

The northern and southern borders of the Tibetan Plateau are outlined on the k-max map (Fig. 13d). The boundary between the Nubian and Somali Plates along the East African Rift is also visible (Chu 1999). One of the most striking features, however, is one that is not seen well on the wavelet transform maps. This is the Central Indian Suture Zone (29) of Proterozoic age separating the Bundelkhand Craton to the north from the Bhandra Craton to the south (Mishra et al. 2000). This suture separates a gravity high in the north from a gravity low to the south (Mishra et al. 2000). This clearly emphasizes the amazing ability of k-max to detect not only current, but also fossil plate boundaries. Yuen et al. (2002) also found the k-max to be excellent at extracting fossil suture zones on the Australian continent (see below).

China and Southeast Asia

The larger-scale wavelet map covering China and Southeast Asia (Fig. 15a, Table 7) shows a sharp low from the Gobi Desert of Mongolia connecting with the low of the Chinese craton. This low maps the area where the depth to Moho drops from around 66 km at the edge of the eastern Himalayas to about 44 km (Meyerhoff et al. 1991). The high of the Tibetan Plateau is relatively

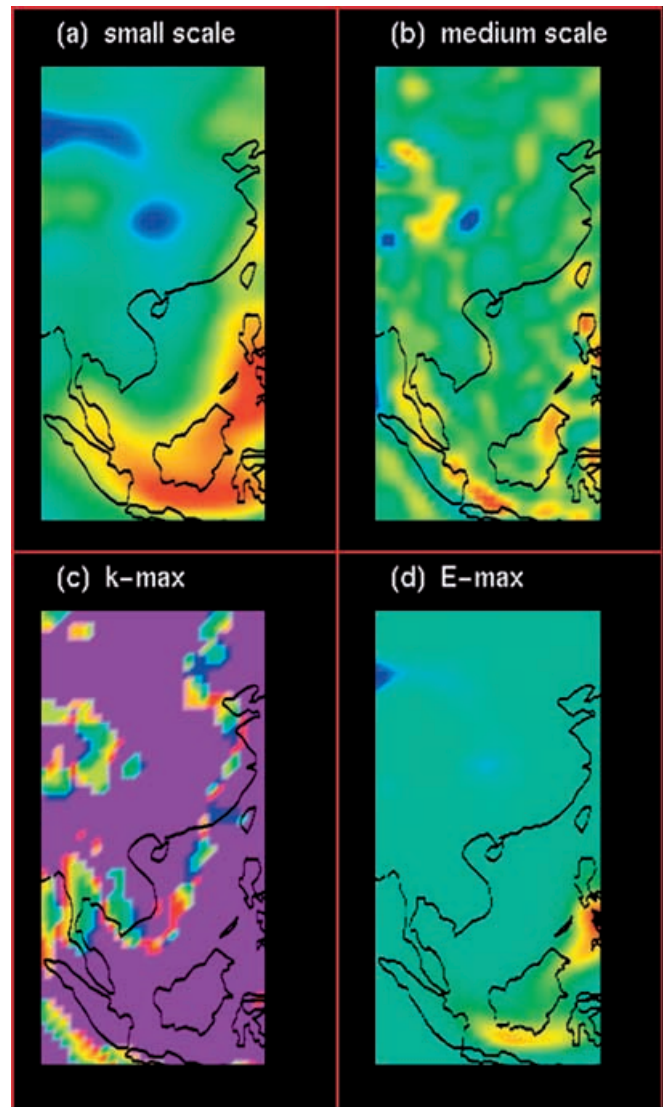


Fig. 15a-d China and Southeast Asia. **a** Wavelet transform at a resolution of 600 km. **b** Wavelet transform at a resolution of 200 km. **c** k-max map. **d** E-max map. The color maps are the same as in Figs. 1 and 3

small at this scale compared to the high under Indonesia and the Philippines.

The smaller-scale wavelet map (Fig. 15b) shows the low of the Java Trench running parallel to the high of the Indonesian Arc. The Sichuan Basin (Fig. 16, 1) separated from the low of the Brahmaputra River Valley (2) in Assam, India by the Gongga Shan (3) of China. The high of the Bayan Har Shan (4) and Qilian Shan (5) surround the low of the Qaidam Basin (6). The low of the Tengger Shamo (7) can be seen to the northeast of the Qilian Shan. The geoid highs and lows seen here in the small-scale wavelets match quite well with the depth to Moho of the basement in the eastern Himalayan Region (Meyerhoff et al. 1991).

The linear low in eastern China extending south from the Yellow Sea is associated with the Tanlu fracture zone.

Table 7 Wavelet coefficients at a given resolution and E-max values for China and Southeast Asia

	Resolution		
	600 km	200 km	E-max
Sichuan Basin, China (1 in Fig. 16)	-0.22	-0.041	-0.17
Assam, India (2 in Fig. 16)	-0.10	-0.041	-
Gongga Shan, China (3 in Fig. 16)	0.05	0.022	-0.06
Qilian Shan, China (5 in Fig. 16)	-	0.020	-
Tengger Shamo, China (7 in Fig. 16)	-	-0.270	-
Java Sea, Indonesia	0.27	0.035	0.40

This is a 300–400 km wide lithospheric scale feature bounded on its western edge by a sharp gravity gradient and thinned lithosphere (Kumarapeli et al. 1990). It appears to be an ancient transform boundary with several pull-apart basins, one of which lies in the Yellow Sea (Kumarapeli et al. 1990). The Tanlu fault is probably Archean in age as shown by the 2.5-Ga-old ophiolites sequences about 250 km north of Beijing representing an ancient suture (Kusky et al. 2001). The Tanlu fault acted as a dextral strike-slip fault in the Triassic when the East Shandong terrane migrated southwest to dock with the West Shandong terrane (Lingzhi et al. 1990).

There are several prominent features in the k-max map of China (Fig. 15c). The first is the circle outlining the Tibetan Plateau along with an area showing the relatively shallow depth to the Moho in the Sichuan Basin (Meyerhoff et al. 1991). A line across eastern China follows the Tanlu fault through eastern China (Xiong 1990; Davis et al. 1996) and then connects with the ancient spreading ridge that formed the South China Sea (Hall 1996). The delineation of the Tanlu fault by the k-max shows the ability of the k-max to quickly pick out areas of lithospheric scale faults formed by ancient suture zones, which are often weak zones in the crust subject to reactivation. The connection of the Tanlu fault with the spreading area of the South China Sea indicates that this spreading might have started in the area of weak lithosphere along the Tanlu fault.

Australia

The medium-scale wavelet transform of the Australian region (Fig. 17a, Table 8) shows the broad highs of New Guinea and south Indonesia separated by the sharp low of the Banda Sea (Fig. 18, 1). The outer edge of the continent of Australia is surrounded by the highs of the Hamersley Range (2), the Kimberley Plateau (3), and the Great Dividing Range (4). These highs surround a low in the Great Victoria Desert (5). A sharp low is seen south of the continent in the South Australian Basin (6). Prominent highs are also seen in the Kermadec (7), Tonga (8), and Fiji (9) regions. The high geoid anomalies from this subduction system are so prominent that they dominate the E-max map (Fig. 17d).

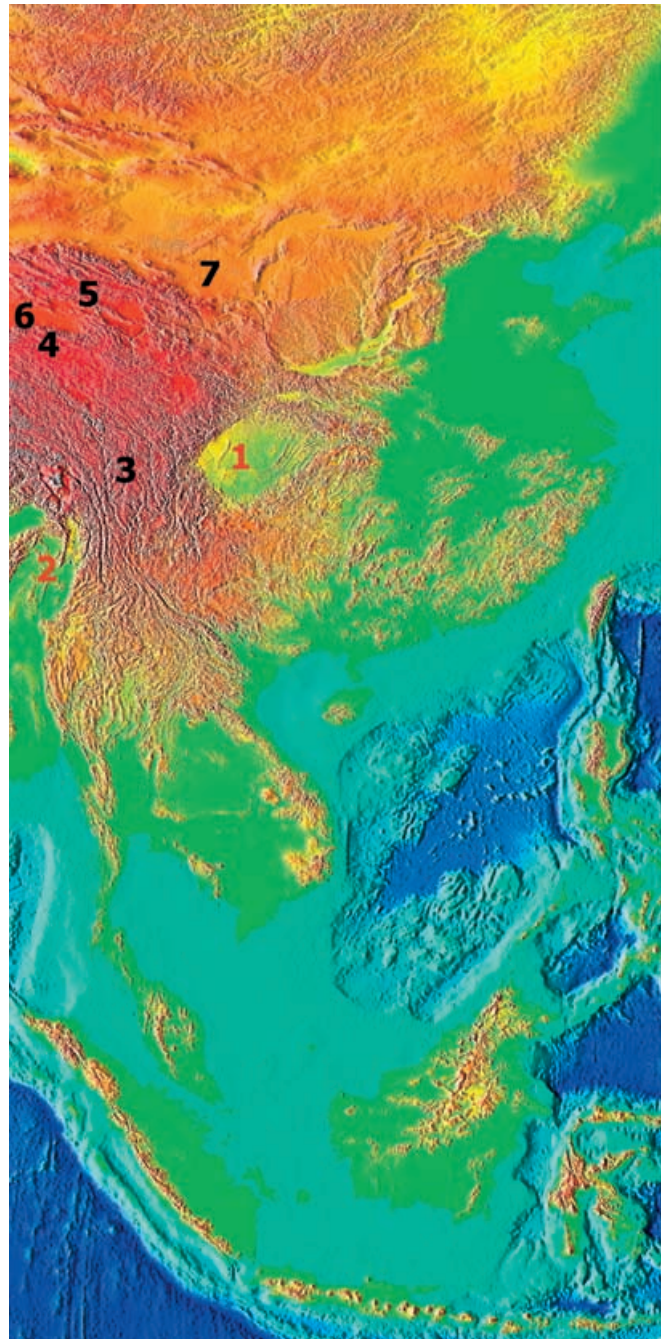


Fig. 16 Topography of China and Southeast Asia (Smith and Sandwell 1997). Numbered areas are discussed in text. The color map is the same as in Fig. 2

The small-scale wavelet transform picks out individual details of the mountains of Australia including the Great Dividing Range and the North Flanders Range (10) (Fig. 17b). A low area is also visible in the Great Victoria Desert. The pattern of alternating circles of highs and lows in central Australia indicate the horst and graben structure of this area, which is characterized by Moho offsets of more than 20 km (Simons et al. 2000). The parallel highs and lows of the Kermadec (7),

Tonga (8), and New Hebrides (12) subduction systems are also clearly shown along with the eastern hook of the Java Trench.

Table 8 Wavelet coefficients at a given resolution and E-max values for Australia

	Resolution		E-max
	600 km	200 km	
Kimberley Plateau, Australia (3 in Fig. 18)	0.16	–	–
Great Victoria Desert, Australia (5 in Fig. 18)	–0.14	–	–
South Australia Basin (6 in Fig. 18)	–0.21	–	–0.22
Kermadec Islands (7 in Fig. 18)	–	0.058	–
Kermadec Trench (7 in Fig. 18)	–	–0.057	–
Tonga Islands (8 in Fig. 18)	0.21	0.069	0.28
Fiji (9 in Fig. 18)	0.23	–	0.33
New Hebrides Arc (12 in Fig. 18)	–	0.041	–
Lau Ridge (13 in Fig. 18)	–	0.055	–

The plate tectonic boundaries in the ocean east of Australia are even more clearly revealed in the k-max map. The Kermadec and Tonga subduction zones are mapped. The Lau Ridge (13), the Norfolk Ridge (14), and the Lord Howe Rise (15) are also visible. The north edge of the Fiji Basin (16) is outlined. The lack of a gradual continental margin along the eastern Australia coast (Smith and Sandwell 1997) is also seen. The hook shape of the eastern edge of the Java Trench is visible.

The most striking feature about the k-max map, however, is the delineation of the Australian continent into three crustal blocks. This structure has been previously detected in tomography studies (Simons et al. 1999). The three regions correspond to the western Archean craton, the Central Australian Proterozoic block, and the eastern Phanerozoic ranges (Simons et al. 2000). Simons et al. (2003) have used a multi-spectrogram method to study the 2-D coherence between the Bouguer gravity anomalies and topography for the

Fig. 17a–d Australian region. **a** Wavelet transform at resolution of 600 km. **b** Wavelet transform at a resolution of 200 km. **c** k-max map. **d** E-max map. Color maps are same as in Figs. 1 and 3

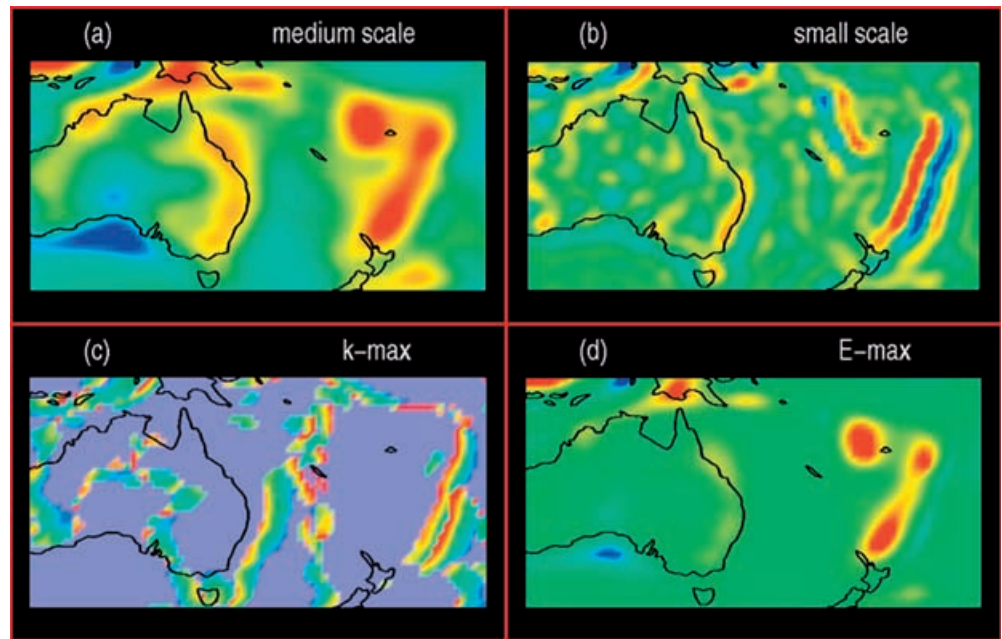
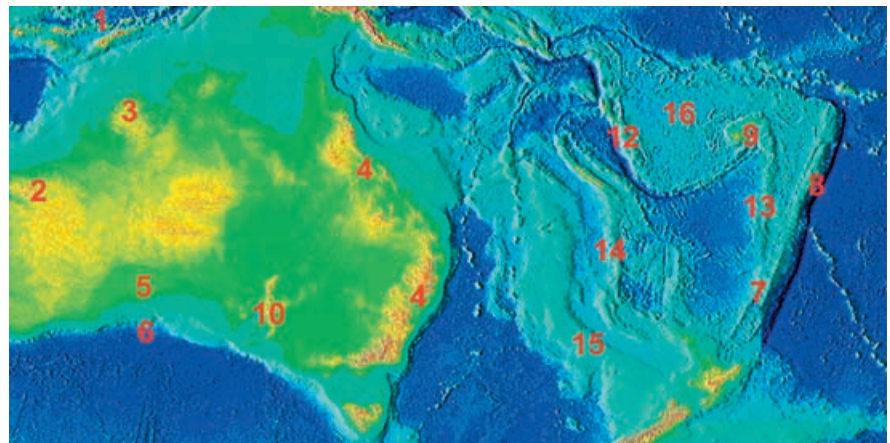


Fig. 18 Topography of Australian region (Smith and Sandwell 1997). Numbered areas are discussed in text. The color map is the same as in Fig. 2



Australian continent. Their study shows that the Precambrian section of the continent consists of stable cratons separated by mechanically weaker boundaries, which have accumulated more gravitational anomalies in relation to their topography than the isotropic average.

Discussion and concluding remarks

We have analyzed the geoid, out to spherical harmonic degree 90, with emphasis on features shorter than 1,000 km, which have some geological significance. We have not concerned ourselves with the influence of the side boundaries on the computation of the wavelets. Our use of the Mercator projection of the spherical field is justified for the latitudes lower than 60–65°. The problem due to geometrical distortion from the Mercator projection and the use of the Fourier transform above 60–65° latitude can be overcome by rotating the data and putting the new equator along the desired great circle. Another possibility would be the use of a quasi-analytical spherical wavelet filter (Kido et al. 2002).

The input dataset, the geoid, was available in the spherical harmonic decomposition. The former method for examining small-scale features, truncation of the spherical harmonic expansion, was compared with the wavelet filter. While it was sufficient to compute only one mode (or wavenumber, k) of the wavelet spectrum, we had to try several truncation levels to uncover as many small-scale features hidden in the field as possible. Problems with false signals in places of low magnitude signals resulted due to the global character of the spherical harmonics. The truncation method is thus very unsuitable for the detection of small-scale features. Moreover, we emphasize that a small number of the local wavelet bases will give much better resolution than the global spherical harmonic decomposition of high order in the case of such inhomogeneous fields as the geoid. The computation of the wavelet transform is also around 200 times faster than using a moving weight function or window to localize regional features with spherical harmonics, and the trial and error method associated with finding a suitable weighting function is not needed in the case of the wavelet transform (Yuen et al. 2002).

In the last sections we focused on the comparison of regional small-scale structures with the geology of the Earth. The comparison was done rather carefully, as the wavelet resolution used was close to the highest degree of the spherical harmonic decomposition used, and the effect of false signals due to truncation of the original spherical harmonic decomposition from 360 to 90 degrees could occur. Nevertheless, the Gaussian wavelet filter could detect fine features of the geoid at convergent zones, ridges, and fracture zones. The k -max maps were able to detect the plate tectonic history of the Earth, including present plate bound-

aries, as well as much older tectonic boundaries. The ability of k -max to pick out fossil plate sutures is demonstrated especially well in its delineation of the Central Indian Suture Zone (Fig. 13c). The E-max maps provided a powerful tool to separate mountain ranges that have isostatic compensation from mountain ranges without isostatic compensation in areas with high compressional stresses. These maps could be especially useful in interpreting whether mountain ranges seen on topographic maps of other planets are being held up by active differential stresses.

After advancing to a higher resolution geoid dataset, the correlation technique applied on the geoid wavelet spectrum and the wavelet transform of the topography (Kido et al. 2002) will be a powerful tool for fast identification of areas of interest in the analyzed geoid. Wavelet processing of extremely high-resolution geoid and topography data, down to 1 min, together with high-resolution regional seismic tomography models will shed more light on the complex geodynamics under Europe.

Concerning the future perspectives for research, we emphasize that the wavelet analysis might be useful for studying plate tectonic-like phenomena on other planets. Two-dimensional wavelets have been used to study the topography of the coronae on Venus (Mettier 2002). The Venusian gravity field could be a possible target for two-dimensional wavelet filtering, as the field should yield information about the diapiric plume tectonics associated with the coronae (Barriot et al. 1998). Window filters based on spherical harmonics have been used to study gravity data from both Earth and Venus (M. Simons et al. 1997; M. Simons and Hager 1997) and geoid lineations in the central Pacific (Cazenave et al. 1995). Multitaper spectral analysis has been used to study the correlation between topography and gravity anomalies in Australia (Simons et al. 2000). One-dimensional wavelets have proved useful for studying the lithospheric strength on Mars based on the local 1-D admittance function (Shcherbakov 2002; Malamud and Turcotte 2001). Kido et al. (2002) are currently using 2-D spherical wavelets to look at the 2-D local admittance function for the entire Martian surface. The k -max maps of Mars could help to decipher whether plate tectonics existed on this planet in the past. Combining these k -max maps with E-max maps would allow one to tell if structures seen on the k -max maps are associated with modern differential stresses or are ancient isostatically compensated structures. Since gravitational energy on Earth is released in earthquakes along belts with characteristic length scales of about 200 km (Tanimoto and Okamoto 2000), the wavelet transform could be used like a voltmeter for detecting changes in the gravitational potential, which is related to high earthquake risk on both Earth and other planets.

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